Assignment 10

Development of a tracking filter of a moving object when measurements and motion models are in different coordinate systems.

By

Group 19

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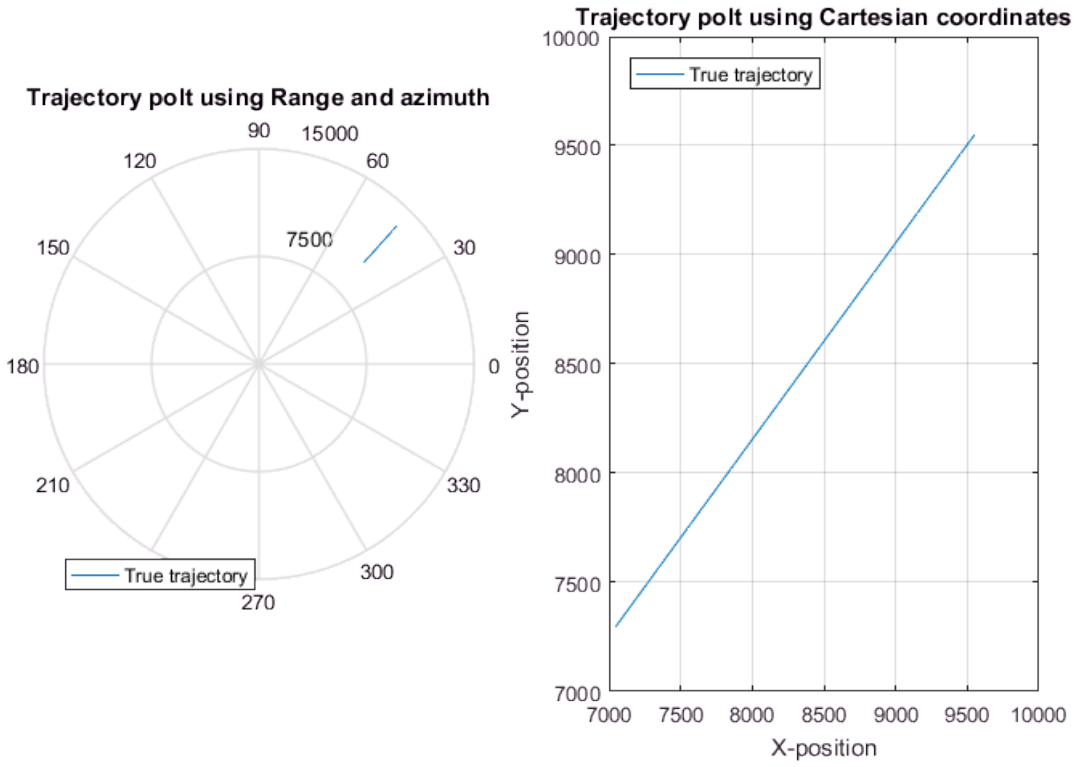
1. Generate a true trajectory *X* of an object that moves uniformly. Trajectory is deterministic, as no random disturbance affect a motion.

2. Generate also true values of range D **and azimuth** β**. Plot generated motion in polar coordinate system.**

```
close all;
clear;

m=1;    %Extrapolation steps
N=26;   %Number of points
NE=N-m; %Number of points for the extrapolation error

X=zeros(1,N);           %X-position
Y=zeros(1,N);           %Y-position
X(1)=13500/sqrt(2);     %X initial value
Y(1)=13500/sqrt(2);     %Y initial value
Vx=-50;     %X velocity
Vy=-45;     %Y velocity
T=2;        %Time step

D=zeros(1,N);             %Range
beta=zeros(1,N);          %Azimuth
D(1)=sqrt(X(1)^2+Y(1)^2); %Range initial value
beta(1)=atan2(X(1),Y(1));  %Azimuth initial value

sigma_d=20;               %Range measurements noise standard deviation
sigma_beta=0.02;          %Azimuth measurements noise standard deviation
%Vectors generation
for i=2:N
    X(i)=X(i-1)+Vx*T;             %X vector generation
    Y(i)=Y(i-1)+Vy*T;             %Y vector generation
    D(i)=sqrt(X(i)^2+Y(i)^2);     %Range vector generation
    beta(i)=atan2(X(i),Y(i));      %Azimuth vector generation
end

%Trajectory plotting for visualization
```

```
figure
subplot(1,2,1)
polar(beta,D)
hold on
title('Trajectory polt using Range and azimuth')
legend({'True trajectory'},'location','southwest')

subplot(1,2,2)
plot(X,Y)
title('Trajectory polt using Cartesian coordinates')
xlabel('X-position')
ylabel('Y-position')
legend({'True trajectory'},'location','northwest')
grid on
set(gcf,'position',[0,0,800,500]);
```



Comment:

In this plot we can see the similaritiy of the trajectory in both coordinates, and since the velocity is with a negative value we know that the trajectory is headed towards the observer in the polar coordinates which would correspond to zero in the cartesian coordinates.

```
%Kalman errors initialization

Errx=zeros(500,N);          %X true estimation error
ErrxE=zeros(500,NE);        %X-extrapolated true estimation error
Erry=zeros(500,N);          %Y true estimation error
ErryE=zeros(500,NE);        %Y-extrapolated true estimation error
ErrD=zeros(500,N);          %D true estimation error
ErrDE=zeros(500,NE);        %D-extrapolated true estimation error
Errbeta=zeros(500,N);       %beta true estimation error
```

```
ErrbetaE=zeros(500,NE);     %beta-extrapolated true estimation error
K500=zeros(500,N);           %kalman filter gain matrix

for M=1:500

    X=zeros(1,N);           %X-position
    Y=zeros(1,N);           %Y-position
    X(1)=13500/sqrt(2);     %X initial value
    Y(1)=13500/sqrt(2);     %Y initial value
    Vx=-50;     %X velocity
    Vy=-45;     %Y velocity
    T=2;        %Time step

    D=zeros(1,N);             %Range
    beta=zeros(1,N);          %Azimuth
    D(1)=sqrt(X(1)^2+Y(1)^2); %Range initial value
    beta(1)=atan2(X(1),Y(1));  %Azimuth initial value
```

3. Generate measurements D^m and β^m of range D and azimuth β.

```
    sigma_d=20;               %Range measurements noise standard deviation
    sigma_beta=0.02;          %Azimuth measurements noise standard deviation

    normaldist=makedist('Normal',0,sigma_d);
    eta_d=random(normaldist,N,1);            %Range measurements noise vector

    normaldist=makedist('Normal',0,sigma_beta);
    eta_b=random(normaldist,N,1);            %Azimuth measurements noise vector
    D_m=D;                                   %Range measurements
    beta_m=beta;                             %Azimuth measurements

    D_m(1)=D(1)+eta_d(1);                    %Range measurements initialization
    beta_m(1)=beta(1)+eta_b(1);              %Azimuth measurements initialization

    %Vectors generation
    for i=2:N
        X(i)=X(i-1)+Vx*T;             %X vector generation
        Y(i)=Y(i-1)+Vy*T;             %Y vector generation
        D(i)=sqrt(X(i)^2+Y(i)^2);     %Range vector generation
        beta(i)=atan2(X(i),Y(i));      %Azimuth vector generation

        D_m(i)=D(i)+eta_d(i);         %Range measurements vector generation
        beta_m(i)=beta(i)+eta_b(i);   %Azimuth measurements vector generation
    end
```

4. Transform polar coordinates D^m and β^m to Cartesian ones and get pseudo-measurements of coordinates x and y.

```
    X_m=D_m.*sin(beta_m);       %X measurements vector generation
    Y_m=D_m.*cos(beta_m);       %Y measurements vector generation
```

5. Create the measurement vector z from pseudo-measurements of coordinates x and y.

```
    Z=[X_m;Y_m];                %Cartesian measurments

    %Kalman filter parameters initialization
    xi=zeros(1,N);
```

```
yi=zeros(1,N);
```

6. Initial conditions for Kalman filter algorithm.

```
Xi=[40000;-20;40000;-20];   %State vector
P=(10^10)*eye(4);           %P matrix
```

7. Create the transition matrix Φ and observation matrix H.

```
%state space matrices
phi=[1 T 0 0;0 1 0 0;0 0 1 T;0 0 0 1];
H=[1 0 0 0;0 0 1 0];
```

