

# Mécanique des solides

## 3. Déformées modales

Reine Fares

[reine.fares@cea.fr](mailto:reine.fares@cea.fr)

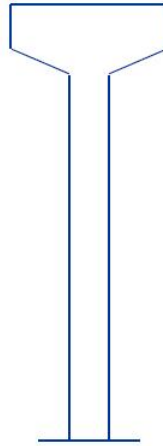
# Cours de Modélisation par Abaqus

- Comportement cyclique des matériaux
  - Élément cubique unitaire pour tester les lois de comportement
- Poutre 1D appuyée et encastree
  - Données : géométrie, charges, matériau, conditions aux limites
  - Résultats : flèche, contrainte max, réactions
- Déformées modales d'une structure
  - Premiers modes de vibration et masses associées
- Portique 3D sous charge dynamique
  - Données : géométrie, matériau, charges statiques et dynamiques
  - Résultats : déformées modales, déplacement dans le temps, contraintes
- Poutre 3D en béton armé
  - Géométrie 3D, éléments finis solides

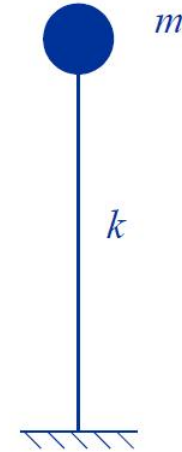
# Oscillateur élémentaire : 1 ddl



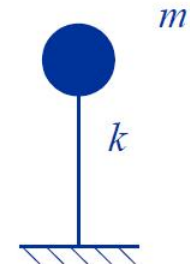
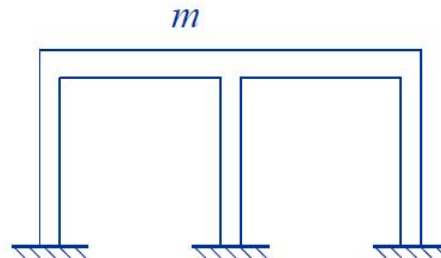
Photo



Schéma



Modèle de calcul



# Condition d'équilibre

Forces en jeu :

- Force d'inertie : Masse [kg], Accélération

$$F(t) = m\ddot{u}(t)$$



- Force élastique : Raideur [N/m], Déplacement

$$F(t) = k u(t)$$



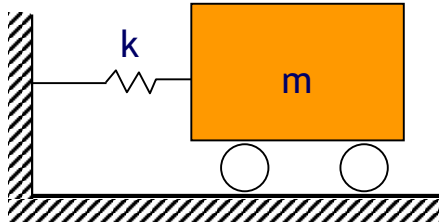
- Force visqueuse : Dissipation [Ns/m], Vitesse

$$F(t) = c \dot{u}(t)$$

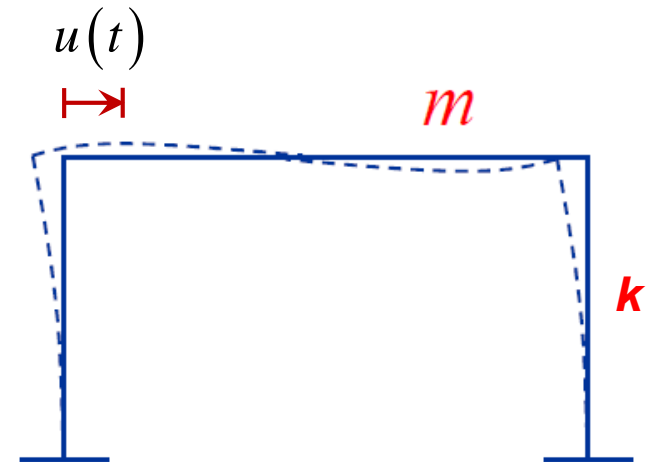
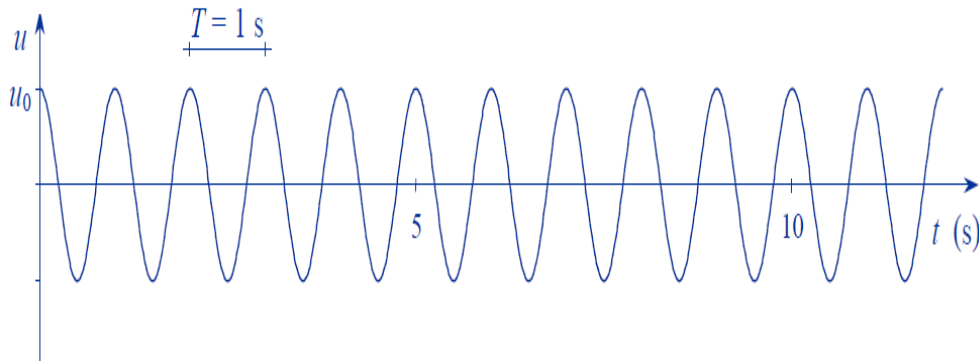


# Oscillateur élémentaire 1/4

## Oscillation libre

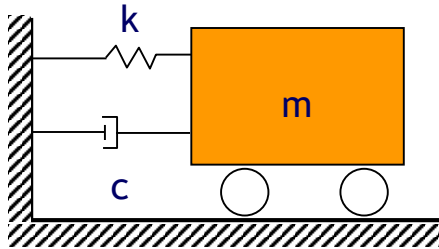


$$m\ddot{u}(t) + ku(t) = 0$$

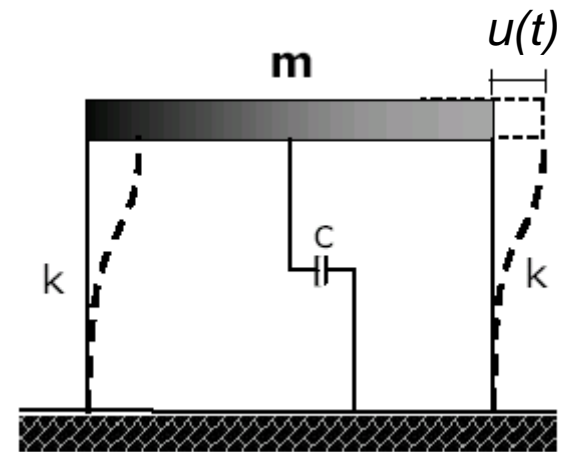
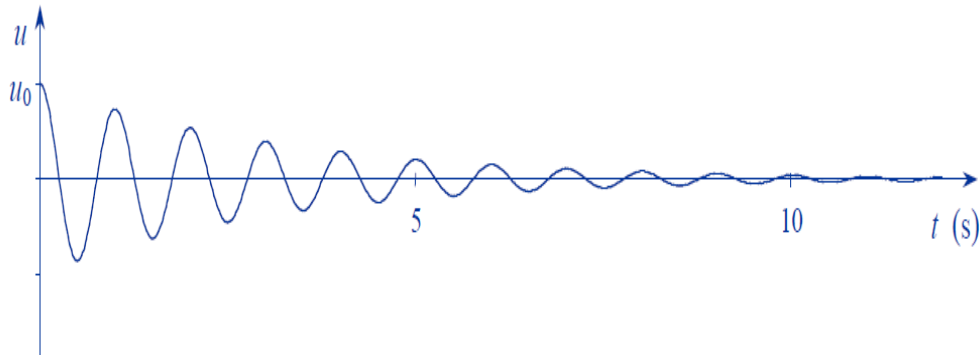


## Oscillateur élémentaire 2/4

### Oscillation amortie

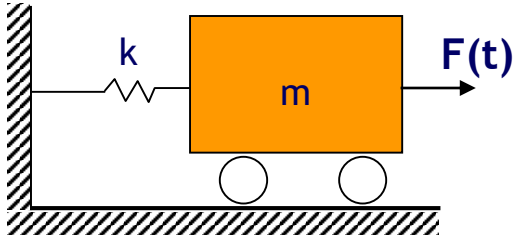


$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = 0$$

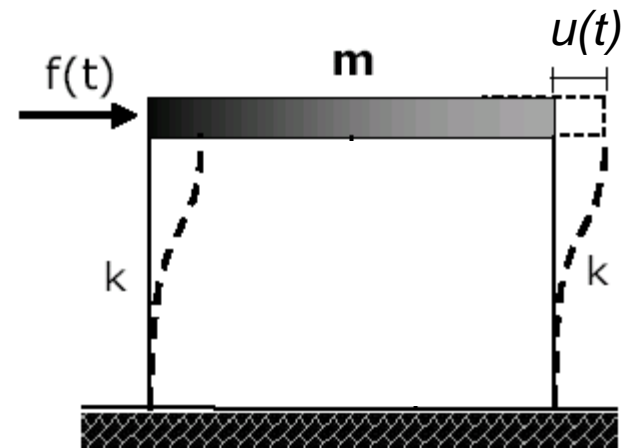
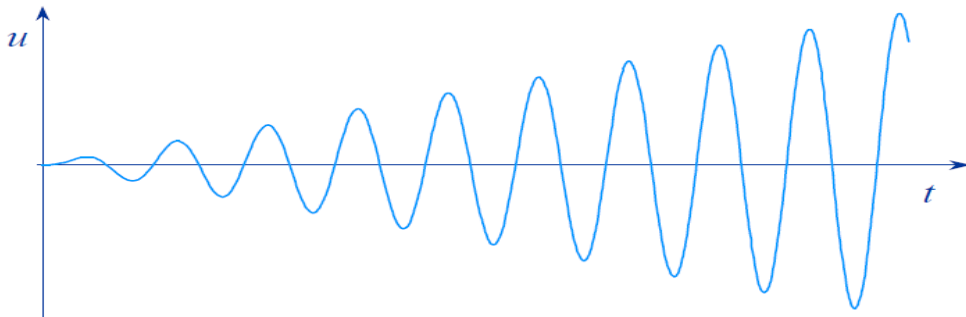


# Oscillateur élémentaire 3/4

## Oscillation forcée

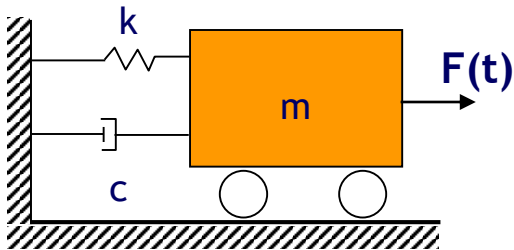


$$m\ddot{u}(t) + ku(t) = F(t)$$



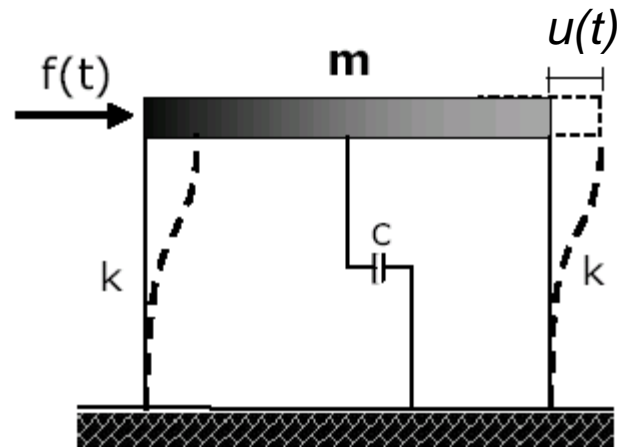
# Oscillateur élémentaire 4/4

## Oscillation amortie et forcée



$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F(t)$$

Hypothèse de  
**plancher rigide**  
et poteaux souples





# Période propre de vibration

## Pulsation propre

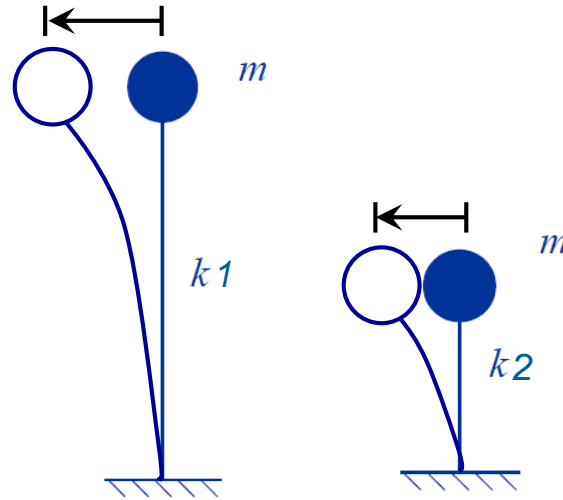
$$\omega_0 = \sqrt{\frac{k}{m}}$$

## Fréquence propre

$$f_0 = \frac{\omega_0}{2\pi}$$

## Période propre

$$T_0 = \frac{2\pi}{\omega_0}$$



$$k_1 < k_2$$

$$T_1 > T_2$$

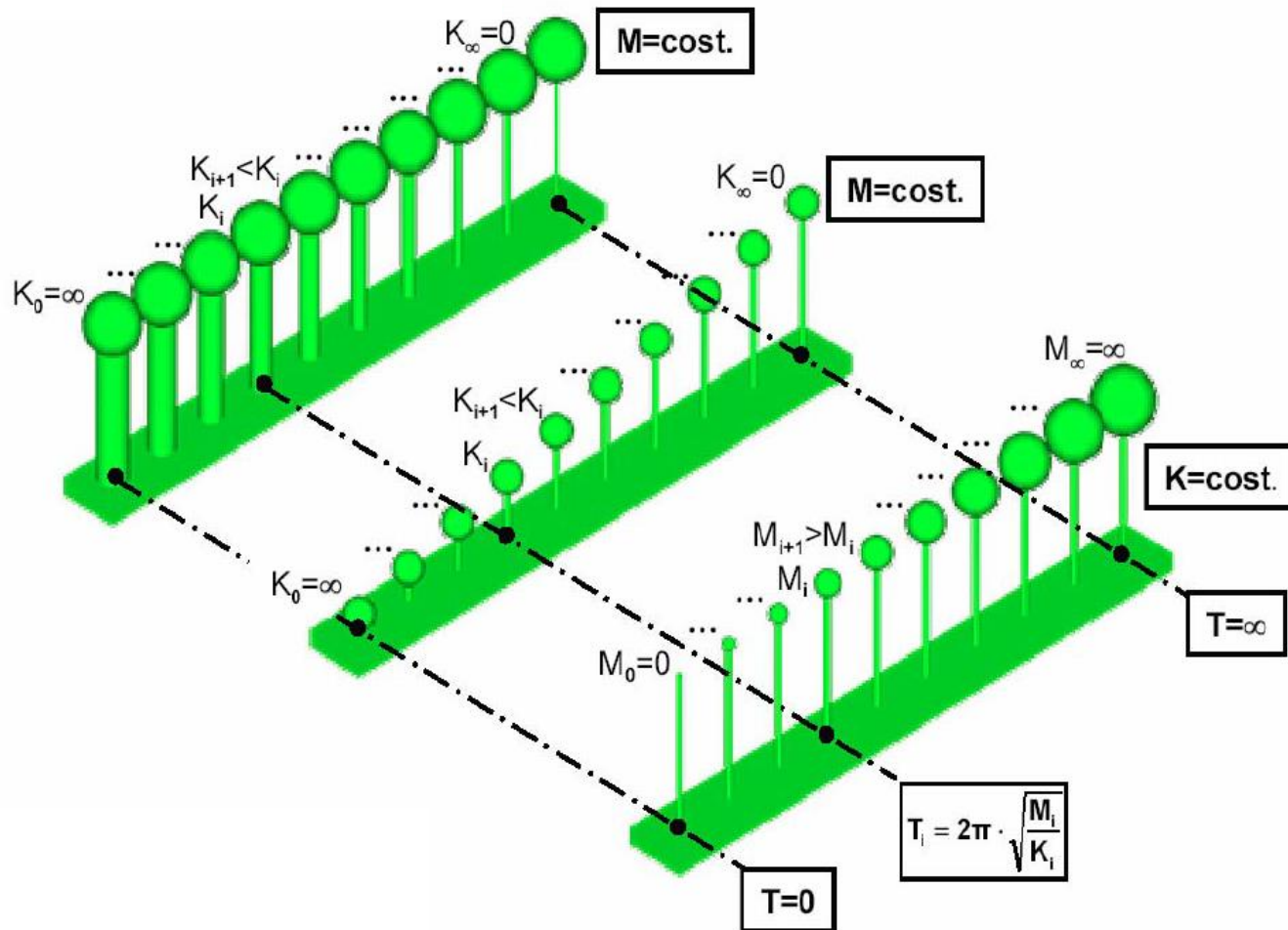
$$f_1 < f_2$$

$$\omega [\text{rad/s}] \quad k [\text{N/m}] \quad m [\text{kg}] \quad f [\text{Hz}] \quad T [\text{s}]$$

