Assignment 13
Vehicle tracking based on GPS and odometry data fusion

Performance –Wednesday October 7, 2020
Due to submit a performance report – Tuesday, October 13, 2020

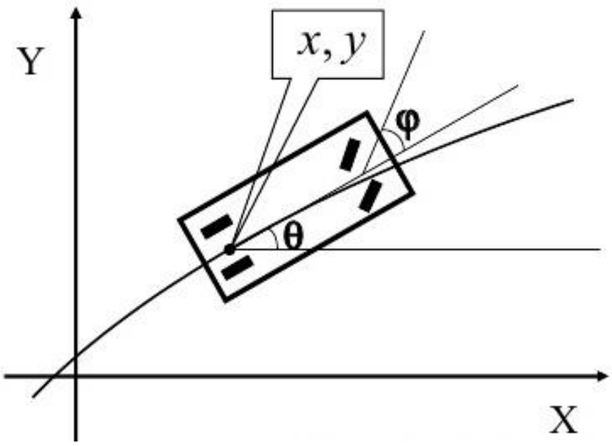
The objective of this laboratory work is to develop a tracking filter to estimate a vehicle's dynamic state by assimilating the navigation data coming from different sources. The task includes the fusion of GPS data (*absolute positioning)* and wheel odometry data (*relative positioning*). Important outcome of this exercise is getting skill to solve the most fundamental data fusion tasks for intelligent traffic applications.

This laboratory work is performed in the class by students as in teams of 4 and the team will submit one document reporting about the performance. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

Here is the recommended procedure:
Part I. Assimilation of GPS data only (absolute positioning)

1. Download the file 'theta.mat' ('theta.txt') from Canvas:
   files/Week 6_October 5_9/Assignment 13/data for lab/

   This is ground truth data for angle Θ describing the orientation of a car.



2. Generate a true trajectory X_i of a moving vehicle

$$x_i = x_{i-1} + V_{i-1}\cos(\Theta_{i-1})T + \frac{a^x_{i-1}T^2}{2}$$
$$y_i = y_{i-1} + V_{i-1}sin(\Theta_{i-1})T + \frac{a^y_{i-1}T^2}{2}$$

In this case (just for explanations)

$$V_x = V_{i-1}\cos(\Theta_{i-1})$$
$$V_y = V_{i-1}\sin(\Theta_{i-1})$$
$$V = \sqrt{V_x^2 + V_y^2}$$

Initial conditions to generate trajectory

(a) Size of trajectory is $N = 500$ points.

(b) $T = 0.05$ seconds – time step.

(c) Initial coordinates
$x_0 = 0;\ y_0 = 0$

(d) Velocity V
Velocity V is constant over the whole observation interval $V = 10$

(e) Variance of noise a_i, $\sigma_a^2 = 1^2$ for both a_i^x, a_i^y

3. Generate measurements of x and y

$$z_i^x = x_i + \eta_i^x$$
$$z_i^y = y_i + \eta_i^y$$

Variances of measurement noises η_i^x, η_i^y are given by
$\sigma_{\eta_x}^2 = 3^2;\ \sigma_{\eta_y}^2 = 3^2$
Measurements z_i^x and z_i^y simulate in this case GPS data (*absolute positioning*).

4. Develop Linear Kalman filter algorithm to localize the moving vehicle on the basis of *GPS data only*.

(a) *State vector*

$$X = \begin{vmatrix} x \\ V_x \\ y \\ V_Y \end{vmatrix}$$

(b) Transition matrix

$$\Phi = \begin{vmatrix} 1 & \mathrm{T} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{T} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

(c) Input matrix

$$G = \begin{vmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{vmatrix}$$

(d) Covariance matrix Q of state noise Ga_{i-1}

$$Q = GG^T\sigma_a^2$$

Important comment:
Use $\boldsymbol{\sigma_a^2 = 5^2}$ (instead of $\boldsymbol{\sigma_a^2 = 1}$) to create the matrix $\boldsymbol{Q}$ in filtration algorithm.
The dependencies $V_x = V_{i-1}\cos(\Theta_{i-1})$ and $V_y = V_{i-1}\sin(\Theta_{i-1})$ are approximate and describe immediate values of V_x and V_y. Angle Θ may change at step from $i-1$ to step i.

Therefore we should increase the covariance matrix of state noise, that will result in a greater filter gain K, and thus more confidence to measurements. In other words, by increasing σ_a^2 we compensate the imperfections of the motion model.

(e) *Initial conditions for Kalman filter algorithm*
Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} z_i^x(2) \\ \frac{z_i^x(2) - z_i^x(1)}{T} \\ z_i^y(2) \\ \frac{z_i^y(2) - z_i^y(1)}{T} \end{vmatrix}$$

Initial filtration error covariance matrix $P_{0,0}$

$$P_{0,0} = \begin{vmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 10^4 \end{vmatrix}$$

(f) Measurement noise covariance matrix R:

$$R = \begin{vmatrix} \sigma_{\eta_x}^2 & 0 \\ 0 & \sigma_{\eta_y}^2 \end{vmatrix}$$

(g) Measurement vector consists of GPS data

$$z = \begin{vmatrix} z^x \\ z^y \end{vmatrix}$$

(h) For convenience you can start your filter from the step $i = 3$ as measurements z_1, z_2 were already used to determine initial conditions.

5. Run Linear Kalman filter on the basis of *GPS data only* over $M = 500$ runs.
   Calculate true estimation errors of
   (a) Errors of extrapolation and filtration estimates of coordinate x
   (b) Errors of extrapolation and filtration estimates of coordinate y
   Please plot these errors on two different plots for the analysis.