8. Create the measurement error covariance matrix R needed for Kalman filter algorithm.

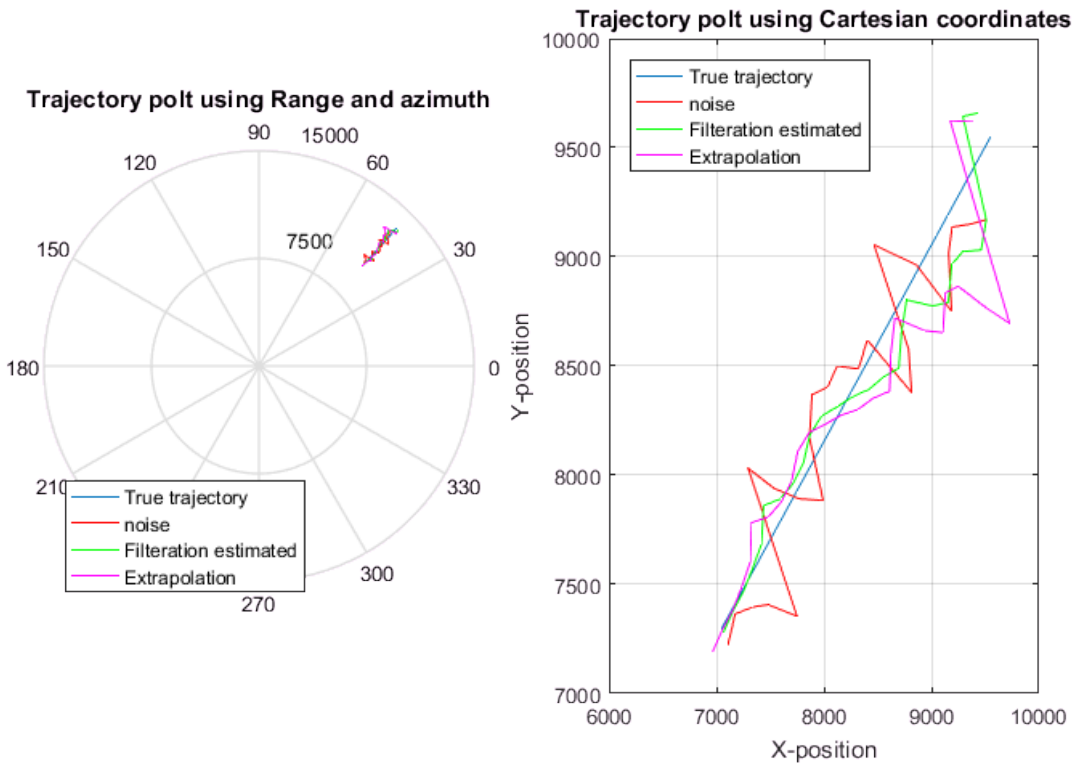
```
%Initial measurement error covariance matrix
R=[(sin(beta_m(1))*sigma_d)^2+(D_m(1)*cos(beta_m(1))*sigma_beta)^2 ...
    sin(beta_m(1))*cos(beta_m(1))*(sigma_d^2-(D_m(1)*sigma_beta)^2); ...
    sin(beta_m(1))*cos(beta_m(1))*(sigma_d^2-(D_m(1)*sigma_beta)^2) ...
    (cos(beta_m(1))*sigma_d)^2+(D_m(1)*sin(beta_m(1))*sigma_beta)^2];
```

9. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration) At every extrapolation and filtration step you will need to calculate range D and Azimuth β from extrapolated and filtered estimates.

```
K=P*H'/(H*P*H'+R);   %Kalman gain initialization

Px=zeros(N,1);       %X Calculation error initialization
Py=zeros(N,1);       %Y Calculation error initialization
xiE=zeros(N,1);      %Extrapolated X
yiE=zeros(N,1);      %Extrapolated Y

Di=zeros(N,1);       %filtered Range
betai=zeros(N,1);    %filtered Azimuth

DiE=zeros(N,1);     %Extrapolated Range
betaiE=zeros(N,1);  %Extrapolated Azimuth

Ki=zeros(1,N);      %array of K(1,1)
CN=zeros(1,N);      %condition number

%Kalman filter
for i=1:N

    Ki(i)=K(1,1);
    R=[(sin(beta_m(i))*sigma_d)^2+(D_m(i)*cos(beta_m(i))*sigma_beta)^2 ...
        sin(beta_m(i))*cos(beta_m(i))*(sigma_d^2-(D_m(i)*sigma_beta)^2); ...
        sin(beta_m(i))*cos(beta_m(i))*(sigma_d^2-(D_m(i)*sigma_beta)^2) ...
        (cos(beta_m(i))*sigma_d)^2+(D_m(i)*sin(beta_m(i))*sigma_beta)^2];
    CN(i)=cond(R);

    Xi=phi*Xi;
    P=phi*P*phi';
    Xi=Xi+K*(Z(:,i)-H*Xi);
    xi(i)=Xi(1);
    yi(i)=Xi(3);
```

```
        K=P*H'/(H*P*H'+R);
        P=(eye(4)-K*H)*P;
        XiE=Xi;

        for mm=m
            XiE=phi*XiE;     %Extrapolated state vector
        end

        %Extrapolated X,Y
        xiE(i)=XiE(1);
        yiE(i)=XiE(3);

        %Range and azimuth estimation (filtered and extrapolated)
        Di(i)=sqrt(Xi(1)^2+Xi(3)^2);
        betai(i)=atan2(Xi(1),Xi(3));
        DiE(i)=sqrt(XiE(1)^2+XiE(3)^2);
        betaiE(i)=atan2(XiE(1),XiE(3));

        %True estimation error calculation
        Errx(M,i)=(Xi(1)-X(i))^2;
        Erry(M,i)=(Xi(3)-Y(i))^2;
        ErrD(M,i)=(Di(i)-D(i))^2;
        Errbeta(M,i)=(betai(i)-beta(i))^2;

        %Covariance matrix error
        Px(i)=sqrt(P(1,1));
        Py(i)=sqrt(P(3,3));

        %Extrapolation error calculation
        if i<N
            ErrxE(M,i)=(XiE(1)-X(i+m))^2;
            ErryE(M,i)=(XiE(3)-Y(i+m))^2;
            ErrDE(M,i)=(DiE(i)-D(i+m))^2;
            ErrbetaE(M,i)=(betaiE(i)-beta(i+m))^2;
        end
    end
    K500(M,:)=Ki;
end

%Trajectory plotting for visualization
figure
subplot(1,2,1)
polar(beta,D)
hold on
polar(beta_m,D_m,'r')
polar(betai,Di,'g')
polar(betaiE,DiE,'m')
title('Trajectory polt using Range and azimuth')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','southwe

subplot(1,2,2)
plot(X,Y)
hold on
plot(X_m,Y_m,'r')
plot(xi,yi,'g')
plot(xiE,yiE,'m')
title('Trajectory polt using Cartesian coordinates')
xlabel('X-position')
ylabel('Y-position')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','northwe
grid on
```

```
set(gcf,'position',[0,0,800,500]);
```



Comment:

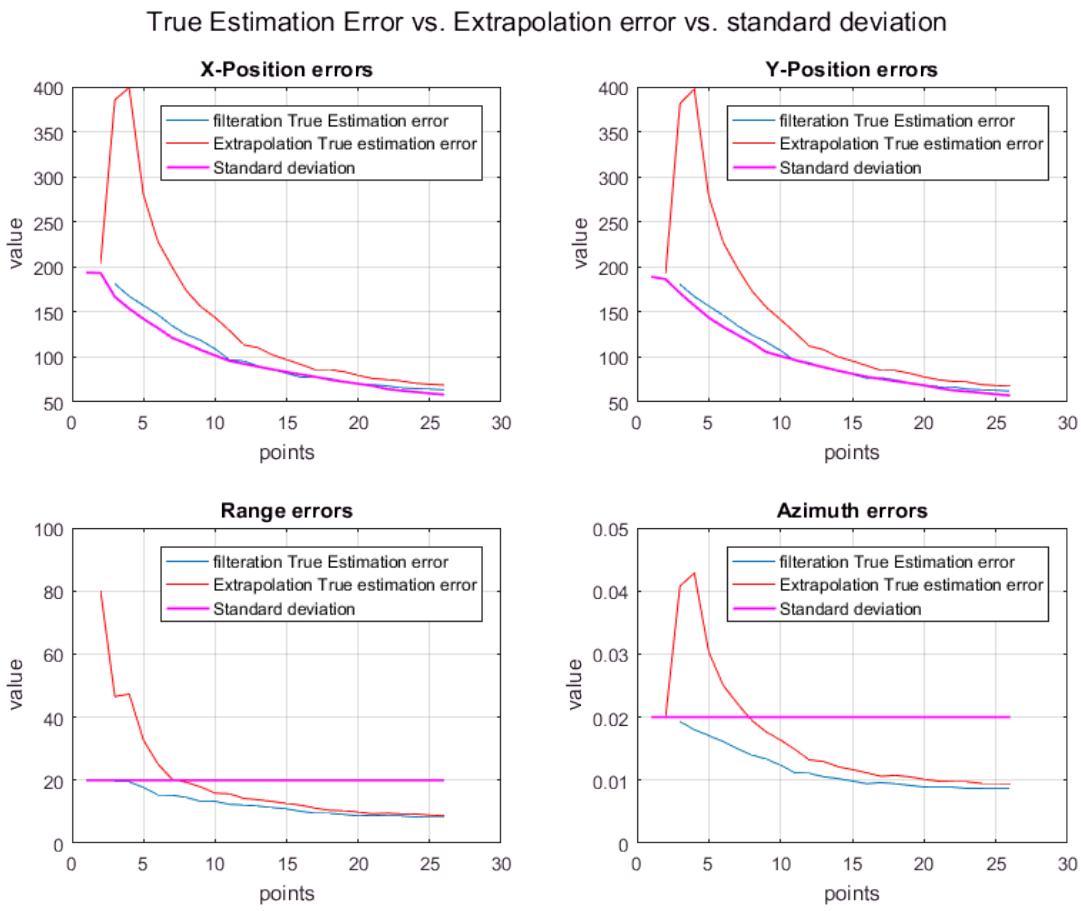
In this figure above we can see the true trajectory, the measurements, the filtered and the extrapolated states for the X,Y in cartesian coordinates, and range and azimuth in polar coordinates. Both the estimation and extrapolation are converging to the true trajectory the closer they approach the observer and we can predict that the error is decreasing.

