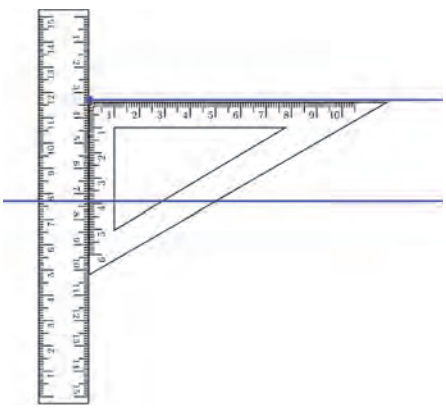
# 1 Parallel Lines

## Let us recall

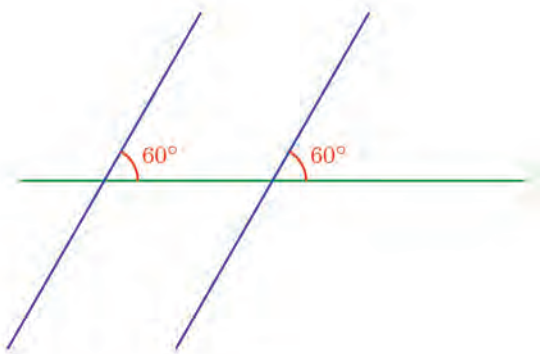


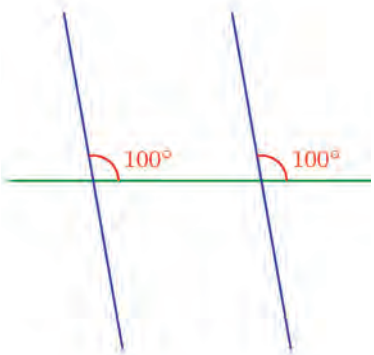
We have heard about parallel lines in class 6.

Lines that don't meet, keeping the same distance between them.

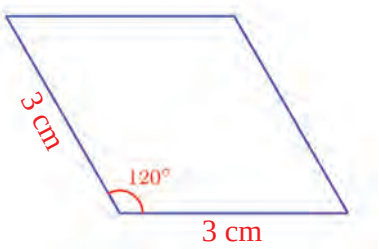
We have also drawn them with a scale and a set square.

Two lines drawn at the same slant to a given line are parallel. We've seen this also.



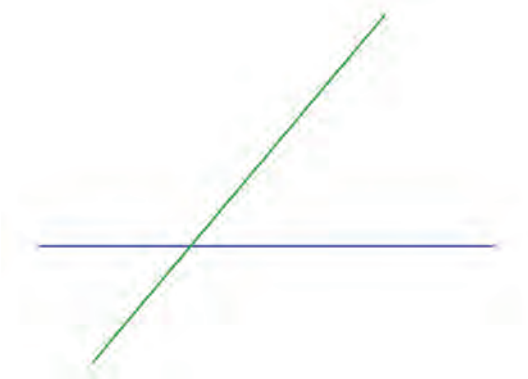




We know that a parallelogram is a quadrilateral in which the two pairs of opposite sides are parallel. Can you draw this parallelogram with measures as given ?

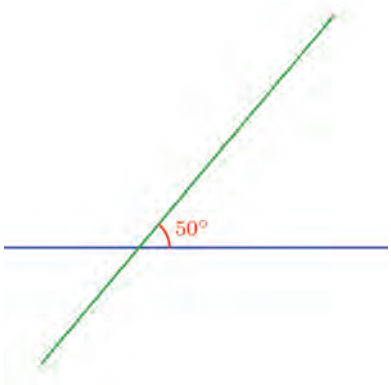


# Lines and angles

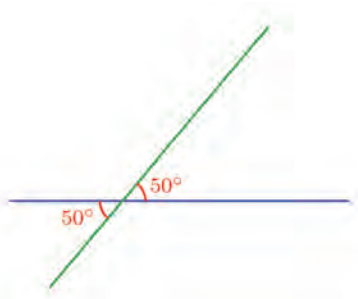
When a line crosses another line, how many angles are formed between them ?

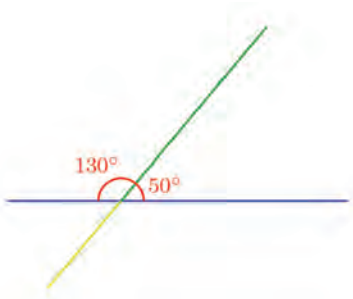


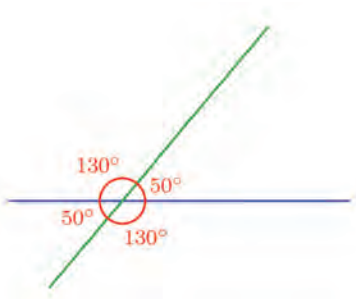
If we know one of these, can we calculate the others ?




This too was seen in class 6.