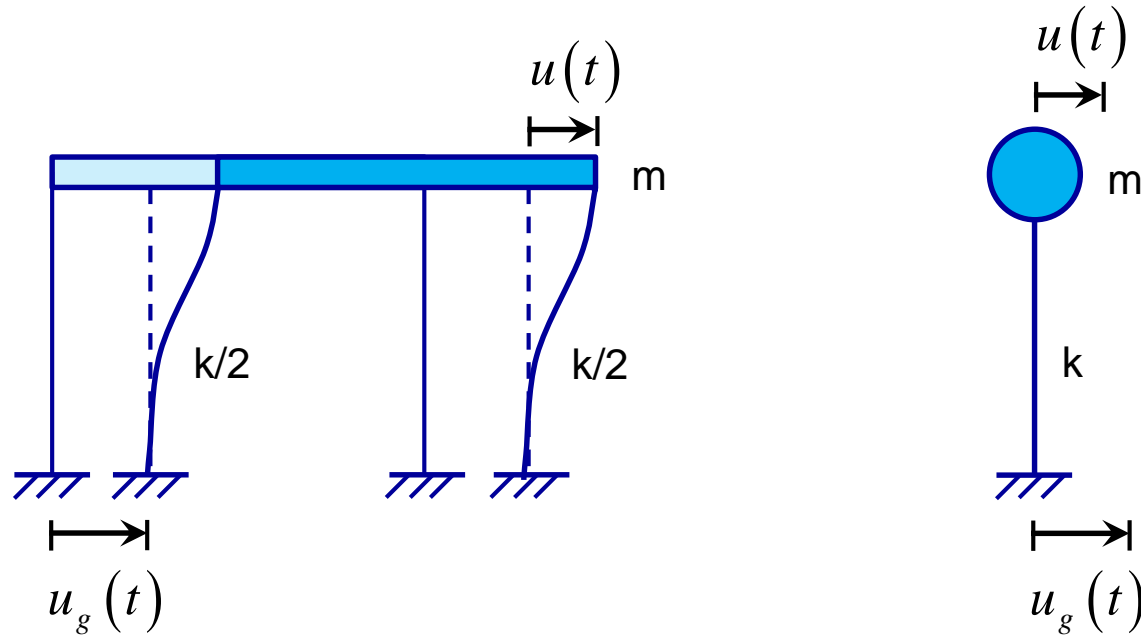
# Analogie avec l'oscillateur élémentaire

+  $T_0 - f_0$  + structure souple

-  $T_0 + f_0$  + structure rigide



# Oscillateur élémentaire 1/3



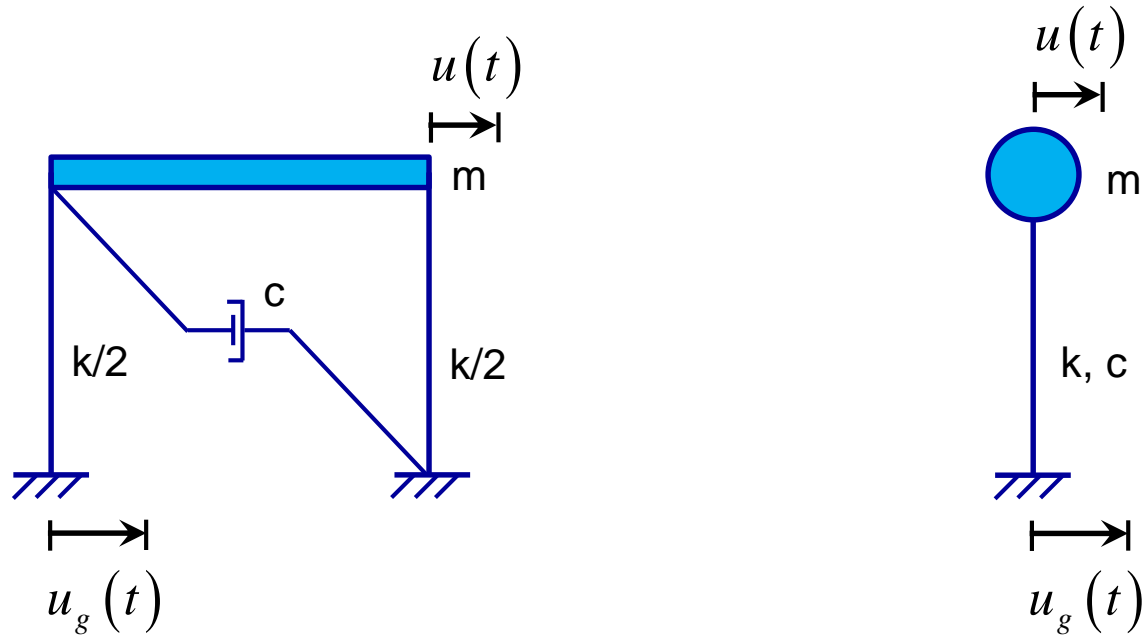
Hypothèses :

- Poteaux : indéformables axialement, masse négligeable
- Plancher rigide

**Équation d'équilibre dynamique d'un oscillateur élémentaire**

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}_g(t)$$

## Oscillateur élémentaire 2/3



Équation d'équilibre dynamique d'un oscillateur élémentaire

$$\ddot{u}(t) + \frac{c}{m} \dot{u}(t) + \frac{k}{m} u(t) = -\ddot{u}_g(t)$$

Rapport d'amortissement  $\zeta_0 = \frac{c}{2\omega_0 m}$

Pulsation propre  $\omega_0^2 = \frac{k}{m}$

## Oscillateur élémentaire 3/3

$$\ddot{u}(t) + 2\zeta_0 \omega_0 \dot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t)$$

$$\dot{u}(0) = 0, \quad u(0) = 0 \quad \textbf{Condition statique initiale}$$

- Amplification des effets :
  - Sol rigide + Bâtiment rigide
  - Sol mou + Bâtiment souple
- Peu de dommages :
  - Sol rigide + Bâtiment souple
  - Sol mou + Bâtiment rigide
- Lien entre le contenu en énergie du séisme aux différentes fréquences et la période fondamentale de vibration de la structure
- La réponse sismique dépend du **signal** et de la **structure**

# Réponse dynamique d'un oscillateur 1/2

## ■ Données :

- Masse :  $m = 18000 \text{ kg}$

- Géométrie **1D** :

section 30 x 30 cm ou 30 x 60 cm

$H = 3 \text{ m}$

- Matériau : béton,  $\rho = 2500 \text{ kg/m}^3$

$E = 31220 \text{ N/mm}^2$ ,  $\nu = 0.2$

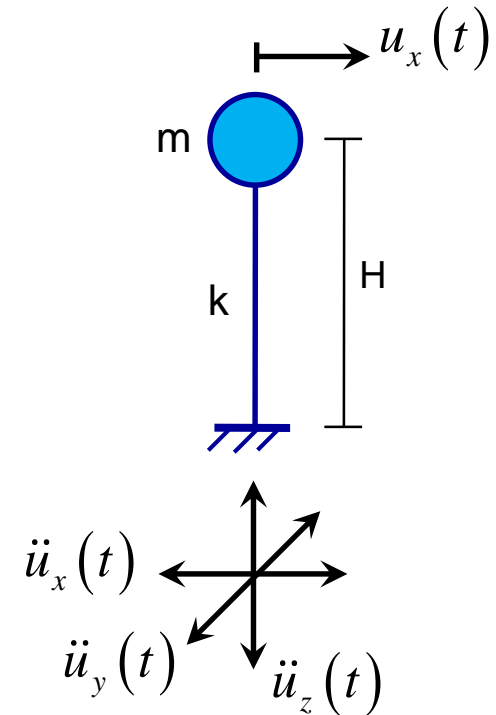
- Loi de comportement :

élastique linéaire

- Conditions aux limites :

1. Base encastrée  $u_x = u_y = u_z = 0$

2. Accélération imposée  $\ddot{u}_x(t), \ddot{u}_y(t), \ddot{u}_z(t)$



# Réponse dynamique d'une structure 3/3

## ■ **Modélisation :**

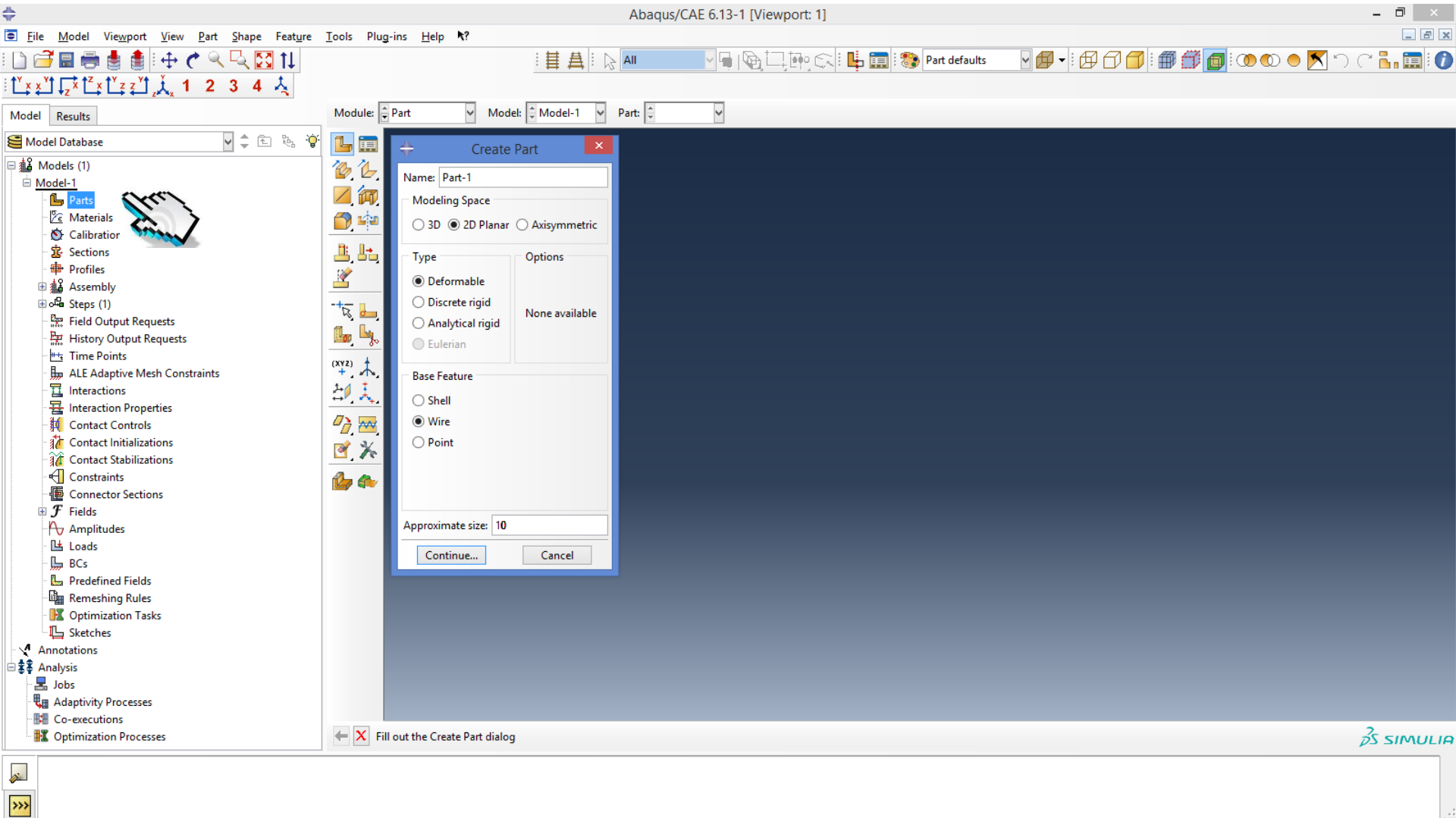
- Masse  $m$ , raideur  $k = 12EI/H^3$
- Fréquence propre  $f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ , période  $T_0 = \frac{1}{f_0}$

## ■ **Résultats :**

- Step 1 : Fréquences propres
- Step 1 : Formes modales
- Step 2 : Histoire temporelle à la base en termes d'accélération [m/s<sup>2</sup>] (input)
- Step 2 : Histoire temporelle au sommet en termes de déplacement [m] (déformation)
- Step 2 : Histoire temporelle au sommet en termes de vitesse [m/s] et accélération
- Déplacement max au sommet

# Model - Parts

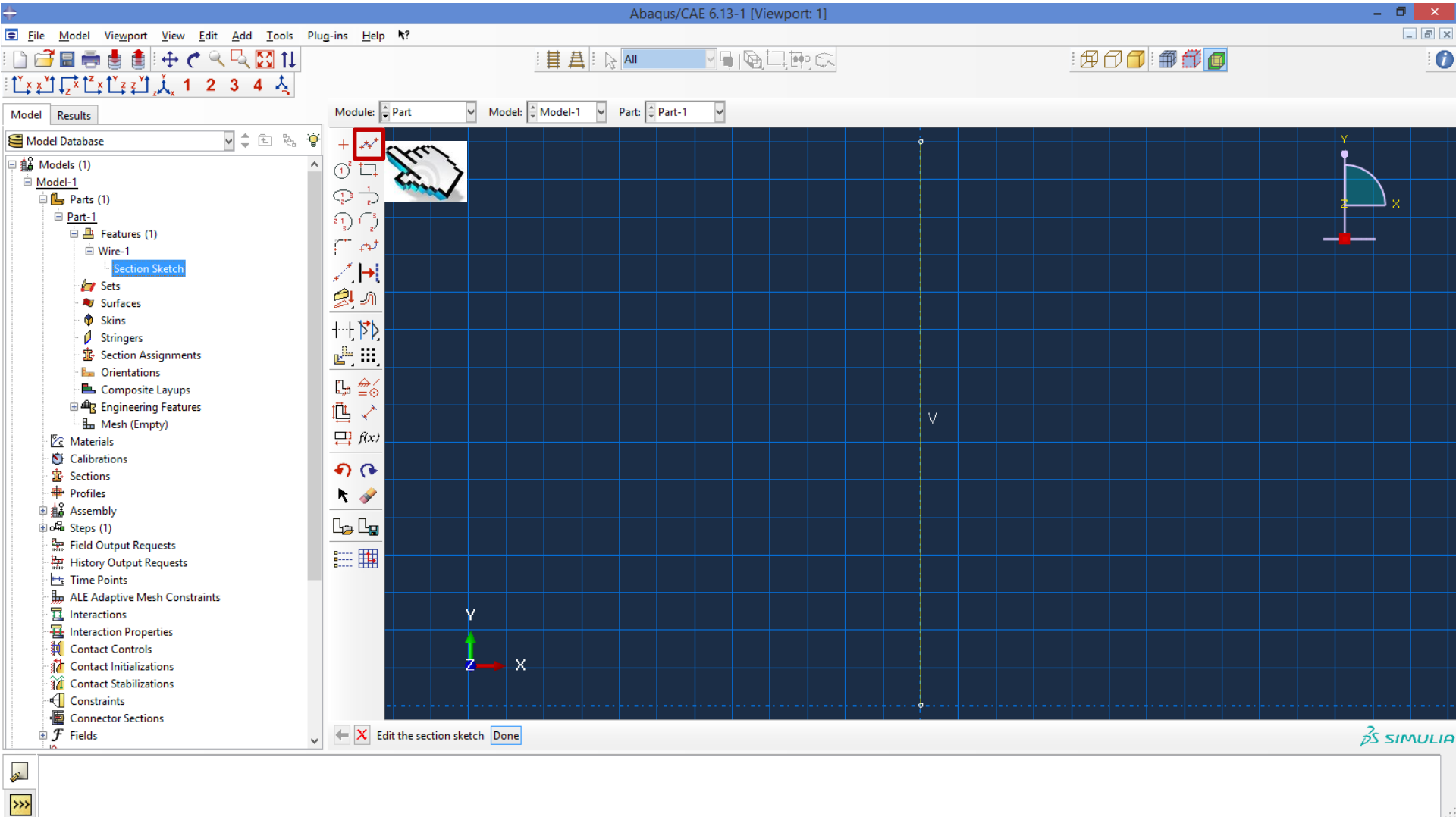
## 1. Géométrie : type d'analyse (1D), type d'élément (poutre)