10. Run Kalman filter algorithm over M=500 runs. Calculate true errors of estimation:

a) Errors of extrapolation and filtration estimates of range D relative to σ_D

b) Errors of extrapolation and filtration estimates of Azimuth β **relative to** σ_β.

```
figure
%Error plotting
subplot(2,2,1)
plotErr(Errx,ErrxE,Px,'X-Position')
subplot(2,2,2)
plotErr(Erry,ErryE,Py,'Y-Position')
subplot(2,2,3)
plotErr(ErrD,ErrDE,sigma_d*ones(1,N),'Range')
subplot(2,2,4)
plotErr(Errbeta,ErrbetaE,sigma_beta*ones(1,N),'Azimuth')
suptitle('True Estimation Error vs. Extrapolation error vs. standard deviation')
```

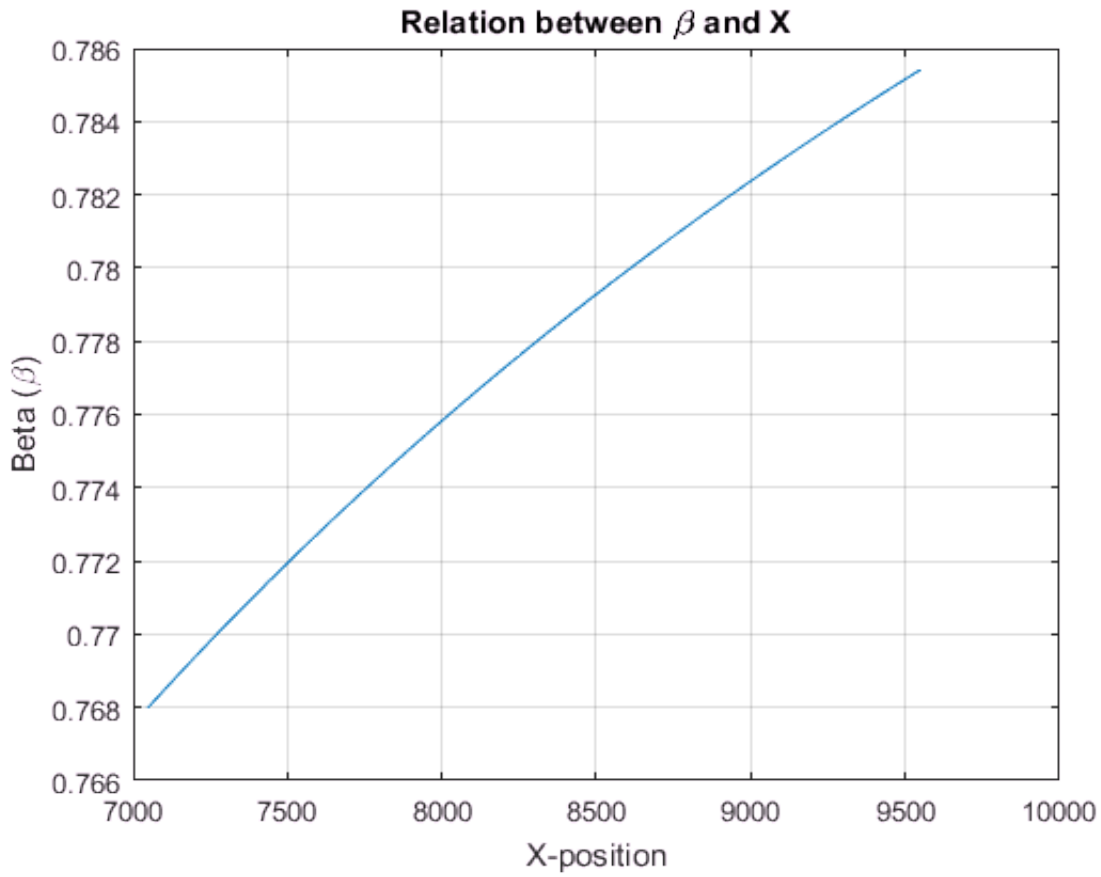


Comment:

The figures above represent the errors in coordinates X,Y and in Range D and azimuth. we can see that all errors are decreasing and converging to a value. for the coordinates X,Y the true estimation error and the calculation error from Kalman are having the same trend and almost the same values. and similarly for the extrapolation at the end of the measurements as it settles for a value close to them but a bit higher. For the range and azimuth, the Azimuth error is having the same tredn of the coordinates error and is close to σ_β and settling for and error value almost 0.1 which is less than the error standard deviation which indicates a good estimation accuracy, and the same for the range except that the extrapolation error has different trend than the coordinates and is starting high then settling with no peak.

11. Analyze dependence of coordinate X on azimuth β.

```
figure
plot(X,beta)
title('Relation between \beta and X')
grid on
xlabel('X-position')
ylabel('Beta (\beta)')
```

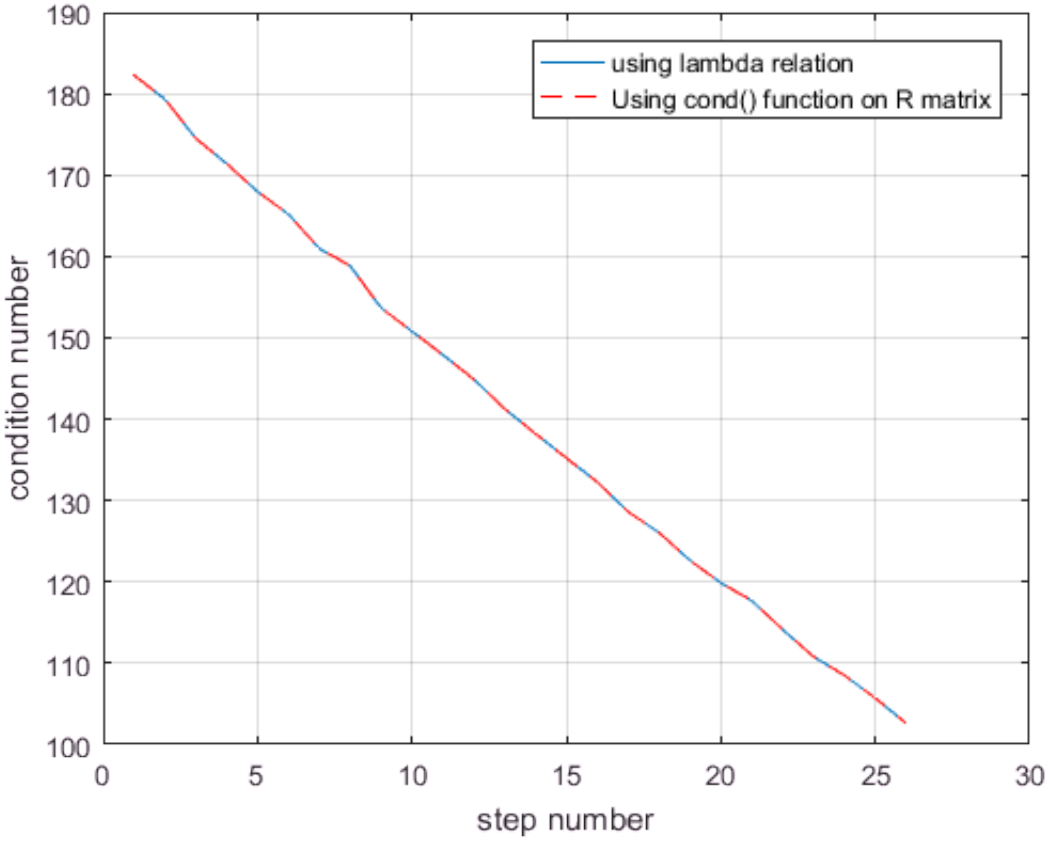


Comment:

Plotting the X versus the azimuth which should be a nonlinear relation according to its equation, but we can visually say that the relation is almost linear withim our range, which mean that this dependence's linearization errors are insignificant. and we can rely on our results.

12. Calculate condition number of covariance matrix R over the observation interval.

```
%condition number
ConditionNo=((D_m.^2).*sigma_beta^2)./sigma_d^2;
if min(ConditionNo)<1
    for j=1:N
        if ConditionNo(j)<1
            ConditionNo(j)=1./ConditionNo(j);
        end
    end
end

figure
plot(ConditionNo)
grid on
hold on
plot(CN,'--r')
title('Conditon number')
legend('using lambda relation','Using cond() function on R matrix')
xlabel('step number')
ylabel('condition number')
grid on
```



Comment:

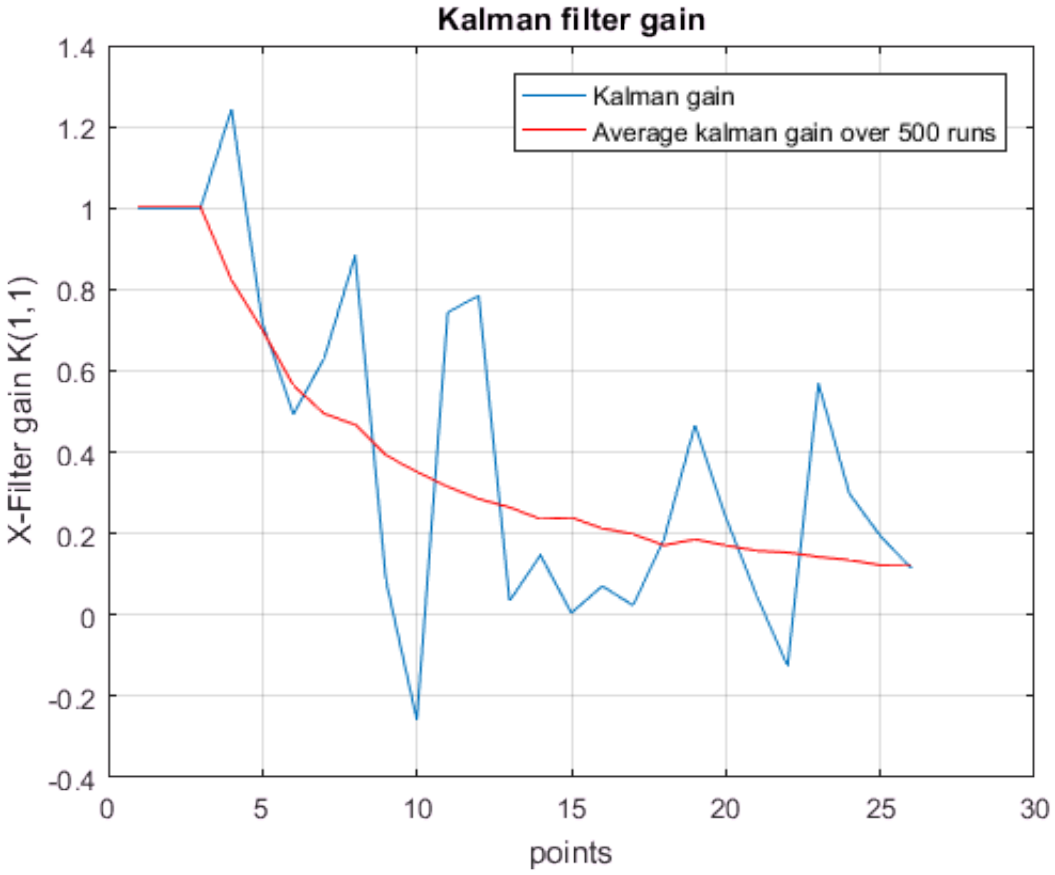
Plotting the condition number over the observation interval we can see that the number is decreasing over time which means that the estimation accuracy is increasing and indicates that our filter estimates would converge eventually; Also we can see, that it's relativly modereate not too big (Conditoin number=1000) or too perfect (Condition number=1), which means that our matrix is well conditioned and we don't have to use the full formulation for error covariance matrix and that our estimation is consistent.

