

Chapter IV

Introduction to Quantum Mechanics – The Schrödinger Equation

I/ The wave properties of the electron

De Broglie's equation says that every moving object has a wavelength.

Louis de Broglie said that a free electron of mass (m) and speed (v) has a wave with a wavelength (λ) given by:

$$\lambda = \frac{h}{mv}$$

h: Planck's
constant

m: the mass

v: the velocity

λ : the wavelength

The wave behavior of a particle is stronger when its wavelength is larger — that means when the particle's mass or energy is smaller.

A moving electron has both particle and wave nature. However, it is considered sometimes as a wave and sometimes as a particle, depending on the phenomenon studied.

The electron is neither a wave nor a particle; it is a quantum particle.

III/ The uncertainty principle

Heisenberg showed that if we know the exact position of the electron, we cannot know its speed precisely; and if we know its energy exactly, we cannot know its position for sure.

Heisenberg concluded that it is impossible to know exactly at the same time the position of an electron and its energy if it is described as a wave.

$$\Delta p \Delta x \geq \frac{h}{2\pi}$$

$$P = mv$$

$$m \Delta v \Delta x \geq \frac{h}{2\pi}$$

h: Planck's constant

m: the mass of the particle

p: the momentum

X: the position of the particle

Consequence of Heisenberg's uncertainty principle for the electron:

- ✓ The trajectory cannot be determined using classical mechanics.
- ✓ The particle (corpuscular) nature cannot be used.
- ✓ The wave nature must be used.

Quantum mechanics replaces classical mechanics by defining the electron through its energy and its probability of being at a given point in space.

Schrödinger's equation (1926): a fundamental relation of quantum mechanics, suitable for particles with very small mass.

Solving it makes it possible to find the energy values available to the electron and the mathematical functions that describe its behavior.

To solve Schrödinger's equation, it is necessary to introduce three quantum numbers: n , l , and m .

V/ The quantum numbers

The principal quantum number, $n = 1, 2, 3, \dots$ can take any whole number value from 1 to infinity.

Its value is the main factor that determines the electron's shell.

The secondary quantum number, $l = 0, 1, 2, \dots, n - 1$ ($l \leq n - 1$)

Electrons in the same shell can be grouped into subshells, which are defined by the secondary quantum number (l).

The magnetic quantum number, $m = 0, -1, +1, -2, +2, \dots, -l, +l$ ($-l \leq m \leq +l$) shows how the orbitals in the same subshell are oriented in space.

They have the same energy but point in different directions.

The quantum number s (or spin quantum number) describes the spin of the electron

Its direction of rotation around its own axis.

It can have only two possible values: $+\frac{1}{2}$ or $-\frac{1}{2}$.

The information given by the quantum numbers

- ✓ The three quantum numbers identify an electron, just like an address.
- ✓ The table below shows the allowed combinations for the three quantum numbers.

Name	Symbol	Allowed Values	Property
Principal	n	positive integers 1,2,3...	Orbital size and energy level
Secondary (Angular momentum)	l	Integers from 0 to $(n-1)$	Orbital shape (sublevels/subshells)
Magnetic	m_l	Integers $-l$ to $+l$	Orbital orientation
Spin	m_s	$+\frac{1}{2}$ or $-\frac{1}{2}$	Electron spin Direction

