National Higher School of Autonomous Systems Technology
Academic year 2025/2026

Physics 1 "Classical Mechanics" Recitation : Problem Sets $N°04$

Chapter 04 : Work and Energy ; Work-Energy Theorem

Exercise 1 – To be solved in Lecture
Consider a point-like particle moving under the action by a force field given by,

$$\vec{F} = (2xy + z^3)\,\vec{i} + x^2\,\vec{j} + 3xz^2\,\vec{k} \qquad (1)$$

Verify if $\overrightarrow{rot}\,\vec{F} = \vec{\nabla} \wedge \vec{F} = 0$, then $\vec{F}$ is derived from potential E_p (called potential energy) that we ask you to find its expression.

Exercise 2 – To be solved in Lecture
Consider a particle moving between two points A(0, 0) and B(3, 3) of a frame of reference $\Re(O; x, y)$: it is subjected to a force

$$\vec{F} = 2xy^2\,\vec{i} + 2x^2y\,\vec{j} + 0\,\vec{k} \qquad (2)$$

1. Find the work done on the particle by the force $\vec{F}$ if the particle describes from the point A to B the following paths :
   (a) the segment AB from A to B.
   (b) the segment BA from B to A.
   (c) along the closed path ABA. The line segment AB from A to B followed by the line segment BA from B to A where physical units are being omitted.
   (d) along the x axis from $(0, 0)$ to $(3, 0)$ followed by another path parallel to y axis until $(3, 3)$.
   (e) along the x axis from $(3, 3)$ to $(0, 3)$ then parallel to the y axis until $(0, 0)$.
2. Find $\overrightarrow{rot}\,\vec{F}$
3. Conclusion.

Week eleven

Exercise 3 – To be solved in Recitation
After its release at the top of the first rise, a roller coaster car moves with negligible friction. The roller coaster has a circular loop of radius $R = 20.0\ m$. The car barely makes it around the loop : at the top of the loop, the riders are upside-down and feel weightless as shown in figure 1.

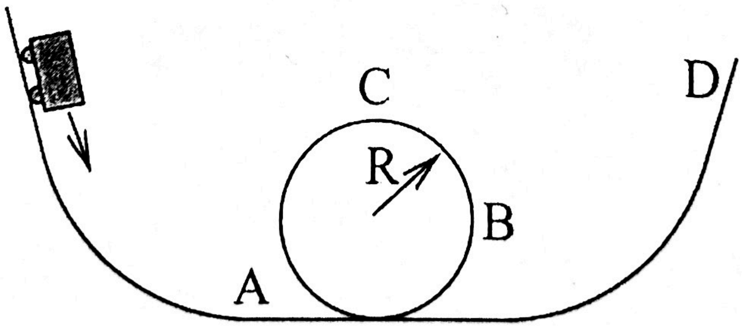

FIGURE 1 – .

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① Find the speed of the roller coaster at the top of the loop (position C).

② Find the speed of the roller coaster at position A, at the bottom.

③ Find the speed of the roller coaster at position B, halfway up the loop.

④) Find the difference in height between positions A and D if the speed at D is $10.0\ m/s$.

Exercise 4 – To be solved in Recitation

A pendulum consists of a bob M of mass $m = 20\ g$ hanging from a massless inextensible string of length $L = 40\ cm$. We pull the string out to an angle $\theta_0 = 45°$ with the vertical and released as shown in figure 2, where its initial velocity is zero. Use $g = 9.81\ ms^{-2}$ and neglect the resistance due to the air.

1. Write and draw the free-body diagram which shows all external forces acting on the object.
2. Find the work done by the weight between the initial position and the vertical position.
3. Find the work done by the tension in the string.
4. Find the angular momentum of the bob relative to O.

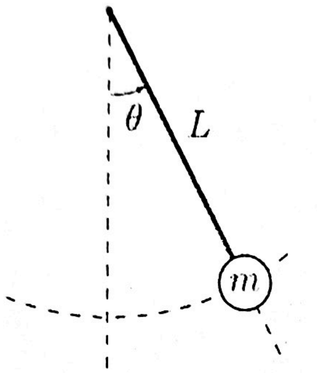

FIGURE 2 – Simple pendulum.

