## Assignment 12
## Joint assimilation of navigation data coming from different sources

Performance – Monday October 5, 2020
Due to submit a performance report – Friday October 9, 2020

      The objective of this laboratory work is to develop a navigation filter by assimilating data coming from different sources. Important outcome of this exercise is getting skill to incorporate all available measurement information into assimilation algorithm and develop a tracking filter for nonlinear models.

This laboratory work is performed in the class by students as in teams of 3-4 on October 18, 2019 and the team will submit one document reporting about the performance till October 21, 2019. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

1. ***Here is the recommended procedure:***
   Generate a true trajectory $X_i$ of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$
$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$
$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

   ***Initial conditions to generate trajectory***
   (a) Size of trajectory is $N = 500$ points.

   (b) $T = 2$ seconds – time step.

   (c) Initial coordinates
       $x_0 = 1000; \ y_0 = 1000$

   (a) Initial components of velocity $V$
       $V_x = 100; V_y = 100;$

   (b) Variance of noise $a_i$, $\sigma_a^2 = 0.3^2$ for both $a_i^x, a_i^y$

2. Generate also true values of range $D$ and azimuth $\beta$
$$D_i = \sqrt{x_i^2 + y_i^2}$$
$$\beta_i = arctg\left(\frac{x}{y}\right)$$

3. Generate measurements of $D^m$ and $\beta^m$ of range $D$ and azimuth $\beta$ provided by first observer that arrive every 4 seconds.
$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$
$$i = 1,3,5, \dots, N - 1 - \textbf{odd} \text{ time steps}$$
   Variances of measurement noises $\eta_i^D, \eta_i^\beta$ are given by

$$\sigma_D^2 = 50^2; \ \sigma_\beta^2 = 0.004^2$$

4. Generate more accurate measurements of azimuth $\beta^m$ provided by second observer that arrive between measurement times of the first observer.

$$\beta_i^m = \beta_i + \eta_i^\beta$$
$$i = 4,6,8, \dots, N - \textbf{even time steps}$$

Variance of measurement noise $\eta_i^\beta$ in this case is given by

$$\sigma_{\beta_{add}}^2 = 0.001^2$$

5. Initial conditions for Extended Kalman filter algorithm
   Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} x_3^m \\ \dfrac{x_3^m - x_1^m}{2T} \\ y_3^m \\ \dfrac{y_3^m - y_1^m}{2T} \end{vmatrix}$$

$$x_1^m = D_1^m \sin\beta_1^m$$
$$x_3^m = D_3^m \sin\beta_3^m$$
$$y_1^m = D_1^m \cos\beta_1^m$$
$$y_3^m = D_3^m \cos\beta_3^m$$

Initial filtration error covariance matrix $P_{0,0}$
First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 10^4 \end{vmatrix}$$

6. Develop Kalman filter algorithm to estimate state vector $X_i$ (extrapolation and filtration). **Start algorithm from time step = 4.**

   6.1. At every filtration step depending on observer, measurement vector $z_i$ and observation function $h(X_i)$ have different form.
   Consult charts, pages 3 – 6, Lab11_Brief_explanations.pdf.

   6.2. The form of measurement noise covariance matrix $R$ also varies:
   1) Observer 1, odd time steps

   $$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix}$$

   2) Observer 2, even time steps

   $$R = \sigma_{\beta_{add}}^2$$

   6.3. Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate
   (a) range $D$
   (b) azimuth $\beta$

6.4. At every filtration step in the algorithm you should linearize measurement equation by determining

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

Consult charts, pages 3 – 6, Lab11_Brief_explanations.pdf.

7. Run Kalman filter algorithm over $M = 500$ runs.
   Calculate true estimation errors of
   (a) Errors of extrapolation and filtration estimates of range $D$
   (b) Errors of extrapolation and filtration estimates of azimuth $\beta$
   Please plot these errors on two different plots for the analysis.

8. Compare estimation results with measurement errors of $D$ and $\beta$.

9. Analyze again estimation errors of range $D$ and azimuth $\beta$.
   Please make conclusions why the accuracy of estimation varies for odd and even time steps for both $D$ and $\beta$.

10. Make conclusions to the Assignment.
    Conclusions should be done in a form of a learning log.
    **A learning log** is a journal which evidences your **own learning and skills development.**
    It is not just a diary or record of **"What you have done"**
    but a record of **what you have learnt, tried and critically reflected upon.**

11. Prepare performance report and submit to Canvas:
    Performance report should include 2 documents:
    1) A report (PDF) with performance of all the items listed above
    2) Code (PDF)


**Notes:**
- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed
- All figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.