Selection of an Optimal Maintenance Strategy Under Uncertain Conditions: An Interval Type-2 Fuzzy AHP-TOPSIS Method

Manoj Mathew, Ripon K. Chakrabortty, and Michael J. Ryan, *Senior Member, IEEE*

***Abstract*—The selection of an optimal maintenance strategy is one of the principal strategic decisions that must be taken in many contexts in order to maintain an asset with minimum deterioration and to deliver maximum output with high quality. When considering maintenance cost, reliability, and safety level of industrial assets, decision makers must select an appropriate maintenance strategy, preferably, with a known degree of uncertainty. This article utilizes a new interval type-2 fuzzy (IT2F) multicriteria decision-making method based on the analytic hierarchy process (AHP) and the technique for order of preference by similarity to ideal solution (TOPSIS), which can effectively handle the degree of uncertainty in group decision making and assist in the selection of the most feasible and optimal industrial asset maintenance strategy. The proposed method is compared with the results obtained from the conventional AHP-TOPSIS and type-1 fuzzy AHP-TOPSIS—the rank obtained from IT2F-AHP-TOPSIS is identical to those methods and can be used as an effective alternative to the type-1 fuzzy method. Compared with conventional AHP-TOPSIS and type-1 fuzzy AHP, IT2F-AHP-TOPSIS has the added advantage that it allows decision makers to define the membership function with greater flexibility and is able to handle uncertainty during decision making.**

***Index Terms*—Industrial asset maintenance strategy, interval type-2 fuzzy (IT2F) AHP-TOPSIS, multicriteria decision making, uncertainty, group decision making.**

I. Introduction

MAINTENANCE is one of the major activities that can absorb up to 70% of total production cost, depending upon the type of industry [1], [2]. Although often considered to be a necessary evil in the past, today it is considered as a contributor to profit in most industries, particularly by slowing the deterioration rate of an industrial asset leading to a subsequent increase in its life. Maintenance of an asset also helps in maintaining the quality of products and leads to improved safety. There are two broad types of maintenance strategies: corrective maintenance and preventive maintenance [1]. Preventative maintenance can be time-based, condition-based, or predictive. The broad classification of maintenance strategies is shown in Fig. 1. When a system failure occurs, corrective maintenance restores the system to a normal working condition [1]. Corrective maintenance is also known as breakdown maintenance, failure-based maintenance, or fire-fighting maintenance. This strategy is mostly used in industries with large profit margins [3].

In preventive maintenance, with the help of inspection and detection, maintenance is proactively undertaken beforehand in order to prevent system failure. There are three categories of preventive maintenance, i.e., time-based preventive maintenance (TBPM), condition-based preventive maintenance (CBPM) and predictive maintenance (PM). In all those preventative maintenance strategies, actions are performed before the failure occurs. In the case of TBPM, the equipment reliability is taken into consideration and maintenance is planned and scheduled periodically to reduce or prevent system failure/breakdown [4]. In the case of CBPM, the system condition is monitored with the help of condition-monitoring technologies—such as vibrational monitoring, temperature monitoring—and maintenance activity is performed as required [1].

CBPM requires high initial capital and technological facilities as compared with TBPM [2]. There is a common misconception that PM and CBPM are similar [3], [5]. However, with the advent of fault prognosis [6] and health-management techniques [7], the two approaches are considered different practices [1], [2]. Table I compares all these aforementioned preventive maintenance strategies.

Fundamentally, maintenance decisions are affected by a number of factors. Among them, safety [1], [2] is the prime factor in scheduling maintenance in industries such as power and chemical industries. Cost is another factor which drives maintenance activity [1], since every maintenance strategy has its own associated cost, which may include technical facility cost (cost of sensors, monitor, software cost) and the cost of training personnel [1], [2]. The added value [1], [2], which includes the benefit (lower inventory holdings for spare parts, smaller production loss) gained from the maintenance activity, is also considered in selecting the best maintenance strategy. Finally, the feasibility [1], [2] of a particular maintenance strategy is also important. For example, the availability of technically skilled labor to apply a particular maintenance strategy can act as an

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Manoj Mathew is with the Department of Mechanical Engineering, Shri Shankaracharya Institute of Professional Management and Technology, Raipur 492015, India (e-mail: mathewmanojraipur@gmail.com).

Ripon K. Chakrabortty and Michael J. Ryan are with the Capability Systems Centre School of Engineering and Information Technology, University of New South Wales, Canberra, ACT 2600, Australia (e-mail: r.chakrabortty@adfa.edu.au; m.ryan@adfa.edu.au).

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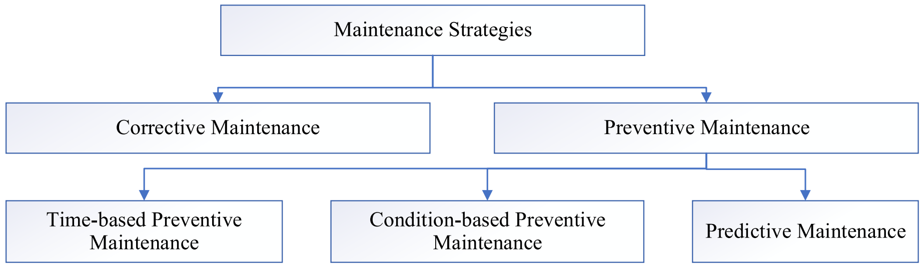

Fig. 1. Classification of maintenance strategies.

TABLE I
COMPARISON OF PREVENTIVE MAINTENANCE STRATEGIES

Maintenance type	Time-Based	Condition Based	Predictive
Maintenance Schedule	Maintenance is based on the time interval	Maintenance is based on condition	Maintenance is based on condition
Action Taken	Based on scheduled maintenance	Based on the measured condition	Based on calculation and predictions.
Time interval	Fixed time interval	Fixed time interval	Based on real-time data

TABLE II
MAJOR FACTORS USED IN THE SELECTION OF AN OPTIMAL MAINTENANCE STRATEGY

Factors/criteria	Explanation	Criteria Type
Cost	Includes initial and working costs required to apply the maintenance strategy.	Non-Beneficial
Safety	Safety of workforce/people working in the industry, machinery, infrastructure, and environment.	Beneficial
Added Value	Added benefits obtained after applying the strategy.	Beneficial
Feasibility	Checking the constraints and possibility of applying the strategy.	Beneficial

additional constraint while deciding the maintenance decision. Table II elaborates on the major factors used in selecting an optimal maintenance strategy.

In selecting an appropriate maintenance strategy, decision makers (DM) (managers and engineers) give importance to the abovementioned factors depending on their backgrounds and responsibilities. For example, a safety manager may give more importance to safety factors, while a production manager may give more importance to production costs. Thus, it is important to select the most appropriate maintenance strategy for particular industrial conditions, taking into consideration the abovementioned factors but also being able to account for the preferences of DM. Furthermore, decisions are often made in the presence of a certain degree of uncertainty, which must also be taken into consideration while selecting an appropriate strategy.

Considering such needs, this article proposes a simple and effective multicriteria decision-making (MCDM) framework for selecting an optimal maintenance strategy among many alternatives. Key features of this proposed method are as follows.

1) The proposed method is simple and effective in handling both vagueness and uncertainty in decision making. In real-life decision making, DM experience some uncertainty in providing a single answer to a specific question; such answers invariably contain some amount of vagueness and uncertainty.
2) Rather than using defuzzified weights, for the development of a weighted decision matrix, this proposed method uses IT2F weights, which keeps intact the uncertainty component during the entire calculation. Some researchers [8] have used defuzzified weights equivalent to crisp numeric scores for calculating the weighted decision matrix, which consequently loses the uncertainty component during calculation [9]. Thus, it is always better to multiply IT2F weights with IT2F values of the decision matrix than simply multiplying crisp weights with IT2F values [9].
3) The use of a single IT2F scale of linguistic variables makes it simpler for the DM to have their preferences incorporated in the decision. Despite that obvious necessity, many researchers (e.g., [10], [11]) have used two IT2F scales of linguistic variables for quantifying preferences. Because of convolutional ways of thinking, experts often find difficulties to express their knowledge or judgment in a single term. However, there is little research in the literature using a single IT2F scale of linguistic variables, which also acts as an additional motivation for this article.

4) A modified formula for defuzzification of the triangular IT2F number is also presented, which is derived from the defuzzified triangular IT2F demonstrated by Kahraman *et al*. [12]. Detailed equations and the potential benefits of that corrected formula are explained in the model formulation and solution approach section.

This article is organized into five sections. The introductory section explains the types of maintenance strategies and the factors affecting the selection process. Section II highlights existing maintenance strategy selection using type-1 and type-2 fuzzy set theories. Section III explains the preliminaries of IT2F set theory and proposes a new IT2F-AHP-TOPSIS method that uses IT2F-AHP for deriving the criteria weights and local priorities of alternative, while IT2F-TOPSIS is used to calculate the final rank of the alternatives. In Section IV, a maintenance strategy selection process is solved using the proposed IT2F-AHP-TOPSIS Method. Finally Section V concludes this article.