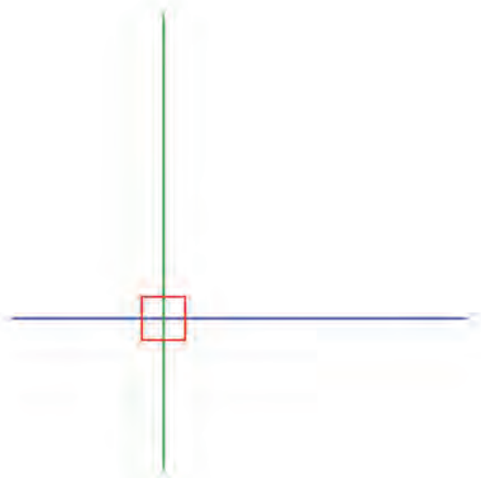


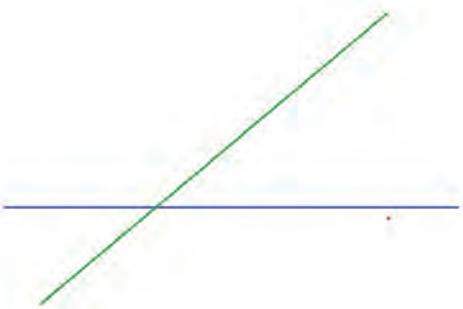




What can we say about the relation between four such angles in general ?

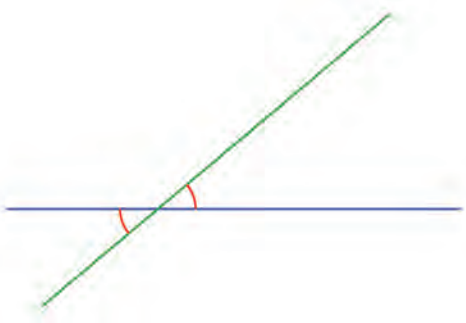
Nothing much to say if the crossing lines are perpendicular. All angles are 90°.

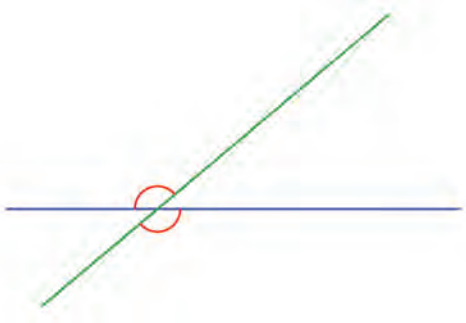


What if one line is a bit tilted ?



Two small angles and two large angles.

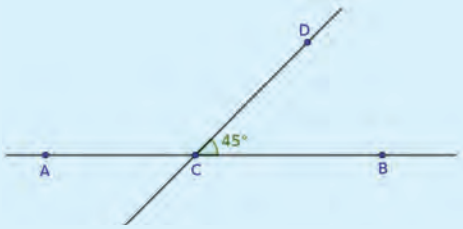




What is the relation between them?

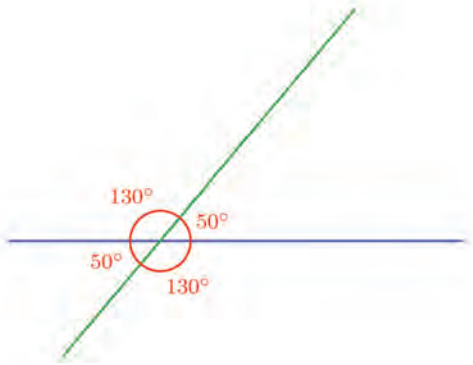
- The two small angles are of the same measure.
- The two large angles are of the same measure.
- The sum of a small angle and a large angle is 180°.

Draw a line *AB*. Mark a point *C* on the line and another point *D* outside the line. Draw a line through *C* and *D*. Find ∠*BCD*. Use the **Angle** *tool* and click on *B, C, D* in order (see what happens if you click in a different order).

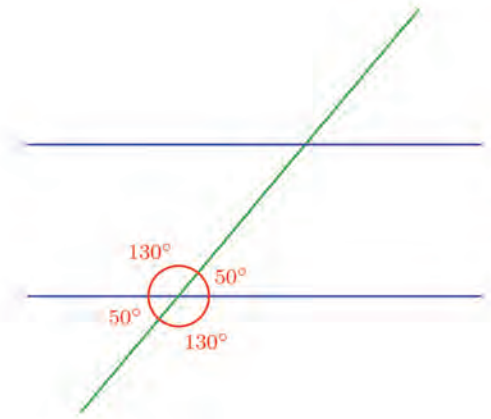



Mark the other angles also like this. What is the relation between the angle measures ? Try changing the position of *D*. Don't you see a change in the angles ? Does the relation change ?

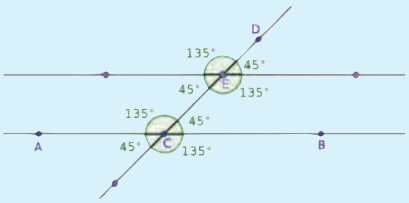
Look at the earlier figure once again:




What if we draw another line above, parallel to the blue line ?




Draw two intersecting lines and mark the angles between them as before. Mark a point *E* on *CD* and draw a line through it parallel to *AB*. Mark the four angles around *E* (You may mark more points on the line for ease of marking the angles).

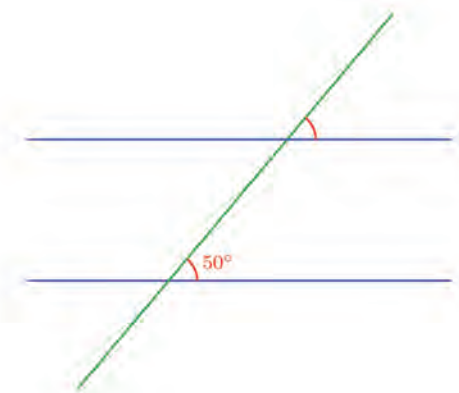



What is the relation between the eight angles you have marked now ? Try changing the position of *D*. Does the relation change when the angle measures change ? Try changing the position of *E*.