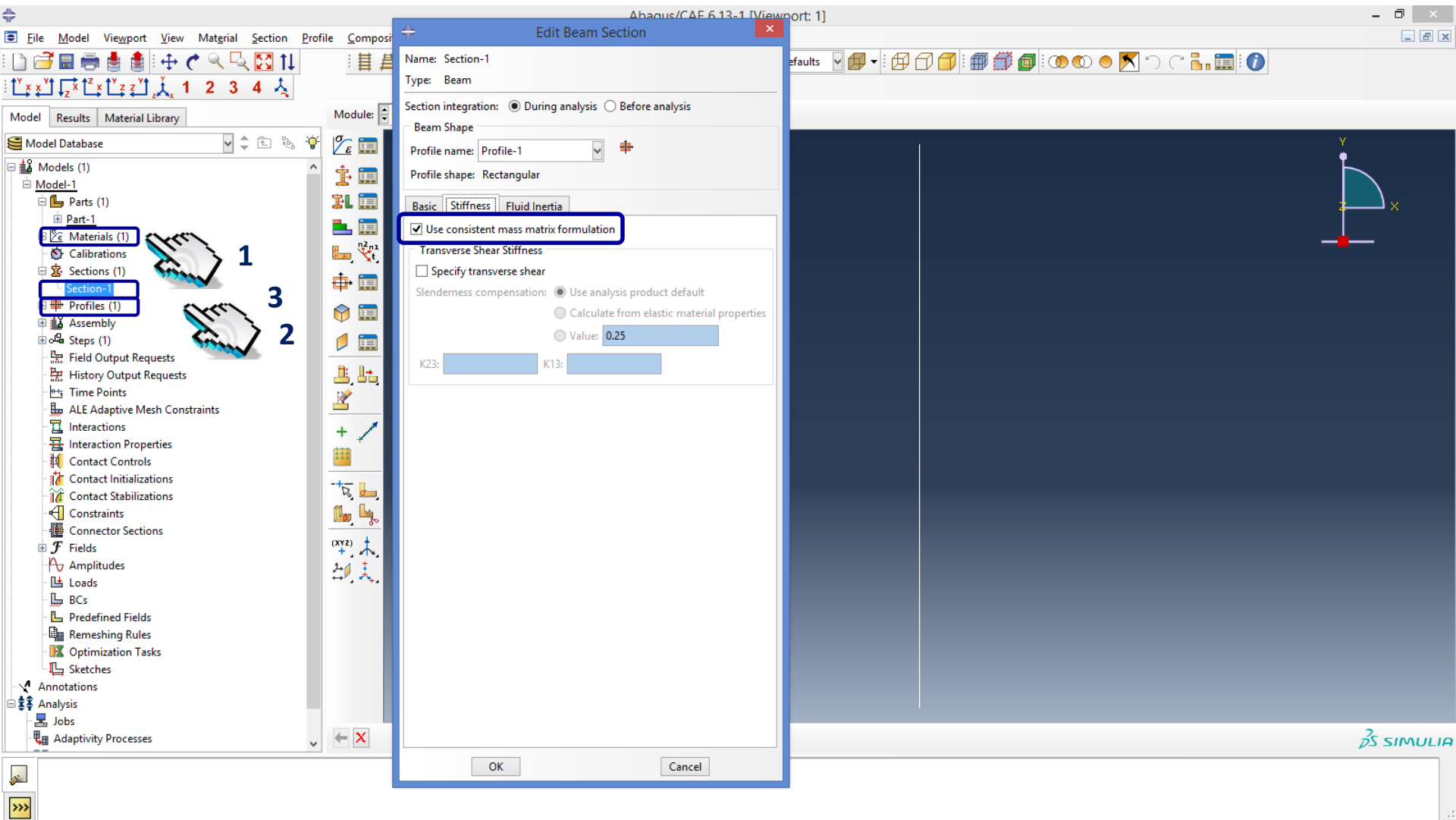
# Model - Parts

## 2. Géométrie : tracer la poutre



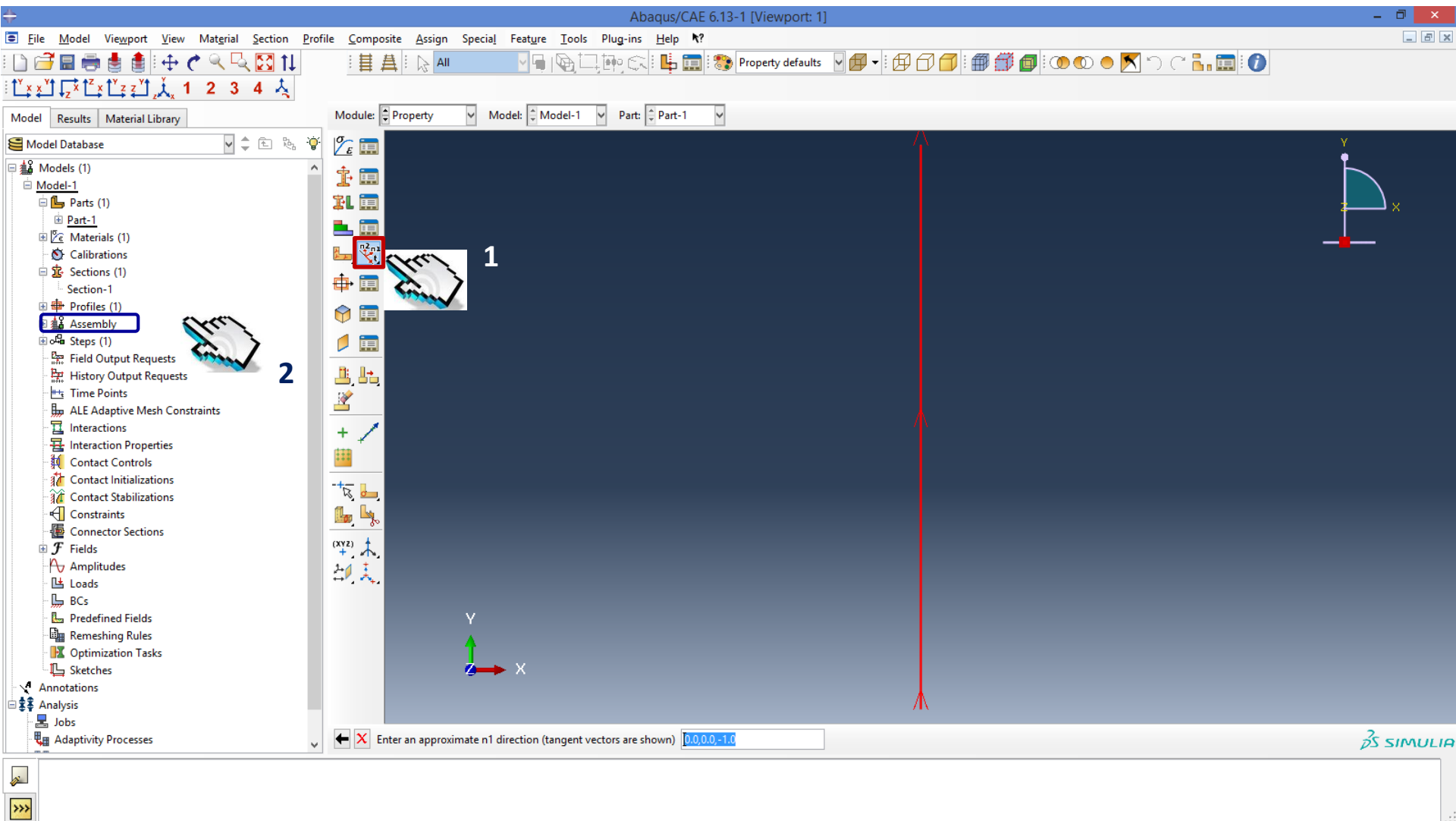
# Model

## 3. Propriétés : Matériau, Profilé R30x30, Section, Attribution de la section



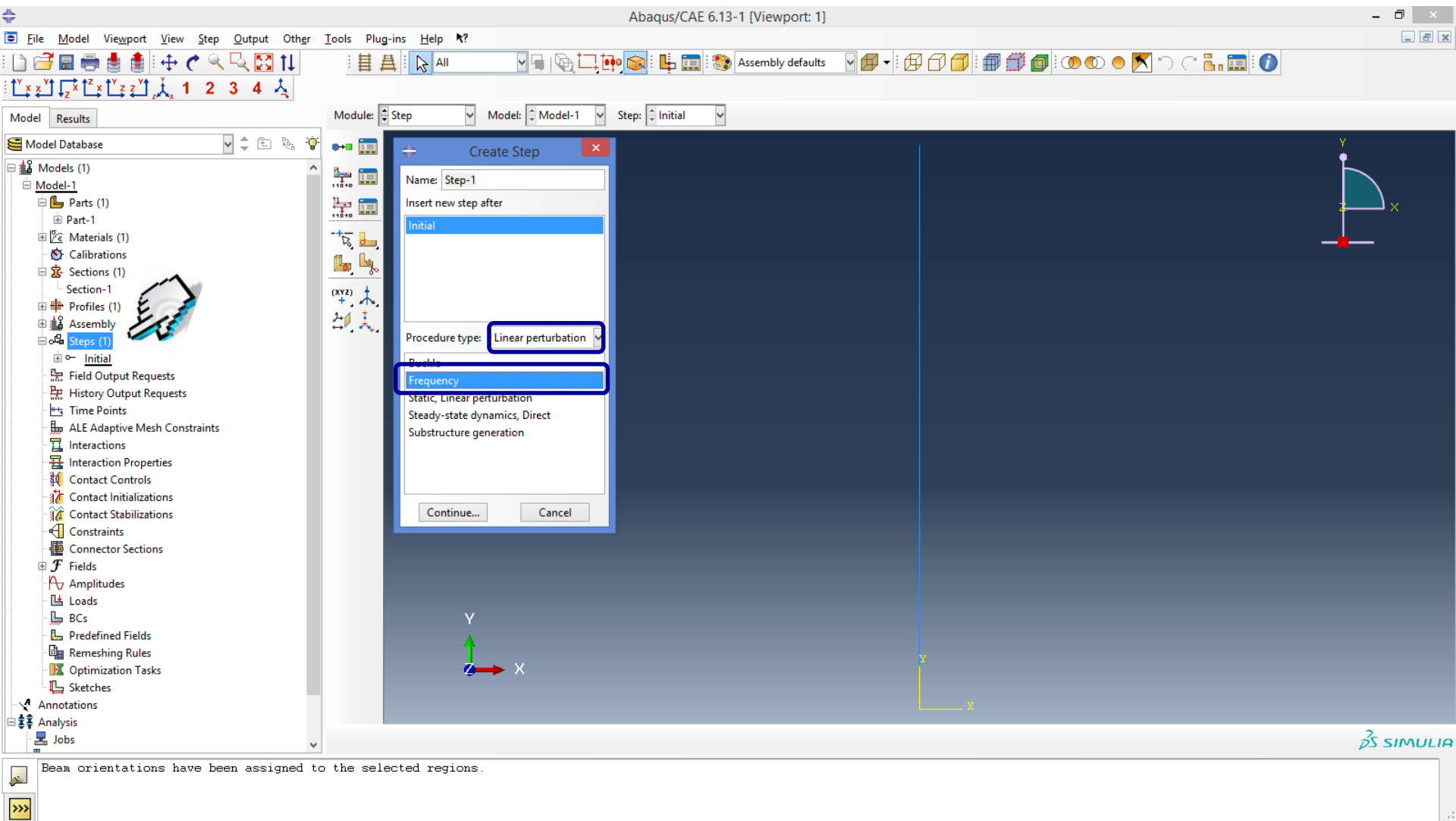
# Model

## 4. Propriétés : Orientation des poutres, Assemblage



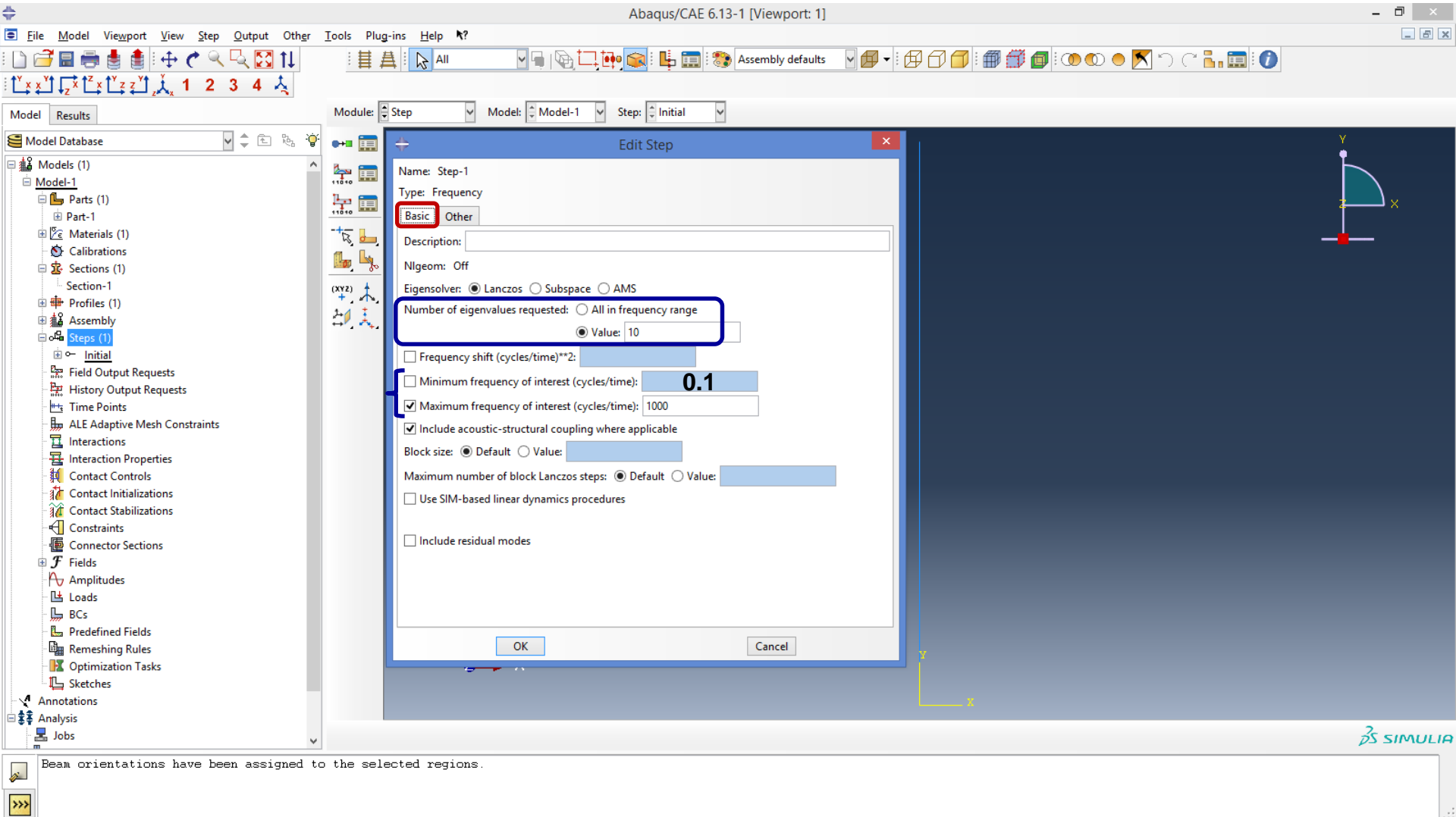
# Model - Steps

## 5. Pas de calcul 1 : fréquences propres



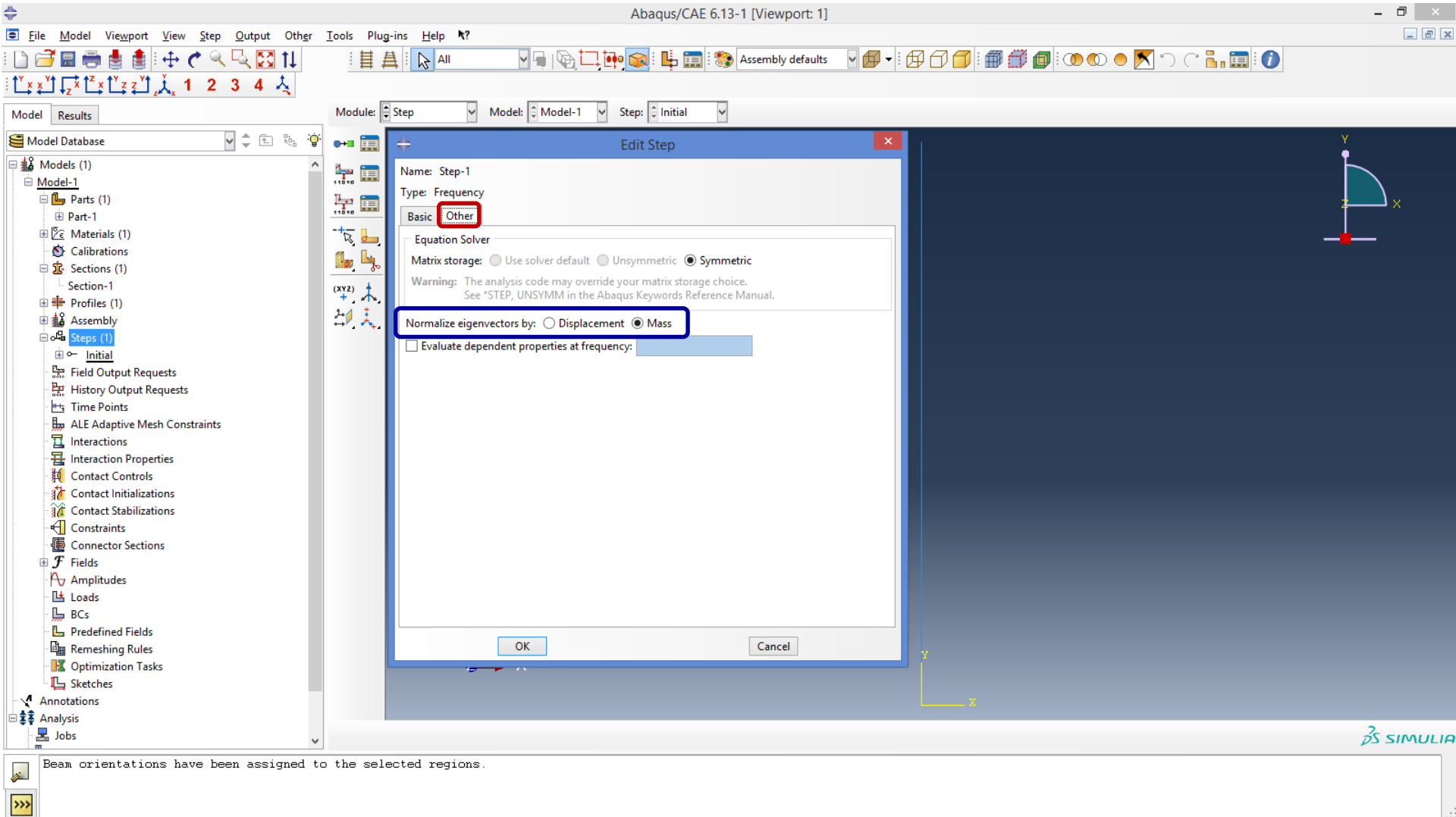