We used too formulations to make verify that our calculations are accurate, we calculated the Condition number directly from the R matrix using MATLAB's cond() function, and the formulation from the slides using the eigen values; and the end result was the same that you can see from the graph that they both are identical and plotted above each other.

13. Analyze filter gain K.

```
Ki2=zeros(1,N);  %Average Kalman gain over 500 runs
for j=1:N;
    Ki2(j)=(1/(M-1))*sum(K500(:,j));
end
figure
plot(Ki)
hold on
plot(Ki2,'r')
grid on
title('Kalman filter gain')
xlabel('points')
```

```
ylabel('X-Filter gain K(1,1)')
legend('Kalman gain' , 'Average kalman gain over 500 runs')
```



Comment:

Ananlyzing the filter gain from the individual run, We can see that the filter gain is not settling and disturbed and have unusal behaviour that it's sometimes exceeding 1 in top and 0 in bottom, and that can be accounted for the R matrix's condition number that it's between 200 and 100, that the gain can't settle for a value but oscillates around almost 0.2. However, the disturbances are decreasing as we approach the observer and that's because the condition number is decreasing too. That behaviour can be seen clearly at the averaged filter gain with the decrease and the settling of the value around 0.2.

14. Run filter again over $M = 500$ runs but use other initial conditions to generate a trajectory

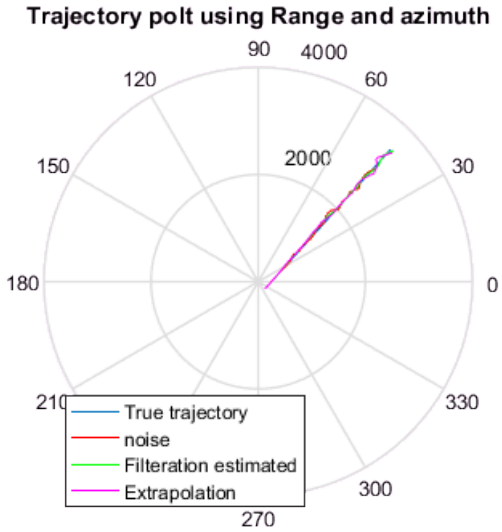
$$x_0 = \frac{3500}{\sqrt{2}}, y_0 = \frac{3500}{\sqrt{2}}$$

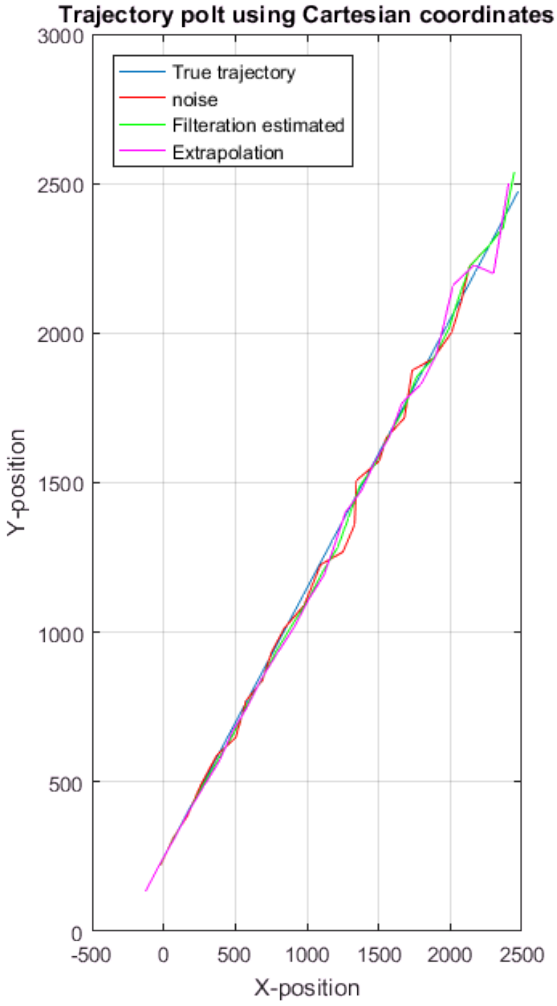
```
clear;
N=26;
m=1;
M=500;
x1=3500/sqrt(2);
y1=3500/sqrt(2);
sigma_d=20;
sigma_beta=0.02;
[ Errx,ErrxE,Erry,ErryE,ErrD,ErrDE,Errbeta,ErrbetaE,Px,Py,...
```

```
    X,Y,xi,yi,xiE,yiE,X_m,Y_m ...
    ,D,beta,D_m,beta_m,Di,betai,DiE,betaiE,Ki,K500,CN] = KalmaPolarMeasurements( x1,y1,N,m,M,s
```

14.1 ploting the trajectory for visualization.

```
%Trajectory plotting for visualization
figure
subplot(1,2,1)

polar(beta,D)
hold on
polar(beta_m,D_m,'r')
polar(betai,Di,'g')
polar(betaiE,DiE,'m')
title('Trajectory polt using Range and azimuth')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','southwe

subplot(1,2,2)
plot(X,Y)
hold on
plot(X_m,Y_m,'r')
plot(xi,yi,'g')
plot(xiE,yiE,'m')
title('Trajectory polt using Cartesian coordinates')
xlabel('X-position')
ylabel('Y-position')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','northwe
grid on
set(gcf,'position',[0,0,800,800]);
```



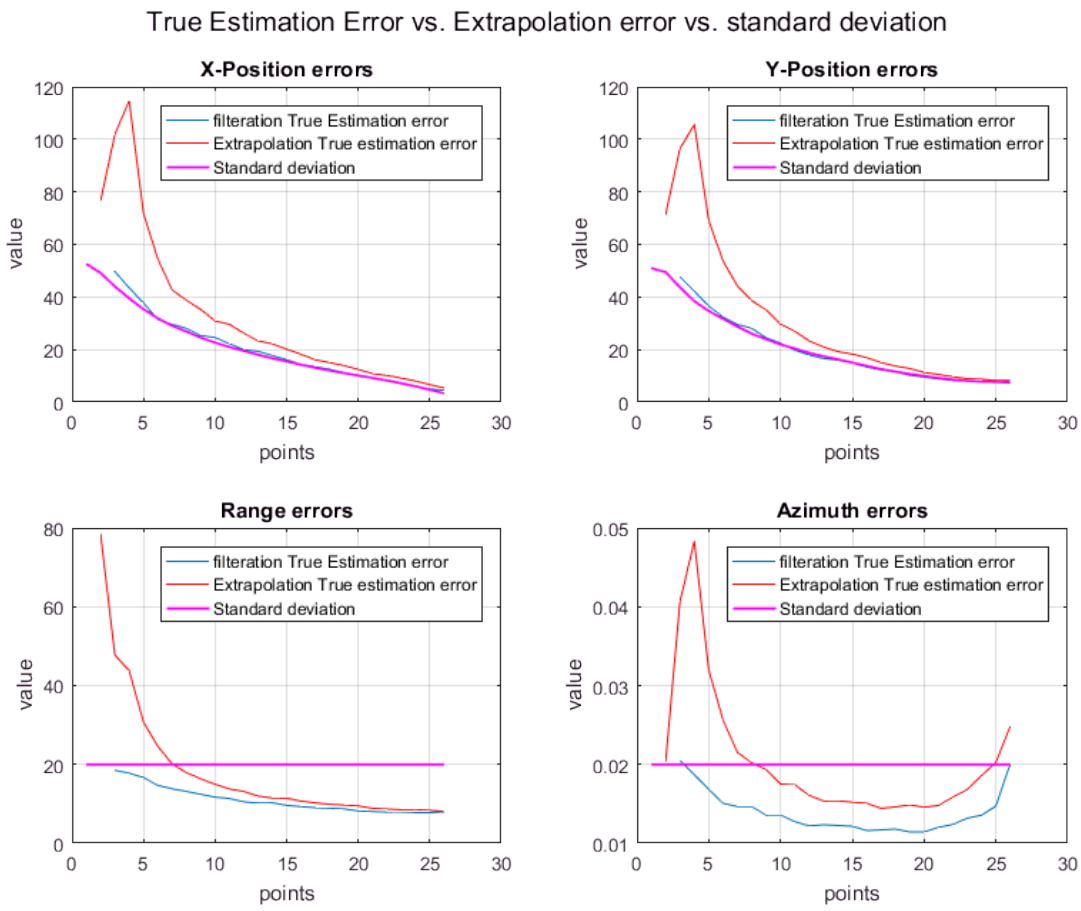


Comment:

In this figure above we can see the true trajectory, the measurements, the filtered and the extrapolated states for the X,Y in cartesian coordinates, and range and azimuth in polar coordinates. we can see faster convergence by the estimation and the extrapolation to the true trajectory compared to the last case with the object far away from the observer. by the end of observation interval the filter could predict the motion to a very good extant.

