Chapter 2 : Numbring Systems

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Information coding

All information Outside the computer are in many forms :

- Numbres, text,
- Photos, Audio, Videos,
- Programmes, Applications, ...

Inside, they always represented in the binary form : sequence of 0 and 1

Information coding is the process of converting external information into an internal binary representation, using precise rules.

Example: The number 35 is the external representation of the number thirty-five. (100011) is the internal representation of 35 in the machine

Numbering system

A numbering system describes the way in which numbers a
represented. It is defined by :

- An alphabet A: a set of symbols or digits,
- Rules for writing numbers: Juxtaposition of symbols

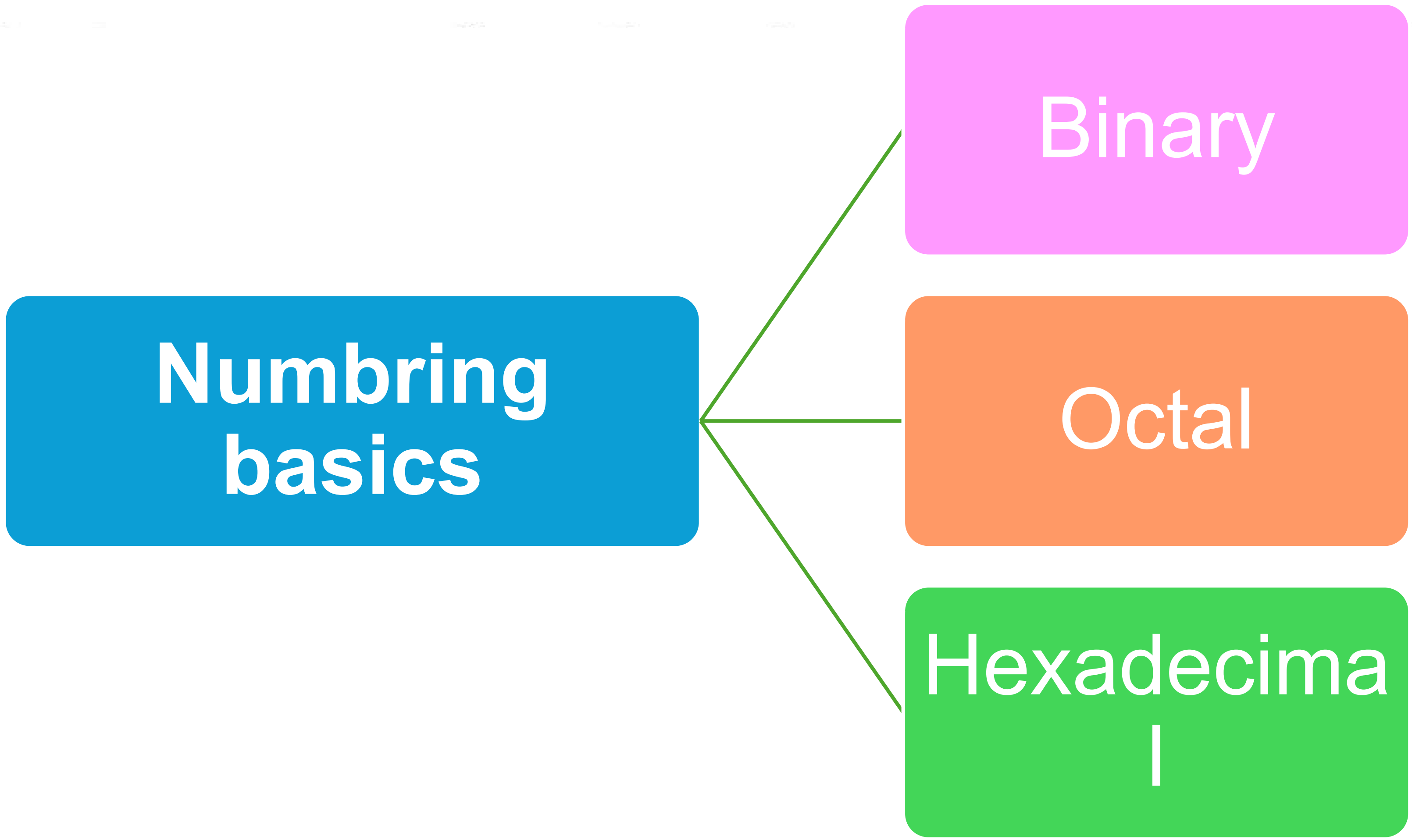
Example of a numbering system: **Decimal system**