# Model - Steps

## 5. Pas de calcul 1 : fréquences propres



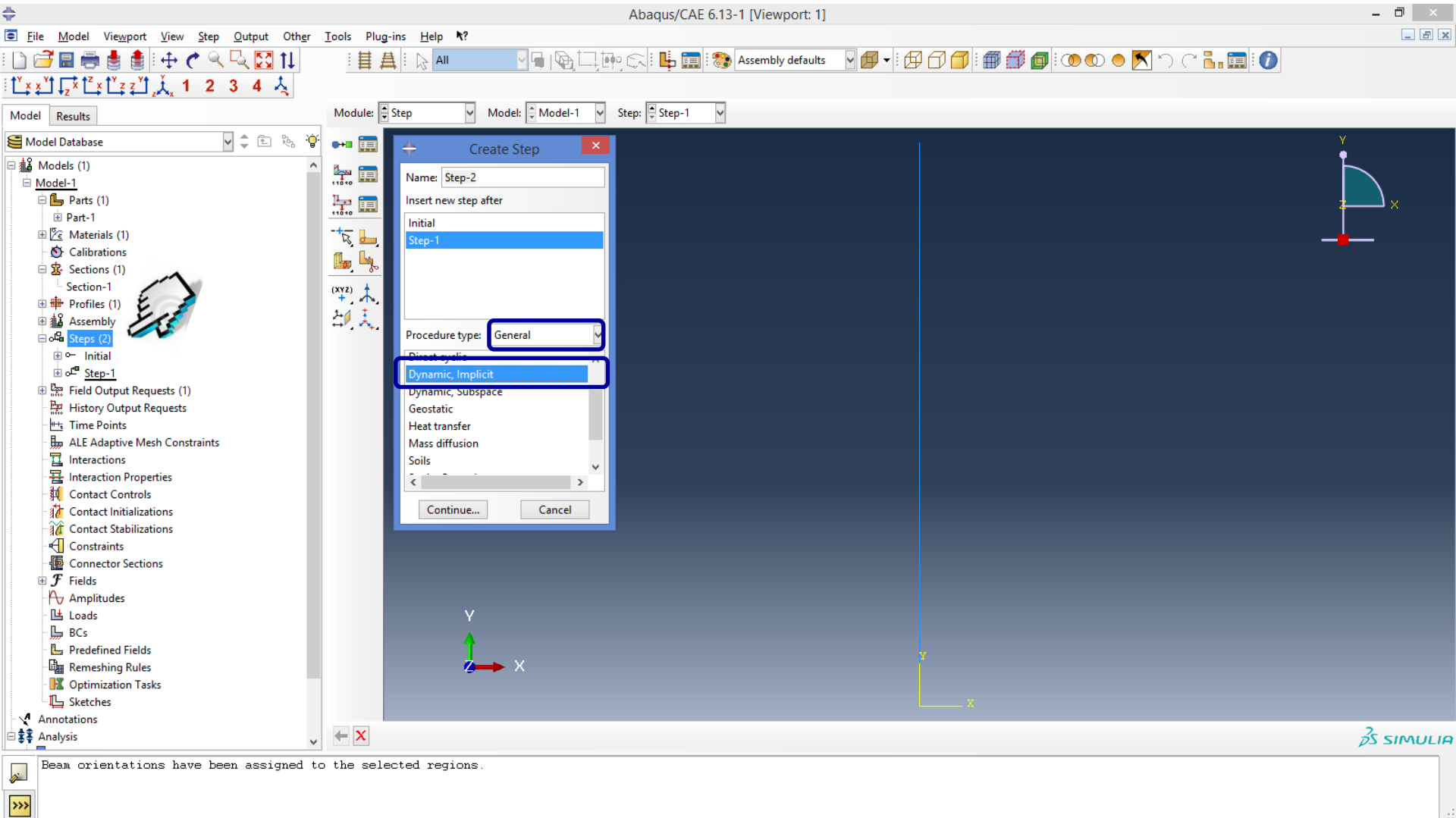
# Model - Steps

## 5. Pas de calcul 1 : fréquences propres



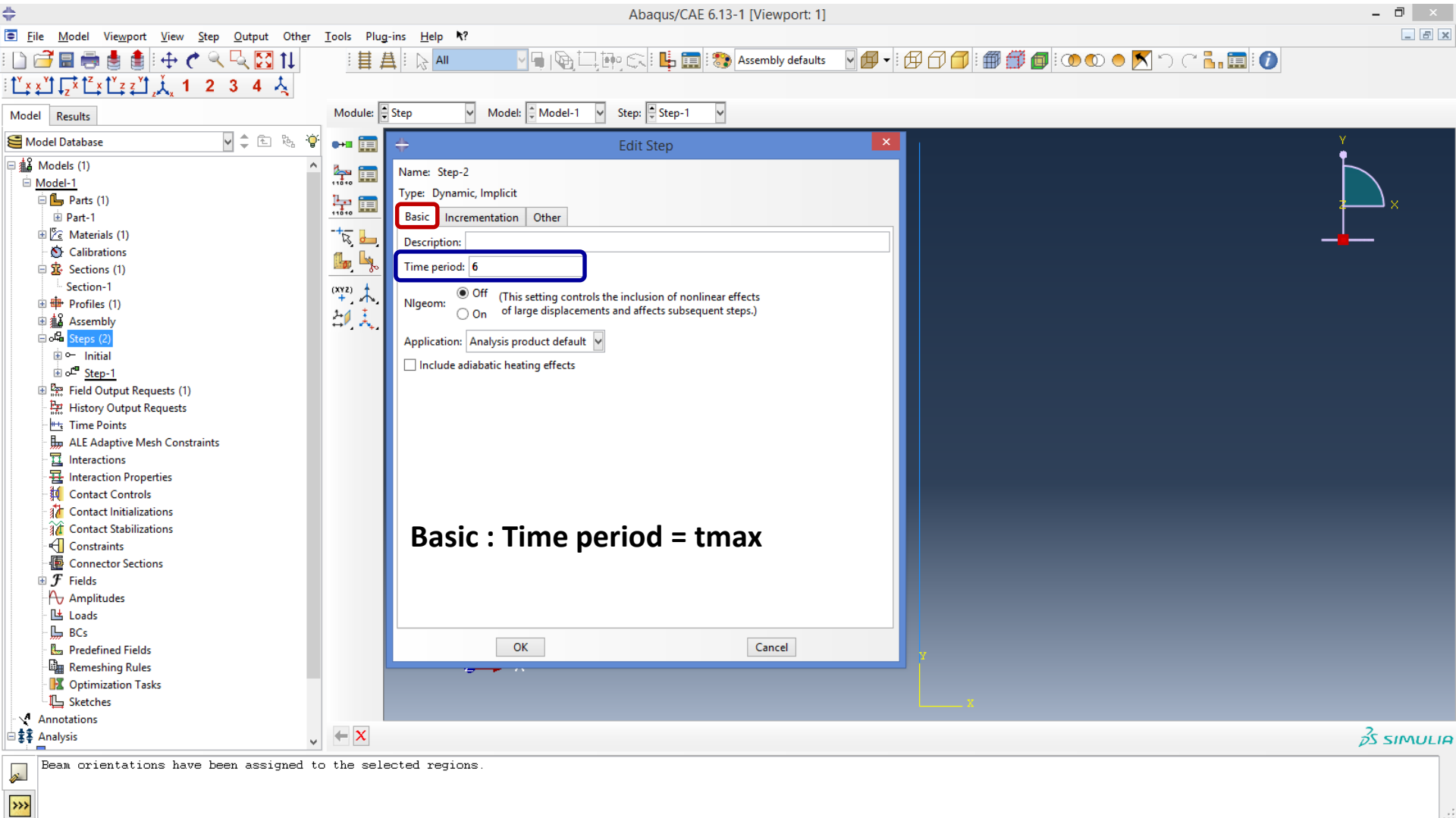
# Model - Steps

## 6. Pas de calcul 2 : mouvement dans le temps



# Model - Steps

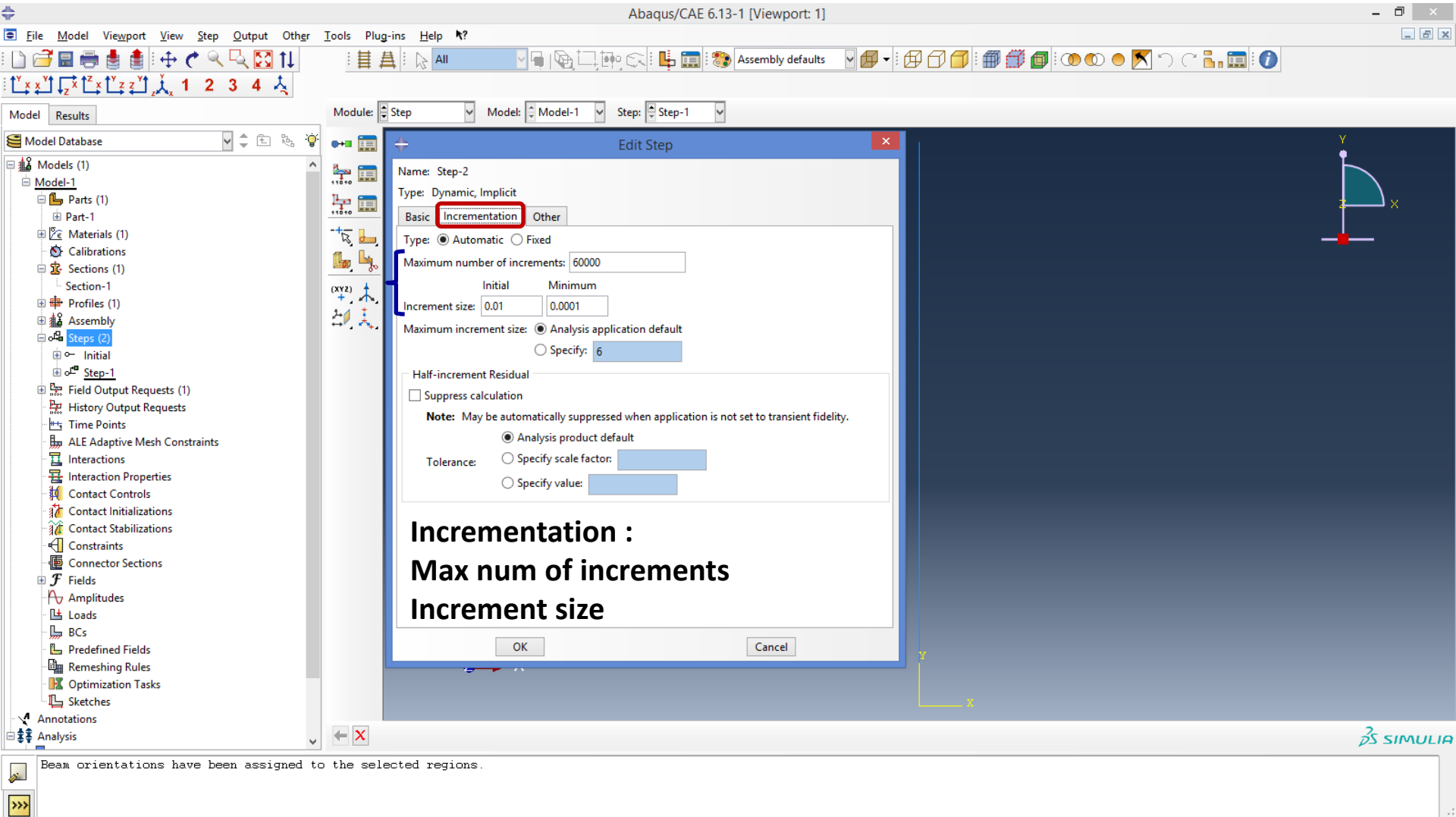
## 6. Pas de calcul 2 : mouvement dans le temps