15. Calculate true errors of estimation

```
figure
%Error plotting
subplot(2,2,1)
plotErr(Errx,ErrxE,Px,'X-Position')
subplot(2,2,2)
plotErr(Erry,ErryE,Py,'Y-Position')
subplot(2,2,3)
plotErr(ErrD,ErrDE,sigma_d*ones(1,N),'Range')
subplot(2,2,4)
plotErr(Errbeta,ErrbetaE,sigma_beta*ones(1,N),'Azimuth')
suptitle('True Estimation Error vs. Extrapolation error vs. standard deviation')
```



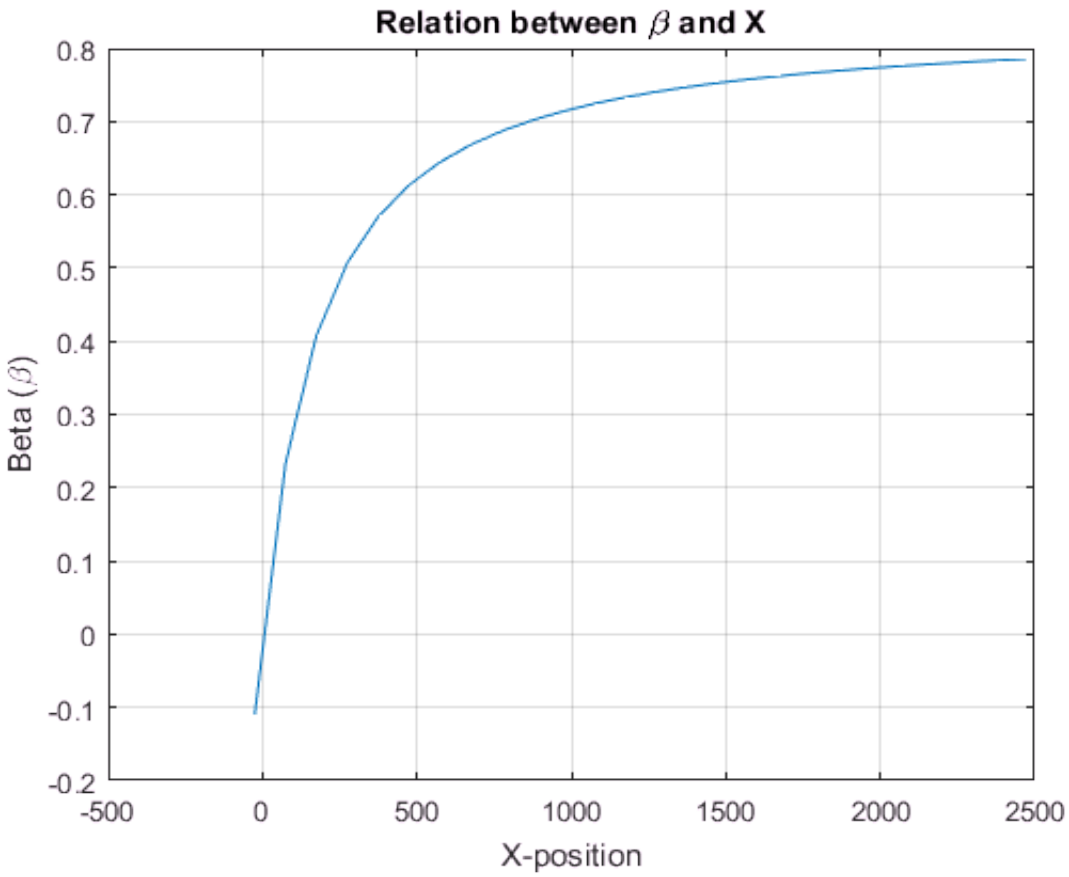
Comment:

The figures above represent the errors in coordinates X,Y and in Range D and azimuth. First we notice that the magnitude of the mean error decreases that the previous case (Far object) in the case of the X,Y positions as the object gets closer to the observer the estimation of the range enhances and we get less error. secon the error of the Azimuth estimation starts to rise up as the object gets closer to the observer (for a far away object the change in the Azimuth with change of the position is not great and the linear assumption works well, however the closer the object the high affect the nonlinear term will have on the Azimuth estimations). We can expect from this a similar behavior on the condition number also the dependance between X position and Azimuth β is expected to show some nonlinearity.

The extrapolation error as the previous time starts to converge to produce good results compared to estimation error and calculated error with steady state values a little bit higher than the estimation error.

16. Analyze dependence of coordinate x on azimuth β.

```
figure
plot(X,beta)
title('Relation between \beta and X')
grid on
xlabel('X-position')
ylabel('Beta (\beta)')
```



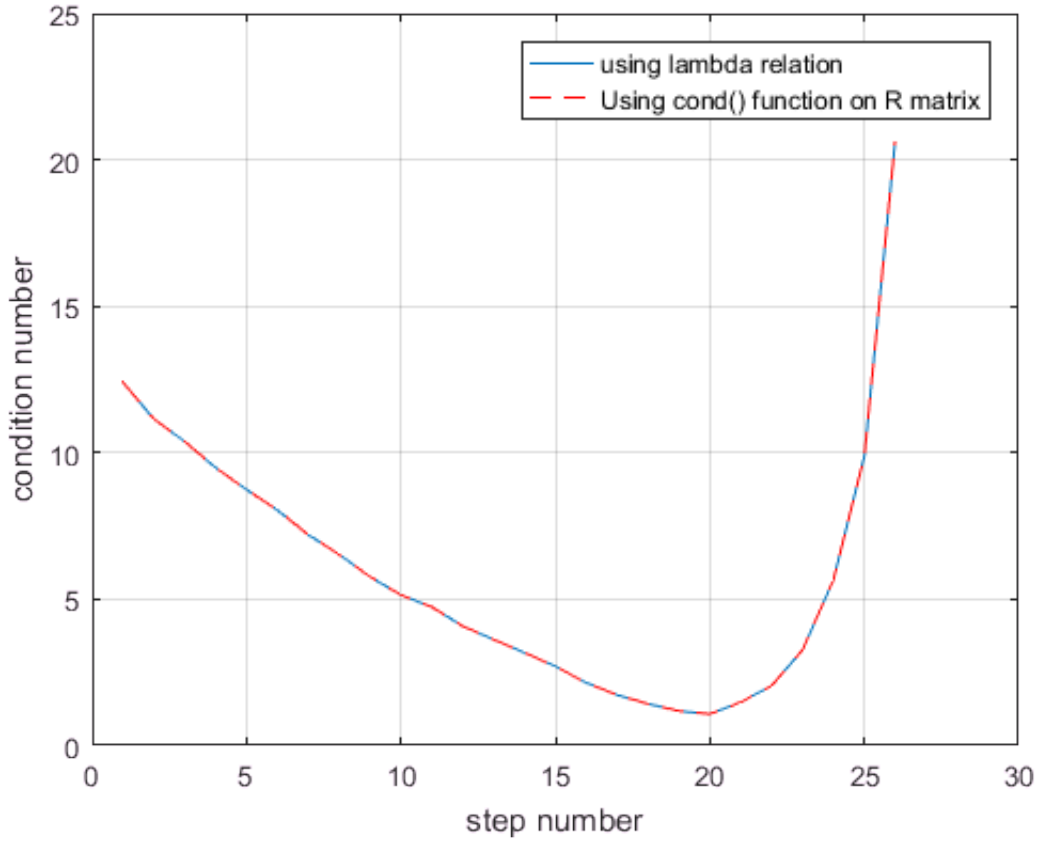
Comment:

As we can see here the relation between the X-position and the Azimuth of the object is getting nonlinear as the object approaches the observer that can lead to ill conditioning of the measurement covariance matrix eventually causing the filter to diverge as the object gets close. the linear assumption is not safe to assume in these situations. Instead we can use nonlinear filters for the cases of close objects to observers.

