“Experimental Data Processing”

Development of a tracking filter of a moving object when measurements and motion models are in different coordinate systems.

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Term 1, September-October 2020
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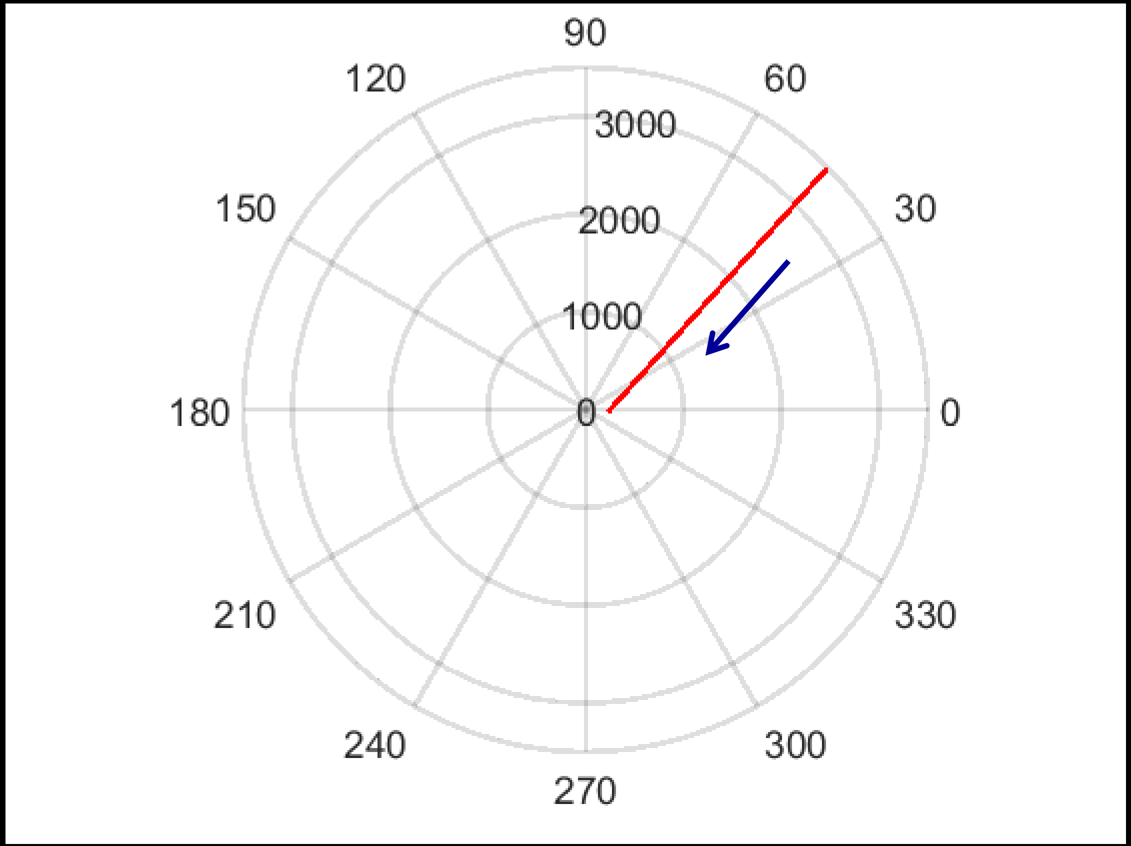
Measurements of navigation parameters are available in polar coordinate system



Range D → Distance from an observer to a moving object

Azimuth β → Angle between direction of North and direction of a moving object

Motion model is in Cartesian coordinate system

Model of uniform motion →

$$x_i = x_{i-1} + V_{i-1}^{x} T$$
$$y_i = y_{i-1} + V_{i-1}^{y} T$$
$$V_i^x = V_{i-1}^x$$
$$V_i^y = V_{i-1}^y$$