Numbering system

Decimal system

The most widely used numbering system. The alphabet is made ten digits : $A = \{0,1,2,3,4,5,6,7,8,9\}$. The number 10 is the basi this numbering system.

Numbering system



Numbering system

- Binary system (b=2) uses two digits: {0,1}. It's the computer's
system.
- Octal system (b=8) uses eight digits: {0,1,2,3,4,5,6,7} Used fo
time in computing. It allows to encode three bits by a one octa
- Hexadecimal system (b=16) uses 16 digits:
{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}. Where: Où : $A=(10)_{10}$, $B=(11)_{1}$
$C=(12)_{10}$, $D=(13)_{10}$, $E=(14)_{10}$ et $F=(15)_{10}$. This base is widely us
computing systems. It allows 4 bits to be encoded by a one he
symbol.

Numbering system

Transcoding / Base conversion :

following conversions are possible :

- Decimal to Binary, Octal and Hexadecimal
- Binary to Decimal, Octal and Hexadecimal
- Octal to Decimal, Binary and Hexadecimal
- Hexadecimal to Decimal, Binary and Octal

Transcoding / Base conversion :

1. From decimal to other bases

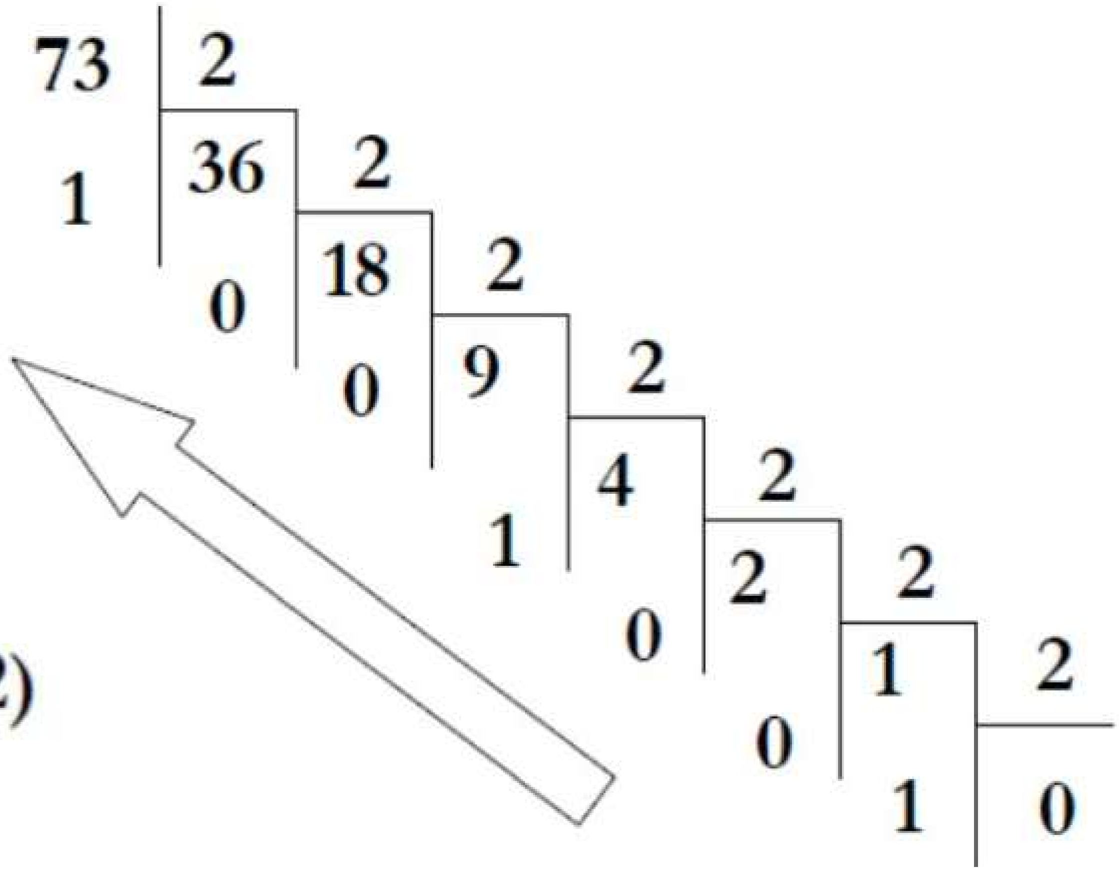
The rule is to follow a successive divisions :