17. Calculate condition number of covariance matrix R over the observation interval for these conditions. Does condition number decrease or increase over time?

```
%condition number
ConditionNo=((D_m.^2).*sigma_beta^2)./sigma_d^2;
if min(ConditionNo)<1
    for j=1:N
        if ConditionNo(j)<1
            ConditionNo(j)=1./ConditionNo(j);
        end
    end
end

figure
plot(ConditionNo)
grid on
hold on
plot(CN,'--r')
title('Conditon number')
legend('using lambda relation','Using cond() function on R matrix')
xlabel('step number')
ylabel('condition number')
```

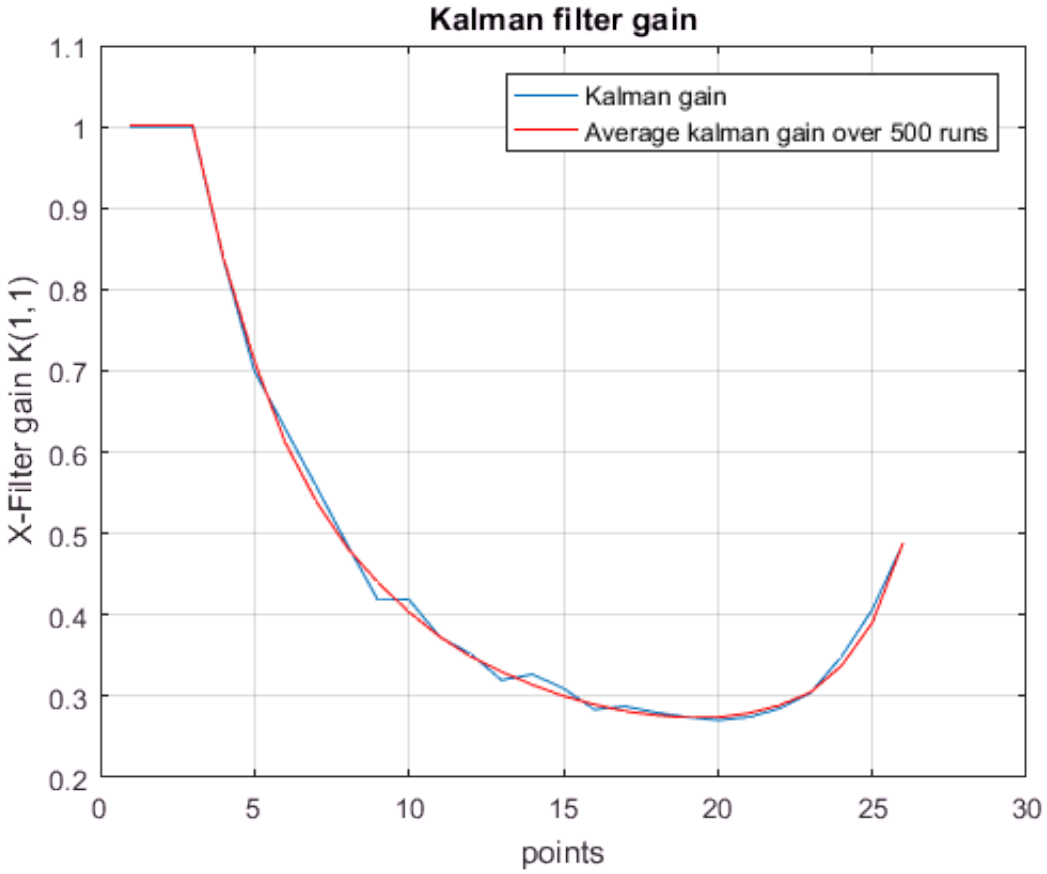
```
grid on
```



Comment:

As we can see here, the condition number of the R matrix is plotted over the observation interval and it can be inferred that with objects moving close to the observer to a certain limit the condition number was getting lower indicating a well conditioned R matrix. but after a certain point it starts to diverge very quickly (in approximately 6 time steps the number has its highest value on the curve). This indicated the deterioration in the linear approximation as the object passed a certain position relative to the observer. has the observation interval got any longer our estimation is the condition number will get very high rapidly causing the filter to diverge.

```
Ki2=zeros(1,N);  %Average Kalman gain over 500 runs
for j=1:N;
    Ki2(j)=(1/(M-1))*sum(K500(:,j));
end
figure
plot(Ki)
hold on
plot(Ki2,'r')
grid on
title('Kalman filter gain')
xlabel('points')
ylabel('X-Filter gain K(1,1)')
legend('Kalman gain' , 'Average kalman gain over 500 runs')
```



Comment:

We se her in the left part of the graph the kalman gain is getting smaller (a typical behaviour as per all previous assignments), and again after the object passes a certain position relative to the observer the gain starting to pick up again indicating a potential divergence as we discussed on the condition number.

19. Run filter again over $M = 500$ runs but use other initial conditions to generate a trajectory ($\sigma_D = 50, \sigma_\beta = 0.0015$)

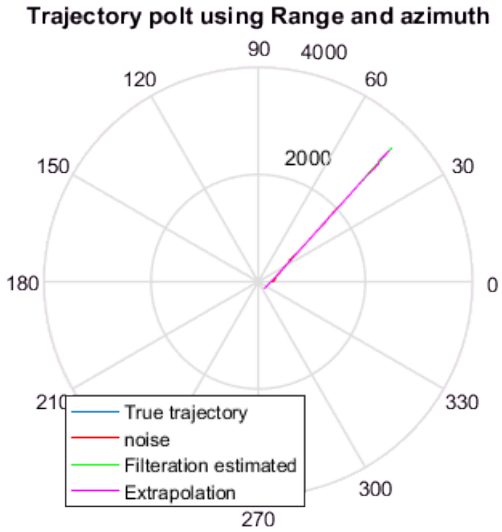
$$x_0 = \frac{3500}{\sqrt{2}}, y_0 = \frac{3500}{\sqrt{2}}$$

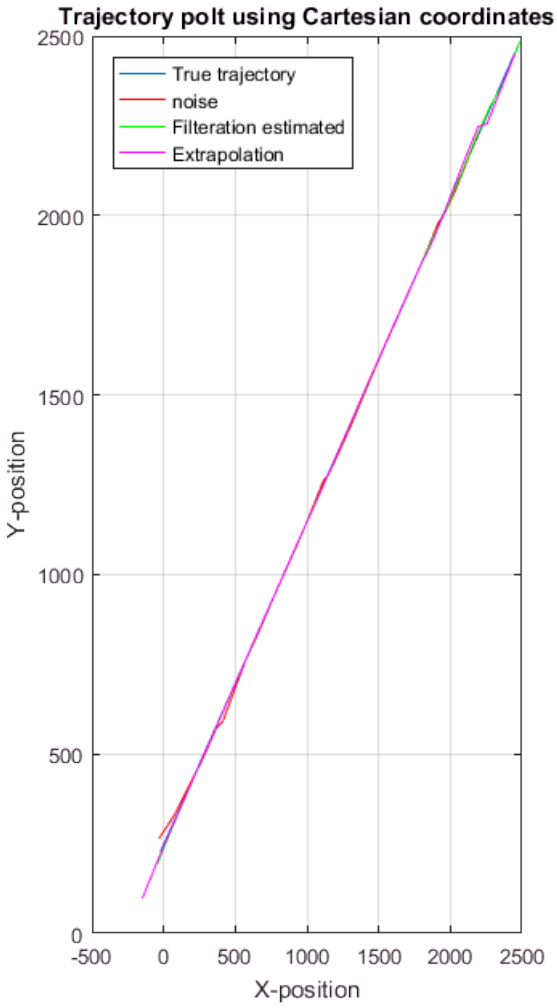
```
clear;
N=26;
m=1;
M=500;
x1=3500/sqrt(2);
y1=3500/sqrt(2);
sigma_d=50;
sigma_beta=0.0015;
[ Errx,ErrxE,Erry,ErryE,ErrD,ErrDE,Errbeta,ErrbetaE,Px,Py,...
    X,Y,xi,yi,xiE,yiE,X_m,Y_m ...
    ,D,beta,D_m,beta_m,Di,betai,DiE,betaiE,Ki,K500,CN] = KalmaPolarMeasurements( x1,y1,N,m,M,s
```

20. Repeat items 14,15,16,17 for these conditions and reply to questions addressed.

20.(14.1) ploting the trajectory for visualization.

```
%Trajectory plotting for visualization
figure
subplot(1,2,1)

polar(beta,D)
hold on
polar(beta_m,D_m,'r')
polar(betai,Di,'g')
polar(betaiE,DiE,'m')
title('Trajectory polt using Range and azimuth')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','southwe

subplot(1,2,2)
plot(X,Y)
hold on
plot(X_m,Y_m,'r')
plot(xi,yi,'g')
plot(xiE,yiE,'m')
title('Trajectory polt using Cartesian coordinates')
xlabel('X-position')
ylabel('Y-position')
legend({'True trajectory','noise','Filteration estimated','Extrapolation'},'location','northwe
grid on
set(gcf,'position',[0,0,800,800]);
```