→ **Cartesian coordinate system**

x_i, y_i → **Cartesian coordinates**

V_i^x, V_i^y → **Components of velocity V_i**

$$V_i = \sqrt{\left(V_i^x\right)^2 + \left(V_i^y\right)^2}$$

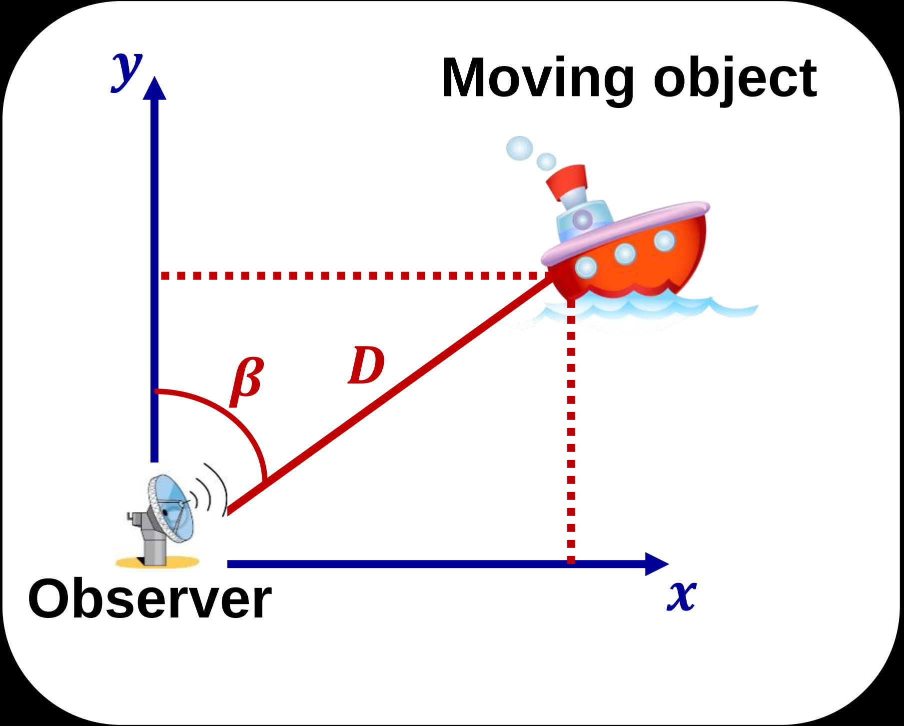
State-space model, state equation

State equation → $$X_i = \Phi_{i,i-1} X_{i-1}$$

State vector → $$X_i = \begin{vmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \end{vmatrix}$$

Transition matrix → $$\Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Transformation from polar to Cartesian coordinate system



$$D = \sqrt{x^2 + y^2}$$

$$\beta = arctg\left(\frac{x}{y}\right)$$

$$x = D sin\beta$$
$$y = D cos\beta$$

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → **Measurements of range D**

β_i^m → **Measurements of azimuth β**

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → Nonlinear relation

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → **Nonlinear relation**

$$Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p \rightarrow Z_i^p = \begin{vmatrix} \sqrt{x_i^2 + y_i^2} \\ arctg\left(\dfrac{x_i}{y_i}\right) \end{vmatrix} + \eta_i^p$$

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → Nonlinear relation

$Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p$ → $Z_i^p = \begin{vmatrix} \sqrt{x_i^2 + y_i^2} \\ arctg\left(\dfrac{x_i}{y_i}\right) \end{vmatrix} + \eta_i^p$

$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$ → Vector of measurement errors of range D and azimuth β

Noises η_i^D, η_i^β are uncorrelated with each other and have variances $\sigma_D^2, \sigma_\beta^2$

Measurement equation
From nonlinear to linear equation

Transform polar measurements D_i^m and β_i^m to Cartesian coordinates →
$$x_i^m = D_i^m sin\beta_i^m$$
$$y_i^m = D_i^m cos\beta_i^m$$

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$ → Pseudo-measurement vector → $Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$

Measurement equation
From nonlinear to linear equation

Transform polar measurements D_i^m and β_i^m to Cartesian coordinates →

$$x_i^m = D_i^m sin\beta_i^m$$
$$y_i^m = D_i^m cos\beta_i^m$$

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$ → Pseudo-measurement vector → $Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$

Measurement equation → $Z_i^c = HX_i + \eta_i^c$ → Linear relation

$$H = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$ → Vector of pseudo-measurement errors of x_i^m and x_i^m

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector of measurement errors of range D_i^m and azimuth β^m

$$\begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p$$

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector of pseudo-measurement errors of x_i and y_i

$$\begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix} = \begin{vmatrix} x_i \\ y_i \end{vmatrix} + \eta_i^c$$

Full differential, example 1

x
Function

$x = f(u)$

Full differential, example 1

x
Function

$x = f(u)$

Δx
Small increment of function

$\Delta x = f(u + \Delta u) - f(u)$

Full differential, example 1

x
Function

$$x = f(u)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u) - f(u)$$

Taylor series

$$f(u + \Delta u) = f(u) + \frac{1}{1!}\frac{dx}{du}\Delta u + \frac{1}{2!}\frac{d^2x}{du^2}(\Delta u)^2 + \frac{1}{3!}\frac{d^3x}{du^3}(\Delta u)^3 + \ \dots +$$

