Linear Programming for College Scheduling

Mohamed Ahmed
Mohamed El-Said
Mohamed Khalid

April 30, 2025

Abstract

This is a project paper for the course "Artificial Intelligence" at Mansura University. We are using Linear Programming to solve the College Scheduling Problem, and this proposal describes the project and the equations we used.

1 Agent Design (PEAS)

- **Performance**: Minimize scheduling conflicts, Achieve the best option to satisfy all parties, Ensure scalability for multiple departments
- **Environment**: Colleges
- **Actuators**: Screen display, exporting schedules, and making folders
- **Sensors**: User input (professors' availability, student courses ...etc)

2 Environment Properties (ODESDA)

- **Fully Observable**: The system can access all the data about the faculty
- **Deterministic**: The algorithm can determine the next step without sudden changes
- **Sequential**: The agent's experience does not divide into episodes
- **Static**: The environment is unchanged while an agent is deliberating.
- **Discrete**: There is a limited number of possible choices
- **Single Agent**: No other agent affects the software algorithm

3 Agents Type

- **Utility-based agents**: Achieving the greatest balance between scheduling solutions (minimize time gaps, preferred teaching times, . . . etc

4 Model Formulation

4.1 Model Variables

Index	Description
e	Environment index
g	Group index
c	Class index
s	Subject index
d	Day index
p	Period index
t	Doctor index
a	Teaching assistant index

Table 1: Indices used in the model.

Parameter	Description
E	List of environments (years)
G_e	List of groups in the environment
S_e	List of subjects in the environment
C_g	List of classes in the group
D	number of days
P	number of periods
H	number of halls
L	number of labs
T	List of doctors
A	List of teaching assistants
TL	Doctor maximum load (load of periods, load of subjects)
AL	Teaching assistant maximum load (load of periods, load of subjects)
$AT_{a,d,p}$	Matrix of preferences periods for teaching assistant a on day d period p
$TT_{t,d,p}$	Matrix of preferences periods for doctor t on day d period p
$AS_{a,s}$	Matrix of preferences subjects for teaching assistant a
$TS_{t,s}$	Matrix of preferences subjects for doctor t

Table 2: Parameters used in the model.

Decision Variable	Description
$X_{e,g,c,s,d,p}$	1 if class c in group g in environment e has a section of subject s on day d period p, 0 otherwise
$Y_{e,g,c,s,d,p}$	1 if class c in group g in environment e has a lecture of subject s on day d period p, 0 otherwise
$BP_{e,g,c,d,p}$	1 if class c in group g in environment e has a bussy period on day d period p, 0 otherwise
$BD_{e,g,c,d}$	1 if class c in group g in environment e has a study day on day d, 0 otherwise
$Load_{e,g,c,d}$	Number of study periods for class c in group g in environment e on day d
$DEV_{e,g,c,d}$	Deviation of the day d (balanced number of study periods) for class c in group g in environment e
$FP_{e,g,c,d}$	index for the first period of study
$LP_{e,g,c,d}$	index for the last period of study
$GAP_{e,g,c,d}$	Number of gaps between study periods
$I_{t,e,g,c,s,d,p}$	1 if doctor t is assigned to class c in group g in environment e on day d period p, 0 otherwise
$J_{a,e,g,c,s,d,p}$	1 if teaching assistant a is assigned to class c in group g in environment e on day d period p, 0 otherwise
$ADS_{a,s}$	1 if teaching assistant a is assigned to subject s, 0 otherwise
$TDS_{t,s}$	1 if doctor t is assigned to subject s, 0 otherwise

Table 3: Decision variables used in the model.