II. Literature Review

In the formulation of an MCDM matrix, DM are asked to provide preferences by assigning a number. Some decision criteria are qualitative in nature, so it is difficult for the DM to assign an exact numerical value/number to such criteria. Researchers have used a number of concepts to deal with this issue: interval-valued decision-making techniques [13], grey set theory [14], [15], and fuzzy set theory [1] are some of the notable concepts which are applied to transfer qualitative data to quantitative values. Of those approaches, the fuzzy set theory has become extremely popular and is used extensively [10].

Fundamental fuzzy theory (type-1) was first introduced by Zadeh in 1965 [16], to handle fuzziness and impreciseness in human judgment and preference. A type-1 fuzzy set in a universe X is generally represented with the help of a membership function $\mu_A(x)$, where $0 \leq \mu_A(x) \leq 1$ and fuzzy set A equals $\{(x, \mu_A(x))|x \in X\}$ [17]. Here, the value of $\mu_A(x)$ is known as degree of membership. The membership function can be represented in the form of a triangular or trapezoidal membership function (MF), shown in the following formula:

for triangular MF$\mu_A(x) =$

$$\mu_A(x; a, b, c) = \begin{cases} 0 & \text{if } x > c \text{ or } x < a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \dfrac{c-x}{c-b} & \text{if } b \leq x \leq c \end{cases}$$

for trapezoidal MF $\mu_A(x) =$

$$\mu_A(x; a, b, c, d) = \begin{cases} 0 & \text{if } x > d \text{ or } x < a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \dfrac{d-x}{d-c} & \text{if } c \leq x \leq d \end{cases}.$$

On the contrary, the type-2 fuzzy set was also introduced by Zadeh in 1975 [18] as an extension of the type-1 fuzzy set, in order to be efficient in dealing with uncertainties. Although the simple type-1 fuzzy set theory is successful in translating the imprecise qualitative information to quantitative terms, i.e., handle the fuzziness of the system, it is unable to address the issue of uncertainty associated with decision making. In type-1 fuzzy, each linguistic term corresponds to a membership function. Every element in the membership function has a degree of membership associated with it, which lies between 0 and 1. In a scenario where DM are unable to assign an exact value to the membership function, the type-2 fuzzy set is used. In the type-2 fuzzy set, the membership grades are a type-1 fuzzy set [19]. Interval type-2 fuzzy is an extension of the type-2 fuzzy set, where the membership function is interval-valued [20]. Researchers have used interval type-2 fuzzy set theory and applied to a range of MCDM problems, for example, Oztaysi [20] selected the best enterprise information system, Abdullah *et al*. [21] solved an ambulance location problem, Mei and Xie [22] selected a best metro station evacuation strategy, Ren *et al*. [23] assessed the risk in the industrial network, Samanta and Jana [24] solved a transportation problem, Sang *et al*. [25] solved a stock selection problem, Wang *et al*. [26] developed a risk prioritization method for failure mode and effect analysis, Wu *et al*. [27] solved a green supplier selection problem, Xu *et al*. [28] selected a sustainable supplier, and Yucesan *et al.* [29] selected a green supplier. Key advantages of using IT2F sets over type-1 fuzzy sets are as follows.

1) The membership functions in IT2F sets are defined with greater flexibility [12].
2) IT2F sets are more appropriate in handling uncertainties than type-1 fuzzy sets [20].
3) The use of IT2F sets requires less computational complexity than conventional type-2 fuzzy sets [12].

Among the well-established MCDM concepts, AHP and TOPSIS are mostly used in decision making and proved as a reliable process. The process of AHP is similar to human thinking [1], whereas TOPSIS uses a distance-based approach to calculate the distance from both positive and negative ideal solutions [13]. Kahraman *et al.* [12] extended Buckley's geometric mean fuzzy AHP approach in IT2F-AHP and proposed a new defuzzification method for triangular and trapezoidal MF. They solved a supplier selection problem with four criteria and two alternatives. Oztaysi [20] solved an enterprise information system using IT2F-AHP. Lee and Chen [30] introduced IT2F extension of the TOPSIS method for group decision making in the selection of a system analysis engineer in a software company. Some other existing application of IT2F-AHP and IT2F-TOPSIS are a hazardous waste management system using IT2F-TOPSIS [31], reverse logistics facility location using IT2F-TOSPSIS [32], solution of a public transport problem using combination of IT2F-AHP and IT2F-TOPSIS to evaluate buses for public transport with lower pollution [33], a multistage facility location using IT2F-TOPSIS [34], and ship-loader selection in a maritime transportation problem by using IT2F-AHP-TOPSIS [35].

Various researchers have introduced a range of MCDM and fuzzy MCDM methods for the selection of the best maintenance

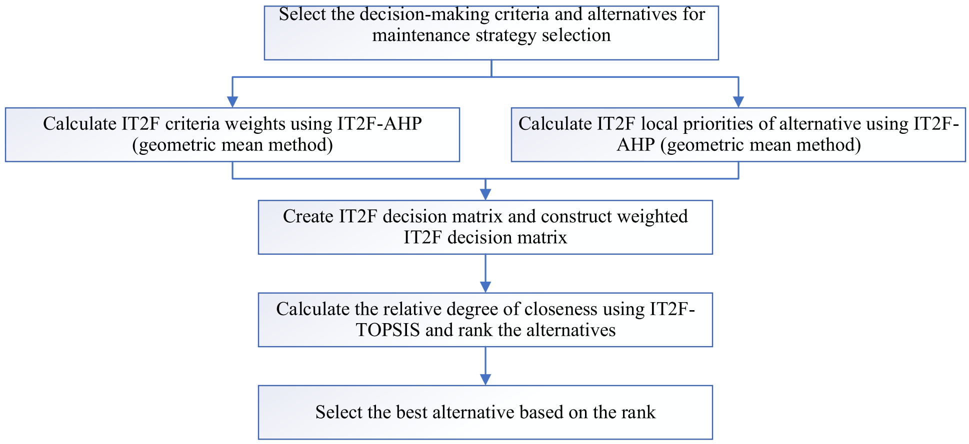

Fig. 2. Flowchart of the solution approach.

strategy. Bevilacqua and Braglia [36] proposed the use of AHP in selecting the best maintenance strategy for an Italian oil refinery company. Sharma *et al.* [3] used fuzzy linguistic modeling to multi-input a single-output model for selecting the best maintenance strategy in the process industry. Wang *et al.* [1] modified the priority calculation and ranking function of fuzzy-AHP to select the best maintenance strategy in a thermal power plant. Jafari *et al*. [37] proposed a combination of fuzzy Delphi and a simple additive weighting method, using a hypothetical example of a maintenance strategy selection problem to explain the method. Bashiri *et al.* [38] proposed a modified fuzzy linear assignment model for selecting the best maintenance strategy and used a hypothetical example to explain the method. Fouladgar *et al*. [39] used fuzzy AHP in combination with the complex proportional assessment method for selecting the best maintenance strategy for a copper mining industry. Nezami and Yildirim [40] proposed a sustainability-based decision-making approach in combination with fuzzy VIKOR. They demonstrated the approach by selecting the best maintenance strategy for a car manufacturing company. Jonge *et al.* [4] studied the effect of parameter uncertainty on the optimum age-based maintenance strategy but did not take decision-making uncertainty into consideration. Ge *et al.* [2] proposed a logarithmic fuzzy preference programming-based method for the selection process.

As outlined above, during the selection of optimal maintenance strategy, researchers have applied type-1 fuzzy set theory only, which is successful in translating the imprecise qualitative information to quantitative terms and handling the fuzziness of the system. However, no researcher has taken into consideration the essential aspect of the uncertainty component of decision making. Hence, to overcome that shortcoming, this research proposes an optimal maintenance strategy selection approach by considering type-2 fuzzy set theory, which can tackle data uncertainties.

III. Model Formulation and Solution Approach

The main objective of this article is to apply IT2F set theory to the selection of a maintenance strategy for an industrial asset, which can effectively handle uncertainty in the decision-making process. This article starts with selecting alternative decision-making criteria for the maintenance selection of an industrial asset. In the second step, weights and local priorities for each of those criteria are calculated by using the basic principle of IT2F set theory. Based on those weights and priorities, a weighted IT2F decision matrix is constructed in the third step. Later, with the application of integrated TOPSIS and IT2F methodologies, the relative degree of closeness among all alternative criteria is measured. Finally, based on those closeness degrees and their corresponding rankings, the best criteria among those alternatives are selected. A generalized methodology is explained with the help of a flowchart, as shown in Fig. 2. For the sake of better understanding to the reader, the preliminaries of IT2F set theory and IT2F-AHP-TOPSIS are discussed in the following sections.