Full differential, example 1

x
Function

$x = f(u)$

Δx
Small increment of function

$\Delta x = f(u + \Delta u) - f(u)$

Taylor series

$$f(u + \Delta u) = f(u) + \frac{1}{1!}\frac{dx}{du}\Delta u + \frac{1}{2!}\frac{d^2x}{du^2}(\Delta u)^2 + \frac{1}{3!}\frac{d^3x}{du^3}(\Delta u)^3 + \ldots +$$

$$\Delta x = f(u + \Delta u) - f(u) \approx \frac{dx}{du}\Delta u$$

$\Delta u \to 0$

Differential dx
Infinitely small increment of function

$$dx = \frac{dx}{du}du$$

Full differential, example 2

x
Function

$$x = f(u, v)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u, v + \Delta \mathrm{v}) - f(u, v)$$

$$\Delta x \approx \frac{dx}{du}\Delta u + \frac{dx}{dv}\Delta v$$

$\Delta u \to 0$
$\Delta v \to 0$

Full differential dx
Infinitely small increment of function

$$dx = \frac{dx}{du}du + \frac{dx}{dv}\mathrm{dv}$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$ ⇒ Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$ ⇒ Vector of pseudo-measurement errors of x_i^m and y_i^m

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector of pseudo-measurement errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector
of measurement errors
of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector
of pseudo-measurement
errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2

$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix} \Rightarrow$$

Vector
of measurement errors
of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} \Rightarrow$$

Vector
of pseudo-measurement
errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2

$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

3

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix} \Rightarrow$$

Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} \Rightarrow$$

Vector of pseudo-measurement errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2

$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

3

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

Rewrite (1) using (2) and (3) for x_i

$$x_i + \eta_i^x = \left(D_i + \eta_i^D\right) sin\left(\beta_i + \eta_i^\beta\right)$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = \left(D_i + \eta_i^D\right) sin(\beta_i + \eta_i^\beta)$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = (D_i + \eta_i^D)\sin(\beta_i + \eta_i^\beta)$$

$$\eta_i^x = (D_i + \eta_i^D)\sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = (D_i + \eta_i^D)sin(\beta_i + \eta_i^\beta)$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - D_i sin\beta_i$$

$$x = Dsin\beta$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = (D_i + \eta_i^D)sin(\beta_i + \eta_i^\beta)$$

$$\downarrow$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

$$\downarrow$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - D_i sin\beta_i$$

$$x = Dsin\beta$$

Or

$$\Delta x = (D + \Delta D)sin(\beta + \Delta\beta) - Dsin\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta \mathrm{x} = (D + \Delta \mathrm{D}) sin(\beta + \Delta\beta) - Dsin\beta$$

$$\mathrm{x} = Dsin\beta$$

$$\Delta \mathrm{y} = (D + \Delta \mathrm{D}) cos(\beta + \Delta\beta) - Dsin\beta$$

$$\mathrm{y} = Dcos\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta \mathrm{x} = (D + \Delta \mathrm{D}) sin(\beta + \Delta\beta) - D sin\beta$$

$$\mathrm{x} = D sin\beta$$

$$\Delta \mathrm{y} = (D + \Delta \mathrm{D}) cos(\beta + \Delta\beta) - D sin\beta$$

$$\mathrm{y} = D cos\beta$$

Taylor series

$$(D + \Delta \mathrm{D}) sin(\beta + \Delta\beta) \approx D sin\beta + \frac{dx}{dD}\Delta \mathrm{D} + \frac{dx}{d\beta}\Delta\beta$$

$$(D + \Delta \mathrm{D}) cos(\beta + \Delta\beta) \approx D cos\beta + \frac{dy}{dD}\Delta \mathrm{D} + \frac{dy}{d\beta}\Delta\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta x = (D + \Delta D)sin(\beta + \Delta\beta) - Dsin\beta$$

$$x = Dsin\beta$$

$$\Delta y = (D + \Delta D)cos(\beta + \Delta\beta) - Dsin\beta$$

$$y = Dcos\beta$$

Taylor series

$$(D + \Delta D)sin(\beta + \Delta\beta) \approx Dsin\beta + \frac{dx}{dD}\Delta D + \frac{dx}{d\beta}\Delta\beta$$

