Assignment 10

Development of a tracking filter of a moving object when measurements and motion models are in different coordinate systems.

Performance – Monday, September 28, 2020
Due to submit a performance report – Friday, October 2, 2020

The objective of this laboratory work is to develop a tracking filter of a moving object when measurements and motion model are in different coordinate systems. This problem is typical for radio navigation systems. Important outcome of this exercise is to detect main difficulties of practical Kalman filter implementation related with instability zone of a tracking filter, and to analyze conditions under which navigation system may become blind and filter diverges. This is important to prevent collisions and for other safety issues.

The first part of this laboratory work is performed in the class by students as in teams of 4 and the team will submit one document reporting about the performance. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

Important information
Please read charts for problem formulation
Tracking_filter_cordinate_transformation_of_measurements.pdf

Here is the recommended procedure:
Instability zone of a tracking filter due to ill-conditioned coordinate transformations of measurements.

1. Generate a true trajectory X_i of an object that moves uniformly. Trajectory is deterministic, as no random disturbance affect a motion.

   Cartesian coordinates x_i, y_i and components of velocity V_i^x and V_i^y are determined by

$$x_i = x_{i-1} + V_{i-1}^x T$$
$$V_i^x = V_{i-1}^x$$
$$y_i = y_{i-1} + V_{i-1}^y T$$
$$V_i^y = V_{i-1}^y$$

   Initial conditions to generate trajectory

   (a) Size of trajectory is $N = 26$ points.

   (b) $T = 2$ – interval between measurements.

   (c) Initial components of velocity V
   $V_x = -50; V_y = -45;$
   It means that an object moves toward an observer (radiolocation station for instance).

   (d) Initial coordinates
   $x_0 = \frac{13500}{\sqrt{2}}$; $y_0 = \frac{13500}{\sqrt{2}}$
   This means that an object starts it motion **at a quite great distance from an observer**. Later we will analyze when an object starts its motion **at a quite close distance from an observer.**

2. Generate also true values of range D and azimuth β

$$D_i = \sqrt{x_i^2 + y_i^2}$$

$$\beta_i = arctg\left(\frac{x}{y}\right)$$

Initial values $D_0 = \sqrt{x_0^2 + y_0^2}$; $\beta_0 = arct\left(\frac{x_0}{y_0}\right)$

Plot generated motion in polar coordinate system.
You can use command polarplot(β, D)

3. Generate measurements D^m and β^m of range D and azimuth β

$$D_i^m = D_i + \eta_i^D$$

$$\beta_i^m = \beta_i + \eta_i^\beta$$

Variances of measurement noises η_i^D, η_i^β are given by

$$\sigma_D = 20$$

$$\sigma_\beta = 0.02$$

Later we will analyze other conditions.

4. Transform polar coordinates D^m and β^m to Cartesian ones and get pseudo-measurements of coordinates x and y
Consult charts, page 31

5. Create the measurement vector z from pseudo-measurements of coordinates x and y
Consult charts, page 31

6. Initial conditions for Kalman filter algorithm
Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} 40000 \\ -20 \\ 40000 \\ -20 \end{vmatrix}$$

Initial filtration error covariance matrix $P_{0,0}$

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix}$$

7. Create the transition matrix Φ and observation matrix H
Consult charts, page 32

8. Create the measurement error covariance matrix R needed for Kalman filter algorithm
Consult charts, page 33

9. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration)
At every extrapolation and filtration step you will need to calculate range D and azimuth β from extrapolated and filtered estimates.

10. Run Kalman filter algorithm over $M = 500$ runs.

Calculate true errors of estimation:
(a) Errors of extrapolation and filtration estimates of range D relative to σ_D.
(b) Errors of extrapolation and filtration estimates of azimuth β relative to σ_β.
Make conclusions if these errors decrease or increase with time.

11. Analyze dependence of coordinate x on azimuth β
According to coordinate transformation x and β has nonlinear relation
$$x = Dsin\beta$$
Plot dependence of coordinate x on azimuth β.
Is it really nonlinear in these conditions?
Or it is close to linear?
If this dependence is close to linear it means that linearization errors are insignificant.

12. Calculate condition number of covariance matrix R over the observation interval.
Does condition number decrease or increase over time?
If condition number close to 1, than matrix R is well-conditioned.
If condition number is relatively great, then matrix R is ill-conditioned.

13. Analyze filter gain K. Dimension of filter gain in this case is $4x2$.
Plot $K(1,1)$. Note that values of $K(1,1)$ over observation interval don't always belong to interval (0,1) .This is related to the fact that matrix R depends on polar measurements that have errors. Ways to adjust filter gain K to be in the required range (0,1) can be analyzed, but this is not the goal of current assignment.

14. Run filter again over $M = 500$ runs but use other initial conditions to generate a trajectory
$x_0 = \frac{3500}{\sqrt{2}}$; $y_0 = \frac{3500}{\sqrt{2}}$
This means that an object starts it motion **at a quite close distance from an observer**.
Check by plotting new polar coordinates.

15. Calculate true errors of estimation:
(a) Errors of extrapolation and filtration estimates of range D relative to σ_D
(b) Errors of extrapolation and filtration estimates of azimuth β relative to σ_β.
Make conclusions if these errors decrease or increase with time.

16. Analyze dependence of coordinate x on azimuth β by plotting
Is it nonlinear or linear in these conditions?
Linearization errors are significant or insignificant?

17. Calculate condition number of covariance matrix R over the observation interval for these conditions. Does condition number decrease or increase over time?

18. Make conclusions how linearization errors affect tracking accuracy and how important for tracking accuracy is starting position of a moving object (close or far from an observer).

19. Run filter again over $M = 500$ runs. Use again initial conditions to generate a trajectory
$$x_0 = \frac{3500}{\sqrt{2}};\ y_0 = \frac{3500}{\sqrt{2}}$$
This means that an object starts it motion **at a quite close distance from an observer**.
But to generate polar measurements D^m and β^m (item 3) use other values of variances of measurement noises η_i^D, η_i^β:
$$\sigma_D = 50$$

$$\sigma_{\beta} = 0.0015$$

20. Repeat items 14,15,16,17 for these conditions and reply to questions addressed.

21. Make final conclusions under which conditions navigation system may become blind and filter may diverge. Which factors has the greatest influence? Linearization errors or ill-conditioned problem? Which solution can help to overcome this particular ill-conditioned problem?

    Conclusions should be done in a form of a learning log. **A learning log** is a journal which evidences your **own learning and skills development.** It is not just a diary or record of **"What you have done"** but a record of **what you have learnt, tried and critically reflected upon.**

22. Prepare performance report and submit to Canvas:
    Performance report should include 2 documents:
    1) A report (PDF) with performance of all the items listed above
    2) Code (PDF)

Notes:

- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed
- All figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.