Introduction

Eléments finis

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Poutre en traction

$$\begin{cases} \boldsymbol{L}, \boldsymbol{S}, \boldsymbol{E} \\ \boldsymbol{dl}?\ \boldsymbol{\sigma}? \end{cases}$$

Poutre en traction

$$\begin{cases} \boldsymbol{L, S, E} \\ \boldsymbol{dl? \, \sigma?} \end{cases}$$

Solution RDM (loi de Hooke $\sigma = E\ \epsilon$) :

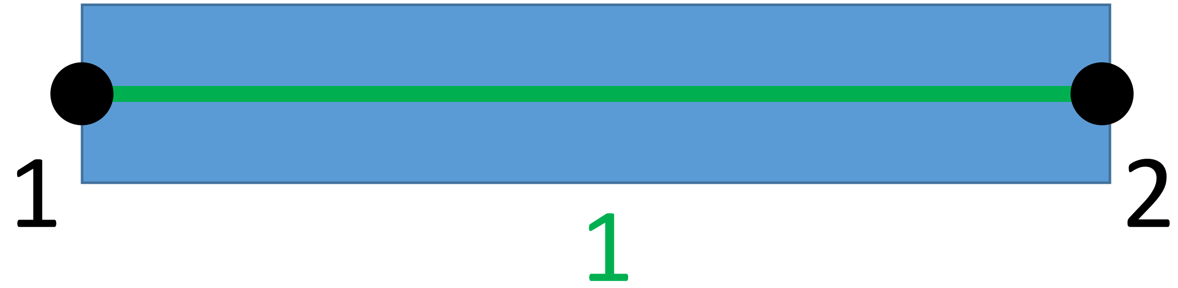
$$\sigma = \frac{F}{S}$$

$$dl = \frac{FL}{ES}$$

Poutre en traction

$$\begin{cases} \boldsymbol{L, S, E} \\ \boldsymbol{dl? \ \sigma?} \end{cases}$$

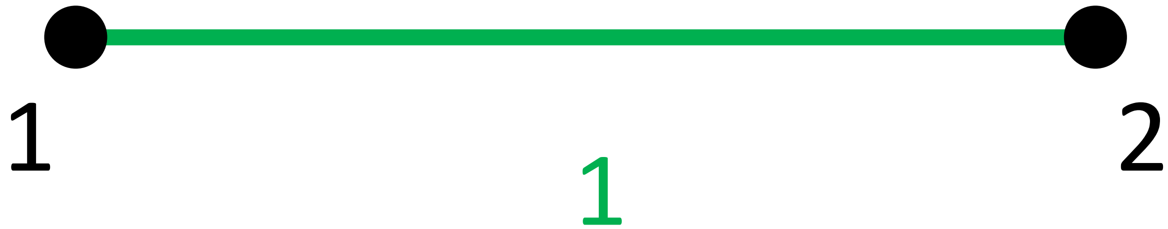
Solution par éléments finis :



Poutre en traction

$$\left\{\begin{array}{l} L, S, E \\ \\ dl?\ \sigma? \end{array}\right.$$

$\mathbf{1}\ \{u_{x1}$



$\mathbf{2}\ \{u_{x2}$

Approximation par éléments finis

Définition: une approximation au sens des éléments finis d 'une variable $u(x)$ sur un élément à deux nœuds, s'écrit :

$$u(x) = N_1(x)\, u_{1x} + N_2(x)\, u_{2x}$$

$N_1(x), N_2(x)$ sont appelées fonctions d'approximation ou fonction de forme. Les fonctions de formes vérifient la relation générale :

$$N_i(x_j) = \begin{cases} 1 \; si \; i = j \\ 0 \; si \; i \neq j \end{cases}$$

Approximation par éléments finis

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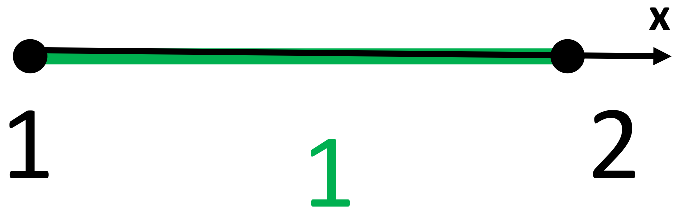
$$\begin{cases} N_1(0) = \\ N_1(L) = \end{cases} \quad et \quad \begin{cases} N_2(0) = \\ N_2(L) = \end{cases}$$