- divide the number by the base b.
- Then divide the quotient by the base and so on unti
obtain 0.
- The sequence of remainders corresponds to the syn
of the base in request.
- The least significant digit is obtained first and the m
significant digit is the last.

Transcoding / Base conversion :

1.1. From decimal to Binary :

Example : Let N = $(73)_{10}$ a number presented in base 10.
The conversion to the base 2 is done as follow :



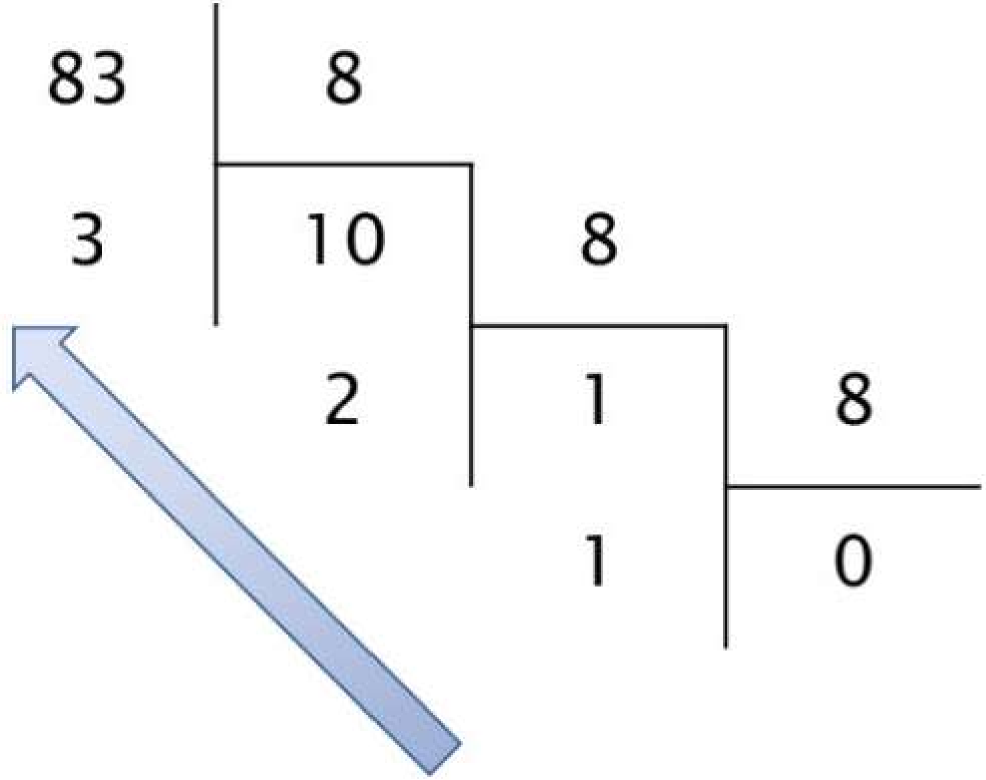
$$73_{(10)} = 1001001_{(2)}$$

Transcoding / Base conversion :

1.2. From decimal to Octal :

Example : Let $N = (83)_{10}$

The conversion to the base 8 octal is done as follow :