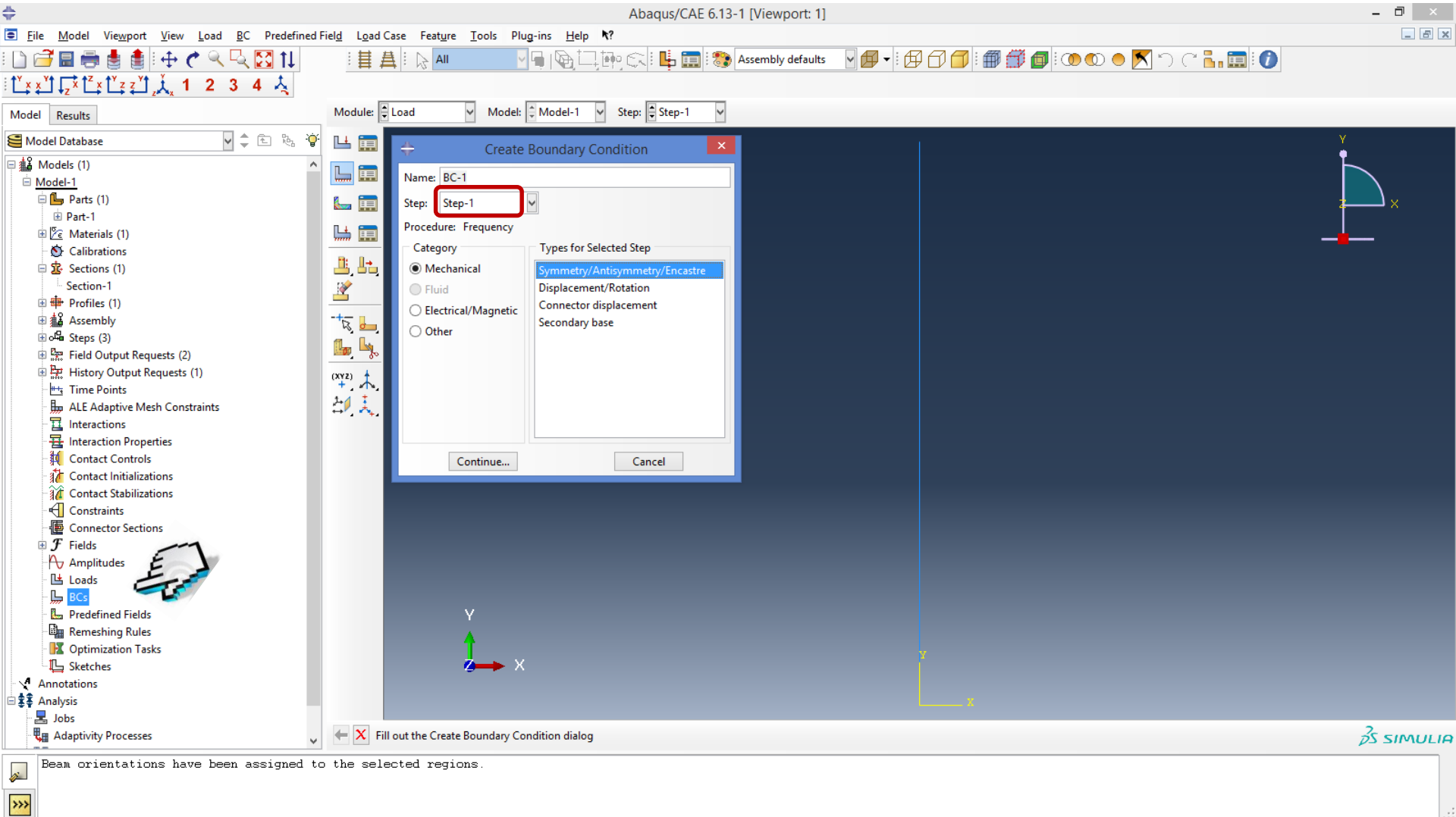
# Model - Steps

## 6. Pas de calcul 2 : mouvement dans le temps



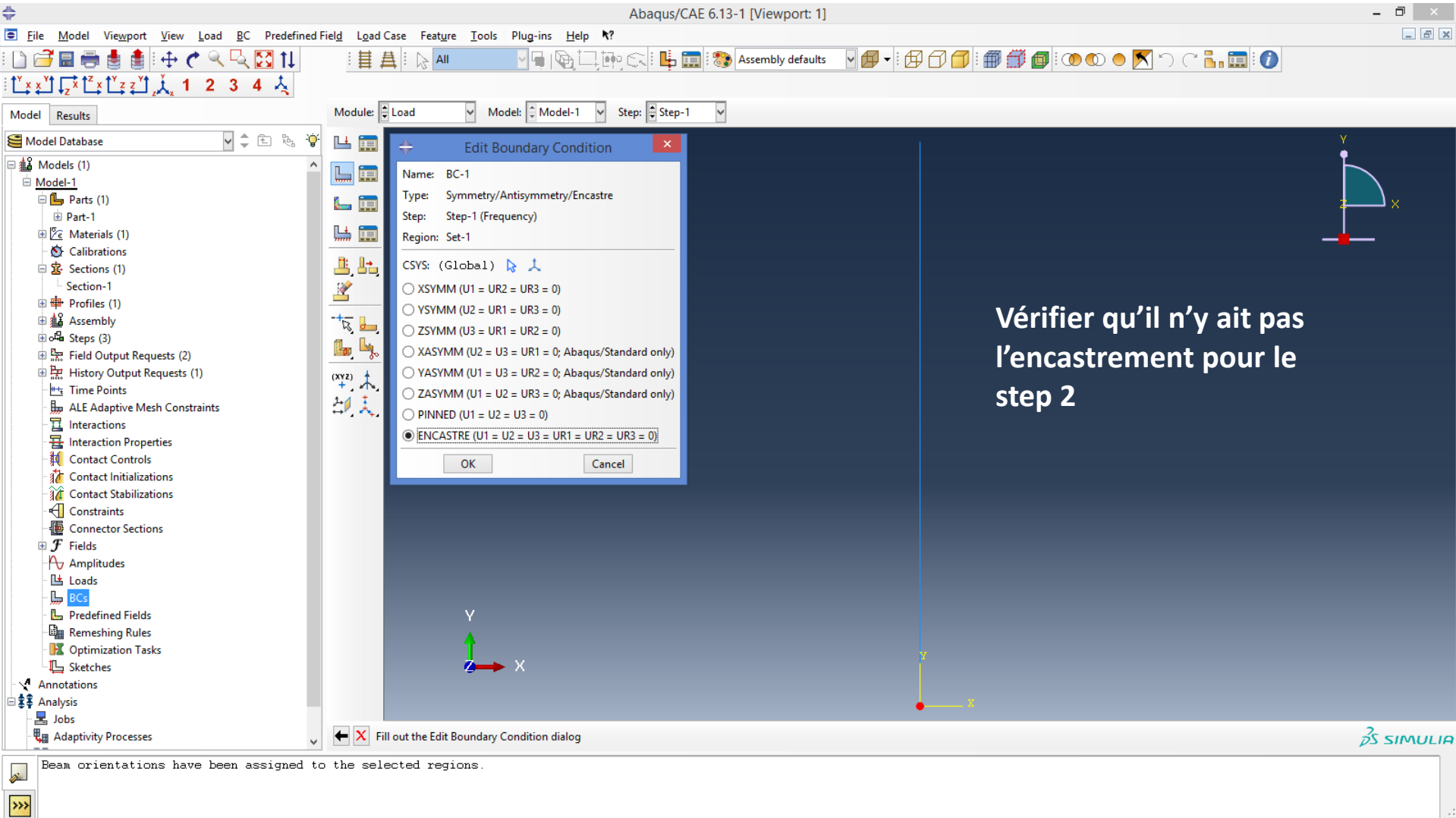
# Model - BCs

## 7. Conditions aux limites : encastrement à la base (step 1)



# Model - BCs

## 7. Conditions aux limites : encastrement à la base (step 1)



Abaqus/CAE 6.13-1 [Viewport: 1]

File Model Viewport View Load BC Predefined Field Load Case Feature Tools Plug-ins Help

Model Database

Models (1)

Model-1

Parts (1)

Part-1

Materials (1)

Calibrations

Sections (1)

Section-1

Profiles (1)

Assembly

Steps (3)

Field Output Requests (2)

History Output Requests (1)

Time Points

ALE Adaptive Mesh Constraints

Interactions

Interaction Properties

Contact Controls

Contact Initializations

Contact Stabilizations

Constraints

Connector Sections

Fields

Amplitudes

Loads

BCs

Predefined Fields

Remeshing Rules

Optimization Tasks

Sketches

Annotations

Analysis

Jobs

Adaptivity Processes

Module: Load Model: Model-1 Step: Step-1

Edit Boundary Condition

Name: BC-1

Type: Symmetry/Antisymmetry/Encastre

Step: Step-1 (Frequency)

Region: Set-1

CSYS: (Global)

☐ XSMM (U1 = UR2 = UR3 = 0)

☐ YSMM (U2 = UR1 = UR3 = 0)

☐ ZSMM (U3 = UR1 = UR2 = 0)

☐ XASMM (U2 = U3 = UR1 = 0; Abaqus/Standard only)

☐ YASMM (U1 = U3 = UR2 = 0; Abaqus/Standard only)

☐ ZASMM (U1 = U2 = UR3 = 0; Abaqus/Standard only)

☐ PINNED (U1 = U2 = U3 = 0)

☒ ENCASTRE (U1 = U2 = U3 = UR1 = UR2 = UR3 = 0)

OK Cancel

Fill out the Edit Boundary Condition dialog

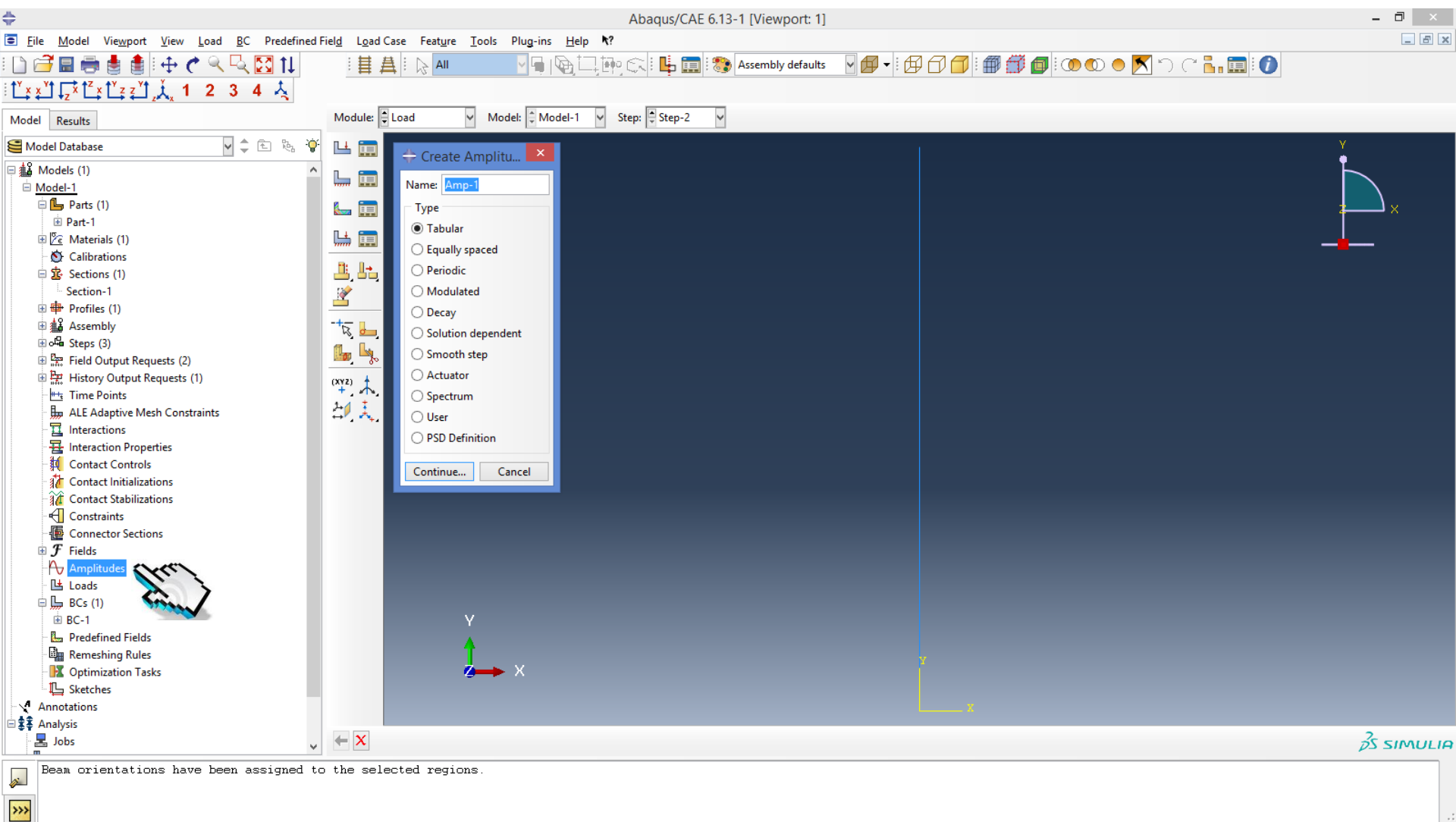
Beam orientations have been assigned to the selected regions.