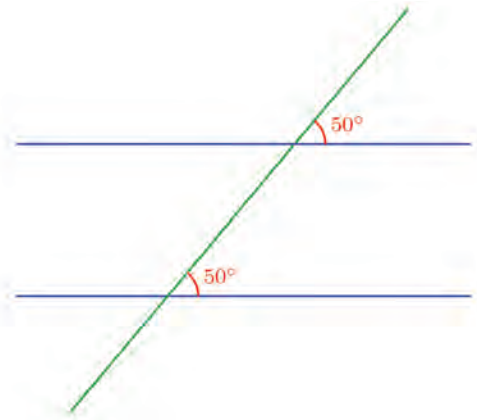
What happens when *E* takes the place of *C* ?

Now the blue line above makes four angles with the green line. What can you say about them?

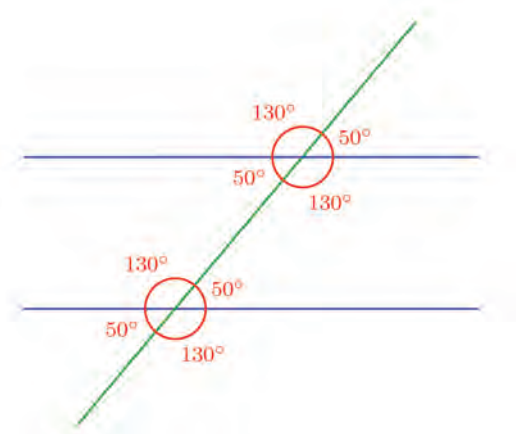
Let's look only at the small angle below and the marked angle above.



The blue lines are parallel. So these two must be of the same measure.




What about the other angles above ?

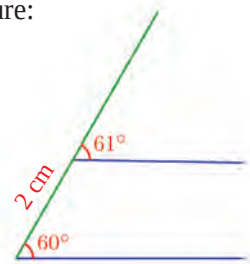



Suppose we start with an angle other than 50°. The measures of the other angles will change. But the relation between the angles will be the same. That is,

A line intersects two parallel lines at angles of the same measure.

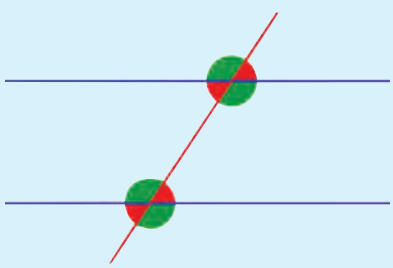
## A change in angle

Suppose there is slight change in the slant of two lines with another line. The two lines won't be parallel. For example, look at this figure:

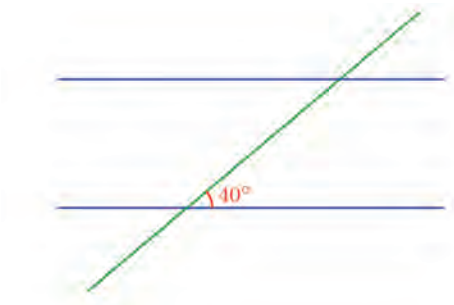


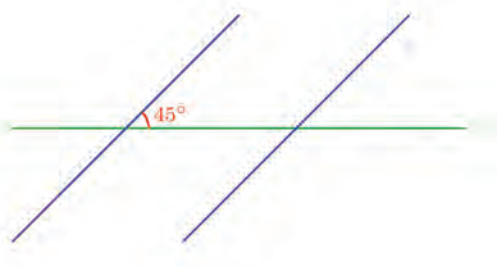

The blue lines in the figure appear to be parallel. As there is a difference of 1° in the slants, they will meet when extended sufficiently. We can calculate how much to be extended. You've to extend them by more than a metre for them to meet!

Draw two parallel lines and an intersecting line as in the earlier activity. Mark the eight angles around the points of intersection. You may hide their measures (Right click and uncheck **Show Label** box). Give the same colour to the four small angles of the same measure **(Right Click → Object Properties → Colour)** Choose the colour you want. You can change **Opacity**. In the same way, give another colour to all the large angles of the same measure.



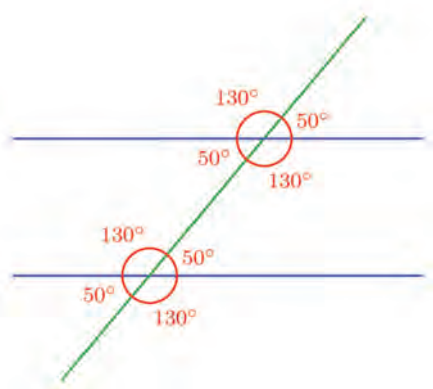
In each of the pictures below, can you calculate the other seven angles which the parallel blue lines make with the green line ?




## Matching angles

We have seen the relations between the four angles made by two intersecting lines. What can we say about the relation between the eight angles formed when a line cuts two parallel lines ?




Let's take another look at this figure which we saw earlier.

We know the relation between the four angles below. Same is the relation between the four angles above.

What if we take an angle from below and an angle from above ?

If both are small angles, each is 50°.

