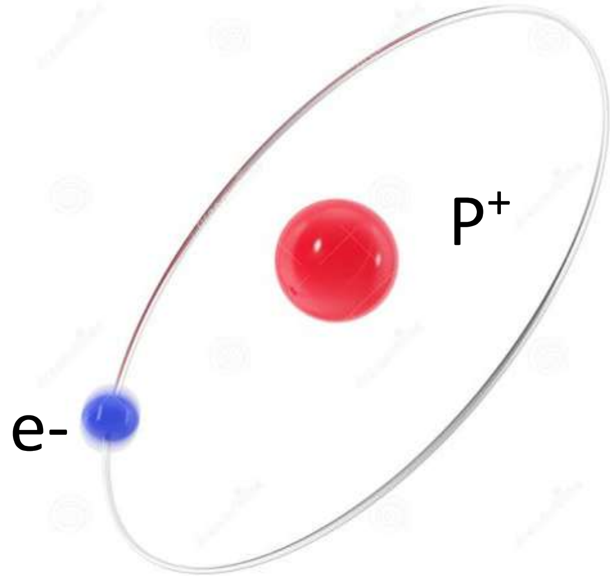
CHAPTER III – PART 2
THE HYDROGEN SPECTRUM
AND BOHR'S MODEL

I/ Rutherford's classical atomic model

We consider a hydrogen atom where the electron, with charge $q_e = -e$ and mass m_e [text cut off at page edge] around the proton with a linear velocity v. The proton's mass m_p is much larger t[text cut off at page edge] electron's.



I.1/ Study of the orbits according to Rutherford's classical model

The electron is subjected to the Coulomb interaction force

$$F_{int} = K\frac{|q_e q_p|}{r^2}$$

q_e : Charge of the electron (q_e = -1.6 10^{-19} C).

q_p : Charge of the proton (q_p = +1.6 10^{-19} C).

r : Radius of the orbit.

K=9.987 $\times 10^9$ kg·m³·A⁻²·s⁻⁴ $\approx 10^{10}(SI)$

The electron is subject to a centrifug

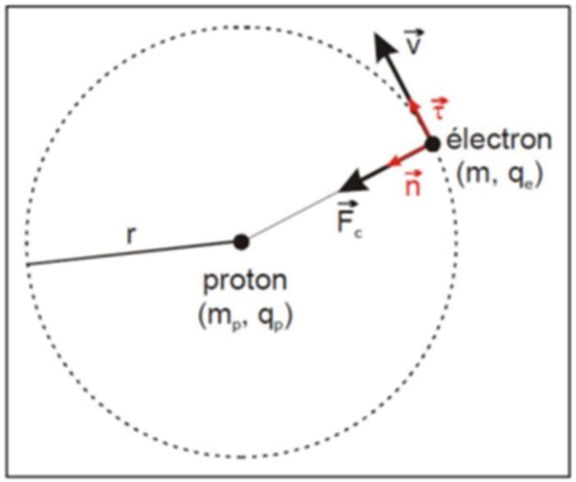
$$F_c = m_e a = m_e\frac{v_e^2}{r}$$

m_e : Mass of the electron. (m_e = 9.1

a : Acceleration of the electron

v_e : Electron velocity.

r : Radius of the orbit.



According to Newton's second law

- $\sum \vec{F} = 0$

$$\overrightarrow{F_{int}} + \overrightarrow{F_c} = 0$$

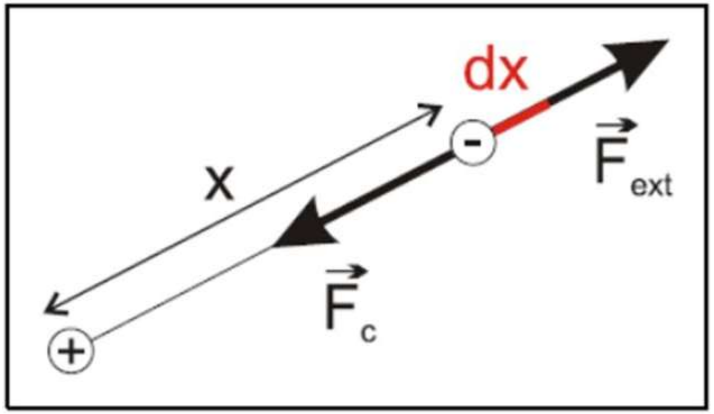
$$r = \frac{Ke^2}{m_e v_e^2}$$

I.2/ Energy study of the hydrogen atom according to Rutherford's model

Electrostatic potential energy of the proton–electron system

We consider a hydrogen atom made up of a proton and an electron.

