Probability and Statistics
Semester 1

Worksheet 3: Probabilities

Exercise 1

Let A be the set of 7-digit numbers containing no digit 1. Determine the number of elements in the following sets:

1. A.

2. A_1: numbers in A having 7 different digits.

3. A_2: even numbers in A.

4. A_3: numbers that are multiples of 5.

5. A_4: numbers in A whose digits form a strictly increasing sequence.

Exercise 2

1. Four people are in an elevator serving 6 floors. Determine the number of possibilities where:

   (a) all 4 people stop on different floors;

   (b) exactly two people stop on the same floor.

2. How many groups of six can be formed from 4 boys and 6 girls if the group must contain exactly 2 boys? Interpret the cases: "given", "only", "at least".

3. We want to arrange on a shelf 3 distinct mathematics books, 5 distinct physics books, and 2 distinct chemistry books. In how many ways can we perform this arrangement:

   (a) without restrictions;

   (b) if books must be stored together by subject;

   (c) if only chemistry books must be stored together.

Exercise 3

A child colors five boxes numbered 1 to 5 using 8 colors at random.

1. How many drawings can be made?

2. How many if no color is used twice?

3. How many if he uses only 2 colors, one appearing three times and the other twice?

Exercise 4

Using the letters of the word CONSEIL, how many different 7-letter words can be formed:

1. in total?

2. if the vowels must appear together?

3. if vowels must be together and consonants must be together?

4. if letters C and S must never be adjacent?

5. if the letter C must be in position 1 or position 2?

Exercise 5

In a group of 20 people, 10 read at least magazine A, 8 read at least magazine B, and 3 read both magazines. How many different samples can be selected if the sample must consist of:

1. five people reading at least one magazine?

2. three reading A only and two reading B only?

3. five people, at least three of whom read magazine A?

Exercise 6

1. How many diagonals does a planar polygon with n vertices have?

2. How many intersection points do these diagonals determine?

3. Which polygon has as many diagonals as sides?

Exercise 7

Let Ω be the sample space of a random experiment and let A, B, C be events. Express the following events using A, B, C:

1. only A occurs;

2. at least one occurs;

3. at most one occurs;

4. all three occur;

5. A and B occur but not C.

Exercise 8

Let A and B be two events. Compute $P(A \cup B)$ in the following cases:

1. A and B are disjoint;

2. A is a subset of B.

Then compute $P(A \cap B)$ and $P(A^c \cap B^c)$ in both cases.

Exercise 9

Three people are born in the same year of 365 days. Compute the probabilities that:

1. all three have different birthdays;

2. exactly two share a birthday;

3. all three share the same birthday.

Exercise 10

We throw three dice successively. Let:

- A: all 3 faces show at least one ace (i.e., each die shows 1);

- B: at least two faces are identical.

Compute $P(A)$, $P(B)$, and $P(A \cap B)$.

Exercise 11

In a lottery, each ticket has a probability $1/100$ of winning.

1. We buy 2 tickets. What is the probability of having at least one winning ticket?

2. For n tickets, compute the probability of having at least one winning ticket.

3. How many tickets must be purchased to be 50% sure of winning at least once?

Exercise 12

Two cities A and B are separated by 3 traffic lights, each of which may be red (R), green (G), or orange (O). Green lights last 40, 30 and 20 seconds respectively. Assuming independence and that the driver respects traffic laws, find the probability of making the trip:

1. without stopping;

2. with exactly one stop;

3. with exactly two stops;

4. with at least one stop.

Exercise 13

In a referendum, two questions were asked. 65% answered "yes" to Q1, 51% answered "yes" to Q2, and 30% answered "no" to both. A person is selected among those who answered both questions. Let:

- A: answered yes to Q1,

- B: answered yes to Q2.

Compute the probability that the chosen person:

1. answered yes to both questions;

2. answered yes to Q1 and no to Q2;

3. answered yes to exactly one question;

4. answered yes to at least one question.

Exercise 14

30% of candidates take exam C1 and 70% take C2. If a candidate takes C1, the probability of failing is 45%. If they take C2, the probability of failing is 20%. Let:

- A: take C1,

- B: take C2,

- E: fail.

1. Represent the situation with a probability tree.

2. Knowing that the candidate took C2, compute the probability of success.

3. Compute the overall probability of failing.

4. Given that the candidate passed, compute the probability that they took C2.

Exercise 15

An aircraft manufacturer equips its four-engine plane with 2 type A engines and 2 type B engines, arranged symmetrically on the wings. Each engine may fail independently. Let p be the failure probability for type A and q for type B. The plane cannot fly if two engines on the same wing fail.

1. List all situations where the plane cannot fly.

2. Compute the probability of each situation.

3. Deduce the probability that the plane can fly.

4. Numerical application: $p = 0.002\%$, $q = 0.01\%$.

Exercise 16

A quarter of the population is vaccinated. Among the sick, one vaccinated person is observed for every four unvaccinated. During the epidemic, one vaccinated person out of 12 became sick. What is the probability of becoming sick for an unvaccinated person? Is the vaccine effective?