If both are large, each is 130°.

If one is small and the other is large; the small one is 50°, the large one 130°; and the sum is 180°.

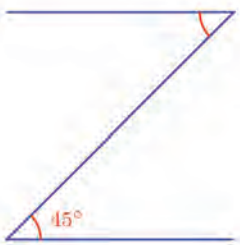
The relations remain the same even if the angles change, right ? So we can say this in general :

Of the angles made when two parallel lines are cut by a slanting line,

- the small angles are of the same measure.
- the large angles are of the same measure.
- a small angle and a large angle add up to 180°.

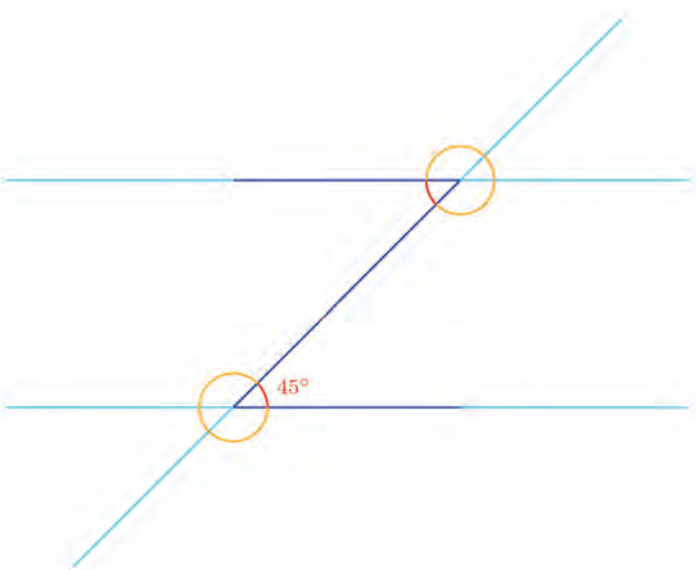
If the intersecting line is perpendicular to one of the parallel lines, it would be perpendicular to the other line too, and all angles would be right angles.

Now, look at this figure.




The top and bottom lines are parallel. What is the measure of angle above ?

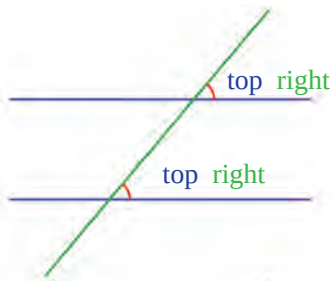
To see all the angles clearly, let's suppose the lines are extended.

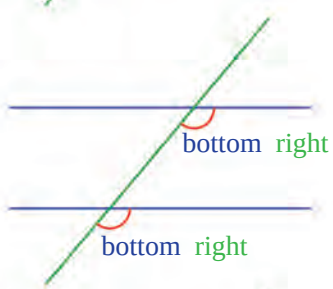


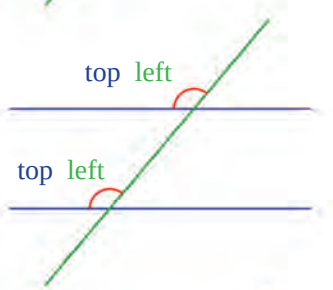

Look at the two small angles formed when the slanting line cuts the top and bottom parallel lines. These are the angles in the first figure. So they are of the same measure.

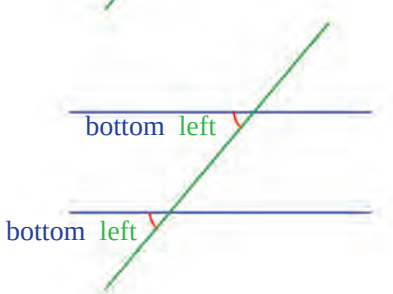
## Position and angle

We may group the angles formed when two parallel lines are cut by another line based on their position. The figures below show angle pairs in the same position.



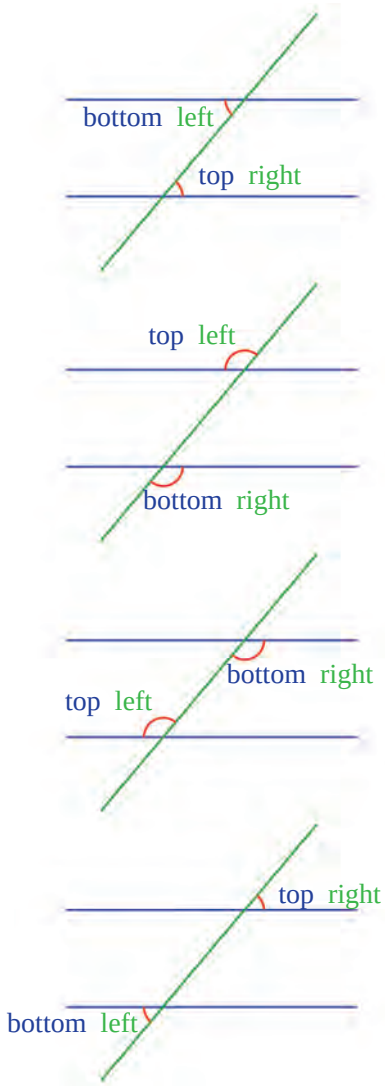




Angles in each such pair are called corresponding angles. The angles in each pair measure the same.