$$E_p = \int_{\infty}^{r} \overrightarrow{f_r}\,\overrightarrow{dr} \qquad E_p = -K\frac{e^2}{r}$$



Kinetic energy

Because the proton's mass is mu
than the electron's mass, the prot
considered stationary

As a result, all the kinetic energy is
to the motion of the electron ar
proton.

$$E_k = \frac{1}{2}m_e v_e^2 \qquad E_k = \frac{1}{2}$$

Total energy

$$E_T = E_K + E_p \quad \Rightarrow \quad \boxed{E_T = -\frac{1}{2}K\frac{e^2}{r}}$$

II/ Bohr's atomic model

II.1/ First Postulate (The Orbit Postulate)

Electrons can move around the nucleus without emitting radiation only in certa orbits. These orbits are defined by the following quantization rule:

$$\boxed{m_e v_e r = n\frac{h}{2\pi}}$$

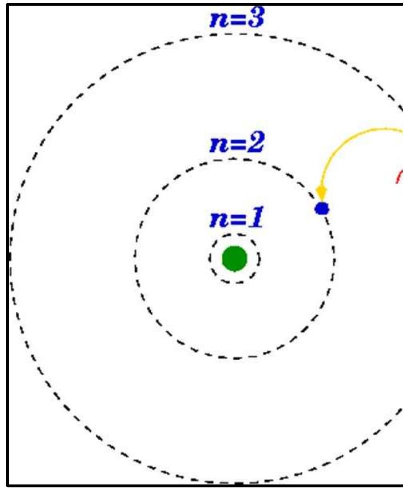
With:

$\boldsymbol{n}$: principal quantum number, $n \in \{1,2,3,\ldots\}$
$\boldsymbol{m_e}$: mass of the electron
$\boldsymbol{r}$: radius of the electron's orbit around the nucleus
$\boldsymbol{v_n}$: linear (tangential) velocity of the electron on its $\boldsymbol{n}_{\text{-th}}$ orbit
$\boldsymbol{h}$: Planck's constant (h=6.626 070 15×10^{-34} J.s)

II.2/ Postulate N°2 (The Postulate of Energy Emission and Absorption)

Each allowed orbit has a specific energy level. When an electron moves from on another, it makes a quantum jump and emits or absorbs a photon with energy:

$$E = \left|E_f - E_i\right| = h\nu.$$

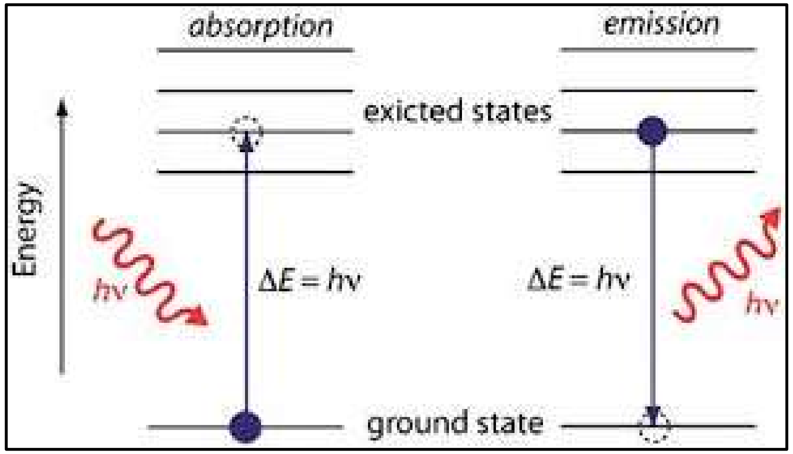


Where:

E_i: energy corresponding to the initial orbit (initial energy level)

E_f: energy corresponding to the final orbit (final energy level)

