Assignment 11
Extended Kalman filter for navigation and tracking

Performance – Wednesday, September 30, 2020
Due to submit a performance report –Tuesday, October 6, 2020

The objective of this laboratory work is to develop Extended Kalman filter for tracking a moving object when measurements and motion model are in different coordinate systems. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation for nonlinear models.

This laboratory work is performed in the class by students as in teams of 4 and the team will submit one document reporting about the performance. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

1. ***Here is the recommended procedure:***
   Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$
$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$
$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

   Initial conditions to generate trajectory
   (a) Size of trajectory is $N = 500$ points.

   (b) $T = 1$ – interval between measurements.

   (c) Initial coordinates
   $x_0 = 1000;\ y_0 = 1000$

   (a) Initial components of velocity V
   $V_x = 10; V_y = 10;$

   (b) Variance of noise a_i, $\sigma_a^2 = 0.3^2$ for both a_i^x, a_i^y

2. Generate also true values of range D and azimuth β

$$D_i = \sqrt{x_i^2 + y_i^2}$$
$$\beta_i = arctg\left(\frac{x}{y}\right)$$

3. Generate measurements D^m and β^m of range D and azimuth β

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

   Variances of measurement noises η_i^D, η_i^β are given by

$$\sigma_D^2 = 50^2$$
$$\sigma_\beta^2 = 0.004^2$$

4. Initial conditions for Extended Kalman filter algorithm
   Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} D_i^m(1)sin\beta\ {}_i^m(1) \\ 0 \\ D_i^m(1)cos\beta\ {}_i^m(1) \\ 0 \end{vmatrix}$$

   Initial filtration error covariance matrix $P_{0,0}$
   First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix}$$

5. Create the transition matrix Φ
   Consult charts, page 27

6. Calculate state noise covariance matrix Q

$$Q = G\mathrm{G}^T\sigma_a^2$$

7. Create the measurement noise covariance matrix R

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix}$$

8. At every filtration step in the algorithm you should linearize measurement equation by determining

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

   Consult charts, page 32

9. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration). Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate
   (a) range D
   (b) azimuth β

10. Run Kalman filter algorithm over $M = 500$ runs.
    Calculate true estimation errors of
    (a) Errors of extrapolation and filtration estimates of range D
    (b) Errors of extrapolation and filtration estimates of azimuth β

11. Compare estimation results with measurement errors of D and β.

12. Make conclusions to the Assignment.
    Conclusions should be done in a form of a learning log. **A learning log** is a journal which evidences your **own learning and skills development.** It is not just a diary or record of **"What you have done"** but a record of **what you have learnt, tried and critically reflected upon.**

13. Prepare performance report and submit to Canvas:

Performance report should include 2 documents:

1) A report (PDF) with performance of all the items listed above
2) Code (PDF)

Notes:

- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed
- All figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.