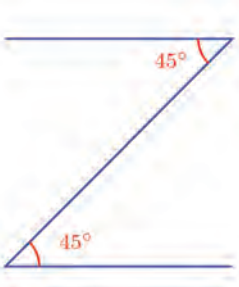
## Opposite positions

The figures below show the angles in opposite positions when two parallel lines are cut by another line

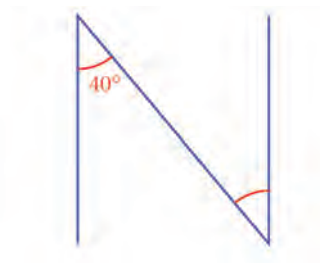



Angles in each such pair are called alternate angles. The angles in each pair measure the same.

That is, the angle above is also 45°.



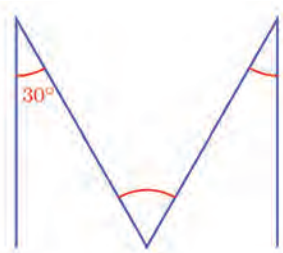

What if the figure looks like this ?



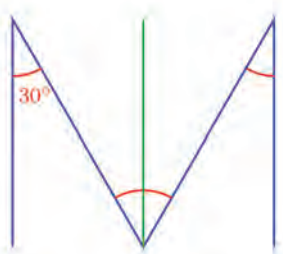

This is the first figure with the angle slightly changed and turned a bit, isn't it ?

What is the measure of the other angle ?

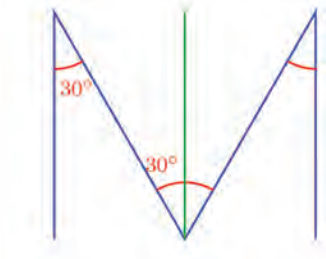
Now look at this figure :




The vertical lines are parallel. But there is no line cutting them. How about drawing another vertical line ?

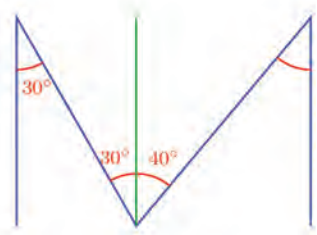


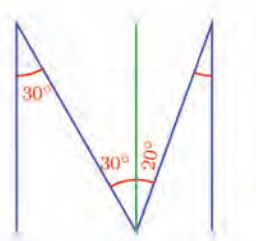
Now the, middle angle is in two parts. We can find the left part.



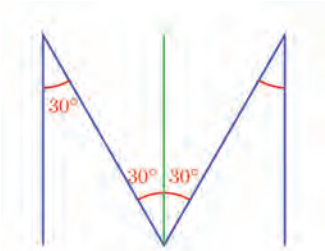

What about the right part ?

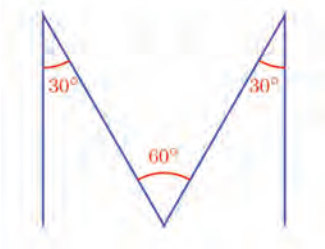
Let's try different measures for this angle:



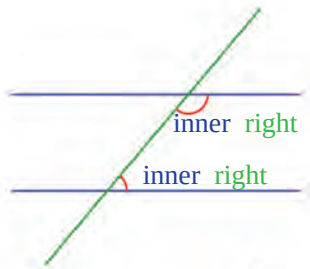

What should be this angle to get a nice figure?

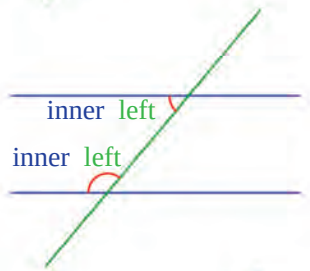


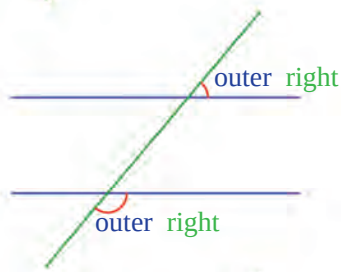



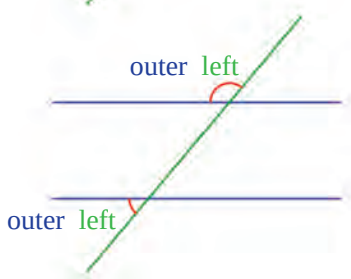

## In and out

The figures below show the interior and exterior angle pairs when two parallel lines are cut by another line.



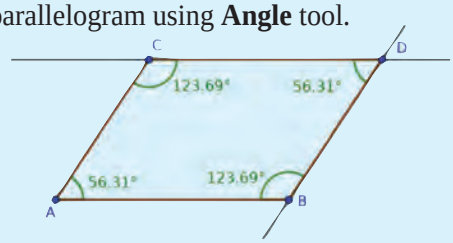




The first two pairs are called co-interior angles and the last two pairs co-exterior angles. The sum of the angles in each such pair is 180°.

Let's draw a parallelogram. Draw two lines *AB* and *AC*. Through *B* draw a line parallel to *AC* and through *C* draw a line parallel to *AB*. The point of intersection of these lines is *D*. Draw parallelogram *ABDC* using **Polygon** tool. We can see all angles if we click in the parallelogram using **Angle** tool.