A. *Fundamentals of IT2F Set Theory*

A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type- 2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$ as, $\tilde{\tilde{A}} = \{((x,u),\mu_{\tilde{\tilde{A}}}(x,u))|\forall x \in X, \forall u \in J_X \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\}$, where $J_X \subseteq [0,1]$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called interval type-2 fuzzy set [41], [12]. Since this article has considered both triangular and trapezoidal membership functions to represent IT2F sets, calculation of their upper and lower bounds are shown below, whereas their arithmetic operations (i.e., addition, subtraction, multiplication, division, reciprocal, exponent and multiplication with a crisp number) are shown in Appendix A.

1) Triangular IT2F Set: If the fuzzy membership functions are represented by triangular IT2F set, then the upper and lower membership functions for such settings are $\widetilde{\widetilde{A_j}} = (\widetilde{A_j^u}, \widetilde{A_j^l}) = ((a_j^u, b_j^u, c_j^u; H(\widetilde{A_j^u})), (a_j^l, b_j^l, c_j^l; H(\widetilde{A_j^l})))$, where $\widetilde{A_j^u}, \widetilde{A_j^l}$ are type-1 fuzzy set, a_j^u, b_j^u, c_j^u and a_j^l, b_j^l, c_j^l are the possible values of upper and lower membership functions of IT2F set, $H(\widetilde{A_j^u})$ and $H(\widetilde{A_j^l})$ are the membership value of b_j^u and b_j^l, respectively. Meanwhile, $H(\widetilde{A_j^u}) \in [0,1]$, $H(\widetilde{A_j^l}) \in [0,1]$ and

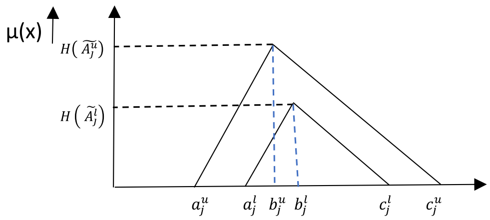

Fig. 3. Illustration of the triangular IT2F set.

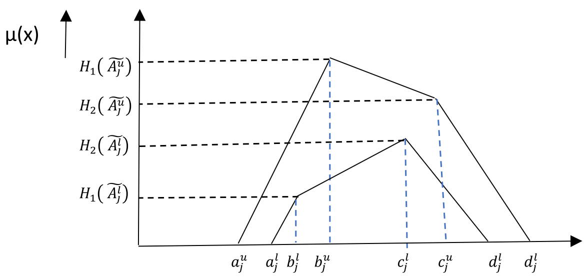

Fig. 4. Illustration of the trapezoidal IT2F set.

$j = 1, 2, \ldots, n$. An illustration of a triangular IT2F set is shown in Fig. 3.

2) Trapezoidal IT2F Set: On the contrary, if the fuzzy membership functions are represented by trapezoidal IT2F set, then the upper and lower membership functions of an interval type-2 fuzzy set are $\widetilde{\widetilde{A}}_j = (\widetilde{A}_j^u, \widetilde{A}_j^l) = (a_j^u, b_j^u, c_j^u, d_j^u;\ H_1(\widetilde{A}_j^u), H_2(\widetilde{A}_j^u)),\quad (a_j^l, b_j^l, c_j^l, d_j^l;\ H_1(\widetilde{A}_j^l), H_2(\widetilde{A}_j^l))$, where $\widetilde{A}_j^u, \widetilde{A}_j^l$ are type-1 fuzzy set, $a_j^u, b_j^u, c_j^u, d_j^u$ and $a_j^l, b_j^l, c_j^l, d_j^l$ are the possible values of upper and lower membership functions of IT2F set, $H_1(\widetilde{A}_j^u), H_2(\widetilde{A}_j^u)$ and $H_1(\widetilde{A}_j^l), H_2(\widetilde{A}_j^l)$ are the membership value of b_j^u, c_j^u and b_j^l, c_j^l, respectively. Meanwhile, $H_1(\widetilde{A}_j^u) \in [0,1]$, $H_2(\widetilde{A}_j^u) \in [0,1]$, $H_1(\widetilde{A}_j^l) \in [0,1]$, $H_2(\widetilde{A}_j^l) \in [0,1]$ and $j = 1, 2, \ldots, n$. An illustration of trapezoidal IT2F is shown in Fig. 4.

a) Defuzzification of triangular and trapezoidal IT2F sets: This article has adopted the defuzzification approach of Kahraman *et al.* [12]. The center of area method is used as an approach for the defuzzification of those considered triangular and trapezoidal IT2F sets and calculated the best nonfuzzy performance value.

If, $\widetilde{\widetilde{A}}_j = (\widetilde{A}_j^u, \widetilde{A}_j^l) = ((a_j^u, b_j^u, c_j^u;\ H(\widetilde{A}_j^u)), (a_j^l, b_j^l, c_j^l;\ H(\widetilde{A}_j^l)))$ and $\widetilde{\widetilde{A}}_j = (\widetilde{A}_j^u, \widetilde{A}_j^l) = ((a_j^u, b_j^u, c_j^u, d_j^u; H_1(\widetilde{A}_j^u), H_2(\widetilde{A}_j^u)), (a_j^l, b_j^l, c_j^l, d_j^l; H_1(\widetilde{A}_j^l), H_2(\widetilde{A}_j^l)))$ be the triangular and trapezoidal IT2F sets, respectively, then, the defuzzified triangular IT2F demonstrated by Kahraman *et al.* [12] equals

$$\frac{\left(\frac{(c_j^u - a_j^u) + (b_j^u - a_j^u)}{3} + a_j^u\right) + \alpha\left(\frac{(c_j^l - a_j^l) + (b_j^l - a_j^l)}{3} + a_j^l\right)}{2}$$

where $\alpha = H(\widetilde{A}_j^l)$.

The value of α is multiplied to $(\frac{(c_j^l - a_j^l) + (b_j^l - a_j^l)}{3} + a_j^l)$, which contradicts the formula of trapezoidal IT2F. In case of the trapezoidal IT2F, the component $H_1(\widetilde{A}_j^l)$ and $H_2(\widetilde{A}_j^l)$ are only multiplied by b_j^l and c_j^l, respectively, instead of the entire components of the lower membership function. Hence, the defuzzification formula of trapezoidal IT2F proposed by Kahraman *et al.* [12] is modified in this article. Here, $\alpha = H(\widetilde{A}_j^l)$ is multiplied with the reference point (b_j^l) of lower membership function, which corresponds to maximum membership value. Rather than only using α, both $H(\widetilde{A}_j^u)$ and $H(\widetilde{A}_j^l)$ are used in the modified formula (membership value of both upper and lower membership functions of triangular IT2F). The modified defuzzified triangular IT2F and trapezoidal IT2F formula are shown at the bottom of this page.

b) Ranking of triangular and trapezoidal IT2F sets: Once the values of $(\widetilde{A}_j^u, \widetilde{A}_j^l)$ are determined for each membership function (and also for each alternative criterion), then those values should be feed into the ranking scheme. The ranking formula for triangular IT2F membership functions is adopted from the work of Chen and Lee [30], and is shown here as follows:

$$\begin{aligned}\text{Rank}\left(\widetilde{\widetilde{A}}_j\right) &= M_1\left(\widetilde{A}_j^u\right) + M_1\left(\widetilde{A}_j^l\right) + M_2\left(\widetilde{A}_j^u\right) + M_2\left(\widetilde{A}_j^l\right)\\ &\quad - \frac{1}{3}\left(S_1\left(\widetilde{A}_j^u\right) + S_1\left(\widetilde{A}_j^l\right) + S_2\left(\widetilde{A}_j^u\right)\right.\\ &\quad \left. + S_2\left(\widetilde{A}_j^l\right) + S_3\left(\widetilde{A}_j^u\right) + S_3\left(\widetilde{A}_j^l\right)\right)\\ &\quad + H\left(\widetilde{A}_j^u\right) + H\left(\widetilde{A}_j^l\right)\end{aligned}$$

where $M_p(\widetilde{A}_j^i) = \frac{(a_{jp}^i - a_{j(p+1)}^i)}{2}$ where $p = 1, 2$.

$$\text{Modified defuzzified triangular } IT2F = \frac{\left(\frac{(c_j^u - a_j^u) + (H(\widetilde{A}_j^u) \times b_j^u - a_j^u)}{3} + a_j^u\right) + \left(\frac{(c_j^l - a_j^l) + (H(\widetilde{A}_j^l) \times b_j^l - a_j^l)}{3} + a_j^l\right)}{2}$$