4.2 Model Constraints

1. Every class has a lecture of each subject on each week

$$\sum_{d=1}^{D}\sum_{p=1}^{P} Y_{e,g,c,s,d,p} = 1 \quad \forall e \in E, g \in G_e, c \in C_g, s \in S_e \tag{1}$$

2. Every class has a section of each subject on each week

$$\sum_{d=1}^{D}\sum_{p=1}^{P} X_{e,g,c,s,d,p} = 1 \quad \forall e \in E, g \in G_e, c \in C_g, s \in S_e \tag{2}$$

3. All classes in the same group take the lecture together

$$\sum_{c=1}^{C_g} Y_{e,g,c,s,d,p} = |C_g| \cdot Y_{e,g,C_g[0],s,d,p} \quad \forall e \in E, g \in G_e, s \in S_e, d \in D, p \in P \tag{3}$$

4. The same doctor is assigned to all classes of the same group

$$\sum_{c=1}^{C_g} I_{t,e,g,c,s,d,p} = |C_g| \cdot I_{t,e,g,C_g[0],s,d,p} \quad \forall t \in T, e \in E, g \in G_e, s \in S_e, d \in D, p \in P \tag{4}$$

5. Number of lectures that is taken in the same period is limited with the number of halls

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{s=1}^{S_e} Y_{e,g,C_g[0],s,d,p} \leq H \quad \forall p \in P, d \in D \tag{5}$$

6. Number of sections that is taken in the same period is limited with the number of labs

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{s=1}^{S_e} X_{e,g,c,s,d,p} \leq L \quad \forall p \in P, d \in D \tag{6}$$

7. Busy periods are the periods that have a lecture or a section

$$BP_{e,g,c,d,p} = \sum_{s=1}^{S_e} (Y_{e,g,c,s,d,p} + X_{e,g,c,s,d,p}) \quad \forall e \in E, g \in G_e, c \in C_g, d \in D, p \in P \tag{7}$$

$$BP_{e,g,c,d,p} \leq 1 \quad \forall e \in E, g \in G_e, c \in C_g, d \in D, p \in P \tag{8}$$

8. Number of study periods for class c in group g in environment e on day d

$$Load_{e,g,c,d} = \sum_{p=1}^{P} BP_{e,g,c,d,p} \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{9}$$

9. Busy day BD is zero if $Load$ is 0, otherwise it is one

$$BD_{e,g,c,d} \leq Load_{e,g,c,d} \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{10}$$

$$BD_{e,g,c,d} \cdot P \geq Load_{e,g,c,d} \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{11}$$

10. DEV = — load - $\frac{P}{2}$ — to achieve balanced number of study periods in the day, but it is equal to zero if it is not a study day

$$DEV_{e,g,c,d} \geq Load_{e,g,c,d} - \lceil \frac{P}{2} \rceil \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{12}$$

$$DEV_{e,g,c,d} \geq BD_{e,g,c,d} \cdot \lceil \frac{P}{2} \rceil - Load_{e,g,c,d} \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{13}$$

$$DEV_{e,g,c,d} \leq BD_{e,g,c,d} \cdot P \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{14}$$

11. Last period of study is greater than or equal to all study periods in the busy day

$$LP_{e,g,c,d} \geq BP_{e,g,c,d,p} \cdot p \quad \forall e \in E, g \in G_e, c \in C_g, d \in D, p \in P \tag{15}$$

$$LP_{e,g,c,d} \leq BD_{e,g,c,d} \cdot P \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{16}$$

12. Fast period of study is greater than or equal to all study periods in the busy day, but to avoid being zero all time, we will use the Big-M method

$$FP_{e,g,c,d} \leq p + M \cdot (1 - BP_{e,g,c,d,p}) \quad \forall e \in E, g \in G_e, c \in C_g, d \in D, p \in P \tag{17}$$

$$FP_{e,g,c,d} \leq BD_{e,g,c,d} \cdot P \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{18}$$

13. Number of gaps between study periods is the difference between the last period of study and the fast period of study

$$GAP_{e,g,c,d} = LP_{e,g,c,d} - FP_{e,g,c,d} - Load_{e,g,c,d} + BD_{e,g,c,d} \cdot P \quad \forall e \in E, g \in G_e, c \in C_g, d \in D \tag{19}$$

