In [ ]:
```python
# importing nessery libraries:
import numpy as np
import matplotlib.pyplot as plt
```

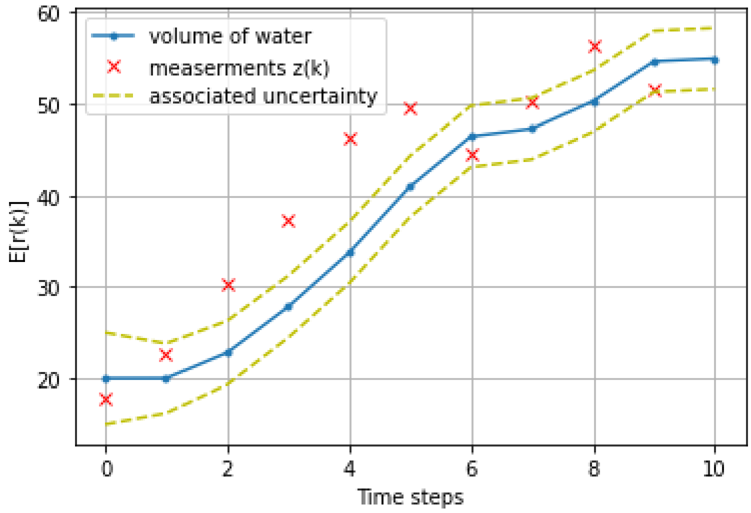
In [ ]:
```python
# the Kalman filter for 10 steps.
n_steps = 10
# At time k = 0, E[r(0)] = 20...
Er0 = 20
# with uncertainty Var[r(0)] = 25.
uncertainty_Vr0 = 25
# d(k) = 10 for all k ≥ 0.
dk = 10
# The consumer is predicted to use a supply m = 7
m = 7
# since d(k) = 10 for all k ≥ 0 and The consumer is predicted to use a supply m = 7
# then uf will always be:
uf = dk - m
# process uncertainty is Var [v(k)] = 9.
process_uncertainty = 9
# sensor uncertainty is Var [w(k)] = 25.
sensor_uncertainty = 25
# We receive the following sequence of measurements z(k):
measurements = [17.8, 22.6, 30.2, 37.3, 46.2, 49.5, 44.6, 50.3, 56.3, 51.6]
```

In [ ]:
```python
water_volume = np.zeros([n_steps+1]) # to store the actual volume of water.
water_volume[0] = Er0 # since E[r(0)] = 20 at time k = 0.

uncertainty = np.zeros([n_steps+1]) # to provide the associated uncertainty for my
uncertainty[0] = uncertainty_Vr0 # since Var[r(0)] = 25 at time k = 0.
```

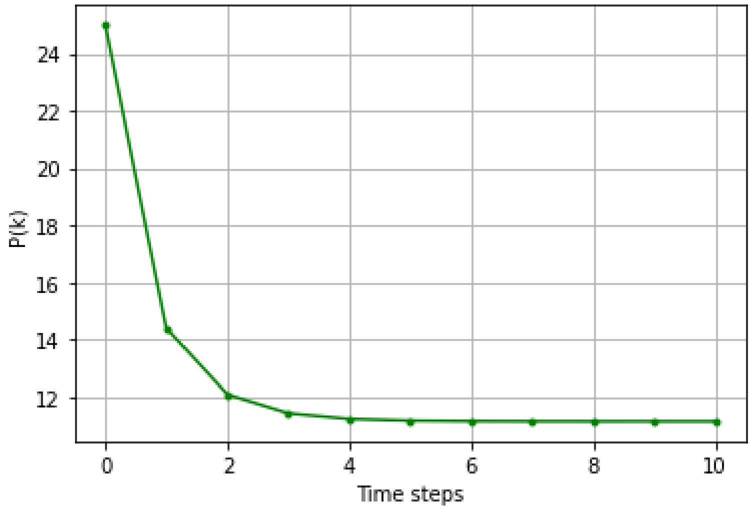
In [ ]:
```python
d = np.mat([[1]]) # dynamic

mm = np.mat([[1]]) # measurement model

mnv = np.mat([[25]]) # measurement noise variance
```

In [ ]:
```python
for k in range(1,n_steps+1):

    Kp = uf + d*Er0
    K_uncertainty = sensor_uncertainty * (d**2) + process_uncertainty

    measurement = measurements[k-1]

    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    Er0 = Kp + K @ (measurement - mm @ Kp)
    sensor_uncertainty = (np.eye(1) - K @ mm) @ K_uncertainty @ (np.eye(1) - K @ mm

    water_volume[k] = Er0[0]

    uncertainty[k] = sensor_uncertainty[0]
```

In [ ]:
```python
plt.plot(water_volume,'.-',label="volume of water")
plt.plot(measurements,'rx',label="measerments z(k)")
plt.plot(water_volume+np.sqrt(uncertainty),'y--',label="associated uncertainty")
plt.plot(water_volume-np.sqrt(uncertainty),'y--')
plt.ylabel('E[r(k)]')
plt.xlabel('Time steps')
plt.legend()
plt.grid(True)
```



In [ ]:
```
plt.plot(uncertainty,'g.-')
plt.ylabel('P(k)')
plt.xlabel('Time steps')
plt.grid(True)
```



In [ ]:
```
d = np.mat([[1,-1],[0,1]]) # dynamics

mm = np.mat([[1,0]]) # measurement model

mnv = np.mat([[25]]) # measurement noise variance

ii = np.eye(2)
```

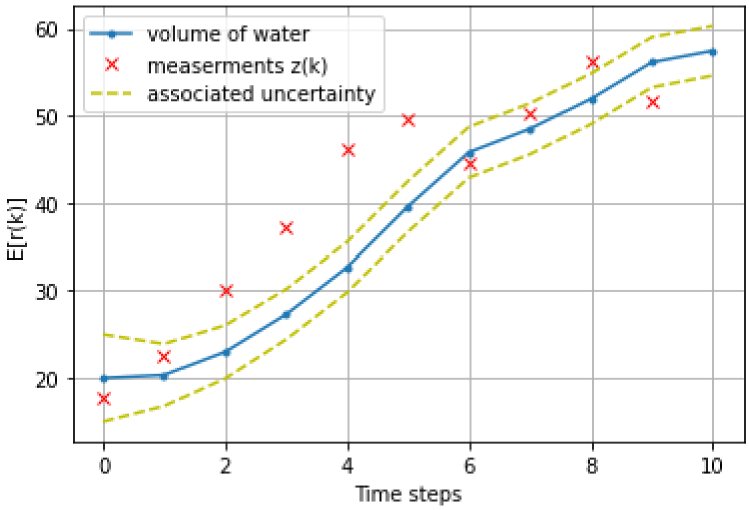
In [ ]:
```
# At time k = 0, E[r(0)] = 20, E[c(0)] = 7...
Er0 = np.mat([[20],[7]])

uf = np.mat([[10],[0]])
# Var [n(k)] = 0.1
process_uncertainty = np.mat([[0,0],[0,0.1]])
# sensor uncertainty is Var [w(k)] = 25, Var [c(0)] = 1
sensor_uncertainty = np.mat([[25,0],[0,1]])
```

In [ ]:
```
water_volume = np.zeros([n_steps+1,2])

water_volume[0,0] = Er0[0,0]
water_volume[0,1] = Er0[1,0]

uncertainty = np.zeros([n_steps+1,2])
```

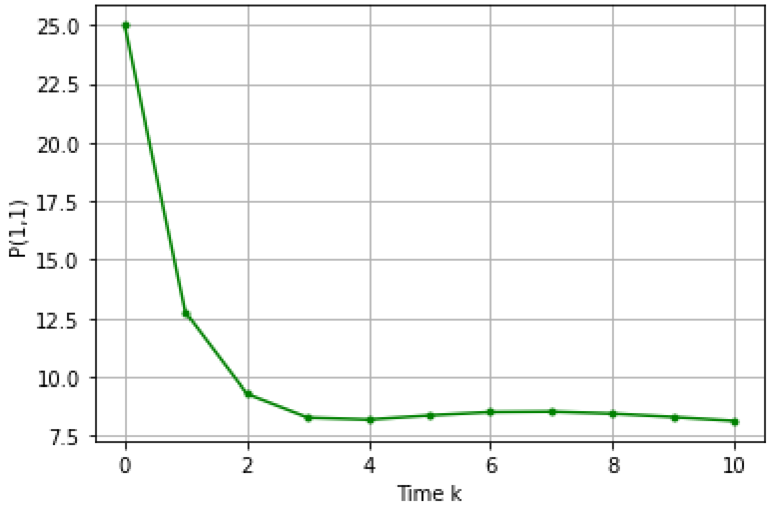
```
uncertainty[0,0] = sensor_uncertainty[0,0]
uncertainty[0,1] = sensor_uncertainty[1,1]
```