Modified defuzzified trapezoidal $IT2F =$

$$\frac{\left(\frac{(d_j^u - a_j^u) + (H_1(\widetilde{A}_j^u) \times b_j^u - a_j^u) + (H_2(\widetilde{A}_j^u) \times c_j^u - a_j^u)}{4} + a_j^u\right) + \left(\frac{(d_j^l - a_j^l) + (H_1(\widetilde{A}_j^l) \times b_j^l - a_j^l) + (H_2(\widetilde{A}_j^l) \times c_j^l - a_j^l)}{4} + a_j^l\right)}{2}$$

TABLE III
IT2F SCALE OF LINGUISTIC VARIABLES [12]

Linguistic Variables	**Triangular IT2F scale**	**Trapezoidal IT2F scale**
Absolutely strong (AS)	(7.5,9,10.5;1), (8.5,9,9.5;0.9)	(7,8,9,9;1,1), (7.2,8.2,8.8,9;0.8,0.8)
Very strong (VS)	(5.5,7,8.5;1), (6.5,7,7.5;0.9)	(5,6,8,9;1,1), (5.2,6.2,7.8,8.8;0.8,0.8)
Fairly strong (FS)	(3.5,5,6.5;1), (4.5,5,5.5;0.9)	(3,4,6,7;1,1), (3.2,4.2,5.8,6.8;0.8,0.8)
Slightly strong (SS)	(1.5,3,4.5;1), (2.5,3,3.5;0.9)	(1,2,4,5;1,1), (1.2,2.2,3.8,4.8;0.8,0.8)
Equal (E)	(1,1,1;1), (1,1,1;1)	(1,1,1,1;1,1), (1,1,1,1;1,1)

$S_q(\widetilde{A_j^i}) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}(a_{jk}^i - \frac{1}{2}\sum_{k=q}^{q+1}a_{ik}^i)^2}$, where $q = 1, 2$.

$S_3(\widetilde{A_j^i}) = \sqrt{\frac{1}{3}\sum_{k=1}^{3}(a_{jk}^i - \frac{1}{3}\sum_{k=1}^{3}a_{ik}^i)^2}$, $H(\widetilde{A_j^i})$ represent the membership value of the element $a_{j(p+1)}^i$ in the triangular membership function where $i \in \{U, l\}$

Meanwhile, the ranking formula for trapezoidal IT2F is also adopted from Chen and Lee [30], and is reintroduced here as follows:

$$\begin{aligned}\text{Rank}\left(\widetilde{\widetilde{A_j}}\right) = {} & M_1\left(\widetilde{A_j^u}\right) + M_1\left(\widetilde{A_j^l}\right) + M_2\left(\widetilde{A_j^u}\right) \\ & + M_2\left(\widetilde{A_j^l}\right) + M_3\left(\widetilde{A_j^u}\right) + M_3\left(\widetilde{A_j^l}\right) \\ & - \frac{1}{4}\left(S_1\left(\widetilde{A_j^u}\right) + S_1\left(\widetilde{A_j^l}\right)\right. \\ & \quad + S_2\left(\widetilde{A_j^u}\right) + S_2\left(\widetilde{A_j^l}\right) + S_3\left(\widetilde{A_j^u}\right) \\ & \quad \left. + S_3\left(\widetilde{A_j^l}\right) + S_4\left(\widetilde{A_j^u}\right) + S_4\left(\widetilde{A_j^l}\right)\right) \\ & + H_1\left(\widetilde{A_j^u}\right) + H_1\left(\widetilde{A_j^l}\right) \\ & + H_2\left(\widetilde{A_j^u}\right) + H_2\left(\widetilde{A_j^l}\right)\end{aligned}$$

where $M_p(\widetilde{A_j^i}) = \frac{(a_{jp}^i - a_{j(p+1)}^i)}{2}$ where $1 \leq p \leq 3$.

$S_q(\widetilde{A_j^i}) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}(a_{jk}^i - \frac{1}{2}\sum_{k=q}^{q+1}a_{ik}^i)^2}$, where $1 \leq q \leq 3$.

$S_4(\widetilde{A_j^i}) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}(a_{jk}^i - \frac{1}{4}\sum_{k=1}^{4}a_{ik}^i)^2}$, $H_p(\widetilde{A_j^i})$ represents the membership value of the element $a_{j(p+1)}^i$ in the trapezoidal membership function where $1 \leq p \leq 2$, $i \in \{U, l\}$.

B. IT2F-AHP-TOPSIS Method

This research employs two different MCDM techniques to weight alternative criteria, which are then integrated with the proposed IT2F set theory to determine the best (or optimal) maintenance strategy for an industrial asset. At first, this research uses IT2F-AHP for deriving the criteria weights and local priorities of alternatives. The criteria weights and local priorities are kept in the IT2F set and are directly used to calculate the IT2F weighted decision matrix in TOPSIS. During calculation, no defuzzifying of the weights is carried out, which helps to keep intact the uncertainty component in the entire calculation. The final rank of the alternatives is calculated in IT2F-TOPSIS. Overall, this proposed methodology is termed as IT2F-AHP-TOSIS. Here to mention that this IT2F-AHP-TOPSIS method is derived from the AHP, and was proposed by Mu and Pereyra-Rojas [42], IT2F-AHP method proposed by Kahraman *et al.* [12], and IT2F-TOPSIS method proposed by Chen and Lee [30]. The proposed IT2F-AHP-TOPSIS method encompasses two primary stages: criteria quantification and ranking. Key steps associated with those stages are highlighted below.

1) Stage 1—Criteria Quantification: The criteria weights are calculated by constructing the pairwise comparison ($n \times n$) matrix using the IT2F scale of linguistic variables as shown in Table III, in which both triangular and trapezoidal IT2F scales are shown. Depending upon the fuzziness of the system, the DM can use either triangular or trapezoidal membership functions. As per Donato and Barbieri [43] "In trapezoidal membership function, a small change in the crisp input near the center of the membership should produce a small change in the degree of membership, in contrast to the triangular memberships, which produces bigger changes in the degree of membership." So, under normal conditions, a triangular membership function should be used, but if the DM is not confident or uncertain in giving the preference then, the trapezoidal membership function should be used. Here "*n*" is the number of evaluation criteria. The consistency of the matrix should be checked with the help of matrix A, which is the crisp equivalent of $\tilde{\tilde{A}}$. If matrix A is consistent, then $\tilde{\tilde{A}}$ will also be consistent [12]

$$\tilde{\tilde{A}} = \begin{bmatrix} 1 & \widetilde{\widetilde{A_{12}}} & \cdots & \widetilde{\widetilde{A_{n1}}} \\ \widetilde{\widetilde{A_{21}}} & 1 & \cdots & \widetilde{\widetilde{A_{n2}}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\widetilde{A_{1n}}} & \widetilde{\widetilde{A_{2n}}} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \widetilde{\widetilde{A_{12}}} & \cdots & 1/\widetilde{\widetilde{A_{1n}}} \\ 1/\widetilde{\widetilde{A_{12}}} & 1 & \cdots & 1/\widetilde{\widetilde{A_{2n}}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\widetilde{A_{1n}}} & \widetilde{\widetilde{A_{2n}}} & \cdots & 1 \end{bmatrix}.$$

In case of group decision making the combined matrix is obtained by aggregating the IT2F set, using the geometric mean: $\widetilde{\widetilde{X_{ij}}} = [\widetilde{\widetilde{X_{ij}^1}} \otimes \widetilde{\widetilde{X_{ij}^2}} \ldots . \widetilde{\widetilde{X_{ij}^k}}]^{\frac{1}{k}}$, where *k* is the number of DM.

Then, the IT2F criteria weights are also calculated using the geometric mean method [12]

$\tilde{\tilde{w}}_j = \frac{\tilde{\tilde{r}}_j}{\tilde{\tilde{r}}_1 \otimes \tilde{\tilde{r}}_2 \ldots\ldots \tilde{\tilde{r}}_n}$ where $\tilde{\tilde{r}}_j = [\tilde{\tilde{a}}_{j1} \otimes \tilde{\tilde{a}}_{j2} \ldots . \tilde{\tilde{a}}_{jn}]^{\frac{1}{n}}$ and $j = 1, 2, \ldots, n$.