Approximation par éléments finis



$$N_i(x_j) = \begin{cases} 1 \; si \; i = j \\ 0 \; si \; i \neq j \end{cases}, \begin{cases} N_1(0) = 1 \\ N_1(L) = 0 \end{cases} \; et \; \begin{cases} N_2(0) = 0 \\ N_2(L) = 1 \end{cases}$$

1- Choisir l’ordre d’approximation : deux nœuds (ordre 1, élément linéique)

$$N_1(x) = a_1 x + b_1, N_2(x) = a_2 x + b_2$$

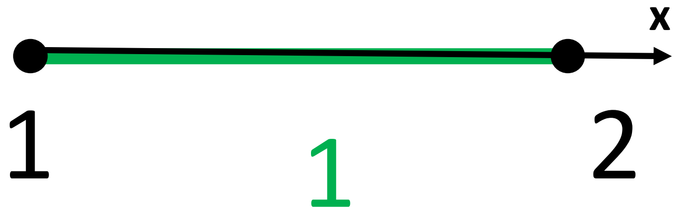
2 – Construction des deux systèmes d’équations

$$\begin{cases} N_1(0) = a_1 \times 0 + b_1 = 1 \\ N_1(L) = a_1 \times L + b_1 = 0 \end{cases} \; et \; \begin{cases} N_2(0) = a_2 \times 0 + b_2 = 0 \\ N_2(L) = a_2 \times L + b_2 = 1 \end{cases}$$

