# Introduction to Machine Learning - HW2 - Q1

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Q1. We are going to find the decision boundary of a binary classifier with a single input feature.

$$P(x \mid \bar{y}) = e^{-2x} \; for \, x \geq 0 \quad and \quad P(x \mid y) = \frac{9}{4\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2}} \; for \, x \geq 0$$

Assume two classes have equal prior probabilities.

$P(y) = P(\bar{y})$

Solution:

Decision boundary is where the conditional probability of two classes cross each other. Let's find the area where $y$ is more probable than $\bar{y}$. Since there are only two classes, the area where $y$ is not more likely to happen than the other case is the area where $\bar{y}$ is more likely to happen.

$P(y \mid x) \geq P(\bar{y} \mid x)$

Using Bayes Law we have:

$$P(y|x) = \frac{P(x|y)P(y)}{P(x)} \; and \; P(\bar{y}|x) = \frac{P(x|\bar{y})P(\bar{y})}{P(x)}$$

$$\frac{P(x|y)P(y)}{P(x)} > \frac{P(x|\bar{y})P(\bar{y})}{P(x)} \rightarrow P(x|y)P(y) > P(x|\bar{y})P(\bar{y})$$

Knowing that $P(y) = P(\bar{y})$ we can simplify it

$$P(x|y) < P(x|\bar{y}) \rightarrow e^{-2x} < \frac{9}{4\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2}}$$

Taking a logarithm leads us to:

$$ln( e^{-2x}) = -2x \quad and \quad ln(\frac{9}{4\sqrt{2\pi}}) + ln(e^{-\frac{(x-1)^2}{2}}) = -1 - \frac{(x-1)^2}{2}$$

$$2x \geq 1 + \frac{(x-1)^2}{2} \rightarrow (x-1)^2 + 2 - 4x = x^2 - 6x + 3 \leq 0$$

$x = \frac{6 \pm \sqrt{24}}{2} = 3 \pm \sqrt{6}$

$3 - \sqrt{6} \leq x \leq 3 + \sqrt{6} \rightarrow y = 1$

$(0 \leq x \leq 3 - \sqrt{6}) \cup (x \geq 3 + \sqrt{6}) \rightarrow y = 0$