Vérifier qu'il n'y ait pas l'encastrement pour le step 2

SIMULIA

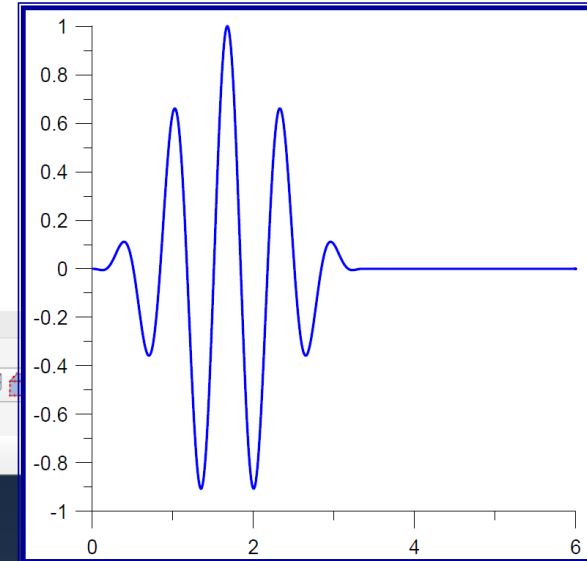
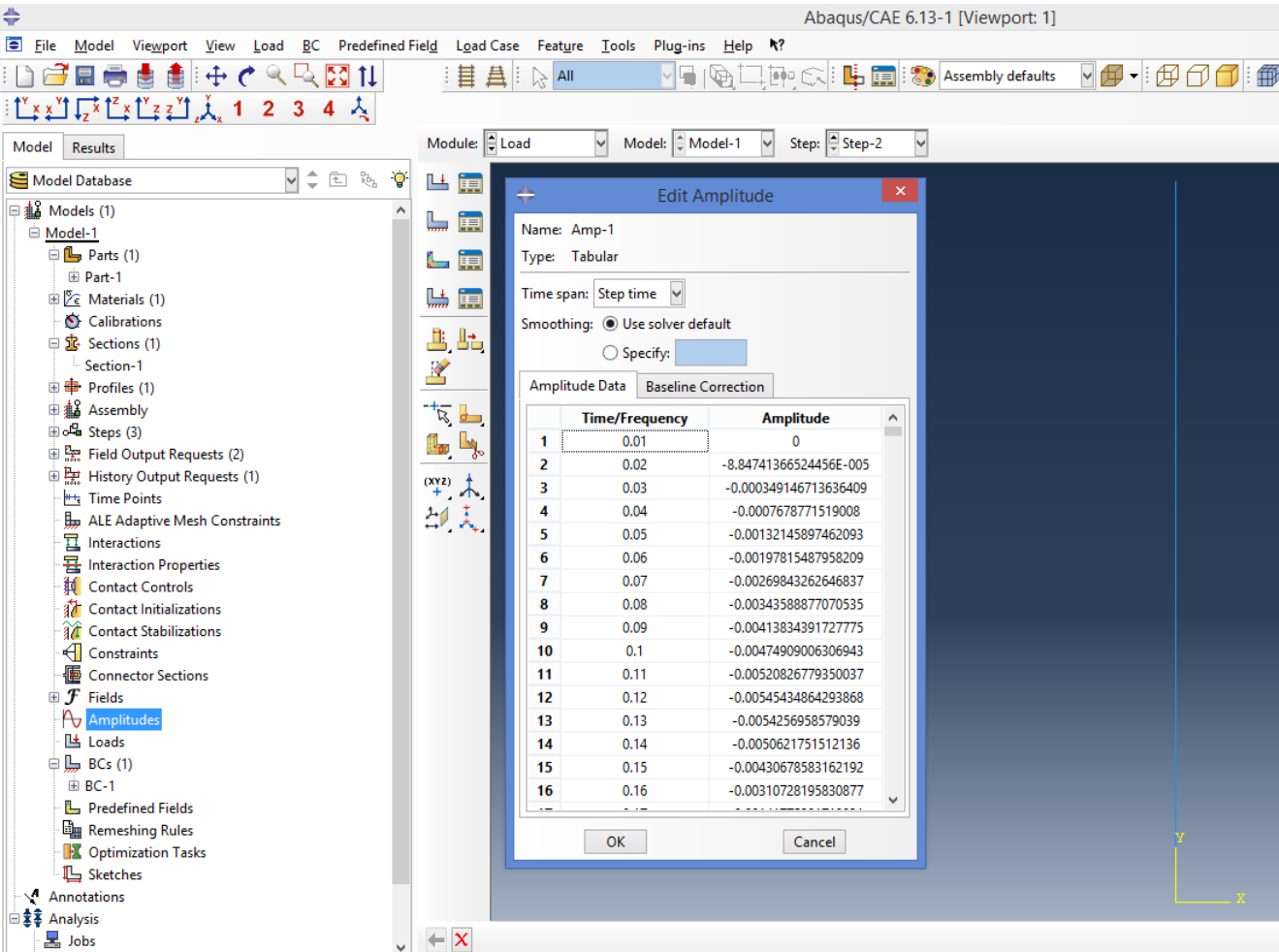
# Model - Amplitudes

## 8. Histoire de charge : amplitude dans le temps



# Model - Amplitudes

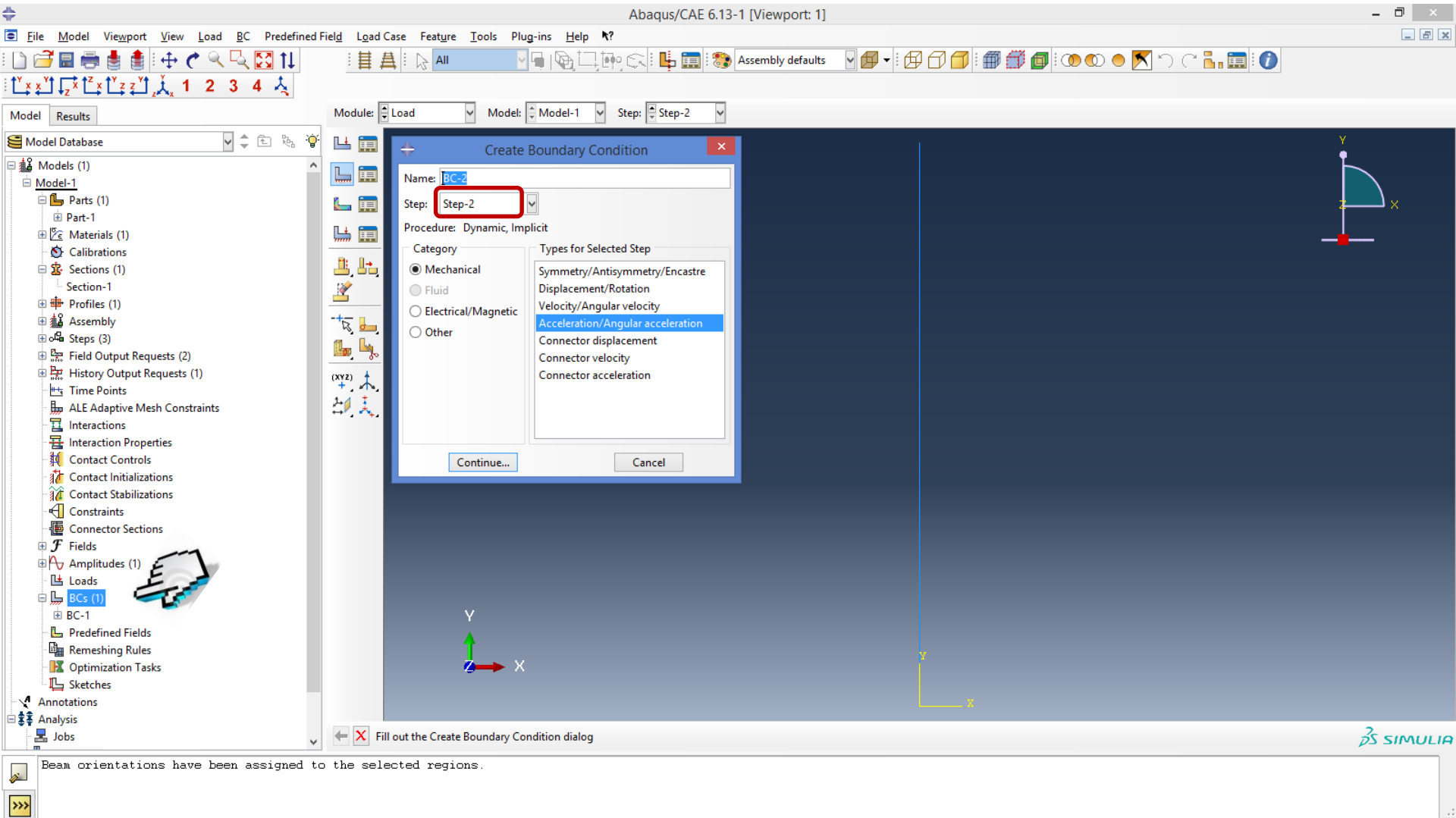
## 8. Histoire de charge : amplitude dans le temps



$$\text{Amplitude} = A(x) / A_{\text{max}}$$

# Model - BCs

## 9. Conditions aux limites : accélération imposée (step 2)



# Model - BCs

## 9. Conditions aux limites : accélération imposée

Abaqus/CAE 6.13-1 [Viewport: 1]

File Model Viewport View Load BC Predefined Field Load Case Feature Tools Plug-ins Help

Model Database

Models (1)

Model-1

Parts (1)

Part-1

Materials (1)

Calibrations

Sections (1)

Section-1

Profiles (1)

Assembly

Steps (3)

Field Output Requests (2)

History Output Requests (1)

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Interaction Properties

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Connector Sections

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Amplitudes (1)

Loads

BCs (1)

BC-1

Predefined Fields

Remeshing Rules

Optimization Tasks

Sketches

Annotations

Analysis

Jobs

Module: Load Model: Model-1 Step: Step-2

Edit Boundary Condition

Name: BC-2

Type: Acceleration/Angular acceleration

Step: Step-2 (Dynamic, Implicit)

Region: Set-2

CSYS: (Global)

Distribution: Uniform  $f(x)$

☒ A1: 3.43 **Amax**

☒ A2: 0

☒ A3: 0 radians/time\*\*2

Amplitude: Amp-1

OK Cancel

Y X

Y X

Fill out the Edit Boundary Condition dialog

Beam orientations have been assigned to the selected regions.

SIMULIA

# Model

## 10. Mouvement dans le temps : Sets, Field Output Request, History Output Request

**Edit History Output Request**

Name: H-Output-1  
Step: Step-2  
Procedure: Dynamic, Implicit

Domain: Set : Part-1-1.base

Frequency: Every n increments n: 1

Timing: Output at exact times

Output Variables

☒ Select from list below ☐ Preselected defaults ☐ All ☐ Edit variables

U1,V1,A1

- ☐ Stresses
- ☐ Strains
- ☒ Displacement/Velocity/Acceleration
  - ☒ U, Translations and rotations
    - ☐ UT, Translations
    - ☐ UR, Rotations
  - ☒ V, Translational and rotational velocities
    - ☐ VT, Translational velocities
    - ☐ VR, Rotational velocities
  - ☒ A, Translational and rotational accelerations
    - ☐ AT, Translational accelerations
- ☐ Output for rebar

Output at shell, beam, and layered section points:

☒ Use defaults ☐ Specify:

☐ Include sensor when available

☒ Use global directions for vector-valued output

OK Cancel

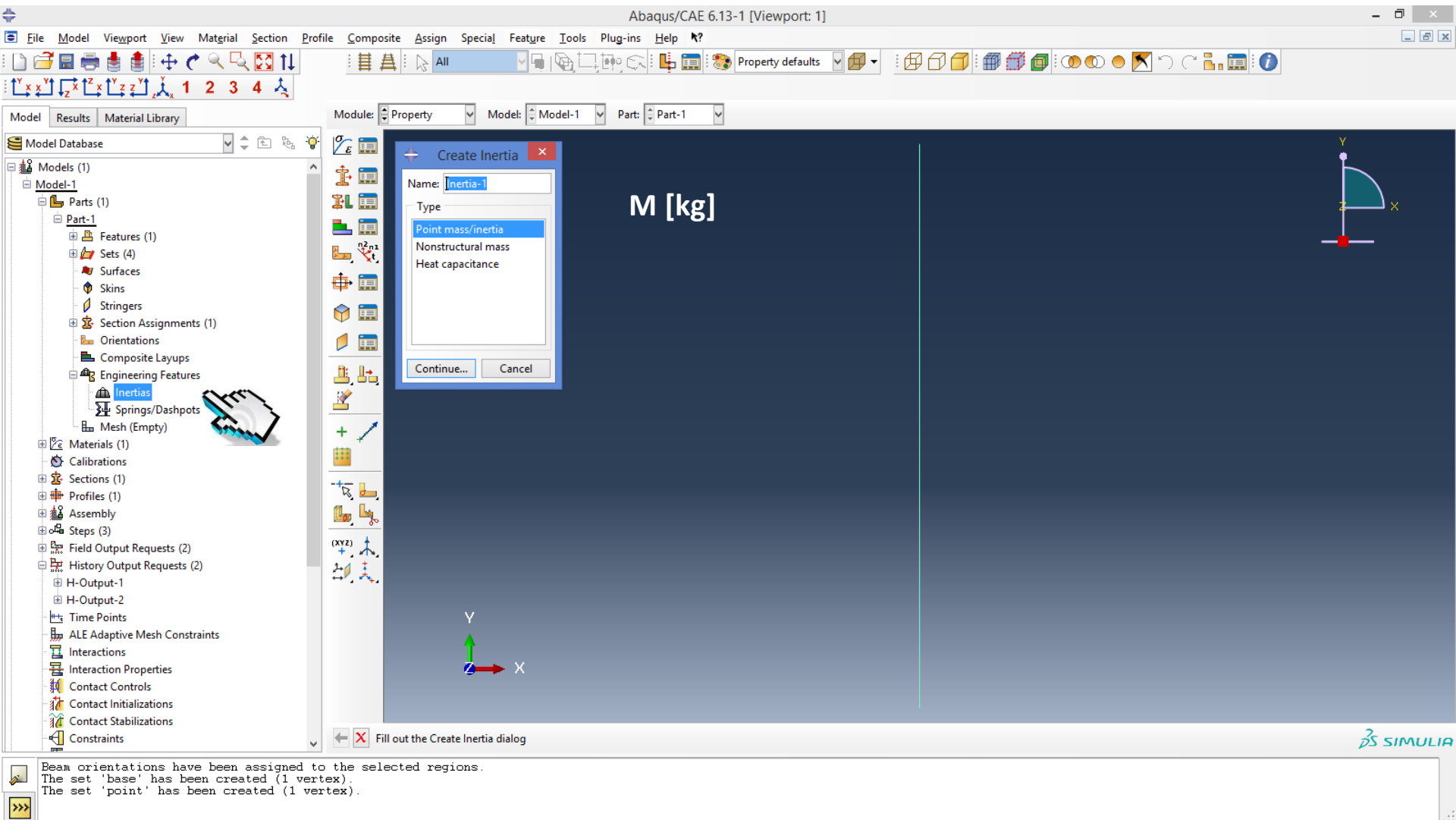
Beam orientations have been assigned to the selected regions.  
The set 'base' has been created (1 vertex).  
The set 'point' has been created (1 vertex).