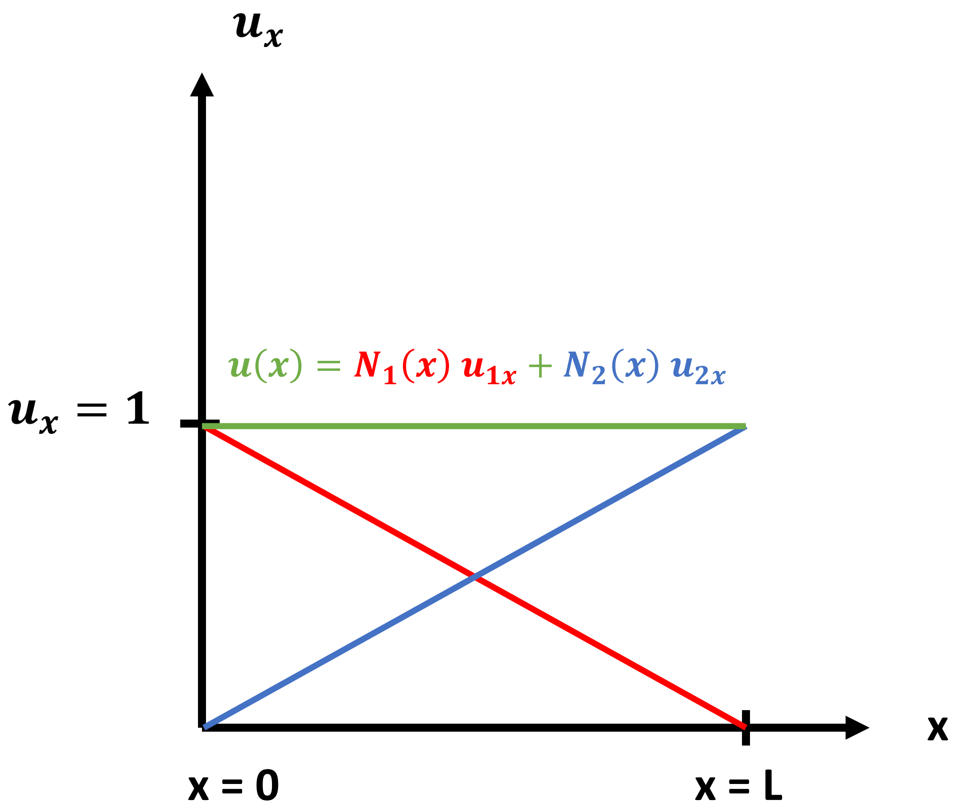
3- Résolution

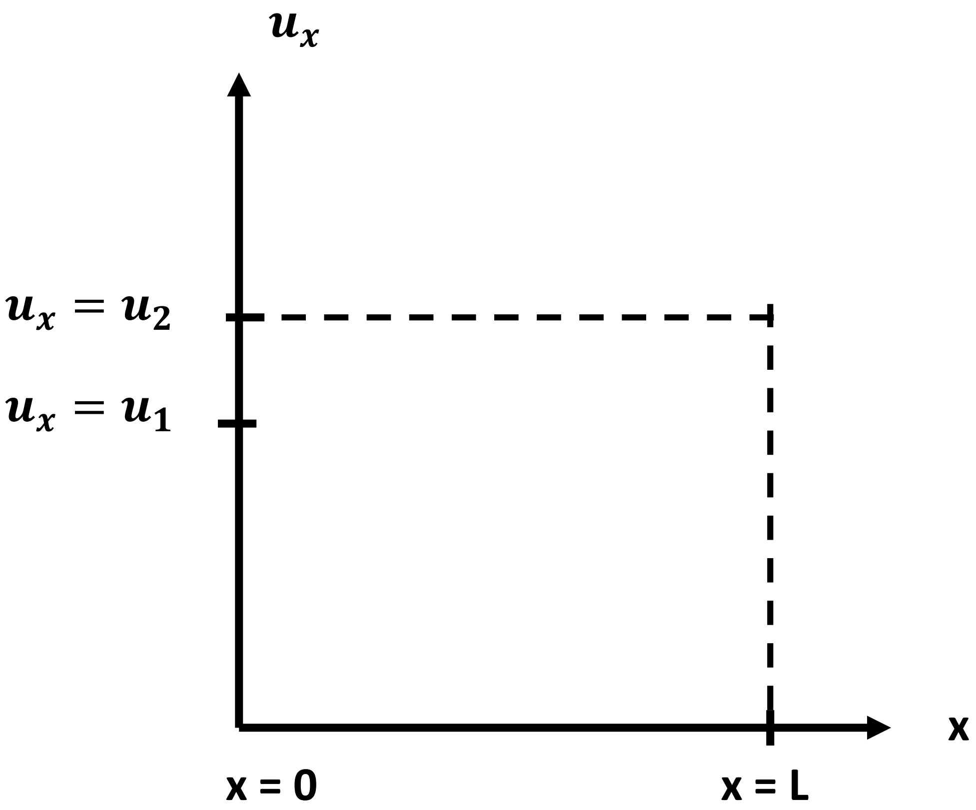
$$N_1(x) = -\frac{x}{L} + 1 \, , N_2(x) = \frac{x}{L}$$

Approximation par éléments finis



$$N_1(x) = -\frac{x}{L} + 1\,, N_2(x) = \frac{x}{L}$$





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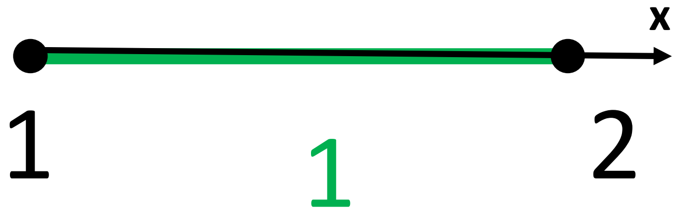

