“Experimental Data Processing”

Assignment 7
Tracking in conditions
of correlated state and measurement noise

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Standard Kalman filter
provides optimal estimate

State noise and measurement noise
are uncorrelated and unbiased

In practice these assumptions
are often not true

Analysis and modifications
of Kalman filter

Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration → $a_i = e^{-\lambda T} a_{i-1} + \zeta_i$

ζ_i → Uncorrelated noise with variance $\sigma_\zeta^2 = \sigma_a^2(1 - e^{-2\lambda T})$

λ → Value that is inverse to correlation interval

$\lambda = 1000$ → a_i - uncorrelated noise

$\lambda = 0.1$ → a_i - correlated noise

T → Time interval between measurements

σ_a^2 → Variance of acceleration

Moving object which trajectory is disturbed by correlated random acceleration

Motion model

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

$$a_i = e^{-\lambda T}a_{i-1} + \zeta_i$$

Moving object which trajectory is disturbed by correlated random acceleration

Motion model →

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

$$a_i = e^{-\lambda T} a_{i-1} + \zeta_i$$

$$X_i = \begin{vmatrix} x_i \\ V_i \\ a_i \end{vmatrix}$$ → **State vector** → **Extension of state vector**

Beside estimation of coordinate x_i and velocity V_i, Kalman filter will also estimate the dynamics of correlated acceleration a_i

State space model

State equation → $X_i = \Phi X_{i-1} + G\zeta_i$

Φ → Transition matrix

G → Input matrix

Measurement equation → $z_i = HX_i + \eta_i$

$$X_i = \begin{vmatrix} x_i \\ V_i \\ a_i \end{vmatrix}$$

H → Observation matrix