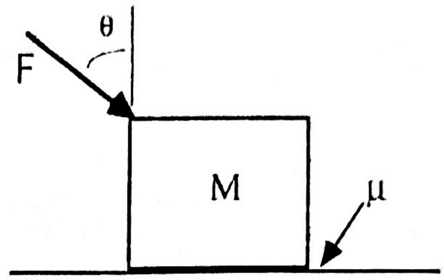

FIGURE 3 – A block M is pushed along a horizontal surface.

Exercise 5 – Homework

Consider the situation shown in the drawing 3 at the right. A block of mass M is pushed along a horizontal surface by a force F making an angle θ with the vertical. The coefficients of static and kinetic friction are identical and equal to μ.

① Make a clear drawing showing the direction of all the forces acting on the mass, and find the magnitude of all the forces in terms of F, M, g, μ, and θ.

② Assume that the object moves a distance B to the right. Find the work done by each force you listed in part (a).

③ If the block has a velocity, V_1, to the right initially, find it's velocity, V_2, after it has moved the distance B to the right.

④ Explain how you would decide if the final velocity, V_2, is larger than, equal to, or smaller than the initial velocity, V_1.

Week twelve

2

Exercise 6 – To be solved in Recitation

We use a spring where one end is fixed at point A as shown in figure 4, to launch a point-like crate M of mass $m = 300\ g$ along a track which consists of two portions. First, along a horizontal rectilinear track AB of length $3R$ and then on curved track $\overset{\frown}{BC}$ which is a forth of vertical circumference of radius R a nd a center O_1. We neglect the air drag force. The spring is ideal with force constant (stiffness constant) of the spring $k = 50\ N/m$ and an unstretched length of $l_0 = R = 30\ cm$. The Ox axis is chosen as reference for gravitational potential energy.

Cont.

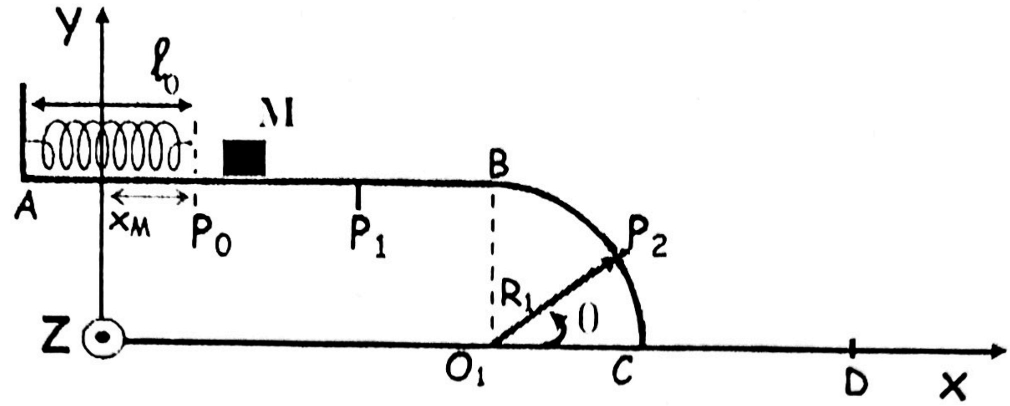

FIGURE 4 – Motion of crate along a rectilinear then along a curved tracks

Assume that the static coefficient and the kinetic (sliding) coefficient of friction between the crate and the horizontal track AB are respectively, $\mu_s = 0,7$ and $\mu_g = 0,3$. However, the friction is neglected between the carte and curved portion $\overset{\frown}{BC}$. The motion is investigated relative to the frame of reference $\Re(O; x, y, z)$ and the acceleration of gravity $g = 10\ ms^{-2}$.