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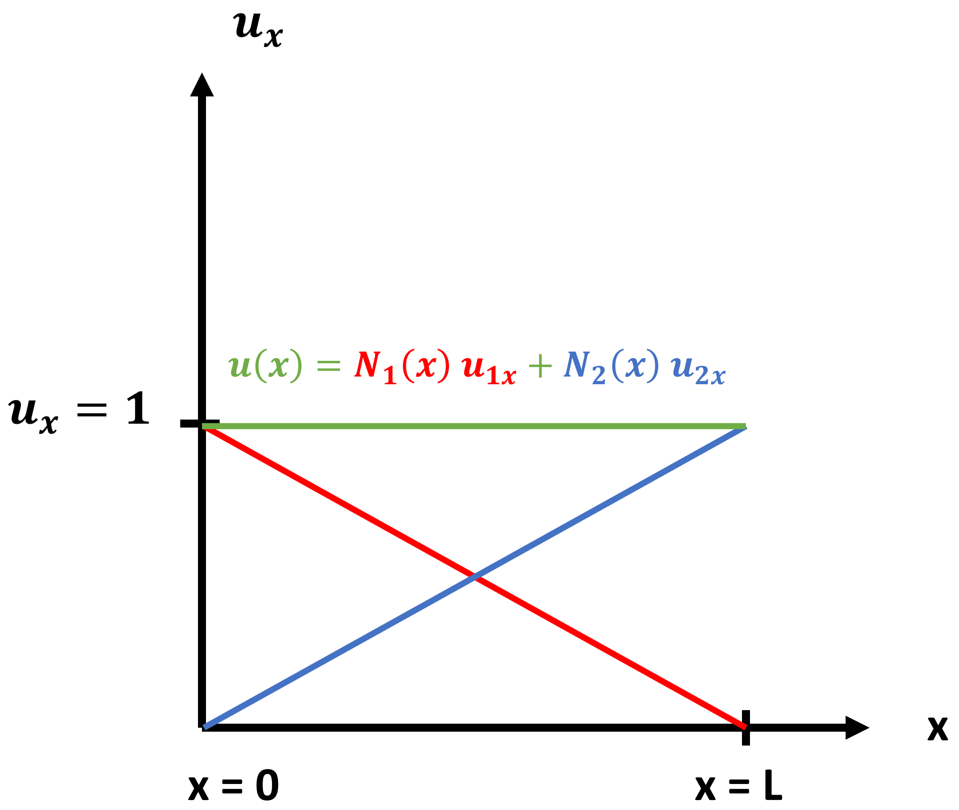