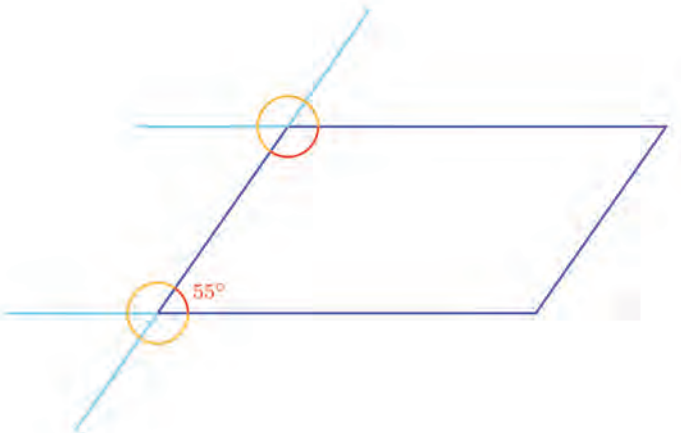



What is the relation between these angles?

Try changing the position of *C*. Do the angles change ? And the relation between them ?

Another question : Can you find out the other angles in the given parallelogram ?

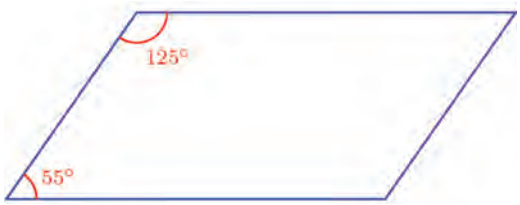

First, take the angle above the 55° angle. To determine this, we shall look at the angles of intersection of the left side with the top and bottom parallel lines.




The 55° angle and the angle above it form a pair of small angle and large angle.

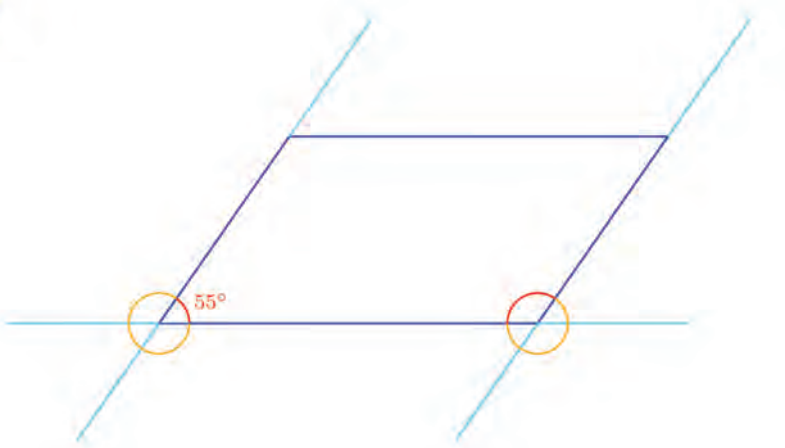
So their sum is 180°.

This means the top angle = 180° – 55° = 125°.



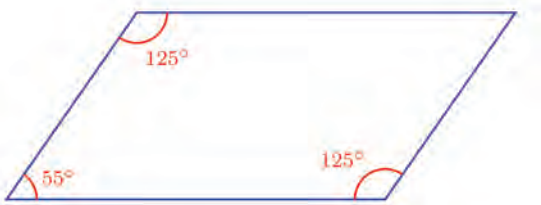

Now look at the angle to the right of the marked angle.

To calculate this, look at the angles made by the left and right parallel sides with the bottom line.



The 55° angle and the angle on its right are a small angle and a large angle of these angles.

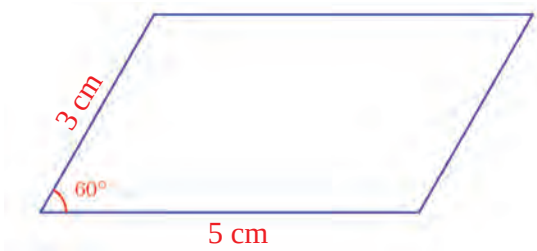
So, this angle also is 125° as calculated earlier.




Can't you find the fourth angle, like this ?

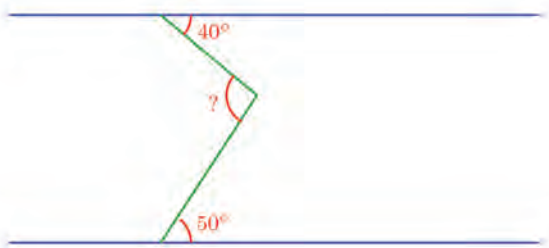
## Now try these problems:

(1) Draw the parallelogram below with the given measures.

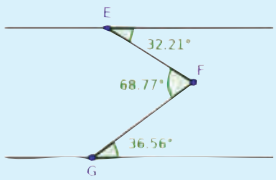



Calculate the other three angles.

(2) The top and bottom blue lines in the figure are parallel. Find the angle between the green lines.



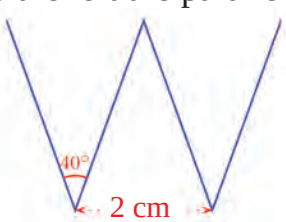

Draw two parallel lines. Mark a point on each. Mark a third point in between them. Draw lines joining the points on the lines with the third point. Mark the angles which these lines make with the parallel lines. Also mark the angle between the lines.




What is the relation between the three angles? Try changing the position of the points. Does the relation between the angles remain the same ?