Now, to construct the decision matrix $(\tilde{\tilde{X}})$ by deriving local priorities $(\tilde{\tilde{X}}_{ij})$ (preference of the alternatives), a pairwise comparison ($m \times m$) matrix is to be developed with respect to each of the criteria. The IT2F scale of linguistic variables as shown in Table III can be used to develop the decision matrix $\tilde{\tilde{X}}$. Now, if m is the number of alternatives, then the decision matrix, $\tilde{\tilde{X}}$ will be

$$\tilde{\tilde{X}} = \begin{bmatrix} \tilde{\tilde{X}}_{11} & \tilde{\tilde{X}}_{12} & \cdots & \tilde{\tilde{X}}_{m1} \\ \tilde{\tilde{X}}_{21} & \tilde{\tilde{X}}_{21} & \cdots & \tilde{\tilde{X}}_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{X}}_{1n} & \tilde{\tilde{X}}_{2n} & \cdots & \tilde{\tilde{X}}_{\mathrm{mn}} \end{bmatrix}.$$

2) Stage 2—Ranking Alternatives: Considering the weight of each criterion and the decision matrix developed in step 1, a weighted IT2F decision matrix is developed by following the formula as follows:

$$\tilde{\tilde{V}} = \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{21} & \cdots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \cdots & \tilde{\tilde{v}}_{mn} \end{bmatrix} \text{ where } \tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_j \otimes \tilde{\tilde{X}}_{ij}.$$

Then, the ranking value of the $\tilde{\tilde{v}}_{ij}$ is to be calculated to construct the ranking weighted decision matrix [11], Y_w

$Y_w = (\mathrm{Rank}(\tilde{\tilde{v}}_{ij}))_{m \times n}$ where $1 \leq i \leq m,\ 1 \leq j \leq n$.

Based on the concepts of TOPSIS, positive ideal solutions and the negative-ideal solutions are to be calculated in the next stage

$$v_i^+ = \begin{cases} \max_j \left\{ \mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) \right\} & \text{if } j \in B \\ \min_j \left\{ \mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) \right\} & \text{if } j \in \mathrm{NB} \end{cases} \quad \text{and}$$

$$v_i^- = \begin{cases} \min_j \left\{ \mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) \right\} & \text{if } j \in B \\ \max_j \left\{ \mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) \right\} & \text{if } j \in \mathrm{NB} \end{cases}$$

where B represents beneficial criteria and NB refers to nonbeneficial criteria.

Once, we have the positive and negative ideal solutions from the previous step, distance $d^+(X_j)$ between each alternative and positive ideal solution, and distance $d^-(X_j)$ between each alternative and negative ideal solution should be calculated using the following equations:

$$d^+(X_j) = \sqrt{\sum\nolimits_{i=1}^{m} \left(\mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) - v_i^+ \right)^2}$$

$$d^-(X_j) = \sqrt{\sum\nolimits_{i=1}^{m} \left(\mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) - v_i^- \right)^2}$$

$$C(X_j) = \frac{d^-(X_j)}{d^-(X_j) + d^+(X_j)}.$$

Finally, after calculating the relative degree of closeness, $C(X_j)$, all those alternative criteria are ranked based on their values. The higher the $C(X_j)$ value, the better is the alternative, hence, it should give rank 1 to the foremost one [11] .

IV. Numerical Illustration and Model Validation

A case study is provided to illustrate the applicability of IT2F-AHP-TOPSIS in selecting the best maintenance strategy. In order to maintain the proper supply of products at peak production hours with good quality, a fabrication unit's managers wanted to review and select the best maintenance strategy for their industry, based on the four main selection criteria: cost, safety, added value, and feasibility. The evaluation is undertaken to select the best maintenance strategy among TBPM, CBPM, CM, and PM. At present, the company is relying on a corrective maintenance strategy because of its controlled maintenance cost and simplicity of implementation. Nevertheless, the breakdown of an essential machine at critical production time slows the operations and increases the downtime, questioning the reliability of the adopted maintenance strategy. Hence, the managers are seeking an alternative approach to mitigate this problem. A review of the current maintenance strategy is completed by the plant managers. Three DM (plant manager with more than 15 years of experience, maintenance manager with 10 years of experience, and operations manager with 12 years of experience) are involved in the decision-making process and provide their preferences. The combined pairwise comparison matrix (shown in Table IV) is formulated for the calculation of weights, using the pairwise comparison matrix given by the three DM, which is shown in Tables V–VII. In this example, a triangular IT2F scale is used for the formulation of the pairwise comparison matrix.

To check the consistency of the pairwise comparison matrix, a matrix is created which is the crisp equivalent of the combined pairwise comparison matrix. The consistency ratio of the crisp equivalent matrix was $0.088 \leq 0.1$, so it can be assumed that the combined pairwise comparison matrix will also be consistent. Using the combined pairwise comparison matrix, IT2F criteria weights (shown in Table VIII) are calculated using the geometric mean method [12]. The formula for the same is: $\tilde{\tilde{w}}_j = \frac{\tilde{\tilde{r}}_j}{\tilde{\tilde{r}}_1 \otimes \tilde{\tilde{r}}_2 \ldots\ldots \tilde{\tilde{r}}_n}$, where $\tilde{\tilde{r}}_j = [\tilde{\tilde{a}}_{j1} \otimes \tilde{\tilde{a}}_{j2} \ldots . \tilde{\tilde{a}}_{jn}]^{\frac{1}{n}}$ and $j = 1, 2, \ldots, n$. The IT2F criteria weights are defuzzified and normalized to obtain the normalized weight percentage. For example, the defuzzified values of (0.571,2.943,13.466; 1) and (1.783,2.943,4.799; 0.9) are calculated using unnumbered equation shown at the bottom of this page.

Next, the decision matrix (shown in Table IX), which has IT2F numbers, is formulated using the combined pairwise comparison matrix with respect to each criterion (shown in Tables X to XIII). The combined pairwise comparison matrix with respect to

TABLE IV
COMBINED PAIRWISE COMPARISON MATRIX FOR WEIGHT CALCULATION

	Cost	Safety	Added Value	Feasibility
Cost	(1,1,1;1) (1,1,1;0.9)	(1.99,3.557,5.087; 1) (3.041,3.557,4.069;0.9)	(2.639,4.217,5.75; 1) (3.699,4.217,4.731;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Safety	(0.197,0.281,0.503; 1) (0.246,0.281,0.329;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Added Value	(0.174,0.237,0.379; 1) (0.211,0.237,0.27;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)
Feasibility	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE V
PAIRWISE COMPARISON MATRIX FOR WEIGHT CALCULATION OF CRITERIA BY DM-1

	Cost	Safety	Added Value	Feasibility
Cost	(1,1,1;1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5;1) (4.5,5,5.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Safety	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Added Value	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)
Feasibility	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE VI
PAIRWISE COMPARISON MATRIX FOR WEIGHT CALCULATION OF CRITERIA BY DM-2

	Cost	Safety	Added Value	Feasibility
Cost	(1,1,1;1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Safety	(0.222,0.333,0.667;1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Added Value	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)
Feasibility	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE VII
PAIRWISE COMPARISON MATRIX FOR WEIGHT CALCULATION OF CRITERIA BY DM-3

	Cost	Safety	Added Value	Feasibility
Cost	(1,1,1;1) (1,1,1;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Safety	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)
Added Value	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)
Feasibility	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.154,0.2,0.286; 1) (0.182,0.2,0.222;0.9)	(0.222,0.333,0.667; 1) (0.286,0.333,0.4;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE VIII
IT2F CRITERIA WEIGHTS AND DEFUZZIFIED WEIGHTS

Parameters	*IT2F criteria weights*	*Defuzzified weights*	*Normalized weight Percentage*
Cost	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	4.368	52.86
Safety	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	2.223	26.90
Added Value	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	1.133	13.71
Feasibility	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)	0.540	6.54

TABLE IX
IT2F SET DECISION MATRIX

Criteria Weights	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Criteria →	**Cost**	**Safety**	**Added Value**	**Feasibility**
CM	(0.073,0.312,1.494;1) (0.196,0.312,0.504;0.9)	(0.066,0.34,1.975; 1) (0.201,0.34,0.58;0.9)	(0.085,0.312,1.28; 1) (0.206,0.312,0.48; 0.9)	(0.594,3.201,15.505; 1) (1.917,3.201,5.288;0.9)
TBPM	(0.137,0.669,3.333; 1) (0.407,0.669,1.101;0.9)	(0.128,0.76,4.862; 1) (0.434,0.76,1.339; 0.9)	(0.256,1,3.905; 1) (0.655,1,1.526; 0.9)	(0.215,1.316,7.485; 1) (0.758,1.316,2.27;0.9)
CBPM	(0.3,1.495,7.278; 1) (0.908,1.495,2.457;0.9)	(0.254,1.495,8.591; 1) (0.865,1.495,2.578;0.9)	(0.317,1.136,4.281; 1) (0.759,1.136,1.708;0.9)	(0.122,0.669,3.757; 1) (0.393,0.669,1.139;0.9)
PM	(0.669,3.201,13.754; 1) (1.983,3.201,5.112;0.9)	(0.41,2.59,13.848; 1) (1.488,2.59,4.434;0.9)	(0.632,2.817,10.747; 1) (1.8,2.817,4.342; 0.9)	(0.071,0.355,2.082; 1) (0.212,0.355,0.604;0.9)