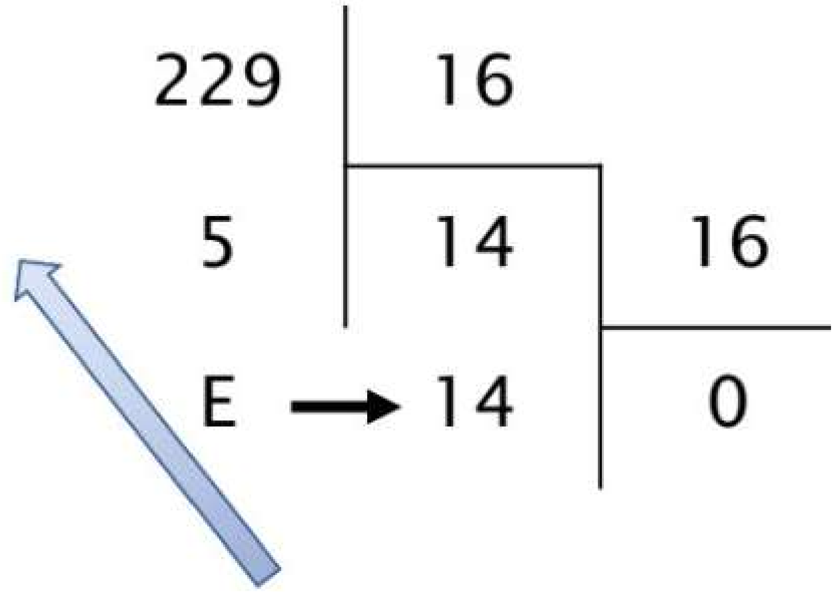
$$(83)_{10} = (123)_8$$

Transcoding / Base conversion :

1.3. From decimal to Hexadecimal :

Example : Let N = $(229)_{10}$
The conversion to the base 16 hexadecimal is done as follow

$(229)_{10} = (E5)_{16}$



Transcoding / Base conversion :

2. From other bases to decimal :

Example : Find the decimal representation of these numbers
$(1001001)_2$, $(123)_8$, $(E5)_{16}$

$$(\overset{6}{1}\overset{5}{0}\overset{4}{0}\overset{3}{1}\overset{2}{0}\overset{1}{0}\overset{0}{1})_2 = 1*2^0 + 0*2^1 + 0*2^2 + 1*2^3 + 0*2^4 + 0*2^5 + 1*$$

$$= 1 + 0 + 0 + 8 + 0 + 0 + 64$$

$$= \mathbf{(73)_{10}}$$

$$(\overset{2}{1}\overset{1}{2}\overset{0}{3})_8 = 3*8^0 + 2*8^1 + 1*8^2$$

$$= 3 + 16 + 64$$

$$= \mathbf{(83)_{10}}$$

$$(\overset{1}{E}\overset{0}{5})_{16} = 5*16^0 + E*16^1 = 5 + 14*16^1$$

$$= 5 + 224$$

$$= \mathbf{(229)_{10}}$$

Transcoding / Base conversion :

3. Binary to octal / hexadecimal:

Binary to octal :

- Grouping bits by three bits from rigtht to left,
- Then replace each group with the corresp symbol in the octal base.

Example : $(1011100)_2 = \underset{1}{\underline{001}}\ \underset{3}{\underline{011}}\ \underset{4}{\underline{100}} = (134)_8$

Transcoding / Base conversion :

3. Binary to octal / hexadecimal:

Binary to hexadecimal :

- Grouping bits by four bits from rigtht to left,
- Then replace each group with the corresp symbol in the hexadecimal base.

$$(1011100)_2 = \underbrace{0101}_{5}\ \underbrace{1100}_{C} = (5C)_{16}$$

3. Binary to octal / hexadecimal:

Correspondence tables

For Octale

Octal	Binaire
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

For Hexadécimal

Hexa	Binaire
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Hexa	B
8	[illegible]
9	[illegible]
A	[illegible]
B	[illegible]
C	[illegible]
D	[illegible]
E	[illegible]
F	[illegible]