1. At $t = 0$, the spring is compressed from equilibrium and is then released with no initial velocity.
   (a) calculate the maximum compression of the spring x_{M1} for which the mass m remains at rest when it is released.
   (b) Use a convenient scale to Free-body diagram of the forces acting on the crate M.
2. Under a compression x_M, the crate M arrives at point B with velocity $v_B = 1.5\ (m/s)$.
   (a) Apply the work-energy theorem to find the expression of the kinetic energy of M.
      i. for any position M of the crate, located before the P_0. Deduce then the expression of the kinetic energy of M (E_{c,P_0}) at point P_0:
      ii. now for any position P_1 of the crate located between P_0 and B. Deduce then the value of the compression x_M for which the crate M will arrive at the point B with velocity $v_B = 2\ (m/s)$.
   (b) Find the work by force of friction between P_0 and B.
3. We suppose that the opposite path could happen, find the work by force of friction between B and P_0.
4. Calculate the work done by this force of friction along the closed path P_0BP_0. Conclusion.
5. Write Newton's second law of motion for any position P_2 of the crate between B and C.
6. Project this equation in the polar coordinates and determine the normal force $C_\perp$ of the track $\overset{\frown}{BC}$ on the crate M.
7. Find again this result by applying the work-energy theorem.
8. From where (position determined by the angle $\theta_q = (\overrightarrow{O_1C}, \overrightarrow{O_1P_2})$) the crate lose contact with the $\overset{\frown}{BC}$.
9. In the case where $v_B = 0$, what is the value θ_{q0} of this angle?
10. Find the value of the minimum velocity $v_{B,M}$ for which the crate M leaves the surface $\overset{\frown}{BC}$, as it passes by B, it describes a parabolic trajectory in the air and then falls on the plane CD.
11. What will be the ulterior motion of the crate?
12. Calculate the maximum range and the time when the crate falls on the plane CD.
13. Deduce its velocity at that time.
14. Find again this result by applying the principle of conservation of mechanical (total) energy.

Exercise 7 – To be solved in Lecture

① An object of mass, $M = 2\ kg$, is attached to a spring of spring constant k=50 N/m which is compressed a distance d=20 cm and then released at rest as shown in figure 5. Find the speed of the object when it has gone past the point where the spring is uncompressed and now the spring is stretched a distance of 10 cm. Assume that the mass is moving on a horizontal, frictionless surface.

Cont.

② Write an equation for the position of the mass as a function of time with t=0 being the instant that the mass was first released from rest. Use this equation to find out how long it takes the mass to get from the initial point with the spring compressed by 20 cm to the point where the spring is stretched by 10 cm. (Hint : Think carefully about the units of ω when doing the trig functions on your calculator.)

③ Write an equation for the velocity as a function of time, using the same definition of t=0 as in part (b). Use this equation to find the velocity (magnitude and direction) when the time is $t = 3\frac{T}{4}$ where T is the period of motion of the mass on the spring.

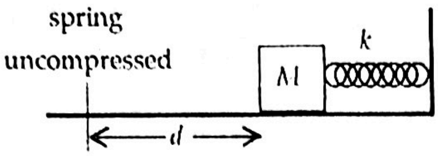

FIGURE 5 – Spring mass system.

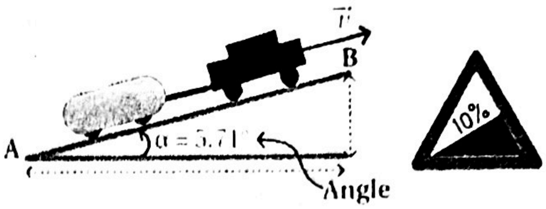

FIGURE 6 – Towing a caravan uphill.

Exercise 8 – Homework
A four by four car is towing up a hill steep 10% slope (which is inclined at an angle $\theta = 5.71°$ to the horizontal), with a constant driving force of $F = 3000\ N$, as shown in figure 6. Given that the mass $m = 500\ kg$ of the caravan is $m = 500\ kg$. Use $g = 10\ N.kg^{-1}$.
Find the work done by the car and by the caravan weight during a displacement of $AB = 100\ m$ along the slope in upward direction.

Exercise 9 – Homework
A 10.0-kg block is released from point A in Figure 7. The track is frictionless except for the portion between B and C, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant $k = 2250\ N/m$, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between B and C.

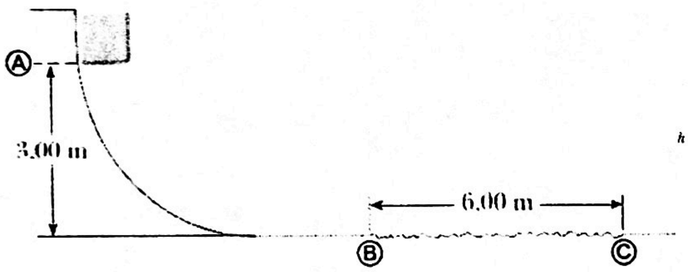

FIGURE 7 – Spring mass system.