In [ ]:
```
for k in range(1,n_steps+1):

    # Kalman filter prediction:
    Kp = d @ Er0 + uf
    # Kalman filter prediction uncertainty:
    K_uncertainty = d @ sensor_uncertainty @ d.T + process_uncertainty

    measurement = measurements[k-1]

    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    Er0 = Kp + K @ (measurement - mm @ Kp)
    sensor_uncertainty = (ii-K @ mm)@ K_uncertainty @(ii-K @ mm).T + K @ mnv @ K.T

    # store the variables for plotting:
    water_volume[k,0] = Er0[0,0]
    water_volume[k,1] = Er0[1,0]

    uncertainty[k,0] = sensor_uncertainty[0,0]
    uncertainty[k,1] = sensor_uncertainty[1,1]
```

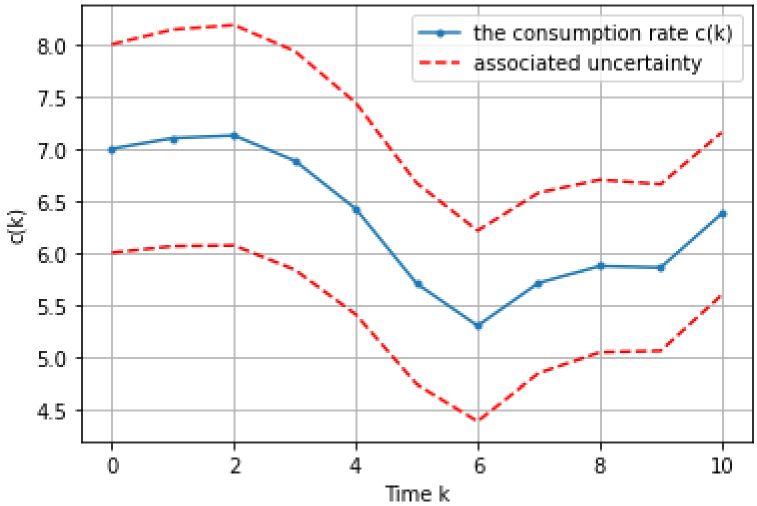
In [ ]:
```
plt.plot(water_volume[:,0],'.-',label="volume of water")
plt.plot(measurements,'rx',label="measerments z(k)")
plt.plot(water_volume[:,0]+np.sqrt(uncertainty[:,0]),'y--',label="associated uncer
plt.plot(water_volume[:,0]-np.sqrt(uncertainty[:,0]),'y--')
plt.ylabel('E[r(k)]')
plt.xlabel('Time steps')
plt.legend()
plt.grid(True)
```



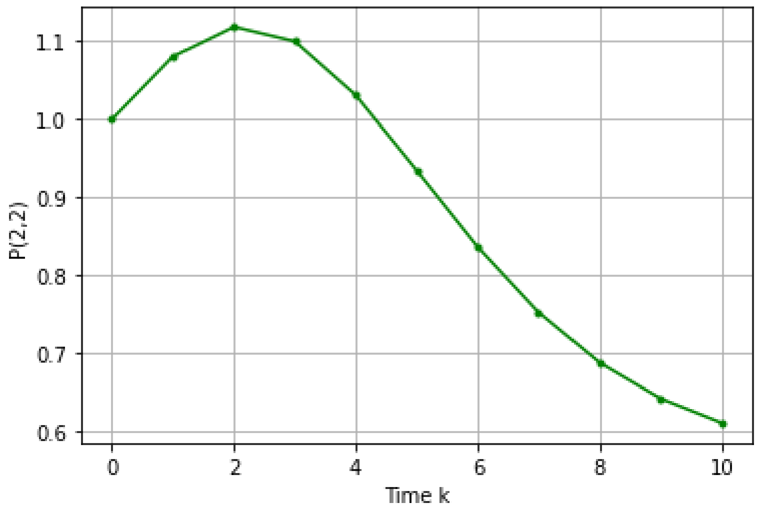
In [ ]:
```
plt.plot(uncertainty[:,0],'g.-',label="P(k)(1,1)")
plt.xlabel('Time k')
plt.ylabel('P(1,1)')
plt.grid(True)
```



In [ ]:
```
plt.plot(water_volume[:,1],'.-',label="the consumption rate c(k)")
plt.plot(water_volume[:,1]+np.sqrt(uncertainty[:,1]),'r--',label="associated uncer
plt.plot(water_volume[:,1]-np.sqrt(uncertainty[:,1]),'r--')

plt.xlabel('Time k')
plt.ylabel('c(k)')
plt.grid(True)
plt.legend();
```



In [ ]:
```
plt.plot(uncertainty[:,1],'g.-',label="P(k)(2,2)")
plt.xlabel('Time k')
plt.ylabel('P(2,2)')
plt.grid(True)
```



In [ ]:
```
α = 0.3
d = np.matrix([[1-2*α, α, α, 0, -1, 0, 0, 0],
               [α, 1-2*α, α, 0, 0, -1, 0, 0],
               [α, α, 1-3*α, α, 0, 0, -1, 0],
               [0, 0, α, 1-α, 0, 0, 0, -1],
               [0, 0, 0, 0, 1, 0, 0, 0],
               [0, 0, 0, 0, 0, 1, 0, 0],
               [0, 0, 0, 0, 0, 0, 1, 0],
               [0,0,0,0,0,0,0,1]])

process_uncertainty=np.diag([0,0,0,0,0.1,0.1,0.1,0.1])

mm = np.eye(4,8)

mnv = np.eye(4)*25
```

In [ ]:
```
z1 = np.array([59.3, 72, 64.4, 83.6, 84.9, 94.3, 84, 86.6, 89, 89.1])
z2 = np.array([39.1, 38.4, 36.2, 43.4, 50.5, 56.3, 40.3, 58.5, 55.4, 59.6])
z3 = np.array([31.1, 31.2, 41.6, 44.4, 41, 41.9, 39.2, 46.3, 43.3, 45.3])
z4 = np.array([38.6, 38, 32.6, 18, 29.4, 23.3, 11, 14.6, 18.4, 20.5])

list_of_measurements = [z1,z2,z3,z4]
measurements = np.mat(list_of_measurements)
```

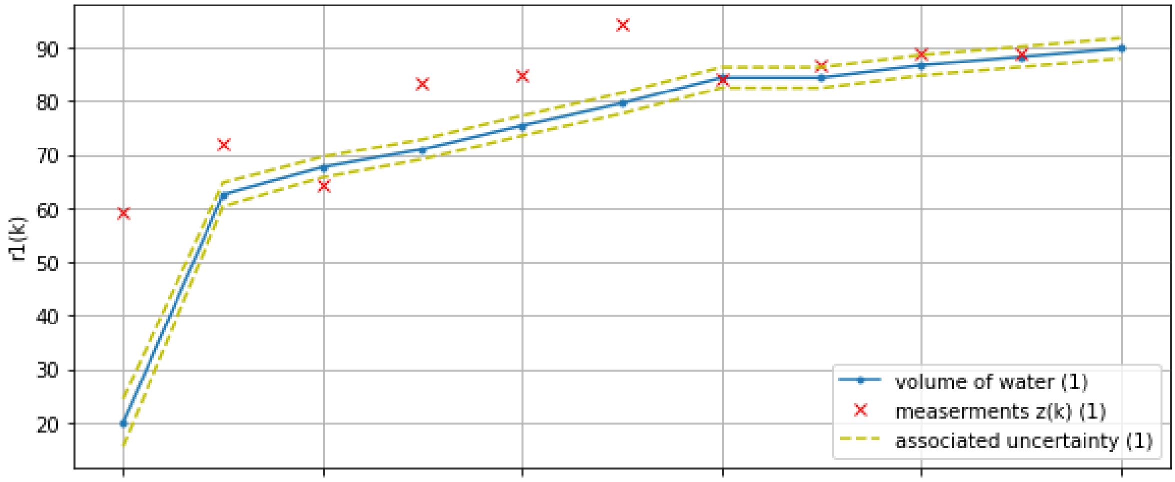
In [ ]:
```
Er0 = np.mat([[20],[40],[60],[20],[7],[7],[7],[7]])

sensor_uncertainty= np.diag([20,20,20,20,1,1,1,1])

uf = [[30],[0],[0],[0],[0],[0],[0],[0]]
```