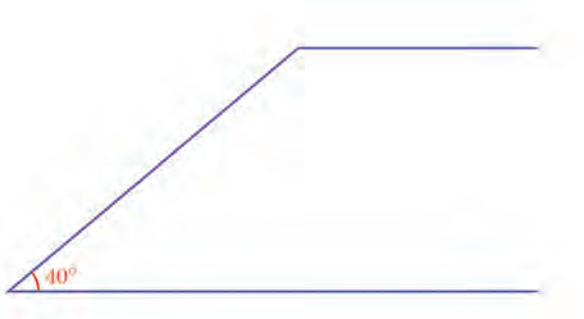
(3) In the figure, the pair of lines slanted to the left are parallel; and also the pair of lines slanted to the right.

Draw this figure :



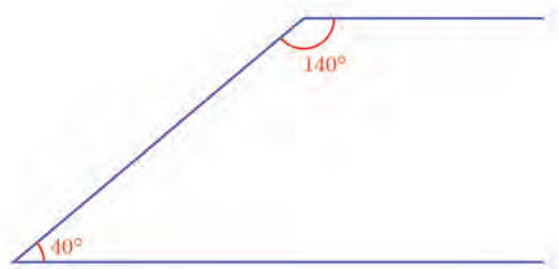
## Triangle sum

Look at this figure :



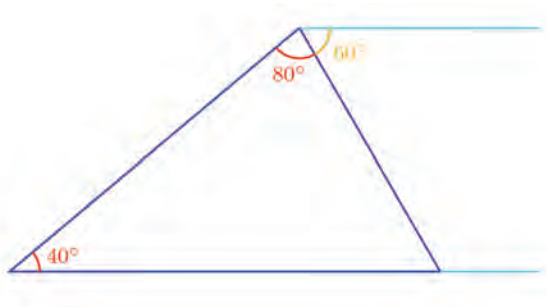

The top and bottom lines are parallel.

So, can you calculate the angle at the top ?




If this angle is drawn less than 140°, then the two lines will meet.

Let's decrease it by 60°.



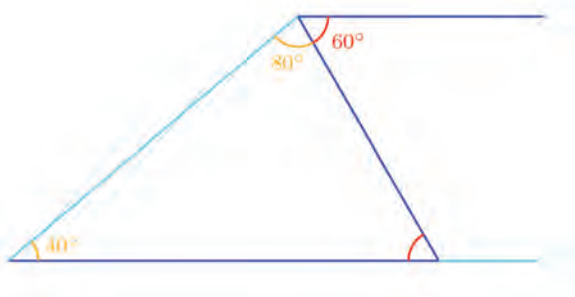

Now we have a triangle.

What are the angles in this triangle ?

The left angle is 40°. The top angle is 140° – 60° = 80°.

What about the third angle ?

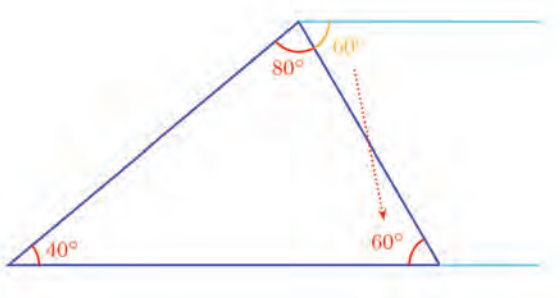
It is one of the small angles which the new slanted line makes with the parallel lines .




Its measure is the same as that of the small angle which this line makes with the top line.

Isn't the top small angle 60° ?

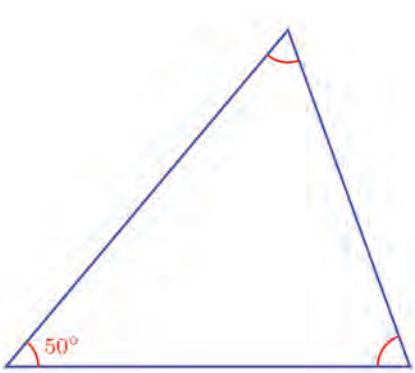
So the bottom small angle is also 60°.




Thus the 60° which we took away from the top reappears at the bottom as an angle of the triangle.

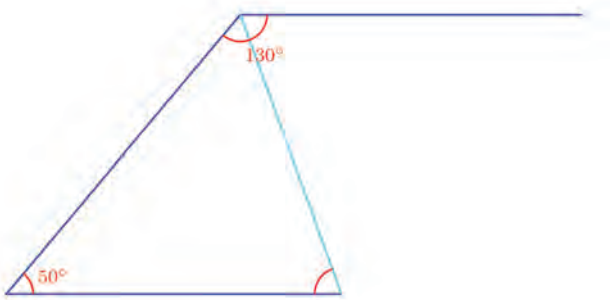
The sum of this angle and the top angle of the triangle 80° + 60° = 140°.

Now look at this figure:

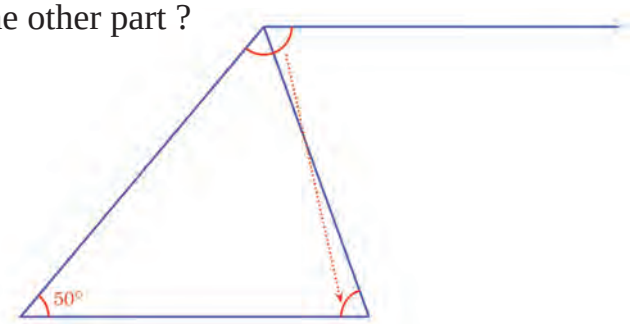


Can you calculate the sum of the other two angles of this triangle ?

