تکلیف سری اول (بروزرسانی اول) درس دینامیک سیالات محاسباتی پیشرفته محمد یوسفی 402126084 https://github.com/mohamadusefi

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse import diags
from scipy.sparse.linalg import  gcrotmk

def S(x):
    return  x * (1 - x)   # definition of source term

K2, K3, H = [], [], []
i = 1
N = 4  # Starting value for N
increment = 3  # Initial increment
```

الگوریتم انتخاب نقاط به نحوی طراحی شده که نقاط ثابت برای مقایسه خطا با کاهش گام طولی وجود داشته باشد.

```
while N <= 1000:
    L = 1  # length of the bar
    h = L / (N - 1)  # step size
    x = np.linspace(0, L, N)
    Phi_left = 0
    Phi_right = 0

    diagonals = [-2 * np.ones(N), 1*np.ones(N-1), 1*np.ones(N-1)]
    A = diags(diagonals, [0, -1, 1], format='csr')
    A[0, 0] = 1
    A[0, 1] = 0
    A[N-1, N-2] = 0
    A[N-1, N-1] = 1

    b = -S(x) * h**2
    b[0] = Phi_left
    b[-1] = Phi_right


    initial_guess = np.zeros(N)

    Phi, info = gcrotmk(A, b, x0=initial_guess)

    Phi[0] = Phi_left
    Phi[-1] = Phi_right
    print(f"N={N}, Solution={Phi}, CG Info={info}")

    K2.append(Phi[1*i])   # values of Phi in x = L/4
    K3.append(Phi[2*i])   # Values of Phi in x = 3L/4
    H.append(h)  # storing values of h into H for log-log plot
```