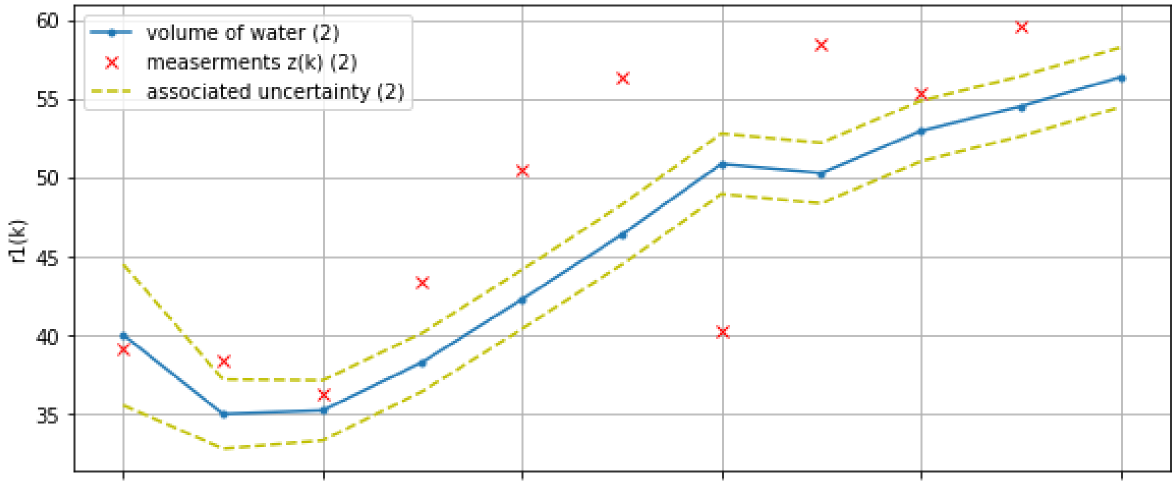
In [ ]:
```
water_volume = np.zeros([n_steps+1,8])

uncertainty = np.zeros([n_steps+1,8])

for i in range(8):
    water_volume[0,i] = Er0[i,0]
    uncertainty[0,i] = sensor_uncertainty[i,i]
```

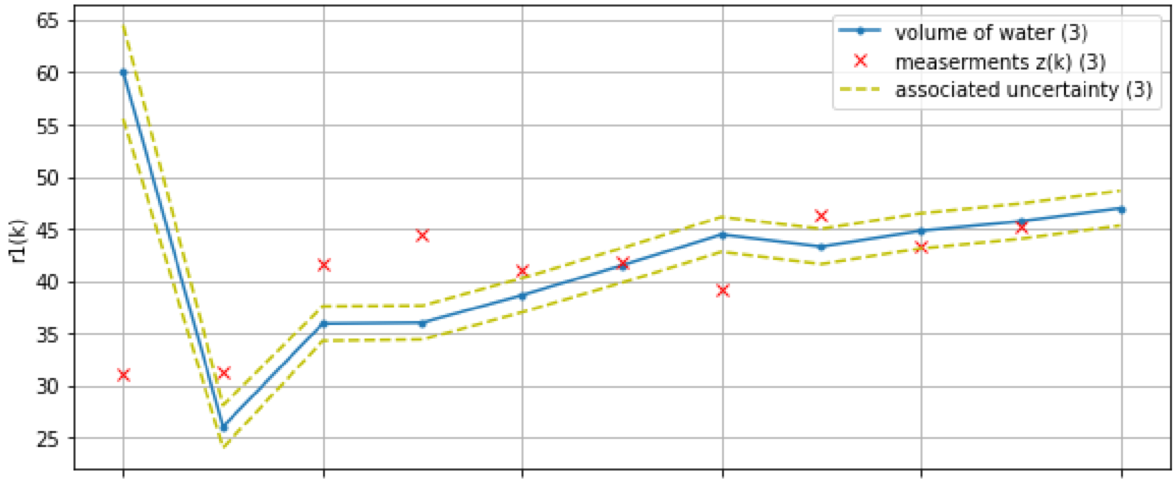
In [ ]:
```
for k in np.arange(1,n_steps+1):

    Kp = d @ Er0 + uf
    K_uncertainty = d @ sensor_uncertainty @ d.T + process_uncertainty

    measurement = measurements[:,k-1]
```

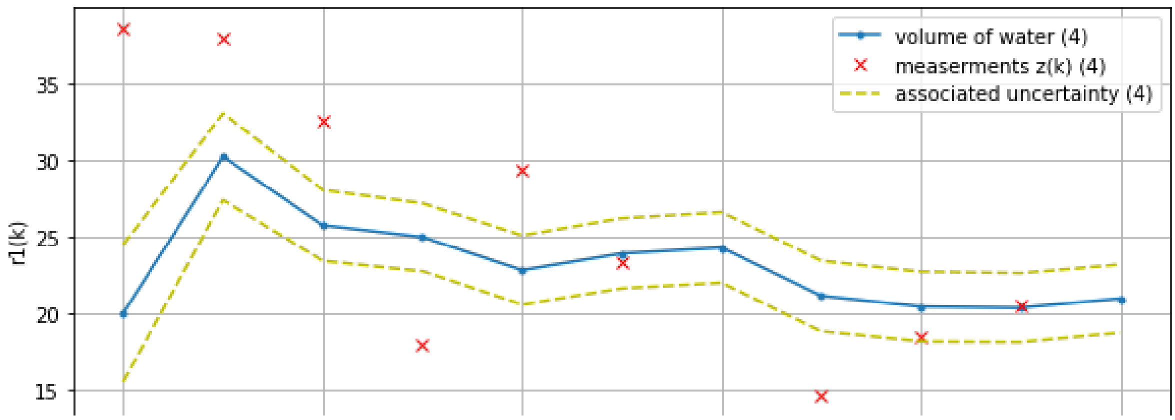
```
    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    Er0 = Kp + K @ (measurement - mm @ Kp)
    sensor_uncertainty = (np.eye(8) - K @ mm) @ K_uncertainty @ (np.eye(8) - K @ mm

    for i in range(8):
        water_volume[k,i] = Er0[i,0]
        uncertainty[k,i] = sensor_uncertainty[i,i]
```

In [ ]:
```
fig, ax = plt.subplots(4,1,sharex=True)
fig.set_size_inches(10,20)

for n in range(4):

    ax[n].plot(water_volume[:,n],'.-',label=f"volume of water ({n+1})")
    ax[n].plot(list_of_measurements[n],'rx',label=f"measerments z(k) ({n+1})")
    ax[n].plot(water_volume[:,n]+np.sqrt(uncertainty[:,n]),'y--',label=f"associate
    ax[n].plot(water_volume[:,n]-np.sqrt(uncertainty[:,n]),'y--',)

    ax[n].set_ylabel('r1(k)')
    ax[n].legend()
    ax[n].grid(True)
```

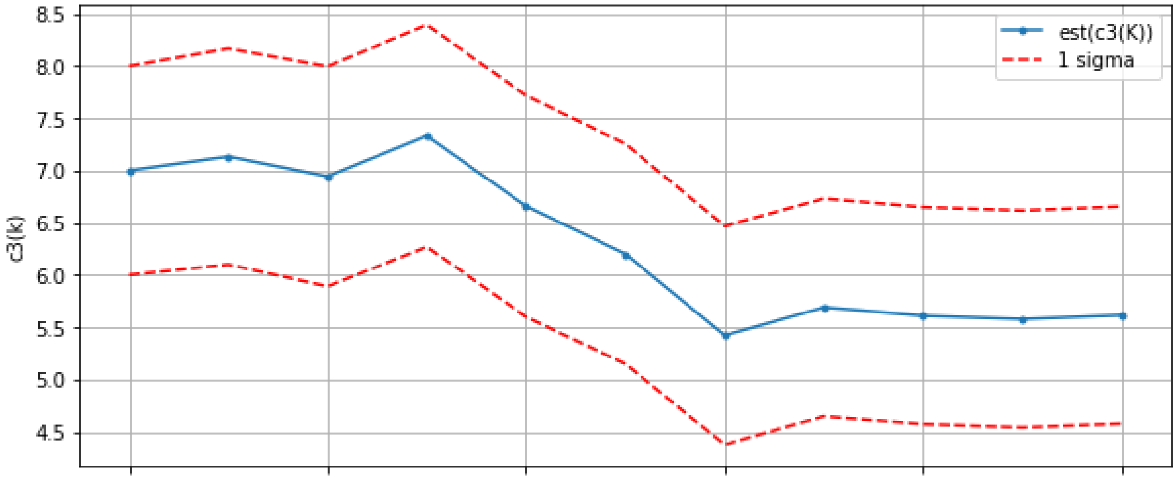

r1(k)
volume of water (1)
measerments z(k) (1)
associated uncertainty (1)