$$(D + \Delta D)cos(\beta + \Delta\beta) \approx Dcos\beta + \frac{dy}{dD}\Delta D + \frac{dy}{d\beta}\Delta\beta$$

$$\Delta x = \frac{dx}{dD}\Delta D + \frac{dx}{d\beta}\Delta\beta$$

$$\Delta y = \frac{dy}{dD}\Delta D + \frac{dy}{d\beta}\Delta\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta x = \frac{dx}{dD}\Delta D + \frac{dx}{d\beta}\Delta\beta \Rightarrow \eta^x = \frac{dx}{dD}\eta^D + \frac{dx}{d\beta}\eta^\beta$$

$$\Delta y = \frac{dy}{dD}\Delta D + \frac{dy}{d\beta}\Delta\beta \Rightarrow \eta^y = \frac{dy}{dD}\eta^D + \frac{dy}{d\beta}\eta^\beta$$

$$\frac{dx}{dD} = sin\beta$$

$$\frac{dx}{d\beta} = Dcos\beta$$

$$\frac{dy}{dD} = cos\beta$$

$$\frac{dy}{d\beta} = -Dsin\beta$$

Summary, scheme of estimation algorithm

1 Measurements are available in polar coordinate system

D_i^m — Measurements of range D

β_i^m — Measurements of azimuth β

2 Transform polar measurements D_i^m and β_i^m to Cartesian coordinates

$$x_i^m = D_i^m sin\beta_i^m$$
$$y_i^m = D_i^m cos\beta_i^m$$

3 Develop Kalman filter with pseudo-measurements in Cartesian coordinates

$$Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$$

Summary, state-space model

State equation

$$X_i = \Phi_{i,i-1} X_{i-1}$$

$$X_i = \begin{vmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \end{vmatrix}$$

$$\Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Measurement equation

$$Z_i^c = H X_i + \eta_i^c$$

$$Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$$

$$x_i^m = D_i^m \sin\beta_i^m$$
$$y_i^m = D_i^m \cos\beta_i^m$$

$$H = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} = \begin{vmatrix} \eta_i^D \sin\beta_i^m + \eta_i^\beta D_i^m \cos\beta_i^m \\ \eta_i^D \cos\beta_i^m - \eta_i^\beta D_i^m \sin\beta_i^m \end{vmatrix}$$

Noises η_i^D, η_i^β - errors of D_i^m and β_i^m

Are characterized by variances $\sigma_D^2, \sigma_\beta^2$

Covariance matrix of measurement error R

Measurement noise → $\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} = \begin{vmatrix} \eta_i^D sin\beta_i^m + \eta_i^\beta D_i^m cos\beta_i^m \\ \eta_i^D cos\beta_i^m - \eta_i^\beta D_i^m sin\beta_i^m \end{vmatrix}$

R Covariance matrix → $R = E[\eta_i^c \cdot (\eta_i^c)^T]$

$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

σ_D^2 → Variance of measurement error of range D

σ_β^2 → Variance of measurement error of azimuth β

Goals of the laboratory work

1 Analyze instability zone of a filter

When object is close to observer → Navigation system may become blind → Related with ill-conditioned matrix R

Condition number of measurement error covariance matrix R

$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + \left(D_i^m\right)^2 cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - \left(D_i^m\right)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - \left(D_i^m\right)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + \left(D_i^m\right)^2 sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

σ_D^2 → Variance of measurement error of range D

σ_β^2 → Variance of measurement error of azimuth β

Eigen values of matrix R → $det(R - \lambda I) = 0$ → $\lambda_1 = \sigma_D^2$, $\lambda_2 = \left(D_i^m\right)^2\sigma_\beta^2$

Condition number of measurement error covariance matrix R

$$R = \begin{vmatrix} \sin^2\beta_i^m \sigma_D^2 + \left(D_i^m\right)^2 \cos^2\beta_i^m \sigma_\beta^2 & \sin\beta_i^m \cos\beta_i^m [\sigma_D^2 - \left(D_i^m\right)^2 \sigma_\beta^2] \\ \sin\beta_i^m \cos\beta_i^m [\sigma_D^2 - \left(D_i^m\right)^2 \sigma_\beta^2] & \cos^2\beta_i^m \sigma_D^2 + \left(D_i^m\right)^2 \sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

σ_D^2 → Variance of measurement error of range D

σ_β^2 → Variance of measurement error of azimuth β

Eigen values of matrix R → $det(R - \lambda I) = 0$ → $\lambda_1 = \sigma_D^2$, $\lambda_2 = \left(D_i^m\right)^2 \sigma_\beta^2$