```
    N += increment
    increment *= 2
    i *= 2
```

```
N=4, Solution=[0.         0.02469136 0.02469136 0.        ], CG Info=0
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0.01350309
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0.02531829
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 0.02607784 0.02586082 0.02521279 0.0241428  0.0226659  0.02080319
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 0.0260439  0.02603034 0.02598966 0.0259219  0.02582713 0.02570541
 0.02555686 0.02538161 0.02517981 0.02495164 0.02469729 0.024417
 0.024111   0.02377957 0.023423   0.02304162 0.02263576 0.02220578
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 0.01463075 0.01391471 0.01318285 0.01243592 0.01167469 0.01089994
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0.02410902 0.02402854 0.02394648 0.02386283 0.02377761 0.02369082
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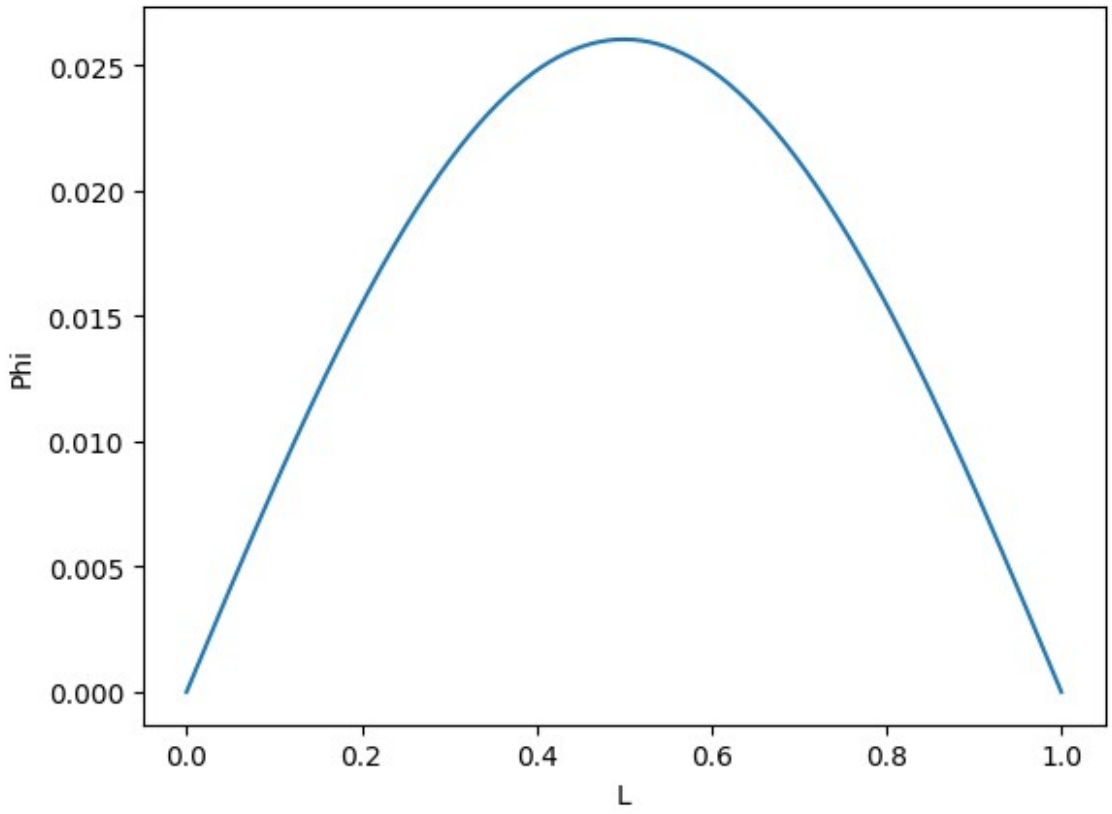
0.02139459 0.02133407 0.02127319 0.02121195 0.02115035 0.0210884
0.02102609 0.02096343 0.02090042 0.02083705 0.02077333 0.02070927
0.02064485 0.02058008 0.02051497 0.02044951 0.02038371 0.02031756
0.02025106 0.02018423 0.02011705 0.02004953 0.01998167 0.01991348
0.01984494 0.01977607 0.01970686 0.01963732 0.01956745 0.01949724
0.0194267 0.01935582 0.01928462 0.01921309 0.01914124 0.01906905
0.01899654 0.01892371 0.01885055 0.01877707 0.01870327 0.01862915
0.01855471 0.01847996 0.01840488 0.01832949 0.01825379 0.01817777

```
 0.01810144 0.0180248  0.01794785 0.01787059 0.01779302 0.01771514
 0.01763696 0.01755848 0.01747969 0.0174006  0.01732121 0.01724153
 0.01716154 0.01708125 0.01700067 0.0169198  0.01683863 0.01675717
 0.01667542 0.01659337 0.01651104 0.01642843 0.01634552 0.01626234
 0.01617886 0.01609511 0.01601108 0.01592676 0.01584217 0.0157573
 0.01567216 0.01558674 0.01550104 0.01541508 0.01532884 0.01524234
 0.01515557 0.01506853 0.01498122 0.01489366 0.01480583 0.01471774
 0.01462939 0.01454078 0.01445191 0.01436279 0.01427341 0.01418379
 0.01409391 0.01400378 0.0139134  0.01382278 0.0137319  0.01364079
 0.01354943 0.01345783 0.013366   0.01327392 0.0131816  0.01308905
 0.01299627 0.01290326 0.01281001 0.01271653 0.01262283 0.0125289