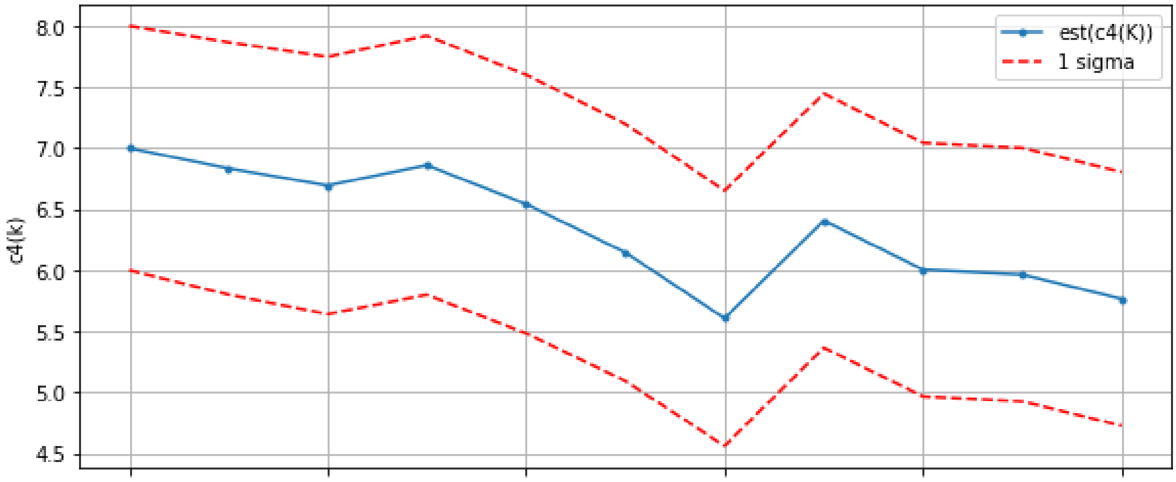

r1(k)
volume of water (2)
measerments z(k) (2)
associated uncertainty (2)

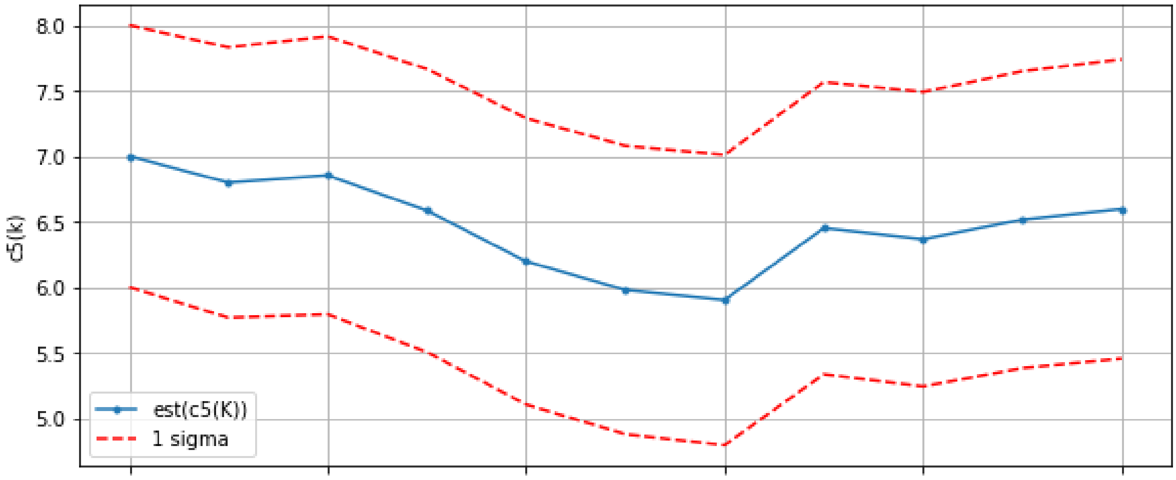

r1(k)
volume of water (3)
measerments z(k) (3)
associated uncertainty (3)

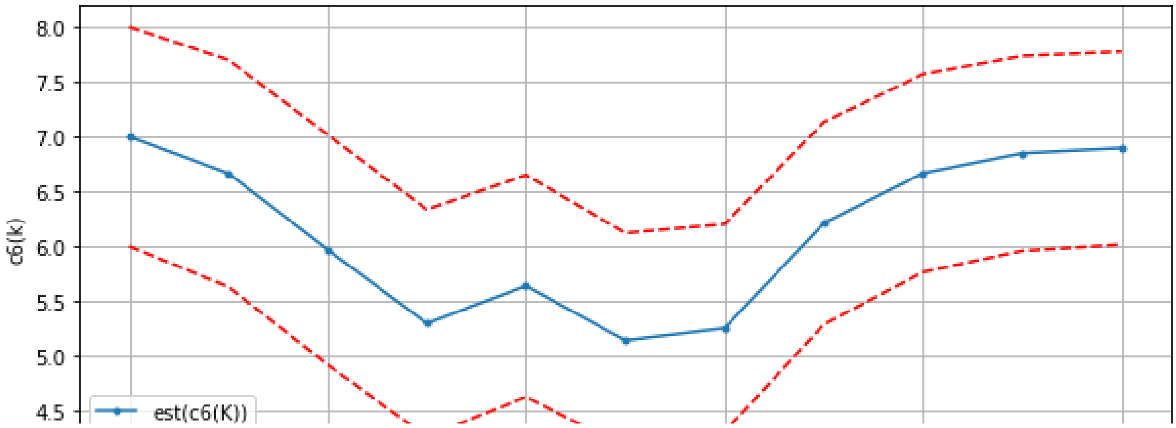

r1(k)
volume of water (4)
measerments z(k) (4)
associated uncertainty (4)

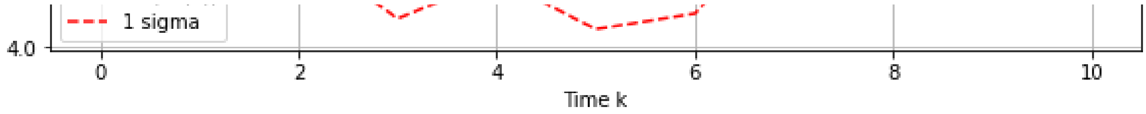
In [ ]:
```
fig, ax = plt.subplots(4,1,sharex=True)
fig.set_size_inches(10,20)

for n in range(4):

    ax[n].plot(water_volume[:,n+4],'.-',label=f"est(c{n+3}(K))")
    ax[n].plot(water_volume[:,n+4]+np.sqrt(uncertainty[:,n+4]),'r--',label="1 sigma
    ax[n].plot(water_volume[:,n+4]-np.sqrt(uncertainty[:,n+4]),'r--',)

    ax[3].set_xlabel('Time k')
    ax[n].set_ylabel(f'c{n+3}(k)')
    ax[n].legend()
    ax[n].grid(True)
```