**u, v, a à la base et au sommet**



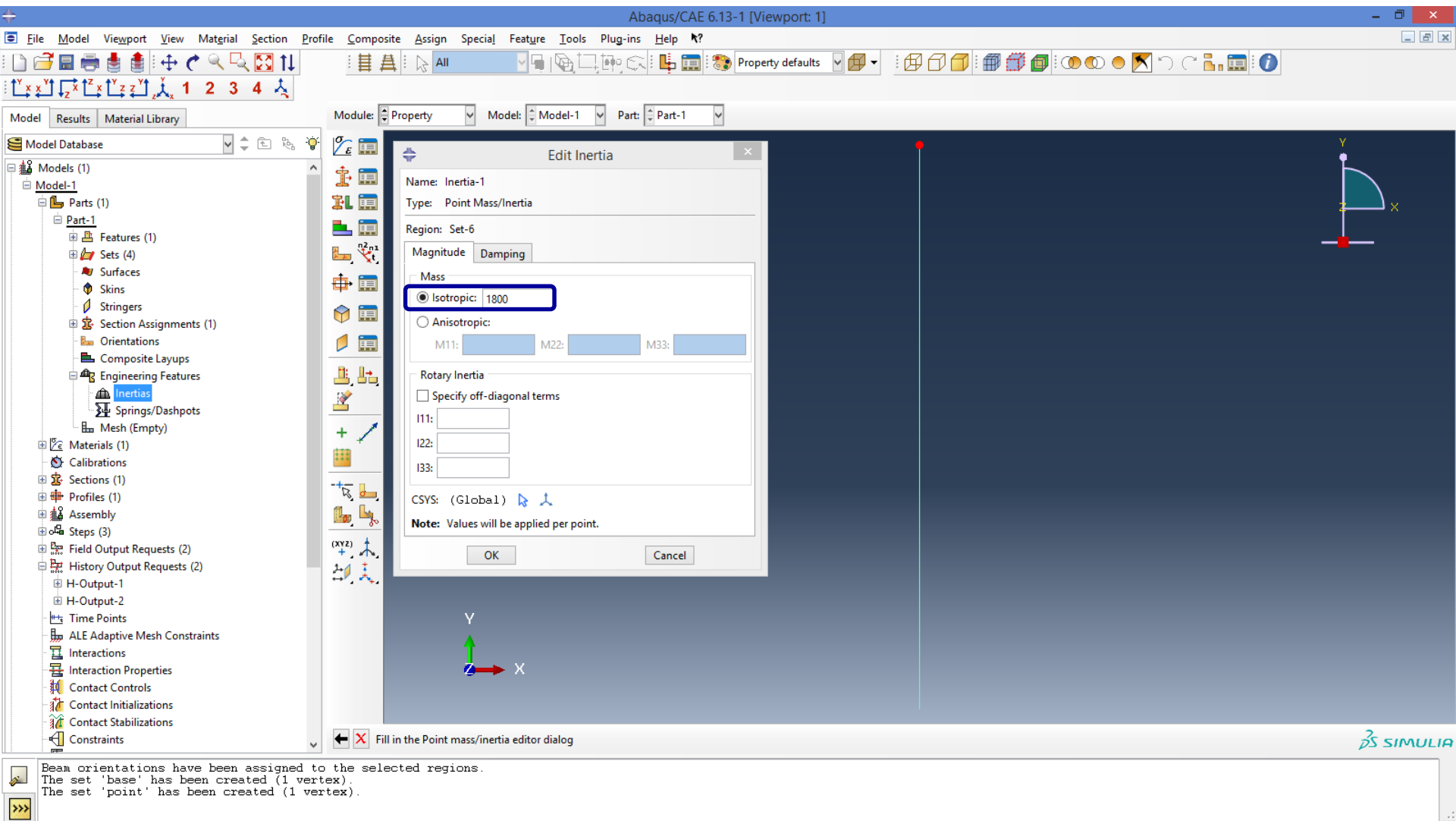
# Model - Parts

## 11. Masse concentrée aux noeuds : éléments de la matrice des masses



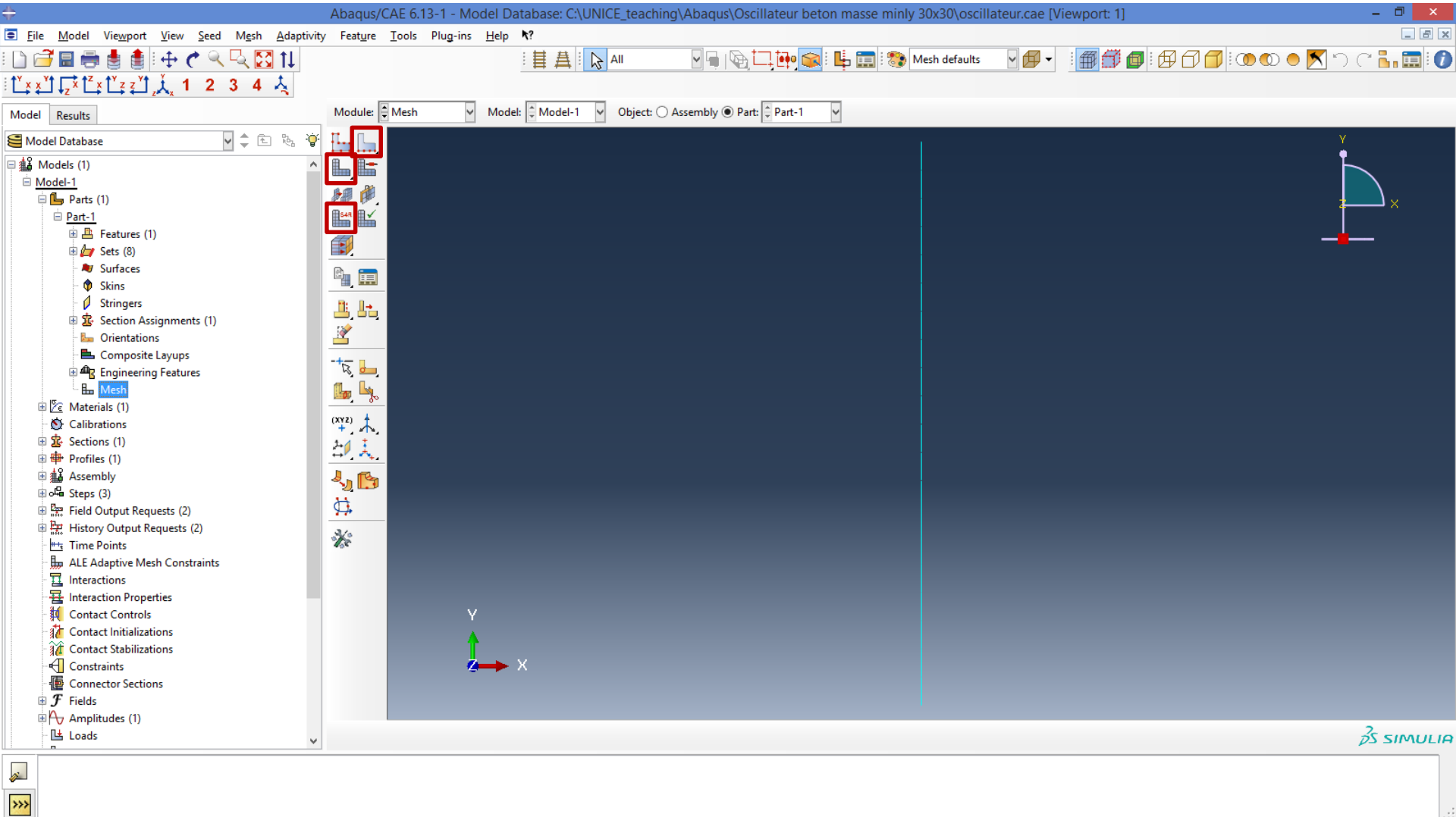
# Model - Parts

## 11. Masse concentrée aux noeuds : éléments de la matrice des masses



# Model

## 12. Maillage (Parts-Mesh) et Calcul (Analysis-Job, Data check, Submit)



# Results

## 1. Fréquences propres

The screenshot displays the Abaqus/CAE 6.13-1 software interface. The main window shows a 3D model of a concrete mass oscillator. The left sidebar contains a tree view of the model's structure, including Output Databases, Steps, Frames, and various sets. The 'Result' menu is open, and the 'Step/Frame...' option is selected. The 'Step/Frame' dialog box is open, showing a table of eigenfrequencies for Step-2. The table lists 9 modes, with the first mode selected. The status bar at the bottom indicates the current step and time.

**Step/Frame Dialog Box:**

Step Name	Description
Step-1	
Step-2	

Index	Description
0	Increment 0: Base State
1	Mode 1: Value = 5.96046E-08 Freq = 3.88562E-05 (cycles/time)
2	Mode 2: Value = 228.84 Freq = 2.4076 (cycles/time)
3	Mode 3: Value = 91590. Freq = 48.166 (cycles/time)
4	Mode 4: Value = 2.58192E+05 Freq = 80.871 (cycles/time)
5	Mode 5: Value = 2.41545E+06 Freq = 247.35 (cycles/time)
6	Mode 6: Value = 9.14663E+06 Freq = 481.34 (cycles/time)
7	Mode 7: Value = 1.38815E+07 Freq = 592.98 (cycles/time)
8	Mode 8: Value = 2.30214E+07 Freq = 763.64 (cycles/time)

ODB: Job-1.odb Abaqus/Standard 6.13-1 Sat Jan 31 20:42:19 Paris, Madrid 2015  
Step: Step-2  
Increment 111: Step Time = 6.000

# Results

## 2. Modes propres

Abaqus/CAE 6.13-1 - Model Database: C:\UNICE\_teaching\Abaqus\Oscillateur beton masse minly 30x30\oscillateur.cae [Viewport: 1]

File Model Viewport View Result Plot Animate Report Options Tools Plug-ins Help

Primary S Mises All Visualization defaults

Model Results

Session Data

- Output Databases (1)
  - Job-1.odb
    - History Output (34)
    - Steps (2)
      - Step-1
        - Frames (9)
          - Frame: 0
          - Frame: 1
          - Frame: 2
          - Frame: 3
          - Frame: 4
          - Frame: 5
          - Frame: 6
          - Frame: 7
          - Frame: 8
        - Step-2
      - Instances (1)
      - Materials (1)
      - Sections (2)
      - Element Sets (5)
      - Node Sets (14)
      - Surface Sets
      - Session Coordinate Systems
      - ODB Coordinate Systems
      - User Data
        - Annotations
        - XYData
- Model Database (1)
- Spectrums (7)
  - XYPlots
  - XYData
- Paths
- Display Groups (1)
- Free Body Cuts

Module: Visualization Model: C:\UNICE\_teaching\Abaqus\Oscillateur beton masse minly 30x30\Job-1.odb

Field Output

Step/Frame  
Step: 1, Step-1  
Frame: 1

Primary Variable Deformed Variable Symbol Variable Status Variable Stream Variable

Output Variable

☐ List only variables with results:

Name	Description (* indicates complex)
E	Strain components at integration points
S	Stress components at integration points
U	Spatial displacement at nodes
UR3	Rotational displacement at nodes

Invariant  
Magnitude

Component  
U1  
U2

Section Points...

OK Apply Cancel

42:19 Paris, Madrid 2015  
cycles/time)

SIMULIA

# Results

## 2. Modes propres

Abaqus/CAE 6.13-1 - Model Database: C:\UNICE\_teaching\Abaqus\Oscillateur beton masse minly 30x30\oscillateur.cae [Viewport: 1]

File Model Viewport View Result Plot Animate Report Options Tools Plug-ins Help

Primary U U1 All Visualization defaults

Model Results

Session Data

- Output Databases (1)
  - Job-1.odb
    - History Output (34)
    - Steps (2)
      - Step-1
        - Frames (9)
          - Frame: 0
          - Frame: 1
          - Frame: 2
          - Frame: 3
          - Frame: 4
          - Frame: 5
          - Frame: 6
          - Frame: 7
          - Frame: 8
        - Step-2
- Instances (1)
- Materials (1)
- Sections (2)
- Element Sets (5)
- Node Sets (14)
- Surface Sets
- Session Coordinate Systems
- ODB Coordinate Systems
- User Data
  - Annotations
  - XYData
- Model Database (1)
- Spectrums (7)
- XYPlots
- XYData
- Paths
- Display Groups (1)
- Free Body Cuts

Module: Visualization Model: C:\UNICE\_teaching\Abaqus\Oscillateur beton masse minly 30x30\Job-1.odb

Step/Frame

Step Name	Description
Step-1	
Step-2	

Frame

Index	Description
0	Increment 0: Base State
1	Mode 1: Value = 5.96046E-08 Freq = 3.88562E-05 (cycles/time)
2	Mode 2: Value = 228.84 Freq = 2.4076 (cycles/time)
3	Mode 3: Value = 91590. Freq = 48.166 (cycles/time)
4	Mode 4: Value = 2.58192E+05 Freq = 80.871 (cycles/time)
5	Mode 5: Value = 2.41545E+06 Freq = 247.35 (cycles/time)
6	Mode 6: Value = 9.14663E+06 Freq = 481.34 (cycles/time)
7	Mode 7: Value = 1.38815E+07 Freq = 592.98 (cycles/time)
8	Mode 8: Value = 2.30214E+07 Freq = 763.64 (cycles/time)

OK Apply Field Output... Cancel

Y

X

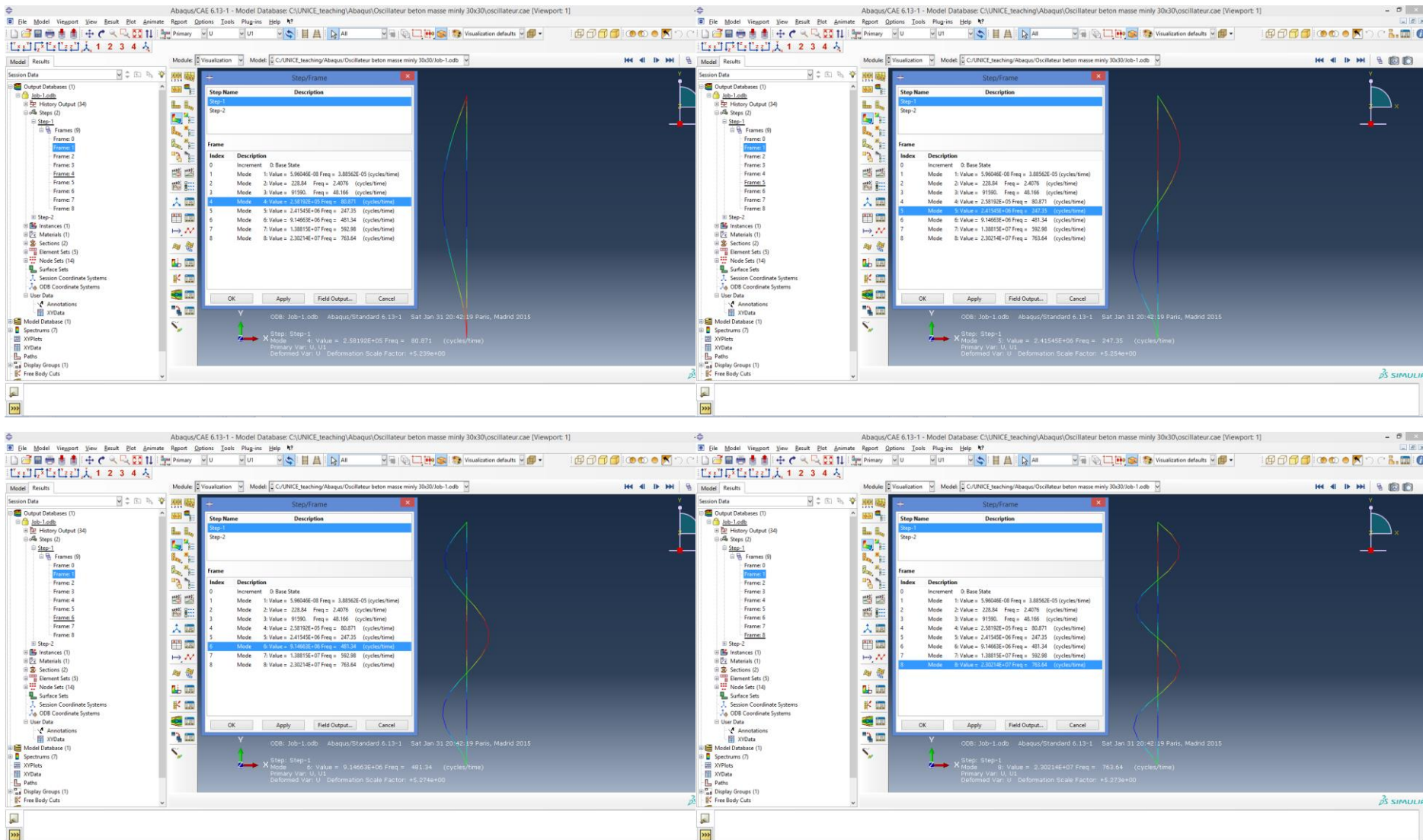
ODB: Job-1.odb Abaqus/Standard 6.13-1 Sat Jan 31 20:42:19 Paris, Madrid 2015

Step: Step-1  
Mode 2: Value = 228.84 Freq = 2.4076 (cycles/time)  
Primary Var: U, U1  
Deformed Var: U Deformation Scale Factor: +3.024e+01