In the first problem, the right side of the triangle was got by drawing a slanted line instead of the parallel line. Let's think in the reverse. A parallel line instead of the slanted right side. What is the angle which this parallel line makes with the left side ?




When we drew the triangle, this angle split into two. One part is the top angle of the triangle. What about the other part ?




That is, one part of the 130° angle is the top angle of the triangle and the other part is the angle on the right in the triangle.

So, the sum of these two angles of the triangle is 130°.

How do we state in general what we've learnt from this problem ?

If we subtract the measure of one angle of a triangle from 180°, we get the sum of the other two angles.

For example, if one angle of a triangle is 60°, the sum of the other two angles:

180° – 60° = 120°

What about the sum of the three angles of the triangle ?

Using the **Polygon** tool, draw a triangle and mark the angles (click inside the triangle using the **Angle** tool). What is the sum of all angles ? Change the vertices of the triangle and check.

This is true for any triangle.

The sum of all angles of a triangle is 180°

Now try this problem :

*One angle of a right triangle is 40°. What is the measure of the angle other than the right angle* ?

Let's think this way. The sum of the angles other than the right angle is

$$180° - 90° = 90°$$

One of them is 40°. Then the other angle is

$$90° - 40° = 50°$$

We can think in another way as well. The sum of the three angles is 180°. The sum of two of them

$$90° + 40° = 130°$$

So the third angle

$$180° - 130° = 50°$$

Another problem :

*One angle of a triangle is 72°. The other two angles are of equal measure. What are their measures ?*

What is the sum of the other two angles ?

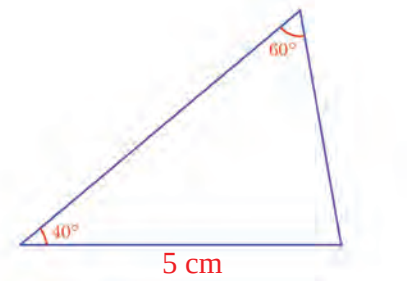
$$180° - 72° = 108°$$

Since the other two angles are equal, each is half the sum, isn't it ? So each is

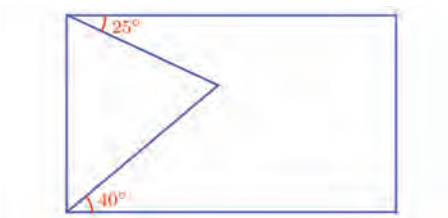
$$\frac{108^\circ}{2} = 54^\circ$$

Now try these problems :

(1) Draw the triangle with the given measures.

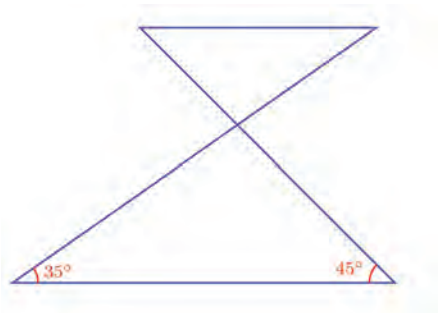



(2) The figure shows a triangle drawn in a rectangle.



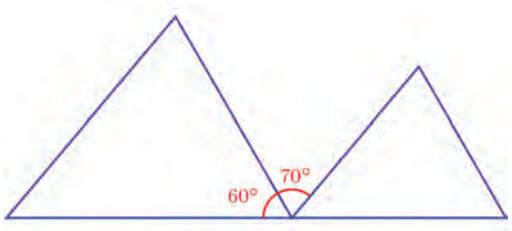

Calculate the angles of the triangle.

(3) The top and bottom lines in the figure are parallel.



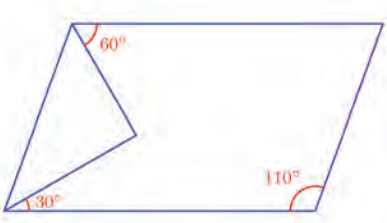

Calculate the third angle of the bottom triangle and all angles of the top triangle.

(4) The left and right sides of the large triangle are parallel to the left and right sides of the small triangle.



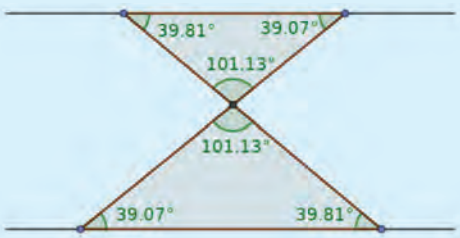

Calculate the other two angles of the large triangle and all angles of the small triangle.

5. A triangle is drawn inside a parallelogram.




Calculate the angles of the triangle.

Draw two parallel lines and mark two points on each. Join them as in the figure and mark the point of intersection. Use the **Polygon** tool to draw two triangles and mark their angles.




What is the relation between the angles of the triangle ?

Try changing the positions of the points.

Draw a triangle and mark a point on one of the sides. Draw a line through this point parallel to another side of the triangle. Mark the point where this line meets the third side. Draw the small triangle with one corner of the triangle and the points on the sides as vertices. Mark the angles of the first triangle and the small triangle. What is the relation between these angles ? Try changing the corners of the triangle.