est(c3(K))
1 sigma
c3(k)
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5


est(c4(K))
1 sigma
c4(k)
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5

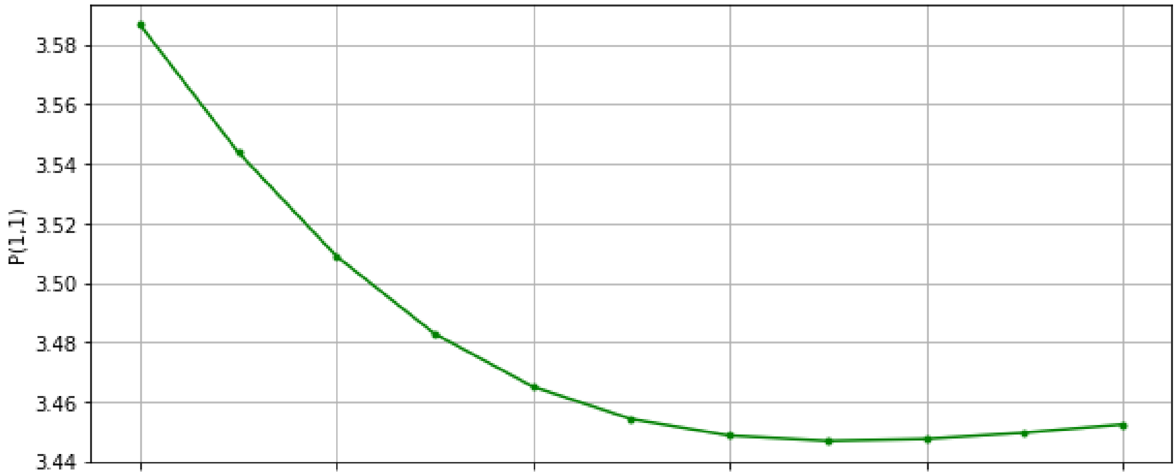

est(c5(K))
1 sigma
c5(k)
8.0
7.5
7.0
6.5
6.0
5.5
5.0

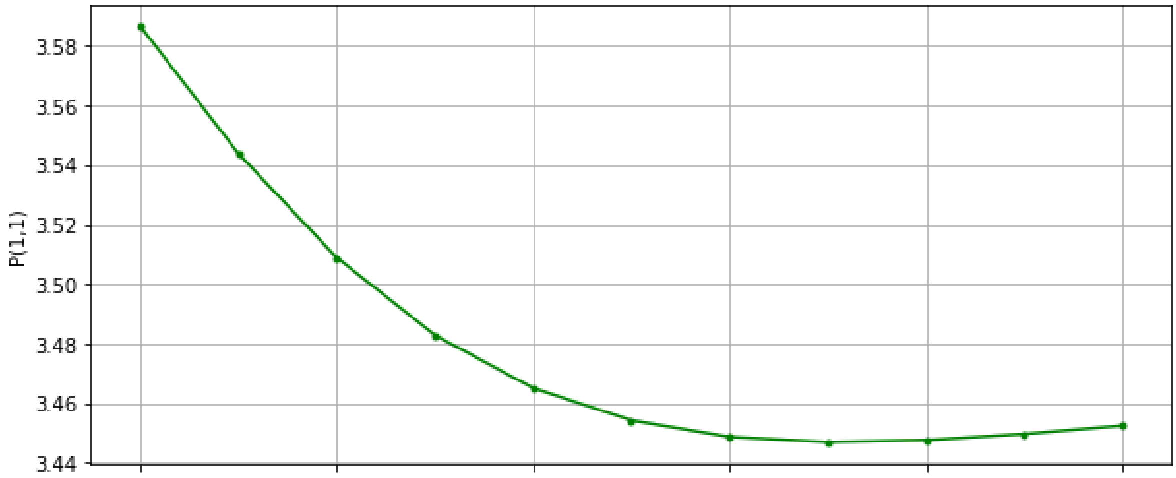

est(c6(K))
c6(k)
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5

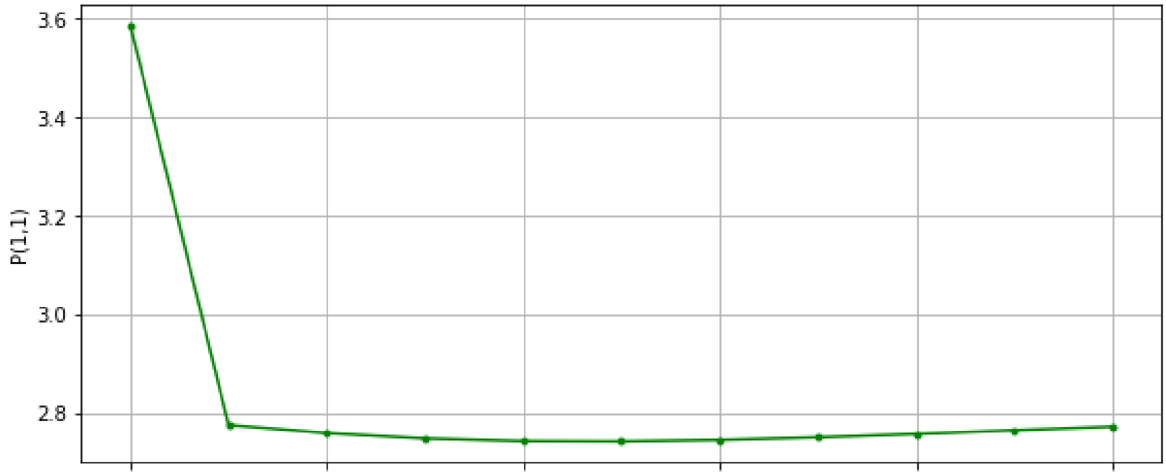


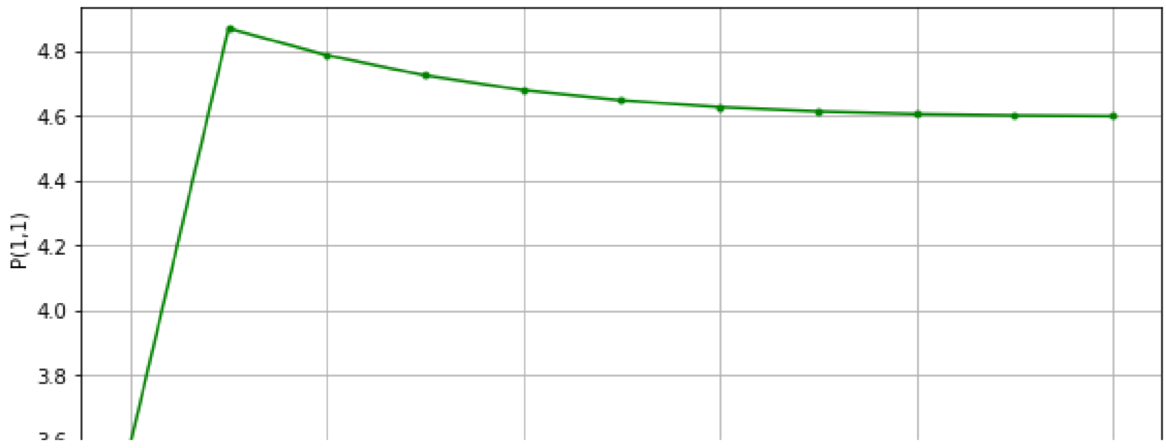
In [ ]:
```
uncertainties = [np.zeros([n_steps+1,1]),np.zeros([n_steps+1,1]),np.zeros([n_steps·

for i in range(4):

    uncertainties[i][0,0] = sensor_uncertainty[0,0]

for s in np.arange(1,n_steps+1):

    K_uncertainty = d @ sensor_uncertainty @ d.T + process_uncertainty

    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    sensor_uncertainty = (np.eye(8) - K @ mm) @ K_uncertainty @ (np.eye(8) - K @ mı

    for i in range(4):
        uncertainties[i][s,0] = sensor_uncertainty[i,i]
```

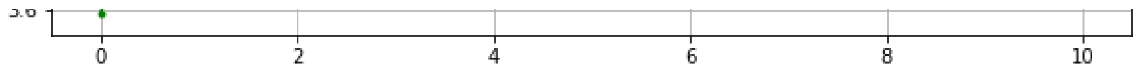
In [ ]:
```
fig, ax = plt.subplots(4,1,sharex=True)
fig.set_size_inches(10,20)

for n in range(4):

    ax[n].plot(uncertainties[n][:,0],'g.-',label="P(1,1)")
    ax[n].set_ylabel('P(1,1)')
    ax[n].grid(True)
```


3.58
3.56
3.54
3.52
3.50
3.48
3.46
3.44
P(1,1)


3.58
3.56
3.54
3.52
3.50
3.48
3.46
3.44
P(1,1)


3.6
3.4
3.2
3.0
2.8
P(1,1)


4.8
4.6
4.4
4.2
4.0
3.8
P(1,1)



