

الجمهورية الجزائرية الديمقراطية الشعبية  
People's Democratic Republic of Algeria

Ministry of Higher  
Education and Scientific  
Research  
National Higher School of  
Autonomous Systems  
Technology



وزارة التعليم العالي والبحث العلمي  
المدرسة الوطنية العليا للكمبيوترجي  
والأنظمة المستنيرة

2025/2026

Probability and Statistics

Semester 1

Worksheet No. 2: Bivariate Statistical Series

Exercise 1:

In an agricultural experiment, the influence of the water factor on sugar beet yields was studied. The following results show X, the quantities of water used, and Y, the corresponding beet yields:

$x_i$ ( $m^3/ha$ )	3	5	6	8	9	11	13	15	17
Yield $y_i$ ( $T/ha$ )	5	7	8	10	12	14	17	20	23

- Calculate the marginal variances and the covariance( $x, y$ ).
- Justify the construction of the regression line and give its equation.
- Provide an interpretation of the results.

Exercise 2:

A survey was conducted on 100 households regarding monthly expenditures X and monthly income Y in thousands of dinars. The following table was obtained:

$X \backslash Y$	[4 - 10[	[10 - 20[	[20 - 40[	
$X$	20	10	0	30
[3 - 5[	10	20	10	40
[15 - 35[	0	10	20	30
	30	40	30	

1. Determine the marginal distributions of  $X$  and  $Y$ .
2. Are  $X$  and  $Y$  independent?
3. Calculate the mean income.
4. Calculate the mean income for households spending less than 15,000 DA.
5. Calculate the correlation coefficient. Comment.

**Exercise 3:**

Consider the following table showing the selling price:  $y$  ( $10^4$  euro) of a used vehicle as a function of its age:  $x$  (in years).

Age $x_i$	1	2	3	4	5	6	7	8
Price $y_i$	2.5	1.7	1.2	1.1	0.9	0.8	0.78	0.4

1. Let  $u = \log(x)$  (decimal logarithm). Calculations will be performed using values to  $10^{-4}$ .
2. Calculate the linear correlation coefficient between  $u$  and  $y$ .
3. Determine the equation of the regression line of  $y$  on  $u$ , using the least squares method.
4. Estimate the selling price of a vehicle aged 10 years.

**Exercise 4:**

Given the following bivariate statistical table for characters  $X$  and  $Y$ :

$X$	$Y$	7	15	30
5	0	1	2	
10	0	2	3	
20	3	3	0	

1. Plot the scatter plot.
2. Are  $X$  and  $Y$  independent? Justify your answer.
3. Determine the conditional distribution of  $X$  given  $Y = 15$  and the conditional mean.
4. Calculate the marginal means and marginal standard deviations of  $X$  and  $Y$ .

**Exercise 6 (Homework):**

Consider the following bivariate statistical series:

<i>X</i>	<i>Y</i>	0	1	2	3	4	Totals
[0 - 6[	5	2					
[6 - 12[	1	5	2				
[12 - 18[		1	7	1			
[18 - 24[			1	6	2		
[24 - 30[						7	
Totals							

1. What is the nature of the variables X and Y?
2. Construct the scatter plot of this bivariate series.
3. Construct the regression curve of Y on X and the associated regression line in the same frame as the scatter plot.
4. Calculate the linear correlation coefficient.
5. Calculate the equation of the regression line of Y on X.
6. Estimate the value of X for Y = 33.