Assignment 2
Comparison of the exponential and running mean for random walk model

Performance –Monday, September 7, 2020
Due to submit a performance report – Friday, September 11, 20202, 23:59 p.m.

The objective of this laboratory work is to compare the errors of exponential and running mean to choose the most effective quasi-optimal estimation method in conditions of uncertainty. Additional important outcome of this exercise is the solution of identification problem of noise statistics that is crucial for reliable estimation.

This laboratory work is performed in the class by students as in teams of 4 and the team will submit one document reporting about the performance. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of two parts:

I. Determination of optimal smoothing constant in exponential mean.
II. Comparison of methodical errors of exponential and running mean.

***Here is the recommended procedure for part I*:**
Determination of optimal smoothing constant in exponential mean

1. Please conduct a simulation experiment. First generate a true trajectory, then generate measurements of this true trajectory.

   1.1. Generate a true trajectory X_i using the random walk model

$$X_i = X_{i-1} + w_i \quad (1)$$

w_i –normally distributed random noise with zero mathematical expectation and variance σ_w^2.
Group 1: $\sigma_w^2 = 8$;
Group 2: $\sigma_w^2 = 9$;
Group 3: $\sigma_w^2 = 10$;
Group 4: $\sigma_w^2 = 11$;
Group 5: $\sigma_w^2 = 12$;
Group 6: $\sigma_w^2 = 13$;
Group 7: $\sigma_w^2 = 14$;
Group 8: $\sigma_w^2 = 15$;
Group 9: $\sigma_w^2 = 16$;
Group 10: $\sigma_w^2 = 17$;
Group 11: $\sigma_w^2 = 18$;
Group 12: $\sigma_w^2 = 19$;
Group 13: $\sigma_w^2 = 20$;
Group 14: $\sigma_w^2 = 8$;
Group 15: $\sigma_w^2 = 9$;
Group 16: $\sigma_w^2 = 10$;
Group 17: $\sigma_w^2 = 11$;
Group 18: $\sigma_w^2 = 12$;
Group 19: $\sigma_w^2 = 13$;
Group 20: $\sigma_w^2 = 14$;

Size of trajectory is
1) 3000 points
2) 300 points

To generate true trajectory use initial condition $X_1 = 10$.

1.2.Generate measurements z_i of the process X_i

$$z_i = X_i + \eta_i \quad (2)$$

η_i – normally distributed random noise with zero mathematical expectation and variance σ_η^2.
Group 1: $\sigma_\eta^2 = 16$;
Group 2: $\sigma_\eta^2 = 12$;
Group 3: $\sigma_\eta^2 = 13$;
Group 4: $\sigma_\eta^2 = 10$;
Group 5: $\sigma_\eta^2 = 9$;
Group 6: $\sigma_\eta^2 = 8$;
Group 7: $\sigma_\eta^2 = 10$;
Group 8: $\sigma_\eta^2 = 9$;
Group 9: $\sigma_\eta^2 = 11$;
Group 10: $\sigma_\eta^2 = 10$;
Group 11: $\sigma_\eta^2 = 12$;
Group 12: $\sigma_\eta^2 = 10$;
Group 13: $\sigma_\eta^2 = 15$;
Group 14: $\sigma_\eta^2 = 16$;
Group 15: $\sigma_\eta^2 = 12$;
Group 16: $\sigma_\eta^2 = 13$;
Group 17: $\sigma_\eta^2 = 10$;
Group 18: $\sigma_\eta^2 = 9$;
Group 19: $\sigma_\eta^2 = 8$;
Group 20: $\sigma_\eta^2 = 10$;

2. Identify σ_w^2 and σ_η^2 using identification method presented on slide 55 (Topic_2_Quasi-optimal approximation under uncertainty.pdf). Perform identification for different size of trajectory (3000 and 300). Compare estimation results with true values of σ_w^2 and σ_η^2. Compare the accuracy of estimation.

3. Determine optimal smoothing coefficient in exponential smoothing

$$\alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2} \quad (3)$$

$$\chi = \frac{\sigma_w^2}{\sigma_\eta^2}$$

4. Perform exponential smoothing with the determined smoothing coefficient. Plot results. For the comparison add measurements, true values of process and exponentially smoothed data.

***Here is the recommended procedure for part II*:**
Comparison of methodical errors of exponential and running mean.

1. Generate a true trajectory X_i using the random walk model (1).
   Size of trajectory is 300 points.
   Initial condition $X_1 = 10$.
   Variance of noise w_i, $\sigma_w^2 = 28^2$

2. Generate measurements z_i of the process X_i using equation (2)
   Variance of noise measurement noise η_i, $\sigma_\eta^2 = 97^2$

3. Determine optimal smoothing coefficient α using equation (3).
   (There is no need to identify it again, just use equation for α from part I).

4. The component of full error that is related to measurement errors is determined as (from slide 37, Topic_2_Quasi-optimal approximation under uncertainty.pdf)

   Running mean (RM):
   $$\sigma_{RM}^2 = \frac{\sigma_\eta^2}{M} \quad (4)$$
   Exponential smoothing (ES):
   $$\sigma_{\mathrm{ES}}^2 = \sigma_\eta^2 \frac{\alpha}{2-\alpha} \quad (5)$$

   Determine the window size M (use round values) that provides equality of σ_{RM}^2 and σ_{ES}^2 using determined smoothing constant α

   $$\sigma_{\mathrm{ES}}^2 = \sigma_{\mathrm{RM}}^2$$

5. Apply running mean using determined window size M and exponential mean (see, for instance, page 30, Topic_2_Quasi-optimal approximation under uncertainty.pdf) using determined smoothing constant α to measurements z_i. Plot true trajectory X_i, measurements z_i, running and exponential mean.

6. Make visual comparison of results. Make conclusions which methods give greater methodical error in conditions of equal errors conditioned by measurement errors for this particular generated trajectory.

7. Make conclusions to the Assignment.
   Conclusions should be done in a form of a learning log.
   A learning log is a journal which evidences your **own learning and skills development.** It is not just a diary or record of **"What you have done"** but a record of **what you have learnt, tried and critically reflected upon.**

8. Prepare performance report and submit to Canvas:
   Performance report should include 2 documents:
   1) A report (PDF) with performance of all the items listed above
   2) Code (PDF)

Notes:

- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed

- **A**ll figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.