In [ ]:
```
for i in range(4):
    print(1+i,':')
    print(uncertainties[i][:,0])
```

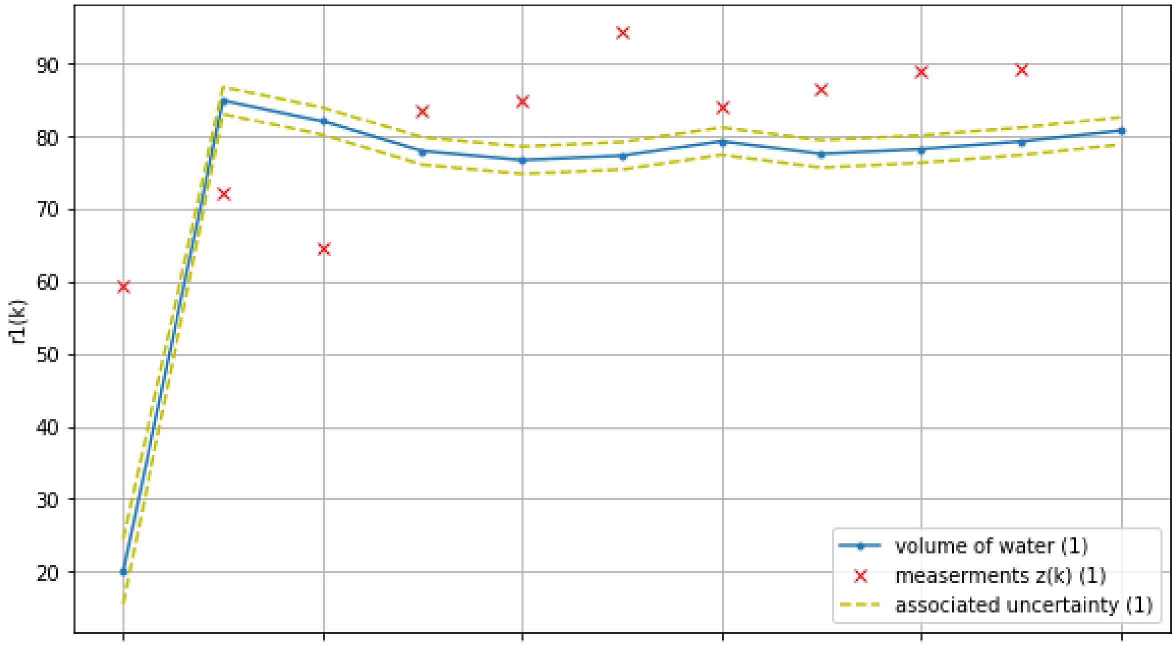
```
1 :
[3.58678078 3.54391255 3.50896257 3.4828954  3.46511245 3.45425204
 3.44870439 3.4469224  3.44758705 3.44967182 3.45244104]
2 :
[3.58678078 3.54391255 3.50896257 3.4828954  3.46511245 3.45425204
 3.44870439 3.4469224  3.44758705 3.44967182 3.45244104]
3 :
[3.58678078 2.77486303 2.75857296 2.74780805 2.74249816 2.74184656
 2.74474987 2.75007506 2.75682296 2.76420467 2.77165648]
4 :
[3.58678078 4.86927961 4.78708804 4.72452577 4.67915383 4.64770437
 4.62687624 4.61374186 4.60591418 4.60157046 4.59939703]
```

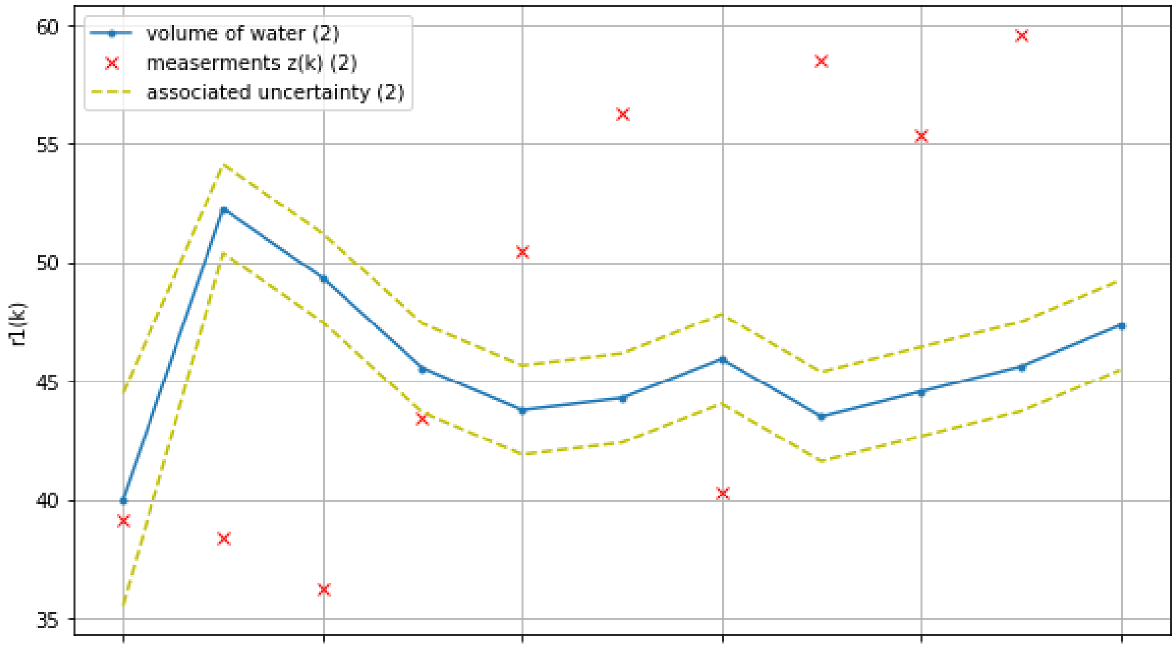
In [ ]:
```
mm = np.eye(3,8) # measurement model

mnv = np.eye(3)*25 # measurement noise variance
```

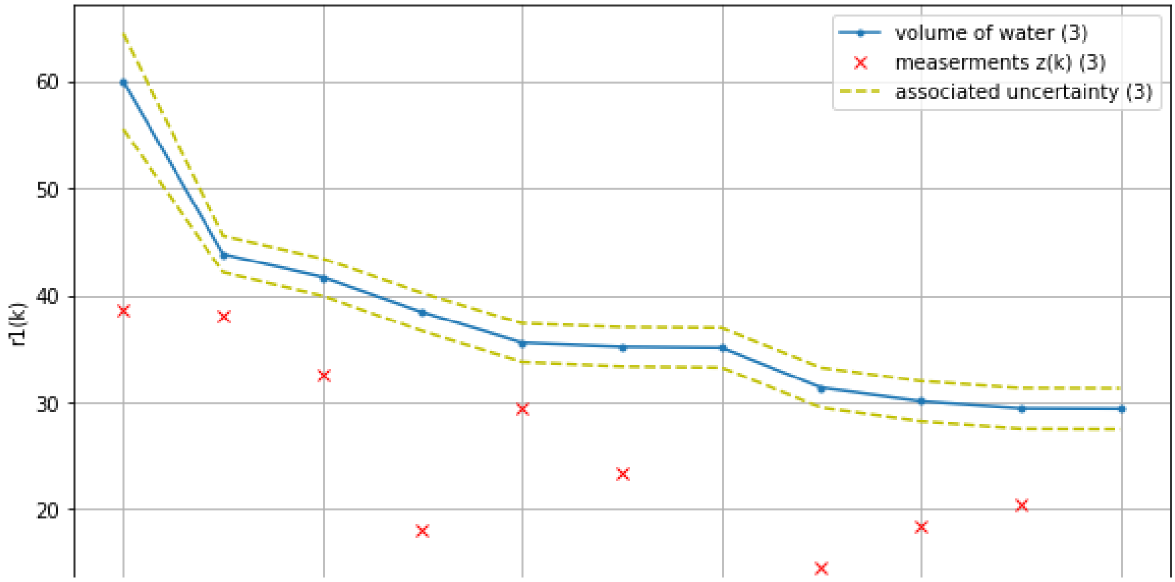
In [ ]:
```
list_of_measurements = [z1,z2,z4]
measurements = np.mat(list_of_measurements)
```