 0.01243474 0.01234036 0.01224576 0.01215094 0.0120559  0.01196064
 0.01186516 0.01176947 0.01167357 0.01157746 0.01148114 0.01138461
 0.01128787 0.01119093 0.01109378 0.01099644 0.01089889 0.01080115
 0.0107032  0.01060507 0.01050673 0.01040821 0.0103095  0.0102106
 0.01011151 0.01001223 0.00991277 0.00981313 0.00971331 0.00961331
 0.00951314 0.00941278 0.00931226 0.00921156 0.00911069 0.00900965
 0.00890845 0.00880708 0.00870555 0.00860385 0.008502   0.00839999
 0.00829782 0.0081955  0.00809302 0.00799039 0.00788761 0.00778469
 0.00768162 0.00757841 0.00747505 0.00737155 0.00726792 0.00716414
 0.00706024 0.0069562  0.00685202 0.00674772 0.00664329 0.00653874
 0.00643406 0.00632926 0.00622434 0.0061193  0.00601414 0.00590887
 0.00580349 0.00569799 0.00559239 0.00548668 0.00538086 0.00527494
 0.00516892 0.0050628  0.00495659 0.00485027 0.00474387 0.00463737
 0.00453078 0.00442411 0.00431735 0.00421051 0.00410358 0.00399658
 0.00388949 0.00378233 0.0036751  0.0035678  0.00346042 0.00335298
 0.00324547 0.0031379  0.00303027 0.00292258 0.00281483 0.00270702
 0.00259916 0.00249125 0.00238329 0.00227529 0.00216724 0.00205914
 0.00195101 0.00184283 0.00173462 0.00162638 0.0015181  0.00140979
 0.00130145 0.00119309 0.00108471 0.0009763  0.00086787 0.00075942
 0.00065096 0.00054249 0.000434   0.00032551 0.00021701 0.00010851
 0.        ], CG Info=0
```

```
fig, ax = plt.subplots()
ax.plot(x, Phi)
plt.xlabel('L')
plt.ylabel('Phi')
plt.show()
```




```
dif1 = []
dif2 = []
error = []
j = 0
for i in range(1, len(H)):
    dif1.append(abs(K2[i] - K2[i-1]))
    dif2.append(abs(K3[i] - K3[i-1]))
    error.append(np.linalg.norm([dif1[j], dif2[j]]))  # compute norm
of errors at K2 and K3 points in each step-size
    j += 1
del H[0]

plt.loglog(H, error, 'o-', label='Error vs H')

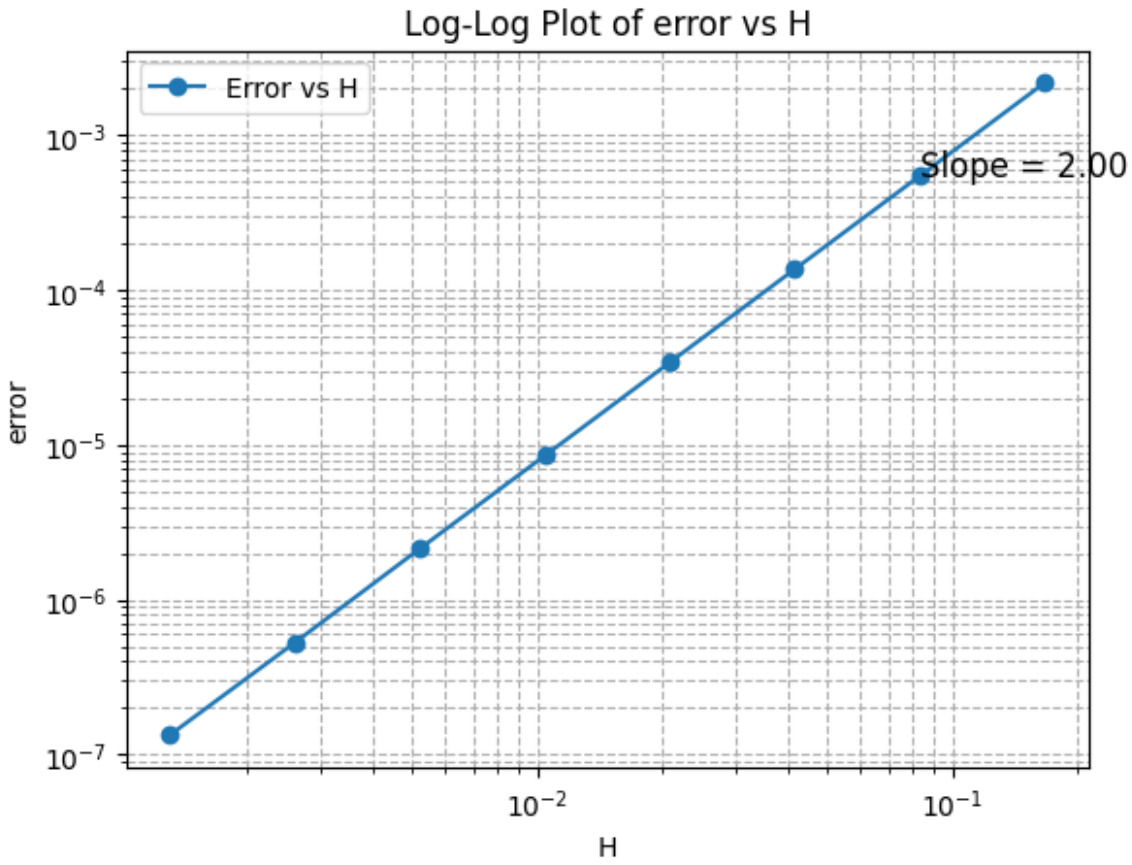
plt.xlabel('H')
plt.ylabel('error')
plt.title('Log-Log Plot of error vs H')
plt.legend()
plt.grid(True, which="both", ls="--")

log_H = np.log(H)
log_error = np.log(error)

slope, intercept = np.polyfit(log_H, log_error, 1)
```

```
plt.text(H[1], error[1], f'Slope = {slope:.2f}', fontsize=12)

plt.show()
```



شکل بالا شیب کلی نمودار خطا در برابر گام طولی را نشان می‌دهد.