ν: frequency of the emitted or absorbed radiation



II.3/ Results of Bohr's Theory

$$r = \frac{h^2}{4\pi^2 K m e^2} n^2 \qquad v_e = \frac{2\pi m K e^2}{nh} \qquad E_T = -\frac{2\pi^2 m_e K^2 e^4}{h^2}\frac{1}{n^2}$$

n: principal quantum number, $n \in \{1,2,3,\ldots\}$

m_e: mass of the electron

r: radius of the electron's orbit around the nucleus

v_n: linear (tangential) velocity of the electron on its $n_{\text{-th}}$ orbit

h: Planck's constant ($h = 6.626\,070\,15 \times 10^{-34}$ J.s)

$K = 10^{10}$ (SI)

For the hydrogen atom and the K sh

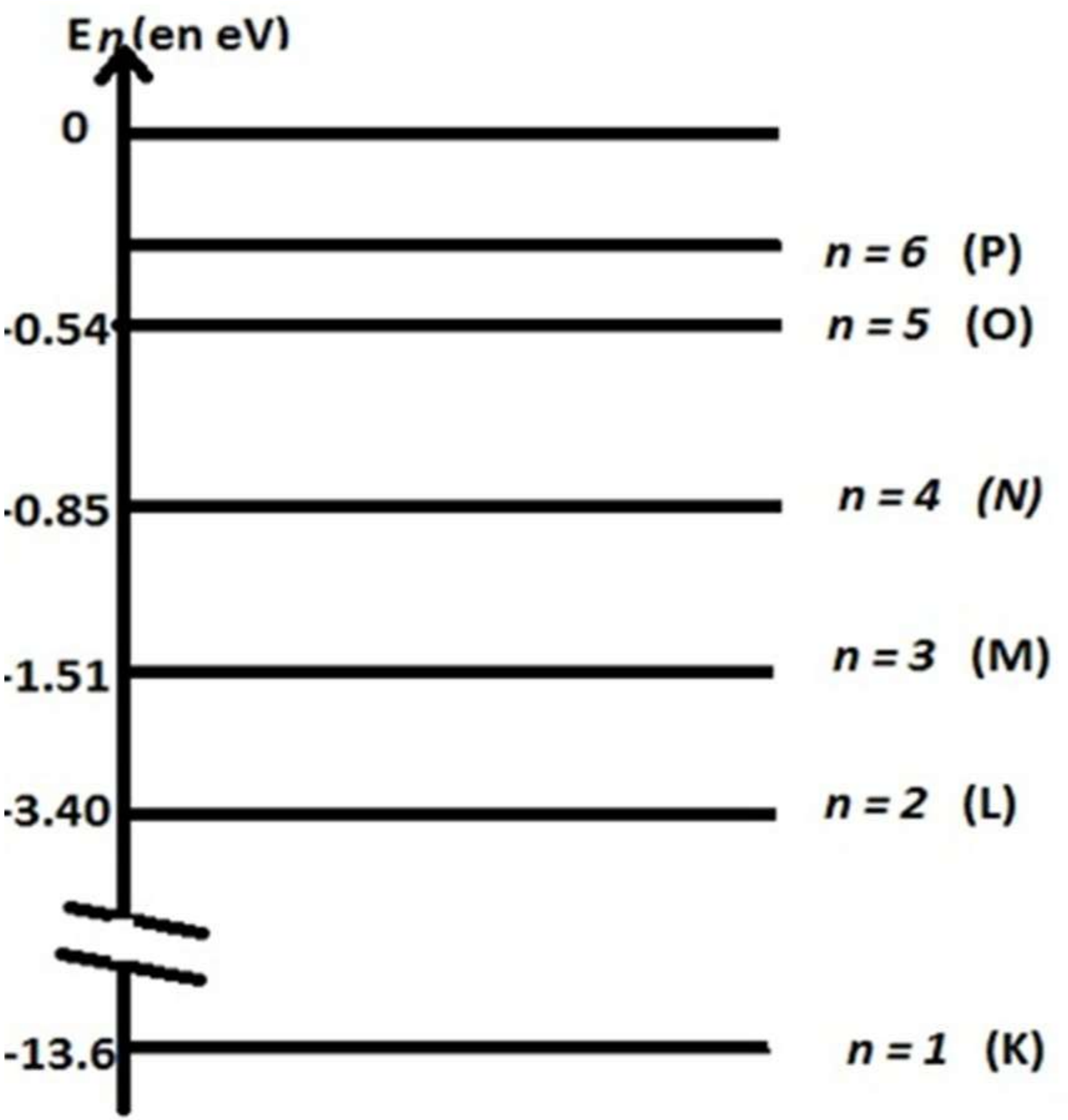
$$r_1 = \frac{h^2}{4\pi^2 K m e^2} = 0,529.10^{-10} m \approx 0.$$

$$E_1 = -\frac{2\pi^2 m_e K^2 e^4}{h^2}\frac{1}{1^2} = -21,18 \times 10^{-19} J = -13,6\ eV$$