In [ ]:
```
for k in np.arange(1,n_steps+1):

    Kp = d @ Er0 + uf
    K_uncertainty = d @ sensor_uncertainty @ d.T + process_uncertainty

    measurement = measurements[:,k-1]

    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    Er0 = Kp + K @ (measurement - mm @ Kp)
    sensor_uncertainty = (np.eye(8) - K @ mm) @ K_uncertainty @ (np.eye(8) - K @ mm

    for i in range(8):
        water_volume[k,i] = Er0[i,0]
        uncertainty[k,i] = sensor_uncertainty[i,i]
```

In [ ]:
```
fig, ax = plt.subplots(3,1,sharex=True)
fig.set_size_inches(10,20)

for n in range(3):

    ax[n].plot(water_volume[:,n],'.-',label=f"volume of water ({n+1})")
    ax[n].plot(list_of_measurements[n],'rx',label=f"measerments z(k) ({n+1})")
    ax[n].plot(water_volume[:,n]+np.sqrt(uncertainty[:,n]),'y--',label=f"associate
    ax[n].plot(water_volume[:,n]-np.sqrt(uncertainty[:,n]),'y--',)

    ax[n].set_ylabel('r1(k)')
    ax[n].legend()
    ax[n].grid(True)
```

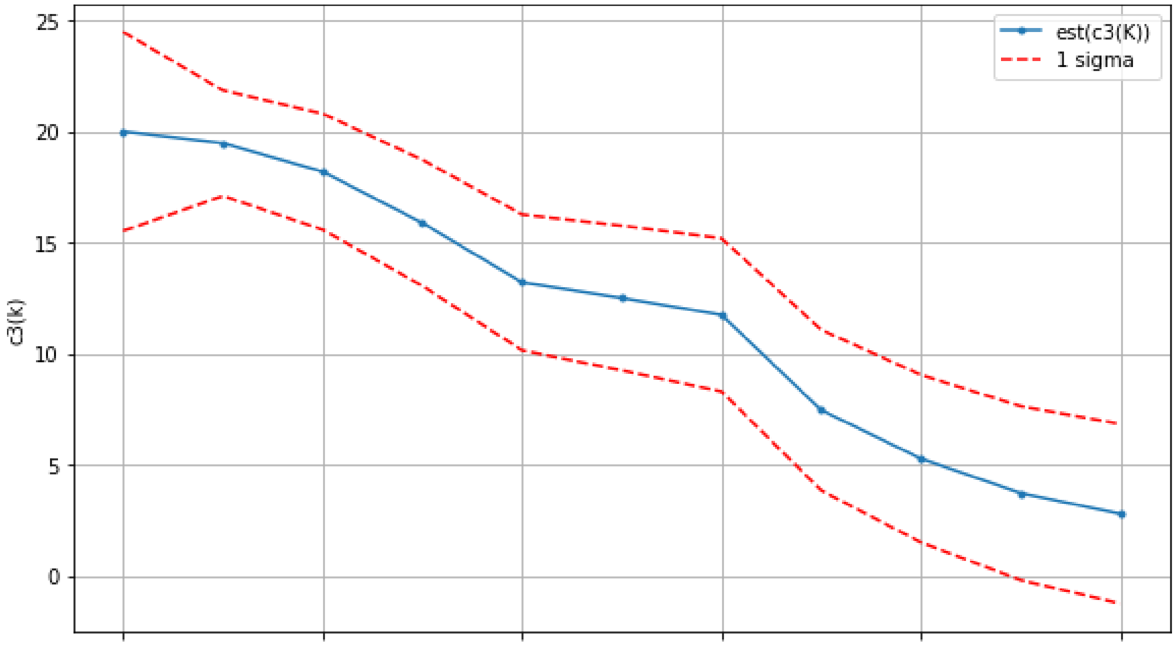

r1(k)
90
80
70
60
50
40
30
20
volume of water (1)
measerments z(k) (1)
associated uncertainty (1)


r1(k)
60
55
50
45
40
35
volume of water (2)
measerments z(k) (2)
associated uncertainty (2)

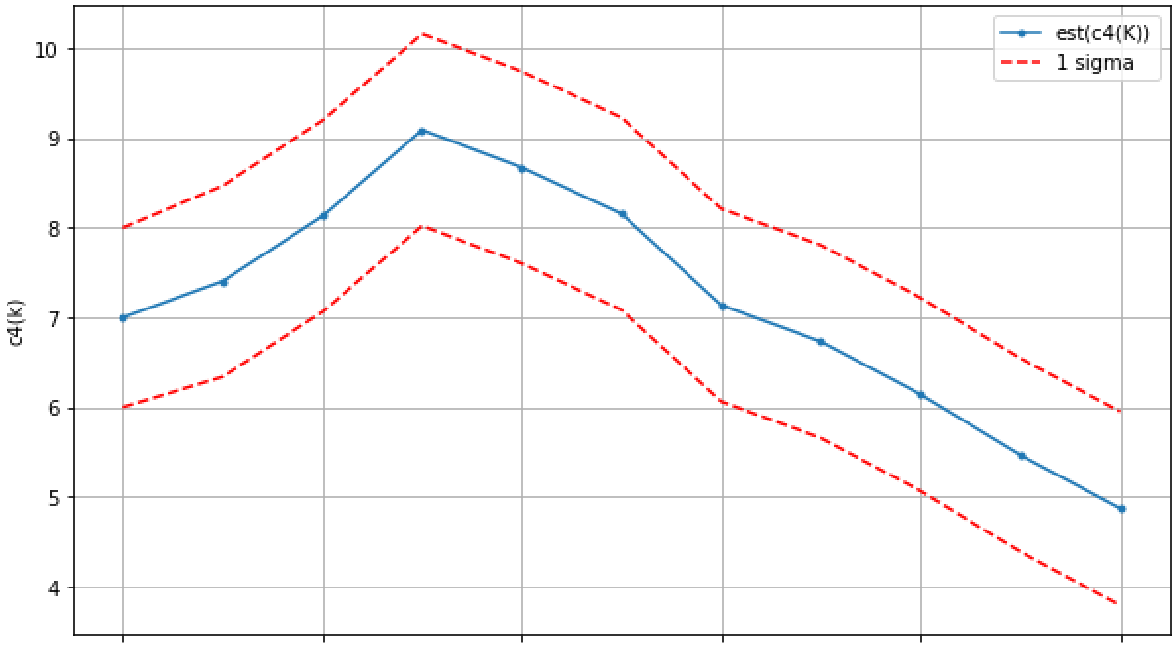

r1(k)
60
50
40
30
20
volume of water (3)
measerments z(k) (3)
associated uncertainty (3)

In [ ]:

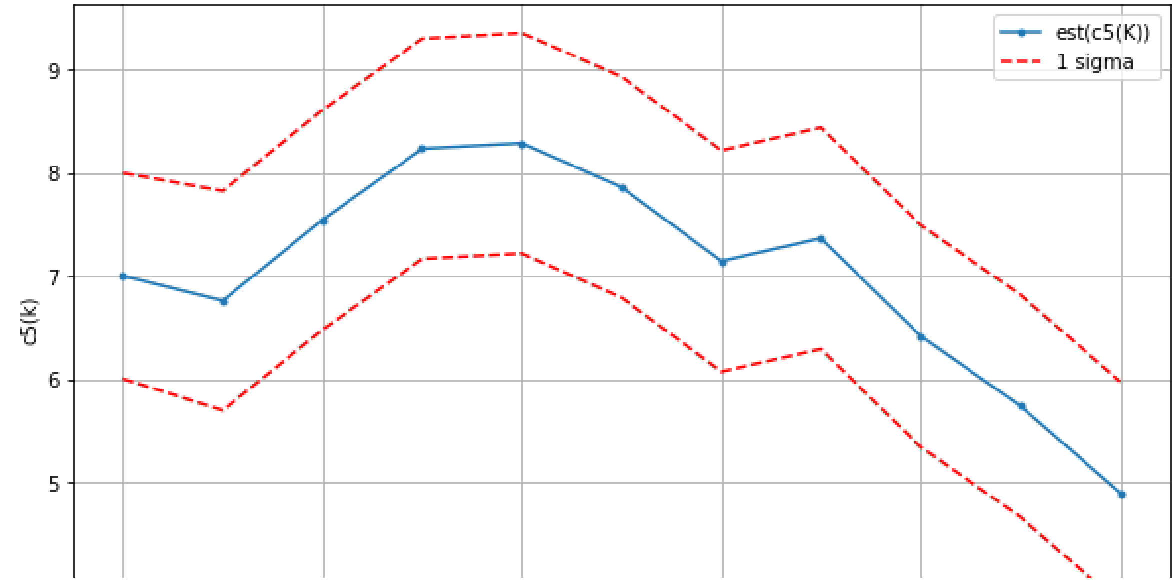
```
fig, ax = plt.subplots(3,1,sharex=True)
fig.set_size_inches(10,20)

for n in range(3):

    ax[n].plot(water_volume[:,n+3],'.-',label=f"est(c{n+3}(K))")
    ax[n].plot(water_volume[:,n+3]+np.sqrt(uncertainty[:,n+3]),'r--',label="1 sigma
    ax[n].plot(water_volume[:,n+3]-np.sqrt(uncertainty[:,n+3]),'r--',)

    ax[2].set_xlabel('Time k')
    ax[n].set_ylabel(f'c{n+3}(k)')
    ax[n].legend()
    ax[n].grid(True)
```



