Assignment 3
Determining and removing drawbacks of exponential and running mean. Task 1
I. Backward exponential smoothing
II. Drawbacks of running mean

Performance - Wednesday, September 9, 2020
Due to submit a performance report – Tuesday, September 15, 2020, 23:59 p.m.

The objective of this laboratory work is to determine conditions for which broadly used methods of running and exponential mean provide effective solution and conditions under which they break down. Important outcome of this exercise is getting skill to choose the most effective method in conditions of uncertainty.

This laboratory work is performed in the class by students as in teams of 4 and the team will submit one document reporting about the performance. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of two parts:

I. Backward exponential smoothing
II. Drawbacks of running mean

Here is the recommended procedure for part I*: *Backward exponential smoothing

1. In the assignment 2 (part II) you have already applied running and exponential mean to random walk model with noise statistics $\sigma_w^2 = 28^2, \sigma_\eta^2 = 97^2$. In this conditions results of exponential smoothing demonstrated significant shift (delay) of estimations.
2. Please apply backward exponential smoothing to forward exponential estimates to further smooth measurement errors.
3. Make visual comparison of results. Plot true trajectory X_i, measurements z_i, running and backward exponential mean. Make conclusions which method provides better accuracy. Compare estimation results of running mean and backward exponential smoothing using deviation and variability indicators (Lab2_Short_discussion.pdf).

Here is the recommended procedure for part II*: *Drawbacks of running mean

First, we will analyze a process which rate of change is changed insignificantly and measurement noise is great. Second, we will study a cyclic process, and measurement noise is small.

First trajectory

1. Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration

$$X_i = X_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}T$$

   Size of trajectory is 300 points.
   Initial conditions: $X_1 = 5; V_1 = 0; T = 0.1$
   Variance of noise a_i, $\sigma_a^2 = 10$

Generate measurements z_i of the process X_i

$$z_i = X_i + \eta_i$$

η_i –normally distributed random noise with zero mathematical expectation and variance $\sigma_\eta^2 = 500$.

2. Determine empirically the window size M of running mean and smoothing coefficient α (forward exponential smoothing) that provide the best estimation of the process X_i using measurements z_i. **As this process is not random walk model you cannot apply equations for optimal smoothing coefficient.**
   Hint: the trajectory is close to the line, but measurement errors are huge. Then which window width is better in this case?

3. Chose better smoothing method using deviation and variability indicators.

Second trajectory

4. Generate cyclic trajectory X_i according to the equation

$$X_i = A_i \cdot \sin(\omega i + 3)$$
$$A_i = A_{i-1} + w_i$$

   Periods of oscillations is T=32 steps.
   Hint: To determine period, please define corresponding angle frequency ω from equation $\omega T = 2\pi$ (radian per one step).

   w_i – normally distributed random noise with zero mathematical expectation and variance $\sigma_w^2 = 0.08^2$.

   Size of trajectory is 200 points.
   Initial conditions: $A_1 = 1$.

5. Generate measurements z_i of the process X_i

$$z_i = X_i + \eta_i$$

   η_i –normally distributed random noise with zero mathematical expectation and variance $\sigma_\eta^2 = 0.05$

6. Apply running mean with window size $M = 13$ to measurements z_i.

7. Determine the period of oscillations for which running mean with given for every group window size M
   a) produces inverse oscillations
   b) leads to the loss of oscillations (zero oscillations)
   c) changes the oscillations insignificantly

   Group 1: $M = 15$; Group 2: $M = 17$; Group 3: $M = 19$; Group 4: $M = 21$;
   Group 5: $M = 23$; Group 6: $M = 25$; Group 7: $M = 27$;

8. Make conclusions about conditions of 7a,b,c.

9. Make conclusions to the Assignment.
   Conclusions should be done in a form of a learning log. **A learning log** is a journal which evidences your **own learning and skills development.** It is not just a diary or record of **“What you have done”** but a record of **what you have learnt, tried and critically reflected upon.**

10. Prepare performance report and submit to Canvas:
    Performance report should include 2 documents:
    1) A report (PDF) with performance of all the items listed above
    2) Code (PDF)

Notes:

- PDF report should contain the names of team members, number of the assignment
- All questions of the assignment should be addressed
- All figures should have a caption, all axes should have labels, a legend to curves should be given, and short conclusions/discussions/results related to figures should be provided.
- The overall conclusion to the assignment should be provided in a form of a learning log.

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2017
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.