14. Number of work periods per week for doctors and assistants is less than their maximum load

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{s=1}^{S_e} \sum_{d=1}^{D} \sum_{p=1}^{P} I_{t,e,g,c,s,d,p} \leq TL[0] \quad \forall t \in T \tag{20}$$

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{s=1}^{S_e} \sum_{d=1}^{D} \sum_{p=1}^{P} J_{a,e,g,C_g[0],s,d,p} \leq AL[0] \quad \forall a \in A \tag{21}$$

15. Number of assigned subjects for doctors and assistants is less than their maximum load

$$\sum_{s=1}^{S_e} TDS_{t,s} \leq TL[1] \quad \forall t \in T \tag{22}$$

$$\sum_{s=1}^{S_e} ADS_{a,s} \leq AL[1] \quad \forall a \in A \tag{23}$$

16. Linking the doctors and assistants to the lectures and sections

$$\sum_{t=1}^{T} I_{t,e,g,c,s,d,p} = Y_{e,g,c,s,d,p} \quad \forall e \in E, g \in G_e, c \in C_g, s \in S_e, d \in D, p \in P \tag{24}$$

$$\sum_{a=1}^{A} J_{a,e,g,c,s,d,p} = X_{e,g,c,s,d,p} \quad \forall e \in E, g \in G_e, c \in C_g, s \in S_e, d \in D, p \in P \tag{25}$$

17. Linking the doctors and assistants to their subjects

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{d=1}^{D} \sum_{p=1}^{P} I_{t,e,g,c,s,d,p} \leq TDS_{t,s} \cdot TL[0] \quad \forall t \in T, s \in S_e \tag{26}$$

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{d=1}^{D} \sum_{p=1}^{P} I_{t,e,g,c,s,d,p} \geq TDS_{t,s} \quad \forall t \in T, s \in S_e \tag{27}$$

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{d=1}^{D} \sum_{p=1}^{P} J_{a,e,g,c,s,d,p} \leq ADS_{a,s} \cdot AL[0] \quad \forall a \in A, s \in S_e \tag{28}$$

$$\sum_{e=1}^{E} \sum_{g=1}^{G_e} \sum_{c=1}^{C_g} \sum_{d=1}^{D} \sum_{p=1}^{P} J_{a,e,g,c,s,d,p} \geq ADS_{a,s} \quad \forall a \in A, s \in S_e \tag{29}$$

18. Doctors and assistants have just 1 subject in the single period

$$\sum_{e=1}^{E}\sum_{g=1}^{G_e}\sum_{s=1}^{S_e} I_{t,e,g,C_g[0],s,d,p} \leq 1 \quad \forall t \in T, d \in D, p \in P \tag{30}$$

$$\sum_{e=1}^{E}\sum_{g=1}^{G_e}\sum_{c=1}^{C_g}\sum_{s=1}^{S_e} J_{a,e,g,c,s,d,p} \leq 1 \quad \forall a \in A, d \in D, p \in P \tag{31}$$

4.3 objective function

$$\begin{aligned} \min \quad & \sum_e \sum_g \sum_c \sum_d (DEV_{e,g,c,d} + BD_{e,g,c,d} + GAP_{e,g,c,d}) \\ & - \sum_t \sum_d \sum_p \left[\sum_e \sum_g \sum_c \sum_s (I_{t,e,g,c,s,d,p}) \cdot TT_{t,d,p} \right] \\ & - \sum_a \sum_d \sum_p \left[\sum_e \sum_g \sum_c \sum_s (J_{a,e,g,c,s,d,p}) \cdot AT_{a,d,p} \right] \\ & - \sum_a \sum_s [ADS_{a,s} \cdot AS_{a,s}] \\ & - \sum_t \sum_s [TDS_{t,s} \cdot TS_{t,s}] \end{aligned} \tag{32}$$

Equation 32 shows that we want to:

1. minimize day deviation,
2. minimize busy days,
3. minimize gaps between study periods.
4. maximize doctors' and assistants' time preferences
5. maximize doctors' and assistants' subject preferences

5 References

An Integer Linear Program for Periodic Scheduling in Universities

Math Department Schedule Optimization