Lab 3 Pendulum motion

Aims:

1-/ Determine the relationship between pendulum length, pendulum mass, and the period of oscillation.

2-/ Determine the value of gravitational acceleration, with the errors of experiment.

3-/ Determine whether mechanical energy is conserved as the pendulum swings.

Apparatus:

1. String (of adjustable length).
2. Small bob (weight).
3. Chronometer.
4. Photogate timer (SpeedGate 197570).
5. Ruler.
6. Protactor.

Theory:

A simple pendulum is a mass (bob) attached to a string or rod, free to swing back and forth under the influence of gravity. The only forces acting on the bob are gravitational force $m\,\vec{g}$ and the tension $\vec{T}$ in the string. Therefore, the bob's trajectory forms a segment of a circle with radius l. In this case, we can break down the forces into a tangential component and a normal component.

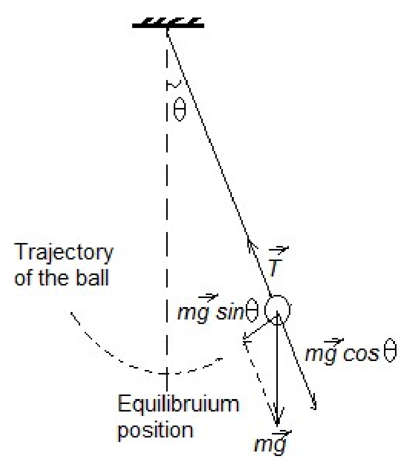
The tangential force F_t acting on the mass m is $F_t = -mg\ sin(\theta$

And the normal force F_t acting on the mass m is $F_N = -mg\ cos(\theta$

For small angles (less than 15°), we can approximate sin(θ)≈θ,

By applying Newton's second law to the tangential axis, we derive

the following differential equation.



$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

To solve this differential equation, we assume a solution of the form $\theta(t)=\theta_0 \sin(\omega t+\phi)$, where $w = \sqrt{\frac{g}{l}}$ is the angular frequency, θ_0 is the maximum angular displacement (amplitude), and ϕ is the phase constant determined by the initial conditions. 5

This analysis reveals that the motion of the pendulum is periodic, with a period given by $T = 2\pi\sqrt{\frac{l}{g}}$

Homework

Let's use Frenet coordinates to find the differential equation of a simple pendulum, with mass m and length l, following these steps:

1. Apply Newton's second law to find the vector equation of the forces.
2. Project this vector equation into the Frenet basis.
3. Use the small angle approximation, $\sin(\theta)\approx\theta$, to find the differential equation of a simple pendulum.

Procedure

1-/ Attach the pendulum to its support and measure the length l of the string at its equilibrium position. Knowing that the length l is between the pivot of rotation and the center of mass of the bob.

2-/ Slightly displace the mass m from its equilibrium position without giving it a push. The angle that the string makes with the vertical must be less than 10°.

Part I Period oscillation versus mass

Use a qualitative experiment to demonstrate that the oscillation period is independent of the mass.

Part II Period oscillation versus length

1-/ Measure the time of 10 oscillations for each length, repeat this measurement three times, and report the results in a table based on different values of length l (a minimum of 4 values).

2-/ Provide the uncertainties for each measured variable (for 4 measurement points).

3-/ Discuss the possible sources of error in this experiment.

Results and discussion

1-/ Plot the square of the period T^2 as a function of the length of the pendulum.

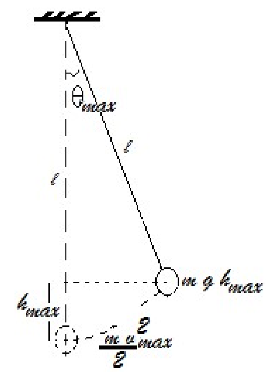
2-/ For each data point, display the uncertainties associated with the various measurements, if possible.

3-/ Use the method of maximum and minimum slopes to calculate the acceleration due to gravity and its uncertainty. Present the result in the proper format, adhering to the significant figure rules.

Part III Conservation of mechanical energy

In an ideal system without air resistance, the mechanical energy of the system is conserved as it continuously converts between potential and kinetic forms during the swing.

First, the gravitational potential energy $E_p = mgh$ depends on the height of the pendulum bob relative to its lowest position. Second, the kinetic energy $E_K = 1/2mv^2$ depends on the speed of the pendulum bob.



Procedure

In this part, the length l and mass m are fixed (values set by the teacher), and adjust the photogate beam strikes the center of the pendulum bob, corresponding to the maximum speed of the bob.

Release the bob with initial angle θ_{max} without pushing it to avoid adding extra energy, and record the value of the speed of the bob displayed by the SpeedGate.

Repeat the measurement for at least 3 different initial angles θ_{max} (10°, 15°, and 20°) to examine the consistency.

Data Analysis:

1-/ For each initial angle, compare the total mechanical energy at the highest point with the total mechanical energy at the lowest point.

2-/ Calculate the difference E_P-E_K for each initial angle θ_{max}.

3-/ Are there any sources of error, and how might they affect the results?

4-/ How could this experiment be modified to minimize these errors?