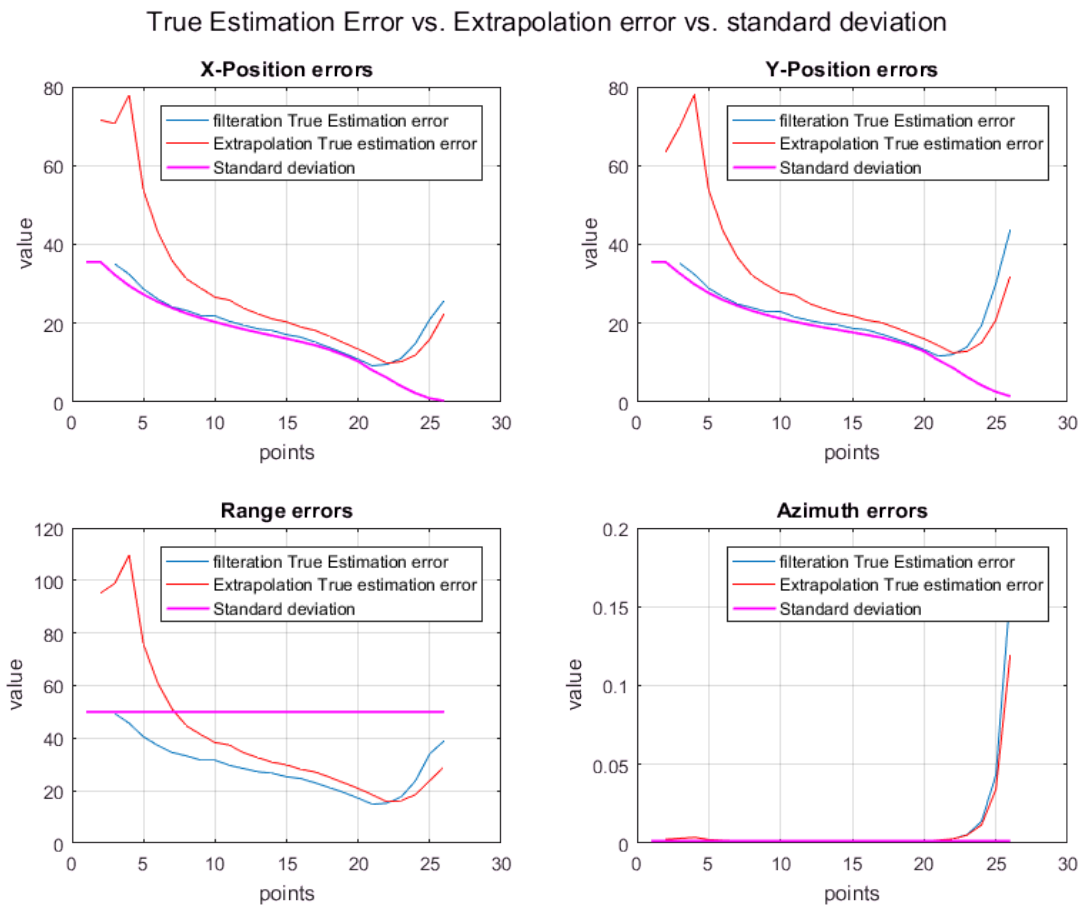


Comment:

In the above graph are the true trajectory, the measurements, the Kalman filter estimation and extrapolation graphed against each other, it can be seen that the estimation values are kind of following the trajectory with some disturbances. but we can see at the end of the trajectory the estimates started to part from the true trajectory after converging to it which we would analyze later in terms of errors as it was starting to diverge.

20.(15) Calculate true errors of estimation

```
figure
%Error plotting
subplot(2,2,1)
plotErr(Errx,ErrxE,Px,'X-Position')
subplot(2,2,2)
plotErr(Erry,ErryE,Py,'Y-Position')
subplot(2,2,3)
plotErr(ErrD,ErrDE,sigma_d*ones(1,N),'Range')
subplot(2,2,4)
plotErr(Errbeta,ErrbetaE,sigma_beta*ones(1,N),'Azimuth')
suptitle('True Estimation Error vs. Extrapolation error vs. standard deviation')
```

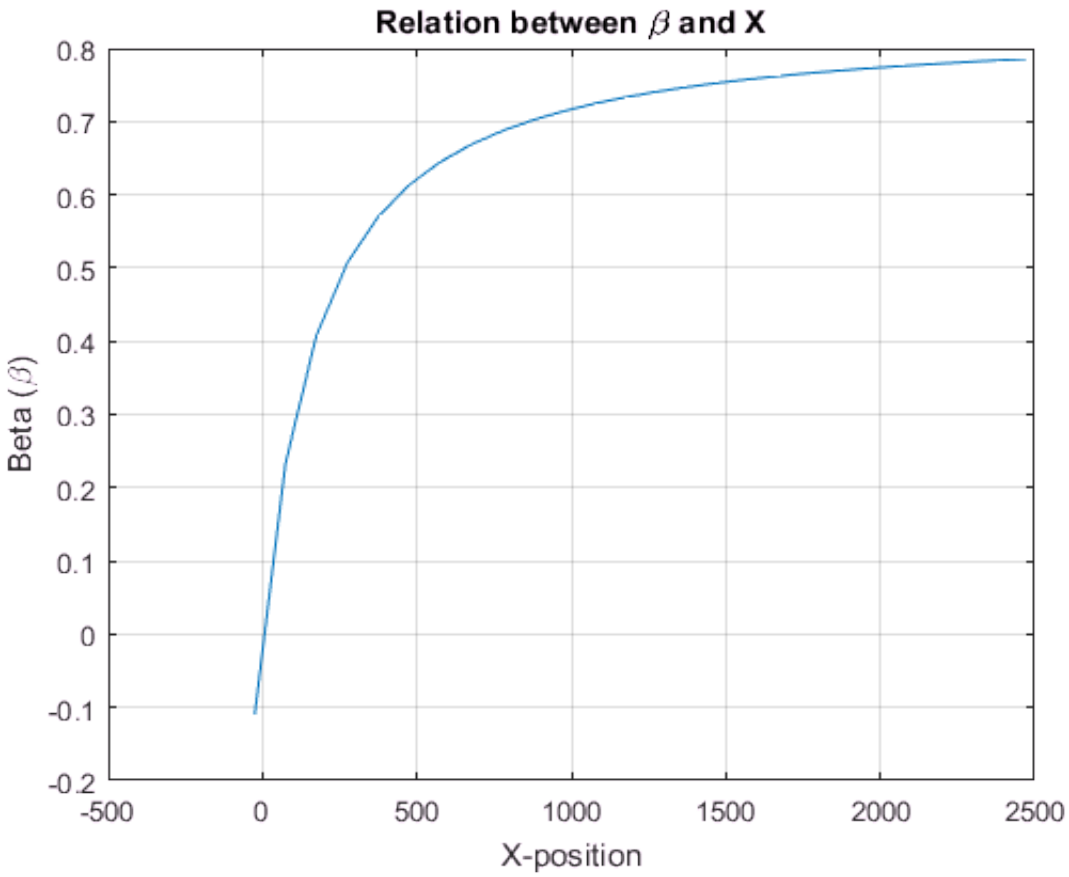


Comment:

In the first two graphs, the true estimation error is plotted against the extrapolation and calculated error for x and y trajectory, as shown both the true estimation error and calculated error have the same behavior at the beginning and settled together but true error diverged at the end. the extrapolation had the same behaviour also but with a bit higher value. in the while with the range, comparing it with the x and y error behaviour we can see that it has the same behaviour. But what we obtained from the azimuth was quite different from previous results, the Azimuth in these conditions diverged, and that can defenitly be accounted for the ill conditioned R-matrix because of the non-linearity of the relation between x and beta and beacause of the high noise in the Range as σ_D was equal to 50 and the very low value for σ_β.

20.(16) Analyze dependence of coordinate x on azimuth β.

```
figure
plot(X,beta)
title('Relation between \beta and X')
grid on
xlabel('X-position')
ylabel('Beta (\beta)')
```



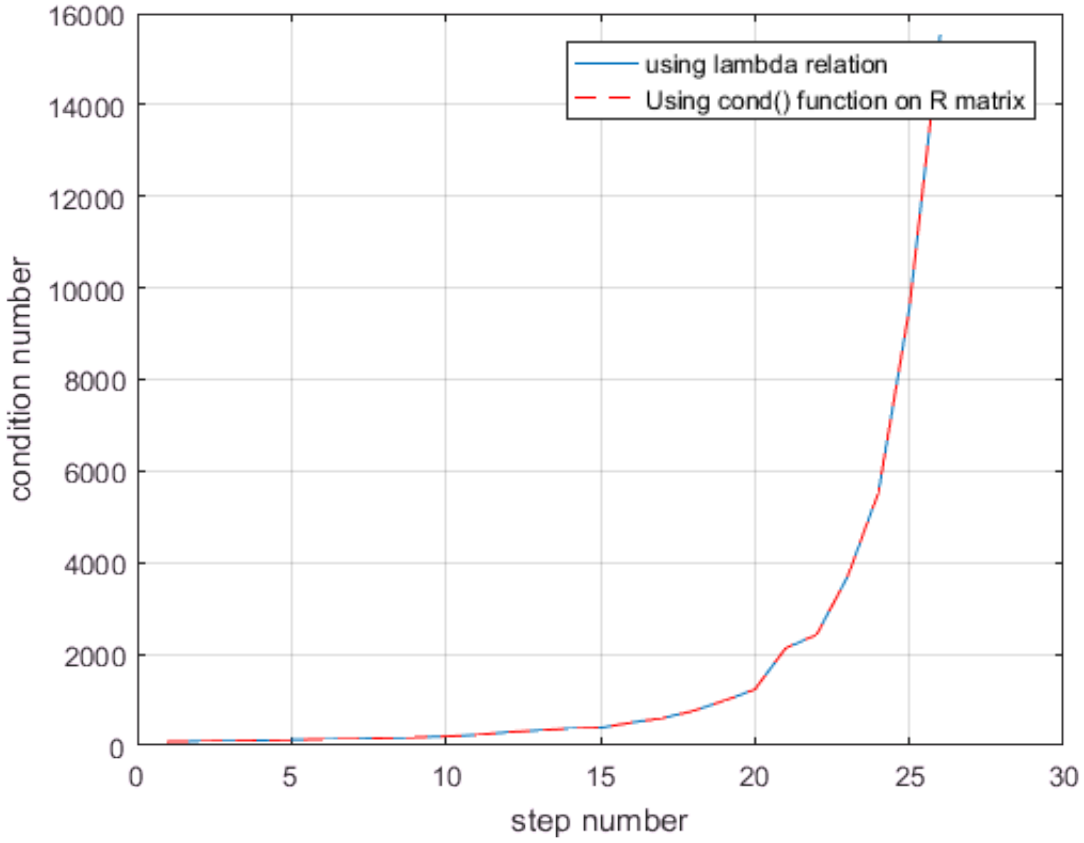
Comment:

In the above graph, the X values are plotted against the Azimuth and according to our linearization equation we can observe the nonlinear relation within our range and this indicates the dependence's linearization errors is significant and this due to the small range which enlarge the effect of variance on Azimuth. So we cannot trust totally our linearization also our results.

