Analysis I

Exercise Set 4 : Elementary Functions

Exercise 1.
• Compute

$$\arccos\left(\cos\left(\frac{2024\pi}{3}\right)\right); \quad \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right); \quad \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) + \arctan\left(\frac{1}{8}\right).$$

Formula:

$$\arctan a + \arctan b = \arctan\frac{a+b}{1-ab}, \quad \forall(a,b) \in [0,1[^2.$$

• Show the following inequality

$$0 < \arccos\left(\frac{3}{4}\right) < \frac{\pi}{4}.$$

• Draw the graph representing of $\arcsin(\sin x)$ and $\sin(\arcsin x)$.

Exercise 2.
• Simplify

$$\cos(\arcsin x); \quad \arcsin(2x\sqrt{1-x^2}); \quad \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\cos^2\left(\frac{1}{2}\arctan(x)\right); \quad \arctan\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) \ (\forall x \in]-\pi,\pi[)$$

$$\text{argsh}(2x\sqrt{1+x^2}); \quad \text{ch}(\text{argth}x); \quad \text{argsh}(3x+4x^3).$$

Show the following identities

- $\forall x \in [0,1] \ \arcsin(\sqrt{x}) = \frac{\pi}{4} + \frac{1}{2}\arcsin(2x-1)$ (One set: $x = \sin^2(u)$)

- $\forall x \in [-1,1] \ \arccos(1-2x^2) = 2\arcsin|x|$

- $\forall x \geq 0 \ \arctan(\text{sh}(x)) = \arccos\left(\frac{1}{\text{ch}(x)}\right)$

Exercise 3.
• Solve the following equations

$$\arccos(x) = 2\arccos\left(\frac{3}{4}\right); \quad \arcsin x = 2\arctan x; \quad \text{ch}x + \cos a = 2\text{sh}x + \sin a.$$

• Solve, by setting $x = \tan(\theta)$, the equation

$$\arcsin\left(\frac{2x}{1+x^2}\right) + \arccos\left(\frac{1-x^2}{1+x^2}\right) = 0.$$