TABLE X
COMBINED PAIRWISE COMPARISON MATRIX WITH RESPECT TO FEASIBILITY

Feasibility	**CM**	**TBPM**	**CBPM**	**PM**
CM	(1,1,1; 1) (1,1,1; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)	(3.5,5,6.5; 1) (4.5,5,5.5; 0.9)	(5.5,7,8.5; 1) (6.5,7,7.5; 0.9)
TBPM	(0.222,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)	(1,1,1; 1) (1,1,1; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)
CBPM	(0.154,0.2,0.29; 1) (0.18,0.2,0.22; 0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)	(1,1,1; 1) (1,1,1; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)
PM	(0.118,0.14,0.18; 1) (0.13,0.14,0.15; 0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)	(1,1,1; 1) (1,1,1; 0.9)

TABLE XI
COMBINED PAIRWISE COMPARISON MATRIX WITH RESPECT TO COST

Cost	**CM**	**TBPM**	**CBPM**	**PM**
CM	(1,1,1;1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)	(0.15,0.2,0.29; 1) (0.18,0.2,0.22;0.9)	(0.12,0.14,0.18; 1) (0.13,0.14,0.15;0.9)
TBPM	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)	(0.15,0.2,0.29; 1) (0.18,0.2,0.22;0.9)
CBPM	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)
PM	(5.5,7,8.5; 1) (6.5,7,7.5;0.9)	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE XII
COMBINED PAIRWISE COMPARISON MATRIX WITH RESPECT TO SAFETY

Safety	**CM**	**TBPM**	**CBPM**	**PM**
CM	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)	(0.15,0.2,0.29; 1) (0.18,0.2,0.22;0.9)	(0.15,0.2,0.29; 1) (0.18,0.2,0.22;0.9)
TBPM	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)
CBPM	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4;0.9)
PM	(3.5,5,6.5; 1) (4.5,5,5.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE XIII
COMBINED PAIRWISE COMPARISON MATRIX WITH RESPECT TO ADDED VALUE

Added value	**CM**	**TBPM**	**CBPM**	**PM**
CM	(1,1,1; 1) (1,1,1; 0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)	(0.15,0.2,0.29; 1) (0.18,0.2,0.22; 0.9)	(0.12,0.14,0.18; 1) (0.13,0.14,0.15; 0.9)
TBPM	(1.5,3,4.5; 1) (2.5,3,3.5;0.9)	(1,1,1; 1) (1,1,1; 0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)
CBPM	(3.5,5,6.5; 1) (4.5,5,5.5; 0.9)	(1,1,1; 1) (1,1,1;0.9)	(1,1,1; 1) (1,1,1;0.9)	(0.22,0.33,0.67; 1) (0.29,0.33,0.4; 0.9)
PM	(5.5,7,8.5; 1) (6.5,7,7.5; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)	(1.5,3,4.5; 1) (2.5,3,3.5; 0.9)	(1,1,1; 1) (1,1,1;0.9)

TABLE XIV
RANKING WEIGHTED DECISION MATRIX Y_w

Criteria ⟶	**Cost**	**Safety**	**Added Value**	**Feasibility**
CM	8.133	5.800	3.383	9.870
TBPM	15.556	11.183	6.517	5.523
CBPM	32.011	18.930	7.044	3.727
PM	61.958	30.077	14.724	2.898

TABLE XV
$d^+(X_j), d^-(X_j)$, THE RELATIVE DEGREE OF CLOSENESS AND RANK OF MAINTENANCE STRATEGIES

	$d^+(X_j)$	$d^-(X_j)$	*The relative degree of closeness*	**Rank**
CM	26.79	54.27	0.67	2
TBPM	22.32	46.89	0.68	1
CBPM	28.13	32.91	0.54	3
PM	54.27	26.79	0.33	4

each criterion is obtained by combining the pairwise comparison matrix with respect to each criterion, given by the three DM, using the geometric mean $\widetilde{\widetilde{X_{ij}}} = [\widetilde{\widetilde{X^1_{ij}}} \otimes \widetilde{\widetilde{X^2_{ij}}} \ldots .\widetilde{\widetilde{X^k_{ij}}}]^{\frac{1}{k}}$, where k is the number of DM.

The weighted decision matrix $\widetilde{\widetilde{v_{ij}}}$ is then calculated and the ranking value of the $\widetilde{\widetilde{v_{ij}}}$ are calculated to construct the ranking weighted decision matrix Y_w, as shown in Table XIV. The positive ideal solution and the negative-ideal solution ranking weighted decision matrix Y_w is found and the distance $d^+(X_j)$ between each alternative and positive ideal solution and distance $d^-(X_j)$ between each alternative and negative ideal solution are calculated in the following step. The relative degree of closeness $C(X_j)$ is then calculated and the corresponding rank of the alternatives based on the relative degree of closeness is determined. The distance of each alternative from the positive ideal solution and negative ideal solution, the relative degree of closeness and rank are shown in Table XV.

As shown in Table VIII, based on the DM's preference, greater priority was given to cost (52.86%) and safety (26.90%). Based on the rank obtained from the relative degree of closeness, it is found that the TBPM strategy has the highest value of the relative degree of closeness and is ranked “1” in Table XV. Thus, TBPM should be utilized in the fabrication unit to tackle the current requirements. The relative degree of closeness values and ranks in Table XV helps DM to select the best maintenance strategy and acts as a decision support tool for the decision-making process. It can also be inferred from Table XV that PM is the lowest ranked strategy as it requires a higher initial cost than all other alternative strategies. To justify the validation of this proposed methodology, results obtained by the proposed IT2F-AHP-TOPSIS method are compared with the results of conventional AHP-TOPSIS (using crisp numeric score) [44] and type-1 fuzzy AHP-TOPSIS [45] by using the crisp and type-1 fuzzy equivalent of a given preference. In conventional AHP-TOPSIS, the weights and local preferences are obtained by using AHP (geometric mean method [46]) and the final rank of the alternative is calculated using the degree of closeness obtained by TOPSIS. In type-1 fuzzy AHP-TOPSIS, the weights and local preferences are obtained using type-1 fuzzy-AHP (geometric mean method) and the final rank of the alternative is calculated using the degree of closeness obtained using TOPSIS. The triangular membership function is used to quantify the preference and the center of the area method is used to defuzzify the fuzzy numbers. Table XVI compares the rank and relative degree of closeness obtained from conventional AHP-TOPSIS, type-1 fuzzy AHP-TOSIS, and IT2F-AHP-TOPSIS for our four considered maintenance strategies. Based on Table XVI, it can be seen that IT2F-AHP-TOPSIS has the same rank as that of conventional AHP-TOPSIS and type-1 fuzzy AHP-TOPSIS. Hence, results obtained by IT2F-AHP-TOPSIS can be claimed as reliable. Compared to conventional AHP-TOPSIS and type-1 fuzzy AHP, IT2F-AHP-TOPSIS has the added advantage that it allows DM to define the membership function with greater flexibility and is able to handle uncertainty during the decision-making process.

To check the robustness of the proposed IT2F-AHP-TOPSIS methodology, a sensitivity analysis is undertaken based on varying weights of criteria [47]. The weights used in the sensitivity analysis is obtained by changing the user-defined weights obtained from IT2F-AHP, i.e., values of the upper (a^u_j, b^u_j, c^u_j) and lower (a^l_j, b^l_j, c^l_j) membership functions of IT2F set are

$$\frac{\left(\frac{(c^u_j - a^u_j) + \left(H\left(\widetilde{A^u_j}\right) \times b^u_j - a^u_j\right)}{3} + a^u_j\right) + \left(\frac{(c^l_j - a^l_j) + \left(H\left(\widetilde{A^l_j}\right) \times b^l_j - a^l_j\right)}{3} + a^l_j\right)}{2}$$

$$= \frac{\left(\frac{(13.466-0.571)+(1\times 2.942-0.571)}{3} + 0.571\right) + \left(\frac{(4.799-1.783)+(0.9\ \times 2.943-1.783)}{3} + 1.783\right)}{2} = 4.368$$

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

TABLE XVI
RANK OF MAINTENANCE STRATEGIES USING DIFFERENT METHODS

	Conventional AHP-TOPSIS		Type-1 AHP-TOPSIS		IT2F-AHP-TOPSIS	
	The relative degree of closeness	Rank	*The relative degree of closeness*	Rank	*The relative degree of closeness*	Rank
CM	0.6627	2	0.6649	2	0.67	2
TBPM	0.6704	1	0.6741	1	0.68	1
CBPM	0.5598	3	0.5533	3	0.54	3
PM	0.3373	4	0.3351	4	0.33	4