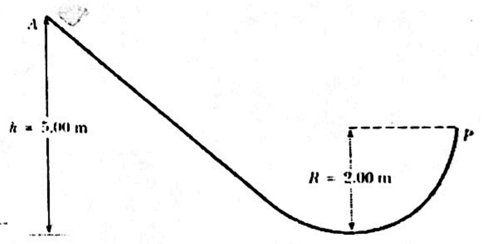

FIGURE 8 – Towing a caravan uphill.

Exercise 10 – Homework
A 6.00 kg block is released from A on the frictionless track shown in Figure 8. Determine the radial and tangential components of acceleration for the block at P.

Exercise 11 – Homework
A bowling ball of mass m and radius R is thrown down an alley with an initial velocity v_0 and no initial angular velocity. The moment of inertia of the ball about the center of mass is $I = 2/5mR^2$. The coefficient of friction between the ball and the alley is μ.

① What is the angular velocity of the ball when the ball just starts to roll without slipping down the alley ?

② What is the kinetic energy of the ball when the ball just starts to roll without slipping down the alley ?

③ What is the change in kinetic energy ?

Cont.

Exercise 12 – Homework

A child's playground slide is 5.0 m in length and is at an angle of 20° with respect to the ground. A child of mass 2.0×10^1 kg starts from rest at the top of the slide. The coefficient of sliding friction for the slide is $\mu_k = 0.2$.

① What is the total work done by the friction force on the child?

② What is the speed of the child at the bottom of the slide?

③ How long does the child take to slide down the ramp?

Exercise 13 – Homework

A skydiver of $m = 8.0 \times 10^1$ kg reaches a terminal velocity of 5.0×10^1 m/s. Suppose the diver falls $1.6 km$. Assume there isa drag force acting on the skydiver.

① How long does it take the skydiver to fall 1.6 km?

② How much work does the gravitational force do on the falling diver?

③ How much work does the drag force do on the falling diver?

④ What is the power corresponding to the work done by the drag force? What is the total energy generated by the drag force? Where does this energy go?

Exercise 14 – Homework

Rubber bands have a spring constant k and are attached to a mass m_1. The rubber bands are intitally stretched spring a distance y from the equilibrium position.

① What is the period of oscillation for this system?

② What is the velocity of the mass when it reaches the equilibrium position?

③ When the rubber bands are completely compressed, a second mass $m2 = 2m_1$ is attached to the first mass, completely inelastically. What is the new period of the system?

④ Will the rubber band-mass now extend to the same distance y? Assume mechanical energy is conserved.

Exercise 15 – Homework

A small block starts from rest and slides down from the top of a fixed sphere of radius R, where $R >> $ size of the block as shown in figure 9. The surface of the sphere is frictionless and constant gravitational acceleration g acts downward.

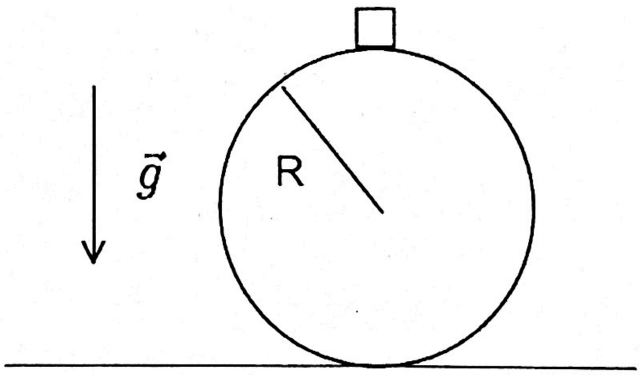

FIGURE 9 – Double pulley system

① Determine the speed of the block as a function of angle from the top while it remains in contact with the sphere.

② At what angle does the block lose contact with the sphere?

5

References

[1] http ://physics.bu.edu/ okctsui/%20Lecture notes/HW2009/HW9.pdf

[2] Halliday, D., Resnick, R. and Walker, J., 2013. Fundamentals of physics. John Wiley & Sons. [3] DiLisi, Gregory A. Classical Mechanics, Volume 3 : Newton's Laws and Uniform Circular Motion. Morgan & Claypool Publishers, 2019.

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The End.