25
20
15
10
5
0
c3(k)
est(c3(K))
1 sigma



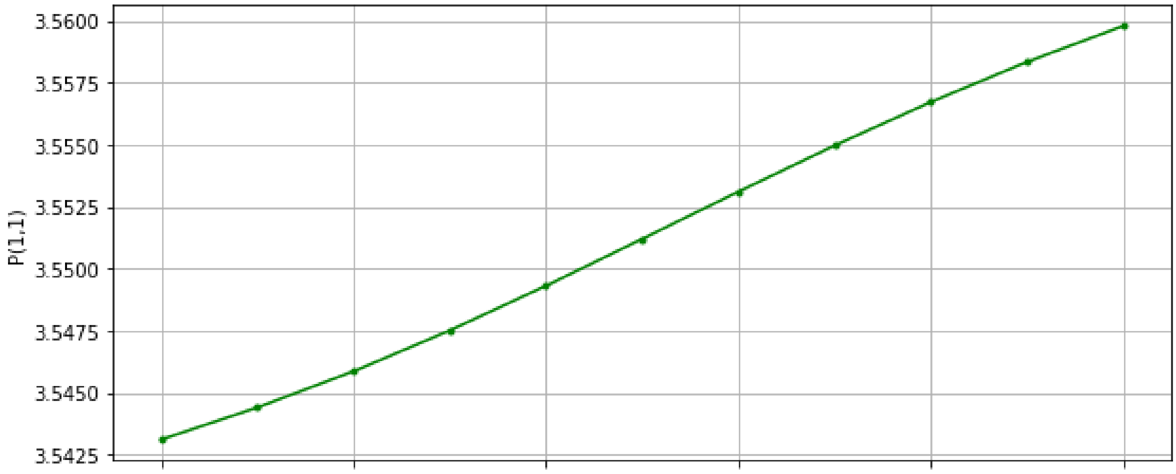
10
9
8
7
6
5
4
c4(k)
est(c4(K))
1 sigma

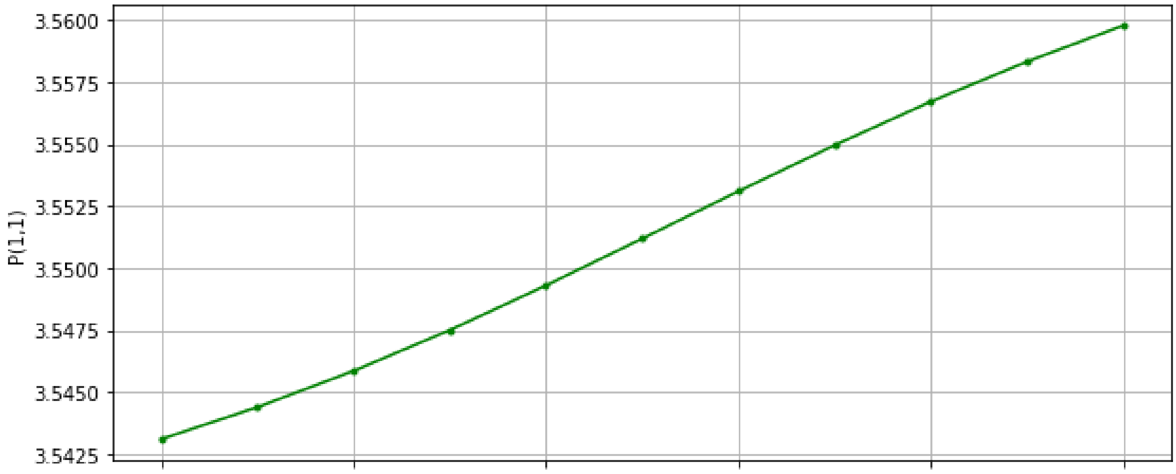


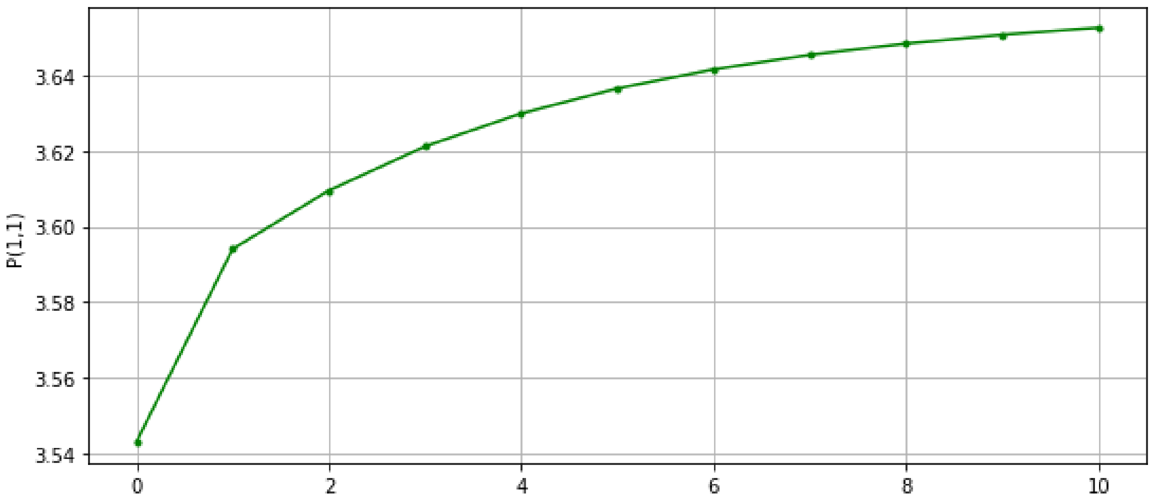
9
8
7
6
5
c5(k)
est(c5(K))
1 sigma

In [ ]:
```
uncertainties = [np.zeros([n_steps+1,1]),np.zeros([n_steps+1,1]),np.zeros([n_steps

for i in range(3):
    uncertainties[i][0,0] = sensor_uncertainty[0,0]

for s in np.arange(1,n_steps+1):

    K_uncertainty = d @ sensor_uncertainty @ d.T + process_uncertainty

    K = K_uncertainty @ mm.T @ np.linalg.inv(mm @ K_uncertainty @ mm.T + mnv)
    sensor_uncertainty = (np.eye(8) - K @ mm) @ K_uncertainty @ (np.eye(8) - K @ mm

    for i in range(3):
        uncertainties[i][s,0] = sensor_uncertainty[i,i]
```

In [ ]:
```
fig, ax = plt.subplots(3,1,sharex=True)
fig.set_size_inches(10,15)

for n in range(3):

    ax[n].plot(uncertainties[n][:,0],'g.-',label="P(1,1)")

    ax[n].set_ylabel('P(1,1)')
    ax[n].grid(True)
```







In [ ]:

```
for i in range(3):
    print(1+i,':')
    print(uncertainties[i][:,0])
```

```
1 :
[3.54310441 3.54439725 3.54585696 3.54750479 3.5493059  3.55119673
 3.55310543 3.55496532 3.55672244 3.5583385  3.55979079]
2 :
[3.54310441 3.54439725 3.54585696 3.54750479 3.5493059  3.55119673
 3.55310543 3.55496532 3.55672244 3.5583385  3.55979079]
3 :
[3.54310441 3.594111   3.60960955 3.62125136 3.63000331 3.63660853
 3.64162738 3.64547634 3.64846159 3.65080658 3.65267365]
```