TABLE XVII
WEIGHTS FOR SENSITIVITY ANALYSIS

Criteria	Cost	Safety	Added Value	Feasibility
Set-1 (Derived from preference)	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-2	(1.571,3.943,14.466;1) (2.783,3.943,5.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-3	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(1.278,2.433,8.101; 1) (1.862,2.433,3.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-4	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(1.135,1.698,4.745; 1) (1.417,1.698,2.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-5	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(1.074,1.34,2.752; 1) (1.208,1.34,1.561;0.9)
Set-6	(0,1.943,12.466; 1) (0.783,1.943,3.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-7	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0,0.433,6.101; 1) (0,0.433,1.373;0.9)	(0.135,0.698,3.745; 1) (0.417,0.698,1.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)
Set-8	(0.571,2.943,13.466; 1) (1.783,2.943,4.799;0.9)	(0.278,1.433,7.101; 1) (0.862,1.433,2.373;0.9)	(0,0,2.745; 1) (0,0,0.173;0.9)	(0.074,0.34,1.752; 1) (0.208,0.34,0.561;0.9)

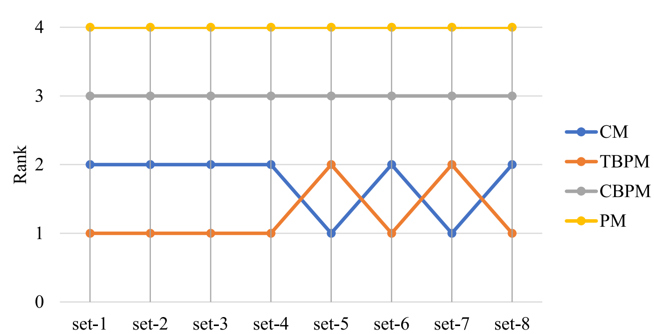

Fig. 5. Ranking results of sensitivity analysis.

increased and decreased by 1 unit to obtain the new weights. The varied weights of criteria are shown in Table XVII. Ranking results for different sets are shown in Fig. 5. It is visible from that figure that, TBPM is ranked one among six different sets, which as a consequence, can be accepted as the best maintenance strategy in the given conditions.

V. CONCLUSION

This article illustrated the use of the IT2F-AHP-TOPSIS method for selecting the best maintenance strategy for an industrial asset, taking into account uncertainty. The degree of uncertainty was effectively tackled by the IT2F-AHP-TOPSIS method. In this article, triangular IT2F sets were used for solving the numerical illustration, for the ease of computation. However, depending upon the fuzziness of the system, the DM can use triangular or trapezoidal membership functions (as is shown in Table III), and the proposed method can be applied for either function. Unlike other approaches that use a different IT2F scale of linguistic variables for IT2F-AHP and IT2F-TOPSIS, this method requires a single IT2F scale of linguistic variables, making it easier for the DM. The ranks obtained from the conventional AHP-TOPSIS and type-1 fuzzy AHP-TOPSIS are comparable with ranks obtained from the proposed IT2F-AHP-TOPSIS approach. IT2F-AHP-TOPSIS is recommended to use for future research, as it allows DM to define the membership function with greater flexibility and can handle the uncertainty associated with the decision.

The review of maintenance strategy by the fabrication unit managers provided better insight into their preferences and priorities. Although greater emphasis was given to the overall maintenance cost, by looking at the overall scenario managers accepted that in order to prevent breakdown at critical production time, management should adopt a time-based preventive maintenance strategy, which requires greater cost than breakdown maintenance but reduces the likelihood of the breakdown of a machine at a critical production time. According to the managers (DM), the tool proposed here is simple and is useful to quantify the preference and priorities of the DM and handling the uncertainty in decision making.

The proposed method can be applied to decision making in other areas such as vendor and supplier selection, material handling equipment selection, robot selection for industrial applications, and many more. There are several new IT2F extensions of conventional MCDM methods which can also be utilized in selecting the best maintenance strategy for industrial assets. The proposed method can also be extended to the second level of hierarchy by adding subcriteria, but this may increase the complexity of decision making and also increase the computation time, making it difficult for the DM. It should be kept in mind that the results of this article have some limitations, as these results are based on the preference value given by the expert DM. Consequently, the result may not be applicable to other industries, as preference values may vary with different expert opinions. It is also assumed that the DM are the experts in their field and have sufficient knowledge of the available maintenance strategies and appropriate selection criteria.

Appendix

A. Arithmetic Operations in Interval Type-2 Fuzzy Set

Arithmetic operations for triangular interval type-2 fuzzy (IT2F) sets are shown as follows:

Let $\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u; H(\widetilde{A_1^u})), (a_1^l, b_1^l, c_1^l; H(\widetilde{A_1^l}))$ and $\widetilde{\widetilde{A}}_2 = (\widetilde{A_2^u}, \widetilde{A_2^l}) = (a_2^u, b_2^u, c_2^u; H(\widetilde{A_2^u})), (a_2^l, b_2^l, c_2^l; H(\widetilde{A_2^l}))$ be two triangular IT2F sets.

Then the addition of two triangular IT2F sets [12] is

$$\widetilde{\widetilde{A}}_1 \oplus \widetilde{\widetilde{A}}_2 = (a_1^u + a_2^u, b_1^u + b_2^u, c_1^u + c_2^u; \min(H(\widetilde{A_1^u}); H(\widetilde{A_2^u})))$$
$$(a_1^l + a_2^l, b_1^l + b_2^l, c_1^l + c_2^l; \min(H(\widetilde{A_1^l}); H(\widetilde{A_2^l}))).$$

Subtraction of two triangular IT2F sets [12] is

$$\widetilde{\widetilde{A}}_1 \ominus \widetilde{\widetilde{A}}_2 = (a_1^u - a_2^u, b_1^u - b_2^u, c_1^u - c_2^u; \min(H(\widetilde{A_1^u}); H(\widetilde{A_2^u})))$$
$$(a_1^l - a_2^l, b_1^l - b_2^l, c_1^l - c_2^l; \min(H(\widetilde{A_1^l}); H(\widetilde{A_2^l}))).$$

Multiplication of two triangular IT2F sets [12] is

$$\widetilde{\widetilde{A}}_1 \otimes \widetilde{\widetilde{A}}_2 = (a_1^u \times a_2^u, b_1^u \times b_2^u, c_1^u \times c_2^u; \min(H(\widetilde{A_1^u}); H(\widetilde{A_2^u})))$$
$$(a_1^l \times a_2^l, b_1^l \times b_2^l, c_1^l \times c_2^l; \min(H(\widetilde{A_1^l}); H(\widetilde{A_2^l}))).$$

Multiplication of a crisp value k ($k > 0$) in a triangular IT2F set [12] is

$$k\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (k \times a_1^u, k \times b_1^u, k \times c_1^u; H(\widetilde{A_1^u}))$$
$$(k \times a_1^l, k \times b_1^l, k \times c_1^l; H(\widetilde{A_1^l})).$$

Division of two triangular IT2F sets [12] is

$$\frac{\widetilde{\widetilde{A}}_1}{\widetilde{\widetilde{A}}_2} = \left(\frac{a_1^u}{c_2^u}, \frac{b_1^u}{b_2^u}, \frac{c_1^u}{a_2^u}; \min(H(\widetilde{A_1^u}); H(\widetilde{A_2^u}))\right)$$
$$\left(\frac{a_1^l}{c_2^l}, \frac{b_1^l}{b_2^l}, \frac{c_1^l}{a_2^l}; \min(H(\widetilde{A_1^l}); H(\widetilde{A_2^l}))\right).$$

Reciprocal and exponent of a triangular IT2F set [12]

$$\frac{1}{\widetilde{\widetilde{A}}_1} = \left(\frac{1}{c_1^u}, \frac{1}{b_1^u}, \frac{1}{a_1^u}; H(\widetilde{A_1^u})\right), \left(\frac{1}{c_1^l}, \frac{1}{b_1^l}, \frac{1}{a_1^l}; H(\widetilde{A_1^l})\right)$$
$$\widetilde{\widetilde{A}}_1^{\,n} = ((a_1^u)^n, (b_1^u)^n, (c_1^u)^n; H(\widetilde{A_1^u}))$$
$$((a_1^l)^n, (b_1^l)^n, (c_1^l)^n; H(\widetilde{A_1^l})).$$

Arithmetic operations for trapezoidal IT2F sets are shown as follows:

Let

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

and

$$\widetilde{\widetilde{A}}_2 = (\widetilde{A_2^u}, \widetilde{A_2^l}) = (a_2^u, b_2^u, c_2^u, d_2^u; H_1(\widetilde{A_2^u}), H_2(\widetilde{A_2^u}))$$
$$(a_2^l, b_2^l, c_2^l, d_2^l; H_1(\widetilde{A_2^l}), H_2(\widetilde{A_2^l}))$$

be two trapezoidal IT2F sets then.