Condition number → $\dfrac{\lambda_1}{\lambda_2} = \dfrac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$ ($\lambda_1 > \lambda_2$) → $\dfrac{\lambda_2}{\lambda_1} = \dfrac{\left(D_i^m\right)^2 \sigma_\beta^2}{\sigma_D^2}$ ($\lambda_2 > \lambda_1$)

Condition number of measurement error covariance matrix R

Condition number

$$\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$$

$\lambda_1 > \lambda_2$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(D_i^m\right)^2 \sigma_\beta^2}{\sigma_D^2}$$

$\lambda_2 > \lambda_1$

If condition number of matrix R ≈ 1

R – well-condtioned matrix

$$\sigma_D^2 \approx \left(D_i^m\right)^2 \sigma_\beta^2$$

Condition number of measurement error covariance matrix R

Condition number → $\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{(D_i^m)^2 \sigma_\beta^2}$ → $\frac{\lambda_2}{\lambda_1} = \frac{(D_i^m)^2 \sigma_\beta^2}{\sigma_D^2}$

$\lambda_1 > \lambda_2$ $\qquad$ $\lambda_2 > \lambda_1$

If condition number of matrix $R \approx 1$ → R – well-condtioned matrix → $\sigma_D^2 \approx (D_i^m)^2 \sigma_\beta^2$

$$R = \begin{vmatrix} sin^2\beta_i^m \sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m \sigma_\beta^2 & sin\beta_i^m cos\beta_i^m [\sigma_D^2 - (D_i^m)^2 \sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m [\sigma_D^2 - (D_i^m)^2 \sigma_\beta^2] & cos^2\beta_i^m \sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

$$\Downarrow$$

$$R = \begin{vmatrix} sin^2\beta_i^m \sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m \sigma_\beta^2 & 0 \\ 0 & cos^2\beta_i^m \sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

Independent assimilation for x, y

Condition number of measurement error covariance matrix R

Condition number

$$\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$$

$\lambda_1 > \lambda_2$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(D_i^m\right)^2 \sigma_\beta^2}{\sigma_D^2}$$

$\lambda_2 > \lambda_1$

If condition number of matrix $R > 1000$

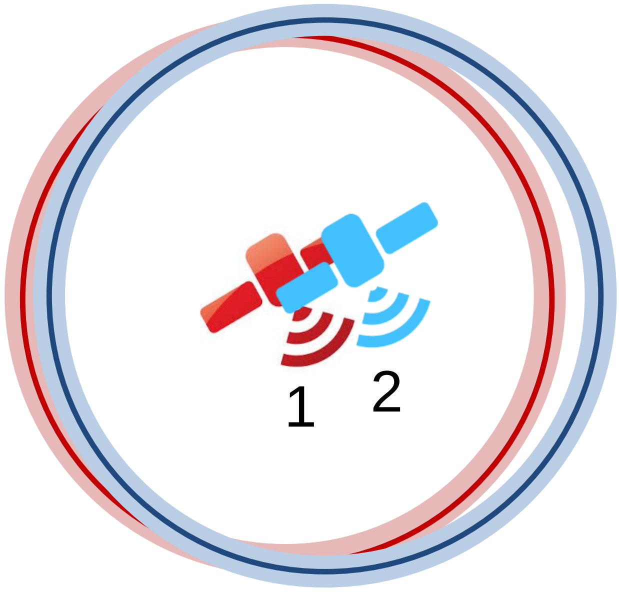
R – ill-condtioned matrix

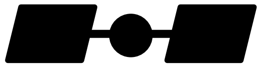
$$R = \begin{vmatrix} sin^2\beta_i^m \sigma_D^2 + \left(D_i^m\right)^2 cos^2\beta_i^m \sigma_\beta^2 & sin\beta_i^m cos\beta_i^m [\sigma_D^2 - \left(D_i^m\right)^2 \sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m [\sigma_D^2 - \left(D_i^m\right)^2 \sigma_\beta^2] & cos^2\beta_i^m \sigma_D^2 + \left(D_i^m\right)^2 sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

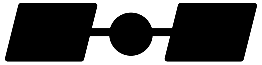
Estimation accuracy is decreased
Filter may diverge

Ill-conditioned problem

Validity of applying a technique







Man-made satellite

Navigation satellite

Il-conditioned problem

Satellite position is undefined!

Thank you for your attention!

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