Binary Arithmetic operations

Addition : Numbers must be aligned correctly, Afte numbers in each column are added, starting fror right by applying the following rules :

0 + 0 = 0 , 1 + 0 = 1 , 1 + 1 = 10, 1 +1 +1 = 11

Example : Do the addition of these numbers $(11011)_2$ and (1

$$\begin{array}{r} \scriptstyle 1\;1\;1\;1\;\; \\ 1\,1\,0\,1\,1 \\ +\;1\,0\,1\,0\,1 \\ \hline 1\,1\,0\,0\,0\,0 \end{array}$$

Binary Arithmetic operations

- **Substraction** : Numbers are written in successive rows in colu they must be aligned correctly, After we do the substraction between two numbers of each column, starting from the righ applying the following rules :

$$0 - 0 = 0 ,\quad 1 - 0 = 1 ,\quad 1 - 1 = 0,\quad 10 - 1 = 1 , 11 - 10 = 1$$

If subtraction is impossible (e.g. 0–1), 1 is acquired and retur the second column.

Example : Do the subtraction between these numbers : $(1101100)_2$ and $(10111)_2$:

Binary Arithmetic operations

Substraction

$$
\begin{array}{cccccccc}
- & 1 & 1 & {}^{1}0 & 1 & {}^{1}1 & {}^{1}0 & {}^{1}\boxed{0} \\
 & & & 1 & 0 & 1 & 1 & \boxed{1} \\
 & & 1+ & & 1+ & 1+ & 1+ & \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}
$$

Direct binary subtraction is restricted, which is why t two's complement method is used.

Binary Arithmetic operations

Multiplication :

- **Numbers must be aligned correctly,**
- **After we multiply each number of the second row**
  all numbers of the first row starting from the right
- **Then we add the results,**
- **The following rules must be applied:**
- **0 * 0 = 0 , 1 * 0 = 0 , 1 * 1 = 1,**

Exemple : Do this operation : $(1101)_2 * (101)_2$:

Binary Arithmetic operations

Exemple : Do this operation : $(1101)_2 * (101)_2$:

```
      1 1 0 1
        1 0 1
  -----------
      1 1 0 1
    0 0 0 0 .
  1 1 0 1 . .
  -----------
1 0 0 0 0 0 1
```

Binary Arithmetic operations

Division : Binary division is applied in the same w
decimal division, with successive subtraction,

- We take the same number of bits on the left as c
right,
- adding one bit if subtraction is impossible,
- then marking a “1” each time subtraction is pos
otherwise marking a “0”.
- To proceed the next operation, the next bit is
and the process is repeated until all the bits on th
have been completed,

Binary Arithmetic operations

Division :