Addition of two trapezoidal IT2F sets [12]

$$\widetilde{\widetilde{A}}_1 \oplus \widetilde{\widetilde{A}}_2 = (a_1^u + a_2^u, b_1^u + b_2^u, c_1^u + c_2^u, d_1^u + d_2^u;$$
$$\min(H_1(\widetilde{A_1^u}); H_1(\widetilde{A_2^u})), \min(H_2(\widetilde{A_1^u}); H_2(\widetilde{A_2^u})))$$
$$(a_1^l + a_2^l, b_1^l + b_2^l, c_1^l + c_2^l, \ d_1^l + d_2^l;$$
$$\min(H_1(\widetilde{A_1^l}); H_1(\widetilde{A_2^l})), \min(H_2(\widetilde{A_1^l}); H_2(\widetilde{A_2^l}))).$$

Subtraction of two trapezoidal IT2F sets [12]

Let

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

and

$$\widetilde{\widetilde{A}}_2 = (\widetilde{A_2^u}, \widetilde{A_2^l}) = (a_2^u, b_2^u, c_2^u, d_2^u; H_1(\widetilde{A_2^u}), H_2(\widetilde{A_2^u}))$$
$$(a_2^l, b_2^l, c_2^l, d_2^l; H_1(\widetilde{A_2^l}), H_2(\widetilde{A_2^l}))$$

be two trapezoidal IT2F sets then

$$\widetilde{\widetilde{A}}_1 \ominus \widetilde{\widetilde{A}}_2 = (a_1^u - a_2^u, b_1^u - b_2^u, c_1^u - c_2^u, d_1^u - d_2^u;$$
$$\min(H_1(\widetilde{A_1^u}); H_1(\widetilde{A_2^u})), \min(H_2(\widetilde{A_1^u}); H_2(\widetilde{A_2^u})))$$
$$(a_1^l - a_2^l, b_1^l - b_2^l, c_1^l - c_2^l, \ d_1^l - d_2^l;$$
$$\min(H_1(\widetilde{A_1^l}); H_1(\widetilde{A_2^l})), \min(H_2(\widetilde{A_1^l}); H_2(\widetilde{A_2^l}))).$$

Multiplication of two trapezoidal IT2F sets [12]

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

and

$$\widetilde{\widetilde{A}}_2 = (\widetilde{A_2^u}, \widetilde{A_2^l}) = (a_2^u, b_2^u, c_2^u, d_2^u; H_1(\widetilde{A_2^u}), H_2(\widetilde{A_2^u}))$$
$$(a_2^l, b_2^l, c_2^l, d_2^l; H_1(\widetilde{A_2^l}), H_2(\widetilde{A_2^l}))$$

be two trapezoidal IT2F sets then

$$\begin{aligned}\widetilde{\widetilde{A}}_1 \otimes \widetilde{\widetilde{A}}_2 = {} & (a_1^u \times a_2^u, b_1^u \times b_2^u, c_1^u \times c_2^u, d_1^u \times d_2^u;\\ & \min(H_1(\widetilde{A_1^u}); H_1(\widetilde{A_2^u})), \min(H_2(\widetilde{A_1^u}); H_2(\widetilde{A_2^u})))\\ & (a_1^l \times a_2^l, b_1^l \times b_2^l, c_1^l \times c_2^l,\ d_1^l \times d_2^l;\\ & \min(H_1(\widetilde{A_1^l}); H_1(\widetilde{A_2^l})), \min(H_2(\widetilde{A_1^l}); H_2(\widetilde{A_2^l}))).\end{aligned}$$

Multiplication of a crisp value k $(k > 0)$ in a trapezoidal IT2F set

Let

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

then

$$\widetilde{\widetilde{kA_1}} = (k \times a_1^u, k \times b_1^u, k \times c_1^u, k \times d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(k \times a_1^l, k \times b_1^l, k \times c_1^l, k \times d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l})).$$

Division of two trapezoidal IT2F sets [12]

Let

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

and

$$\widetilde{\widetilde{A}}_2 = (\widetilde{A_2^u}, \widetilde{A_2^l}) = (a_2^u, b_2^u, c_2^u, d_2^u; H_1(\widetilde{A_2^u}), H_2(\widetilde{A_2^u}))$$
$$(a_2^l, b_2^l, c_2^l, d_2^l; H_1(\widetilde{A_2^l}), H_2(\widetilde{A_2^l}))$$

be two trapezoidal IT2F sets then

$$\begin{aligned}\frac{\widetilde{\widetilde{A}}_1}{\widetilde{\widetilde{A}}_2} = {} & \left(\frac{a_1^u}{d_2^u}, \frac{b_1^u}{c_2^u}, \frac{c_1^u}{b_2^u}, \frac{d_1^u}{a_2^u}; \min(H_1(\widetilde{A_1^u}); H_1(\widetilde{A_2^u})),\right.\\ & \left.\min(H_2(\widetilde{A_1^u}); H_2(\widetilde{A_2^u}))\right),\\ & \left(\frac{a_1^l}{d_2^l}, \frac{b_1^l}{c_2^l}, \frac{c_1^l}{b_2^l}, \frac{c_1^l}{a_2^l}; \min(H_1(\widetilde{A_1^l}); H_1(\widetilde{A_2^l}))\right.\\ & \left.\min(H_2(\widetilde{A_1^l}); H_2(\widetilde{A_2^l}))\right).\end{aligned}$$

Reciprocal and exponent of a trapezoidal IT2F set

Let

$$\widetilde{\widetilde{A}}_1 = (\widetilde{A_1^u}, \widetilde{A_1^l}) = (a_1^u, b_1^u, c_1^u, d_1^u; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))$$
$$(a_1^l, b_1^l, c_1^l, d_1^l; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l}))$$

then

$$\begin{aligned}\frac{1}{\widetilde{\widetilde{A}}_1} &= \left(\frac{1}{d_1^u}, \frac{1}{c_1^u}, \frac{1}{b_1^u}, \frac{1}{a_1^u}; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u})\right)\\ &\quad \left(\frac{1}{d_1^l}, \frac{1}{c_1^l}, \frac{1}{b_1^l}, \frac{1}{a_1^l}; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l})\right).\\ \widetilde{\widetilde{A}}_1^{\,n} &= ((a_1^u)^n, (b_1^u)^n, (c_1^u)^n, (d_1^u)^n; H_1(\widetilde{A_1^u}), H_2(\widetilde{A_1^u}))\\ &\quad ((a_1^l)^n, (b_1^l)^n, (c_1^l)^n, (d_1^l)^n; H_1(\widetilde{A_1^l}), H_2(\widetilde{A_1^l})).\end{aligned}$$

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Manoj Mathew received the B.E. degree in mechanical engineering and the M.Tech. degree in CAD/CAM-Robotics from Chhattisgarh Swami Vivekanand Technical University Bhilai, Chhattisgarh, India, in 2010 and 2013, respectively.

He is currently an Assistant Professor with the Department of Mechanical Engineering, Shri Shankaracharya Institute of Professional Management and Technology Raipur, India, which is affiliated to Chhattisgarh Swami Vivekanand Technical University, a Public State University in Durg, India. His research interests include decision-making process and its application in various engineering and management problems. He has authored many international papers in reputed journals.

Mr. Mathew is a Life Member of Indian Society for Technical Education.

Ripon K. Chakrabortty received the B.Sc. and M.Sc. degrees in industrial and production engineering from Bangladesh University of Engineering & Technology, Dhaka, Bangladesh, in 2009 and 2013, respectively, and the Ph.D. degree in computer science from the University of New South Wales, Canberra, Australia, in 2017.

He is a Lecturer in System Engineering & Project Management with the School of Engineering and Information Technology, University of New South Wales (UNSW Australia), Canberra, ACT, Australia. He has written 2 book chapters and more than 45 technical journal and conference papers. His research interests include a wide range of topics in operations research, project management, supply chain management, and information systems management.

Mike J. Ryan (Senior Member, IEEE) received the B.E. and Master's of Engineering Science degrees in electrical engineering, and the Ph.D. degree in remote sensing/data compression from the University of New South Wales, Australia, in 1981, 1989, and 1996, respectively.

He is an Associate Professor and Director of the Capability Systems Centre (CSC), University of New South Wales (UNSW), Canberra, ACT, Australia. In addition, he has completed two years formal engineering management training in the United Kingdom. He has more than 35 years of experience in communications engineering, systems engineering, project management, and management. Since joining UNSW, he has lectured in a range of subjects including communications and information systems, systems engineering, requirements engineering and project management and he regularly consults in those fields. He has authored/coauthored 12 books, 3 book chapters, and more than 200 refereed journal and conference papers.

Dr. Ryan is a Fellow of Engineers Australia (FIEAust), a Chartered Professional Engineer (CPEng) in electrical and ITEE colleges, a Fellow of the International Council on Systems Engineering (INCOSE), and a Fellow of the Institute of Managers and Leaders (FIML). His research is recognized internationally in a number of domains and has attracted over $4.7 million in external research grants and over $800 thousand in teaching grants. His role as a research supervisor was recognized in 2015 with an ARC Postgraduate Council Supervisor Award. He has acted as a Reviewer for a number of international journals and conferences and was the Editor-in-Chief of an international refereed journal (ERA2010 C), for 18 years. For the last 11 years, he has been the conference Chair of two major annual international conferences.