***Part II. Assimilation of* both *GPS data* (absolute positioning) and *wheel odometry data* (relative positioning).**

1. Generate a true trajectory the same way is in Part I, item 2.

2. Generate measurements of x and y the same way as in Part 1, item 3.

3. Generate measurements of velocity V that in practice come from wheel odometry sensors. Now we make an assumption that V is obtained as average velocity of left and right wheels of a vehicle (front or rear).

$$V_i^m = V + \eta_i^V$$

Variance of measurement noise η_i^V is given by $\sigma_{\eta V}^2 = 0.5^2$.

4. Generate measurements of angle Θ in practice come simultaneously with GPS data (not direct odometry data).

$$\Theta_i^m = \Theta_\mathrm{i} + \eta_i^\Theta$$

Variance of measurement noise η_i^Θ is given by $\sigma_{\eta_\Theta}^2 = 0.02^2$.

5. Develop Extended Kalman filter assimilating both *GPS data* and *wheel odometry measurements.* All initial conditions, state vector X, and matrices Φ, Q and G are the same as in part. However, there are following changes.

   (a) Measurement vector consists of GPS and odometry data

$$z = \begin{vmatrix} z^x \\ z^y \\ V^m \\ \Theta^\mathrm{m} \end{vmatrix}$$

   (b) In this case the measurement equation would be nonlinear

$$z = h(X) + \eta$$

   here

$$h(X) = \begin{vmatrix} x \\ y \\ \sqrt{V_x^2 + V_y^2} \\ \arctan(\frac{V_y}{V_x}) \end{vmatrix}$$

   Observation function $h(X)$ indicates how the measurement vector z is related with the state vector X.

   (c) At every filtration step in the algorithm you should linearize measurement equation by determining the derivative at the point of extrapolation estimate $\hat{X}_{i+1,i}$.

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

   The derivative is

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}} = \begin{vmatrix} \frac{dx}{dx} & \frac{dx}{dV_x} & \frac{dx}{dy} & \frac{dx}{dV_y} \\ \frac{dy}{dx} & \frac{dy}{dV_x} & \frac{dy}{dy} & \frac{dy}{dV_y} \\ \frac{d(\sqrt{V_x^2 + V_y^2})}{dx} & \frac{d(\sqrt{V_x^2 + V_y^2})}{dV_x} & \frac{d(\sqrt{V_x^2 + V_y^2})}{dy} & \frac{d(\sqrt{V_x^2 + V_y^2})}{dV_y} \\ \frac{d(\arctan(\frac{V_y}{V_x}))}{dx} & \frac{d(\arctan(\frac{V_y}{V_x}))}{dV_x} & \frac{d(\arctan(\frac{V_y}{V_x}))}{dy} & \frac{d(\arctan(\frac{V_y}{V_x}))}{dV_y} \end{vmatrix}$$

The derivative at the point of extrapolation estimate $\hat{X}_{i+1,i}$ is

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{V_x}{\sqrt{V_x^2 + V_y^2)}} & 0 & \frac{V_y}{\sqrt{V_x^2 + V_y^2)}} \\ 0 & -\frac{V_y}{V_x^2 + V_y^2} & 0 & \frac{V_x}{V_x^2 + V_y^2} \end{vmatrix}$$

V_x, V_y are taken from the extrapolated vector $\hat{X}_{i+1,i}$

(d) ***Important comment***

In filtration algorithm you should determine a value of observation function $h(\hat{X}_{i+1,i})$. The form of $h(\hat{X}_{i+1,i})$ is given in item (b) of this part. The last component contains arctan function.

In Matlab function "atan" works only in the range $[-\frac{\pi}{2};\frac{\pi}{2}]$

This corresponds to the case when both V_x and V_y are positive.

V_x, V_y are taken from the extrapolated vector $\hat{X}_{i+1,i}$

You should write the proper block to handle different cases

(1) If $V_x > 0$ then arctan is determined as atan$(\frac{V_y}{V_x})$

(2) If $V_y > 0$ and $V_x < 0$ then arctan is determined as atan $\left(\frac{V_y}{V_x}\right) + \pi$

(3) If $V_y < 0$ and $V_x < 0$ then arctan is determined as atan $\left(\frac{V_y}{V_x}\right) - \pi$

(4) If $V_y > 0$ and $V_x = 0$ then arctan is determined as $\frac{\pi}{2}$

(5) If $V_y < 0$ and $V_x = 0$ then arctan is determined as $-\frac{\pi}{2}$

Or as an alternative you can use a proper function that handles this issue.

(e) Measurement noise covariance matrix R:

$$R = \begin{vmatrix} \sigma_{\eta x}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\eta y}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\eta V}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\eta \Theta}^2 \end{vmatrix}$$

6. Run Extended Kalman filter assimilating *GPS data and odometry data* over $M = 500$ runs. Calculate true estimation errors of
   (c) Errors of extrapolation and filtration estimates of coordinate x
   (d) Errors of extrapolation and filtration estimates of coordinate y
   Please plot these errors on two different plots for the analysis.

7. Compare the estimation accuracy in case of assimilating GPS data only and fusion of both *GPS data* and *odometry data*. Make general conclusions.

8. Make conclusions to the Assignment.
   Conclusions should be done in a form of a learning log.
   A learning log is a journal which evidences your **own learning and skills development.**
   It is not just a diary or record of **"What you have done"**
   but a record of **what you have learnt, tried and critically reflected upon.**

9. Prepare performance report and submit to Canvas:
   Performance report should include 2 documents:
   1) A report (PDF) with performance of all the items listed above
   2) Code (PDF)

Notes:

- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed
- All figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.