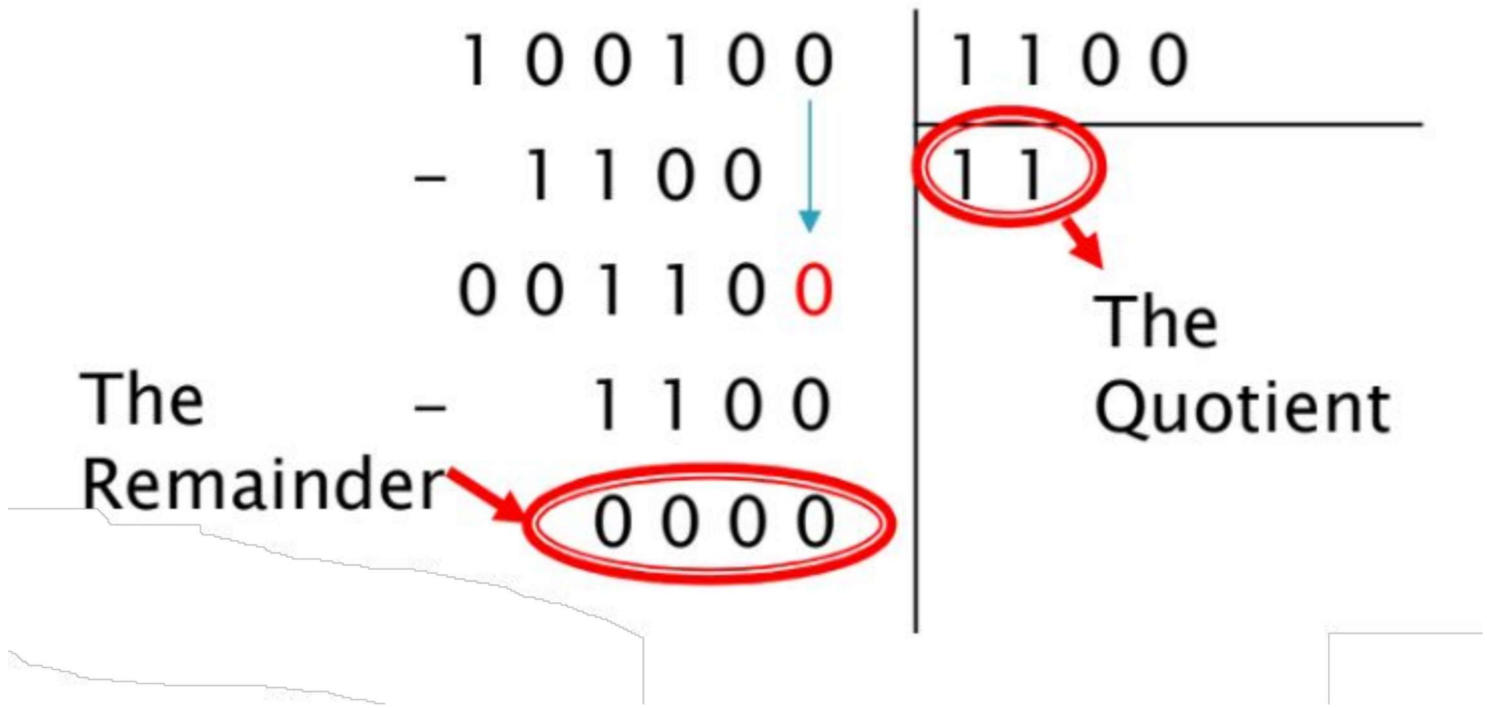
Example : Do these operations : $(100100)_2$ / $(1100)_2$,
$(111100)_2$ / $(1101)_2$

```
   1 1 1 1 0 0 | 1 1 0 1
 − 1 1 0 1     |--------
   -------     | 1 0 0
   0 0 1 0 0 0 |
```

Binary Arithmetic operations

Division :

Example : Do these operations : $(100100)_2$ / $(1100)_2$, $(111100)_2$ / $(1101)_2$



1. Overview : Unsigned binary numbers

m binary digits (bits) of memory can store 2^m d

numbers. They can be positive integers between

$00\ldots00 = \left(0\right)_{10}$ and $11\ldots11 = \left(2^m - 1\right)_{10}$.

For example, using $\boldsymbol{m = 3}$ bits, we can represe

integer between 0 and 7.

2. Signed binary numbers

In order to represent **signed numbers** (i.e., both posi negative numbers) using m bits, we must reserve **on** **indicate the sign** which is the **M**ost **S**ignificant **B**it **(MSB).**

The **MSB** bit indicates the sign (0 = positive, 1 = negative).

The remaining $\boldsymbol{m-1}$ bits represent the magnitude (absolu

This allows us to represent integers from $-\left(\mathbf{2^{m-1}-1}\right)$ to $+\left(\mathbf{2^{m-1}-1}\right)$.

Two’s Complement Representation

- **two’s complement is** obtained by **inverting** a
  bits of the positive number and **adding** 1.

Example : Find the two's complements of the follo

signed binary numbers : **(101100)2 (on 6 bits)**

(01010) (on 5 bits)

Two's Complement Representation

1) **What is the decimal value of (101) In the ca**
   signed bignary numbers and the case of
   unsigned binary numbers ?

2) **In signed binary what are the decimal valu**
   (1010) , (0101)