SIMULIA

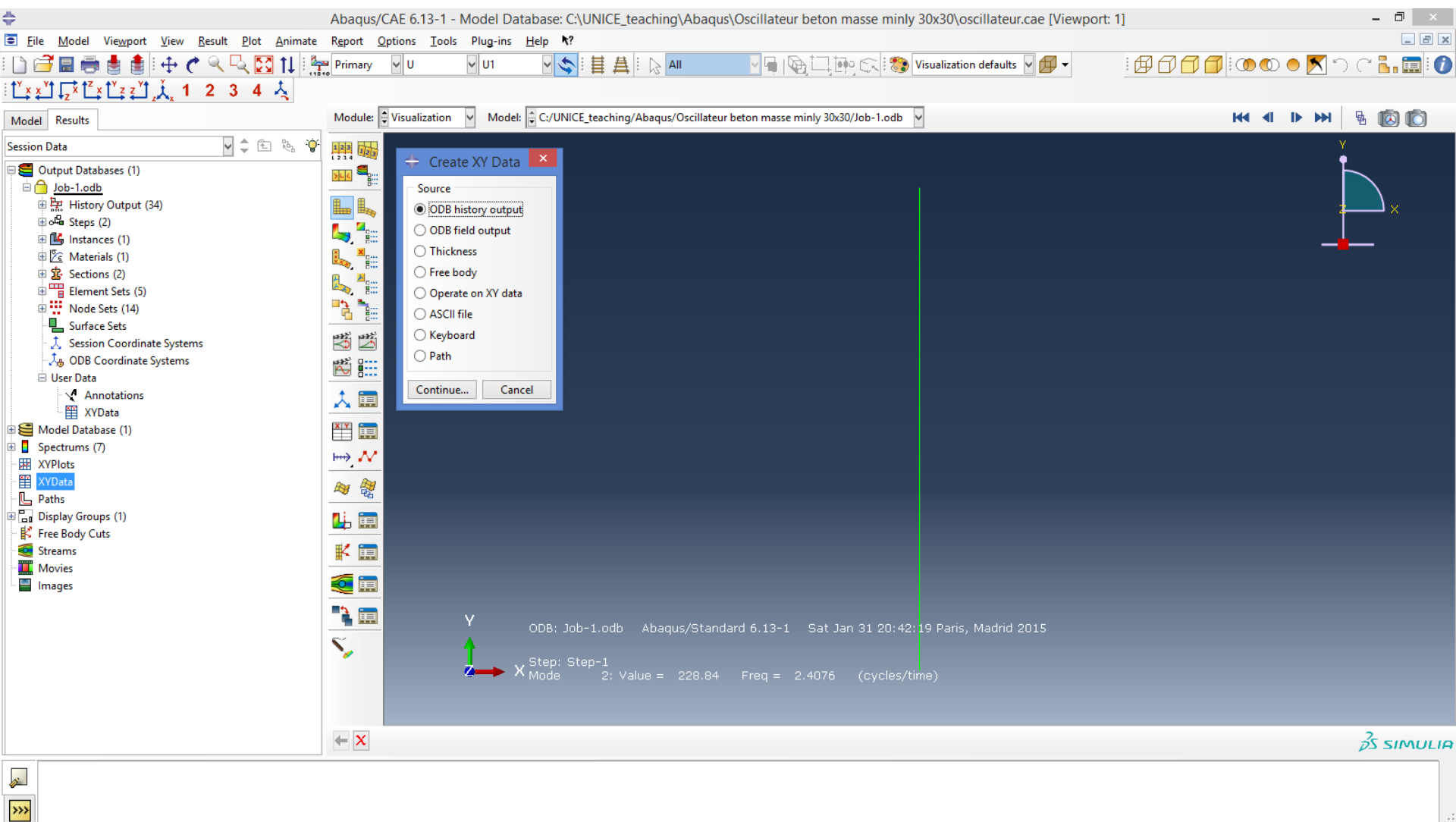
# Results

## 2. Modes propres



# Results - XY Data

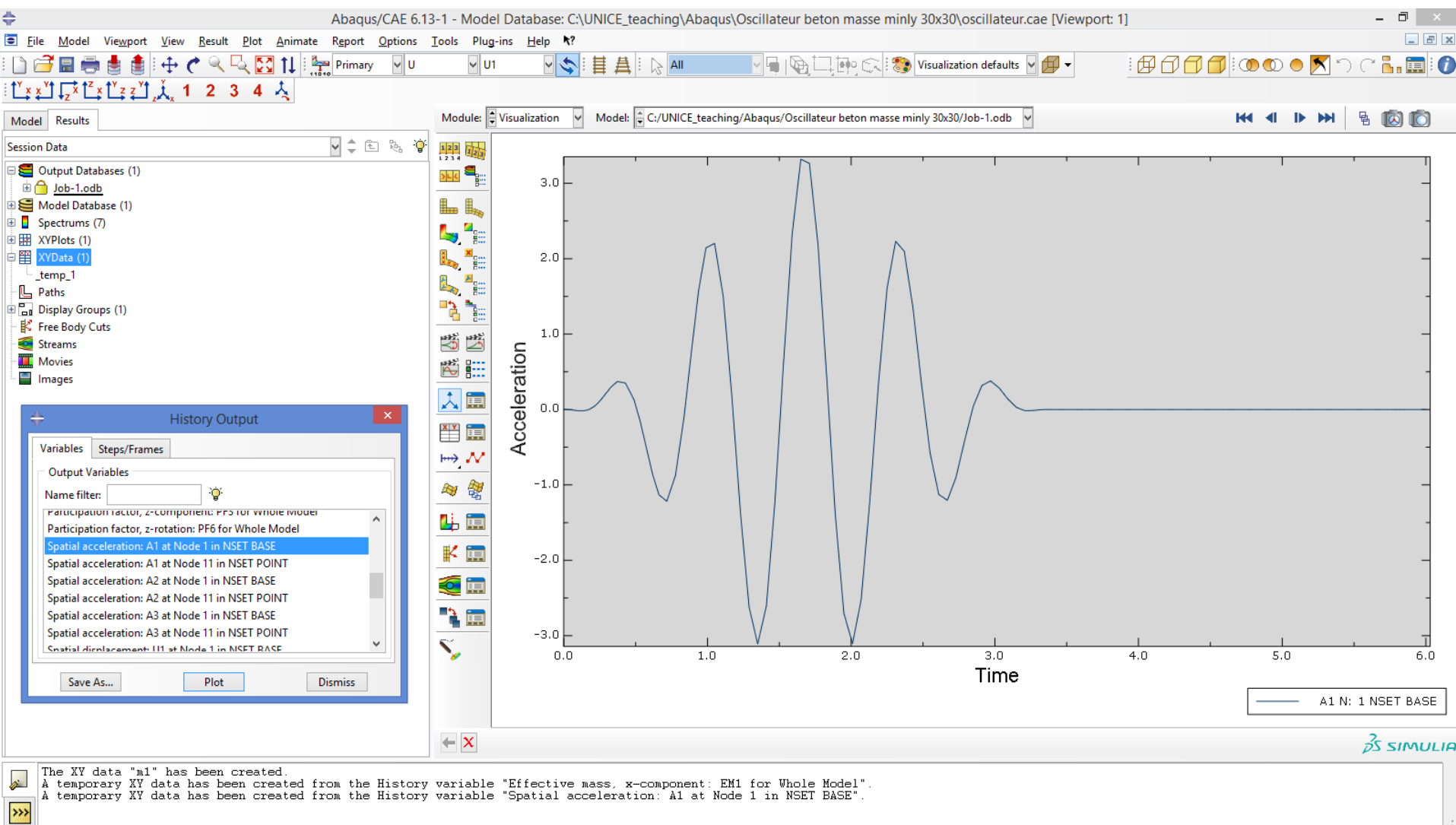
## 3. Courbes : déplacement du sommet dans le temps, vitesse/accélération à la base





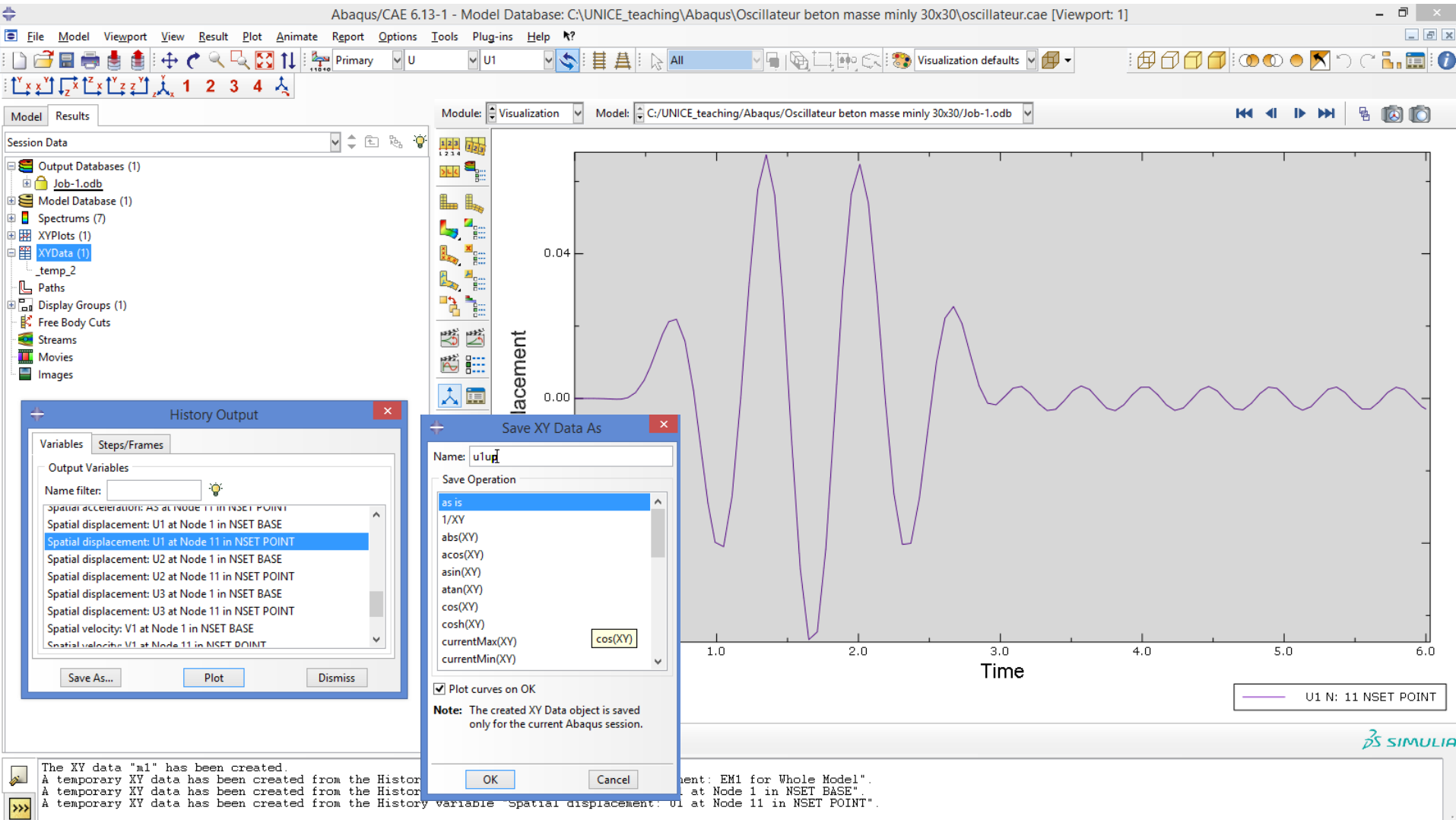
# Results - XY Data

## 4. Courbes : accélération à la base (input)



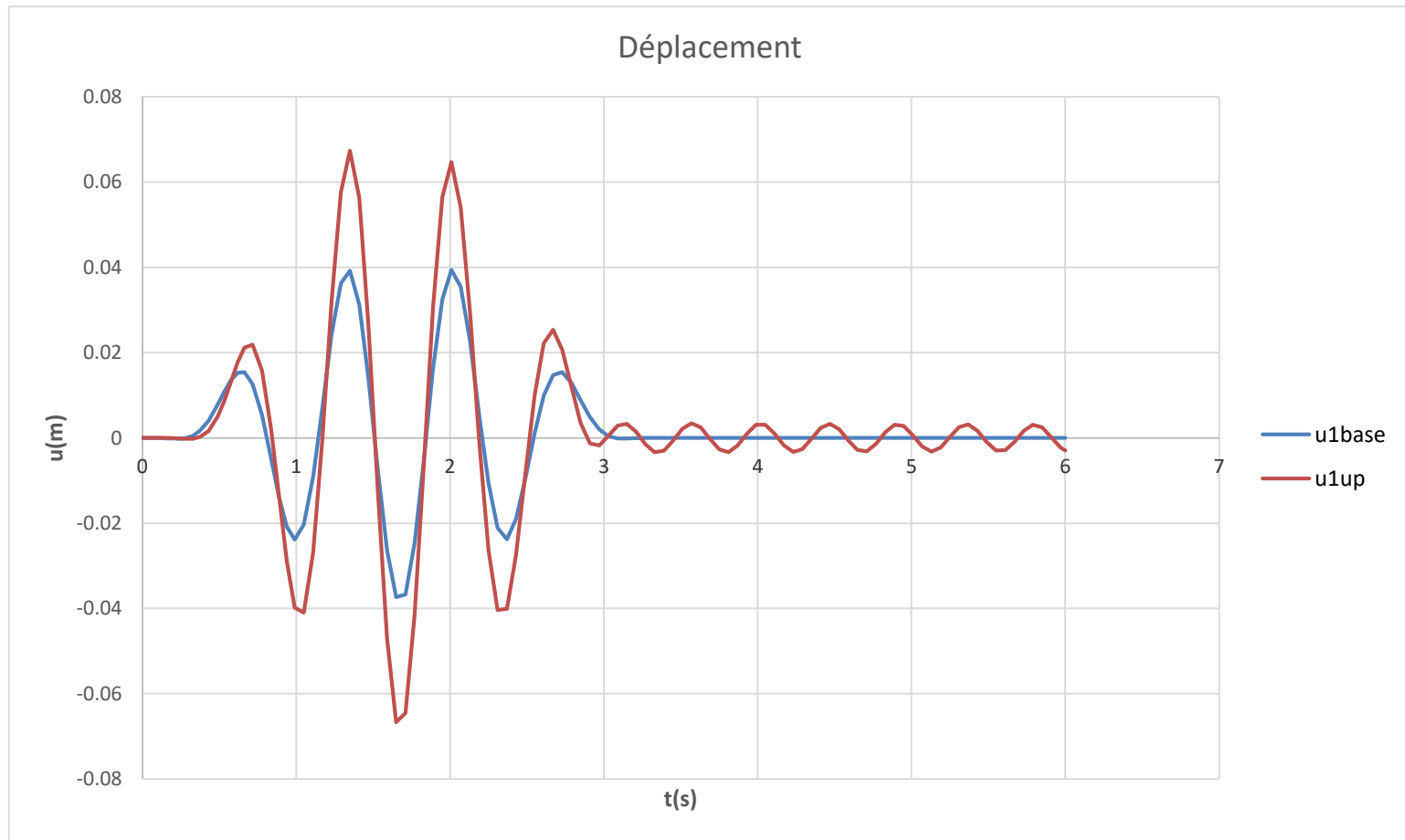
# Results - XY Data

## 5. Courbes : déplacement en tête



# Results

## 6. Courbes : déplacement à la base et en tête



# Fichiers dans le dossier de calcul

Vérifier les données :

- File **\*.cae**
- File **\*.inp**

Résultats :

- File **\*.dat** (Getting Started 7.1.2)
- File **\*.odb**

Messages d'erreurs :

- File **\*.log**
- File **\*.msg**