20.(17) Calculate condition number of covariance matrix R over the observation interval for these conditions. Does condition number decrease or increase over time?

```
%condition number
ConditionNo=((D_m.^2).*sigma_beta^2)./sigma_d^2;
if min(ConditionNo)<1
    for j=1:N
        if ConditionNo(j)<1
            ConditionNo(j)=1./ConditionNo(j);
        end
    end
end

figure
plot(ConditionNo)
grid on
hold on
plot(CN,'--r')
title('Conditon number')
legend('using lambda relation','Using cond() function on R matrix')
xlabel('step number')
```

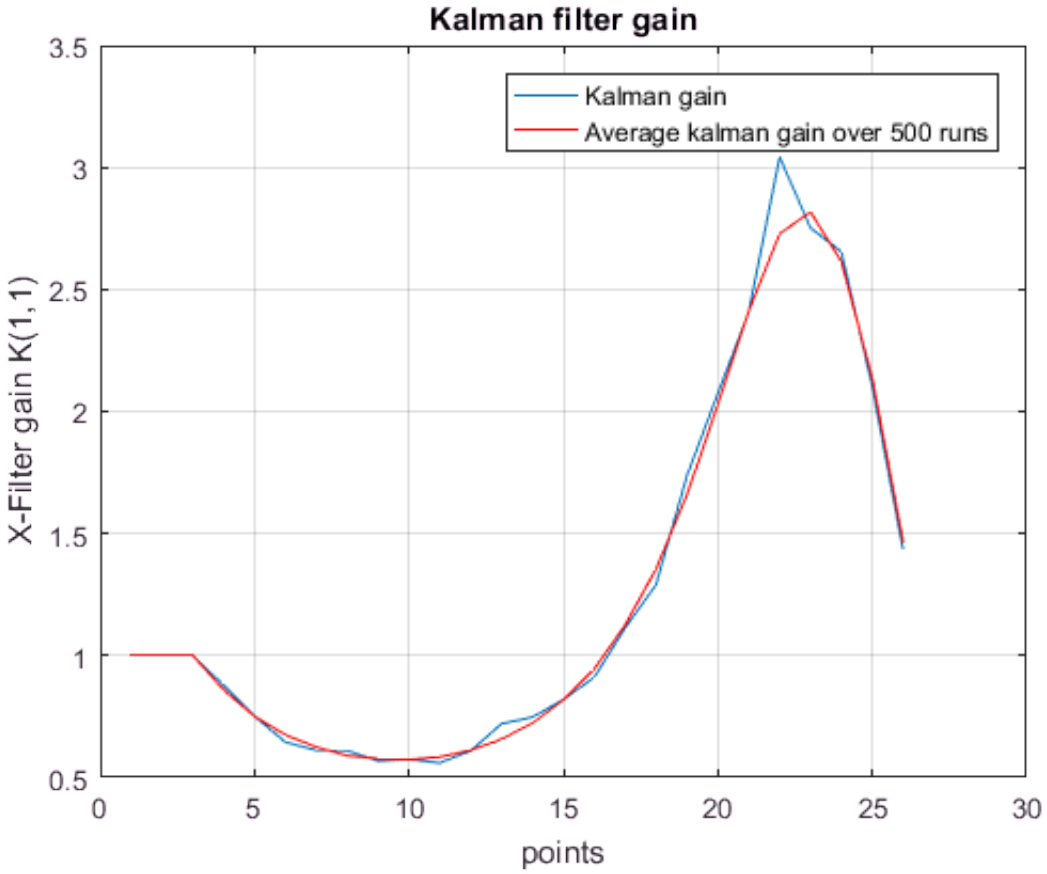
```
ylabel('condition number')
grid on
```



Comment:

Plotting the condition number over the observation interval we can see that the number is increasing over time which means that the estimation accuracy is decreasing and indicates that our filter estimates would diverge eventually,which means that our matrix is ill conditioned.

```
Ki2=zeros(1,N);  %Average Kalman gain over 500 runs
for j=1:N;
    Ki2(j)=(1/(M-1))*sum(K500(:,j));
end
figure
plot(Ki)
hold on
plot(Ki2,'r')
grid on
title('Kalman filter gain')
xlabel('points')
ylabel('X-Filter gain K(1,1)')
legend('Kalman gain' , 'Average kalman gain over 500 runs')
```



Comment:

We tried to explain the behaviour for Kalman filter within our scope of study but yet were stuttered. The filter gain after divergence point due to ill conditions, exceeded the 1 value and continued to do so and that would result in a bad estimation which already happened in our case.

Learning log:

General conclusion:

The further the object is from the observer the better the linearization, because as we see in the last two cases the dependence of beta and X was nonlinear because we were closer, which led to larger estimation errors, than in a first case, when the dependence closer to linear. and what made matters worse was the ill conditioned R matrix because of the values of the sigmas which indicate our measurements accuarcy from the observer (station). and one way to fix it is to change the station's observation parameters to have better compatible variances that would produce better R matrix, of course if we lower the σ_D value would give a more accurate results, but also increasing the σ_β value would give a well conditioned matrix but less accuarate measurements relative to the first suggestion.

What we learned:

in this assignment we develop a tracking filter of a moving object when measurements and motion model are in different coordinate systems. and it had shown us how kalman filter would work in practice assuming these measurements were of a radio station measuring an object moving toward it in different

cases for the object and for the accuracy of the tools in the station, and how when the measurments in the third case resulted in an ill conditioned R matrix, the estimations were diverging and that might cause collisions if we track multiple objects as mentioned in the assignment statement. Eventually we summarizied what we learnt in the following:

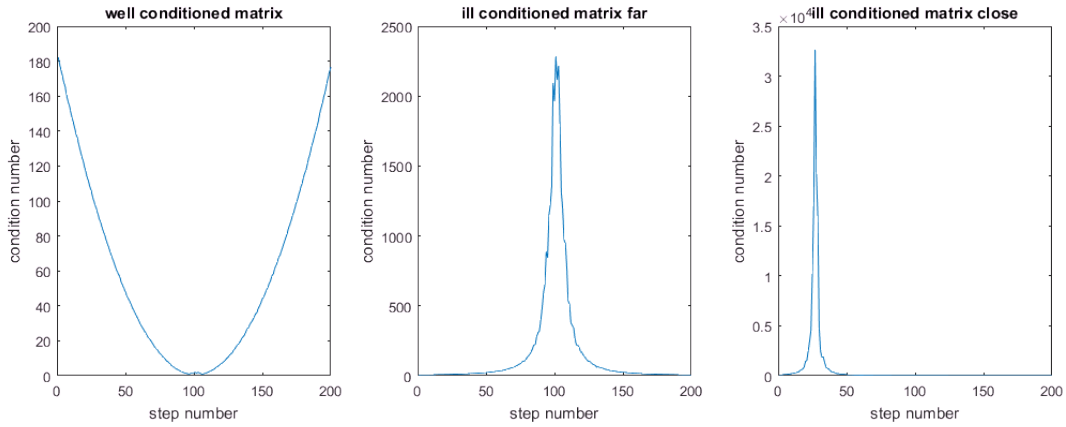
1- As we are using optimal kalman filter which is based on linear systems, the system has limitations when our linearization is bad, which was observed in our cases specially the last case.

2- The condition number for the covariance matrix behaviour is depending on the accuarcy of our measurements that they must be compatible in accuracy that if we have one of the devices producing perfect measurements and the other one isn't the accuracy of our estimation model drops significantly and the more accurate measurements dominate the system and causes the system to diverge following one measurement and not the other one.

3- We wanted to plot the condition number for an object approaching from far to close and see how this behaviour would look like, and the result was:

```
clear;
N=200;
m=1;
M=1;
x1=13500/sqrt(2);
y1=13500/sqrt(2);
sigma_d=20;
sigma_beta=0.02;
[ ~,~,~,~,~,~,~,~,~,~,...
    ~,~,~,~,~,~,~,~...
    ,~,~,~,~,~,~,~,~,~,~,~,CN] = KalmaPolarMeasurements( x1,y1,N,m,M,sigma_beta,sigma_d );

figure
subplot(1,3,1)
grid on
plot(CN)
title('well conditioned matrix')
xlabel('step number')
ylabel('condition number')

x1=13500/sqrt(2);
y1=13500/sqrt(2);
sigma_d=50;
sigma_beta=0.0015;
[ ~,~,~,~,~,~,~,~,~,~,...
    ~,~,~,~,~,~,~,~...
    ,~,~,~,~,~,~,~,~,~,~,~,CN] = KalmaPolarMeasurements( x1,y1,N,m,M,sigma_beta,sigma_d );

subplot(1,3,2)
grid on
plot(CN)
title('ill conditioned matrix far')
xlabel('step number')
ylabel('condition number')

x1=3500/sqrt(2);
y1=3500/sqrt(2);
sigma_d=50;
sigma_beta=0.0015;
[ ~,~,~,~,~,~,~,~,~,~,...
    ~,~,~,~,~,~,~,~...
```

```
    ,~,~,~,~,~,~,~,~,~,~,CN] = KalmaPolarMeasurements( x1,y1,N,m,M,sigma_beta,sigma_d );

subplot(1,3,3)
grid on
plot(CN)
title('ill conditioned matrix close')
xlabel('step number')
ylabel('condition number')
set(gcf,'position',[0,0,1200,400])
```



And the result we obtained above shows us that if the variances were compatiple in the first part the condition number is within the well conditioned region even when it came close to the target, while the seconed one shows us how starting far the matrix with well conditioned and when we approached the matrix was ill-conditioned but yet in a better state than when we started close to the observation station, that the kalman gain wouldn't diverge alot because it was already settled and then started to diverge then went back to being normal. while starting close the gain would be diverging from the begining and would take time to settle for a good value.