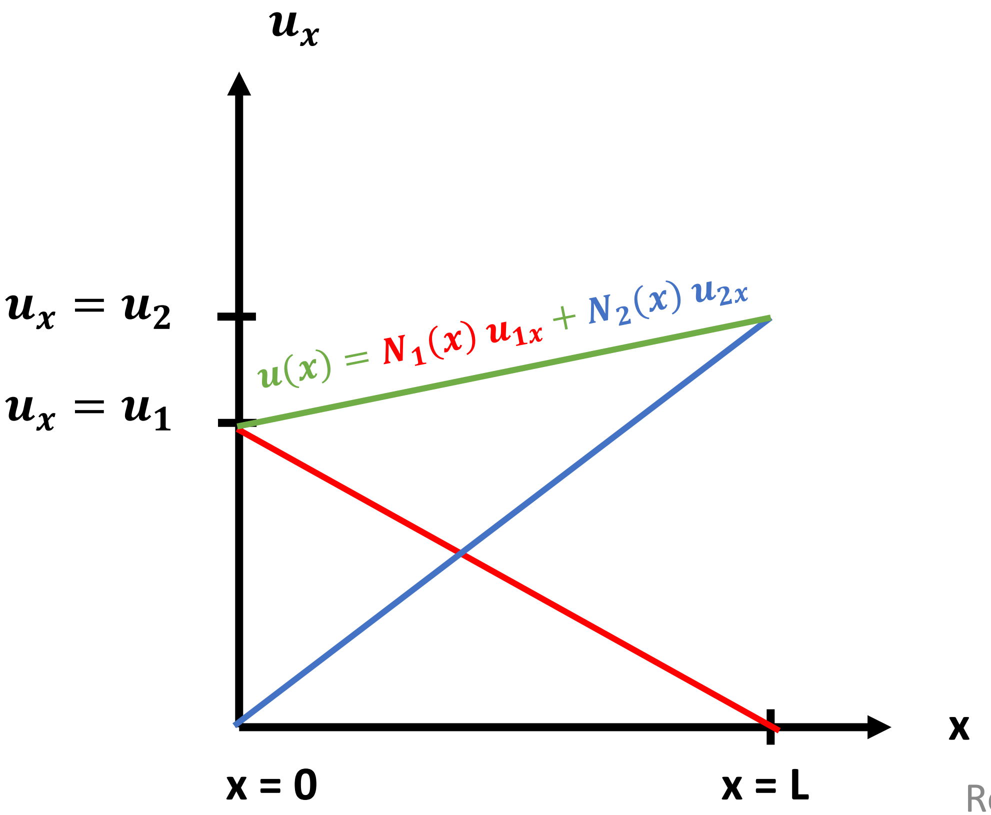

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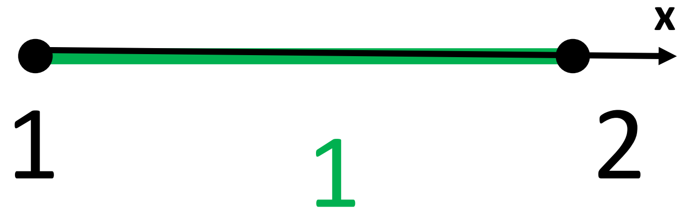


Réécriture du problème :

$$u_e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; \mathrm{u_x} = (1 - \mathrm{x/L})\ \mathrm{u_1} + \mathrm{x/L}\ \mathrm{u_2}$$

$$\mathbf{u} = \mathbf{N}\ \mathbf{U_e}$$

$$\epsilon = \partial u_e/\partial x; \epsilon = (-1/L)u_1 + (1/L)u_2$$

$$\boldsymbol{\epsilon} = \mathbf{B}\ \mathbf{U_e}$$

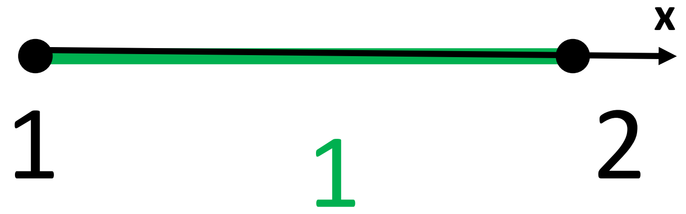
$\sigma = E\epsilon = N/S; \quad \epsilon = N/SE + u(0)$ (N: effort normal)

On associe aux déplacements nodaux u_1 et u_2 les forces nodales f_1 et f_2

$$\begin{cases} f_1 = (SE/L)u_1 + (-SE/L)u_2 = -N(0) \\ f_2 = (-SE/L)u_1 + (SE/L)u_2 = N(L) \end{cases}$$

$$\mathbf{F_e} = \mathbf{K_e}\mathbf{U_e}$$

Approximation par éléments finis



$$\mathbf{F_e} = \mathbf{K_e U_e}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} SE/L & -SE/L \\ -SE/L & SE/L \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F \end{bmatrix} = \begin{bmatrix} SE/L & -SE/L \\ -SE/L & SE/L \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \end{bmatrix} ; u_2 = \frac{FL}{ES} \ et \ F_1 = -F$$

$$\epsilon = \frac{F}{ES} \rightarrow \sigma = \frac{F}{S}$$

Approximation par éléments finis

$$\mathbf{F_e} = \mathbf{K_e U_e}$$

E1 $$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = SE/L \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

E2 $$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = SE/L \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

Assemblage du système à 2 éléments

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = SE/L \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Résolution par éléments finis

- Géométrie et Maillage
- Propriétés des Matériaux
- Assemblage
- Conditions aux limites

ABAQUS

- Algorithme de résolution
- Enregistrement des résultats