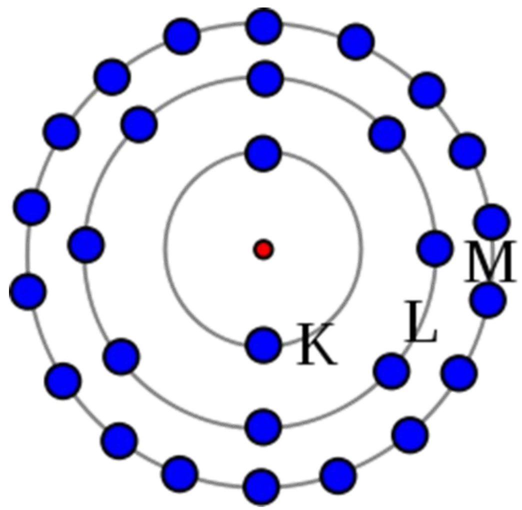
$$\boxed{r_n = r_1 n^2 = 0.53\, n^2 (\text{Å})}$$

$$\boxed{E_n = \frac{E_1}{n^2} = \frac{-13.6}{n^2}\ (eV)}$$

Energy level diagram of the hydrogen atom



Bohr’s atomic model is called the “s
because electrons are arranged i
energy levels, or shells, around the n



n=1 Shell K.

n=2 Shell L.

n=3 Shell M.

II.4/ Extension of the Bohr model to hydrogen-like atoms

Hydrogen-like species are ions containing only one electron, similar to the hydrogen atom

Example: He^{+}, Li^{2+}, Be^{3+}...

In the case of hydrogen-like ions:

q(nucleus) = + Z·e

q(electron) = – e

Accordingly, the extended formulas are:

$$\boxed{E_n = \frac{E_1 Z^2}{n^2} = \frac{-13.6\, Z^2}{n^2}\ (eV)}$$

$$\boxed{r_n = r_1 \frac{n^2}{Z} = 0.53\, \frac{n^2}{Z}\, (\text{Å})}$$

$$\boxed{v_n = 2,19\ 10^6 \frac{Z}{n}\ (m.s^{-1})}$$

Z: The number of protons

n: The principal quantum number (or

II.5/ Definition of Ionization Energy

The ionization energy is the minimum energy required to remove an electron from a[...] atom in its ground state (n = 1) to a point infinitely far from the nucleus (n = ∞, wher[...] the electron is no longer bound to the atom.

Mathematically, for a hydrogen-like atom:

$$\boldsymbol{E_{ion}} = E_{\infty} - E_1 = \frac{\boldsymbol{E_1 Z^2}}{\infty^2} - \frac{\boldsymbol{E_1 Z^2}}{1^2} = 0 - \frac{-13.6\,\boldsymbol{Z^2}}{\boldsymbol{n^2}} = \frac{13.6\,\boldsymbol{Z^2}}{\boldsymbol{n^2}}$$

$$\boxed{\boldsymbol{E_{ion}} = \frac{13.6\,\boldsymbol{Z^2}}{\boldsymbol{n^2}}(\boldsymbol{eV})}$$

III/ Balmer formula

In the case of hydrogen-like atoms

$$\Delta E = |E_f - E_i| = h\nu = h\frac{c}{\lambda}$$

$$|E_f - E_i| = -\frac{2\pi^2 m e^2 K^2 e^4}{h^2 n_f^2} - \left(-\frac{2\pi^2 m e^2 K^2 e^4}{h^2 n_i^2}\right) = h\frac{c}{\lambda}$$

$$\frac{1}{\lambda} = \left|-\frac{2\pi^2 m e^2 K^2 e^4}{h^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right|$$

$$\frac{1}{\lambda} = \left|-R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right|$$

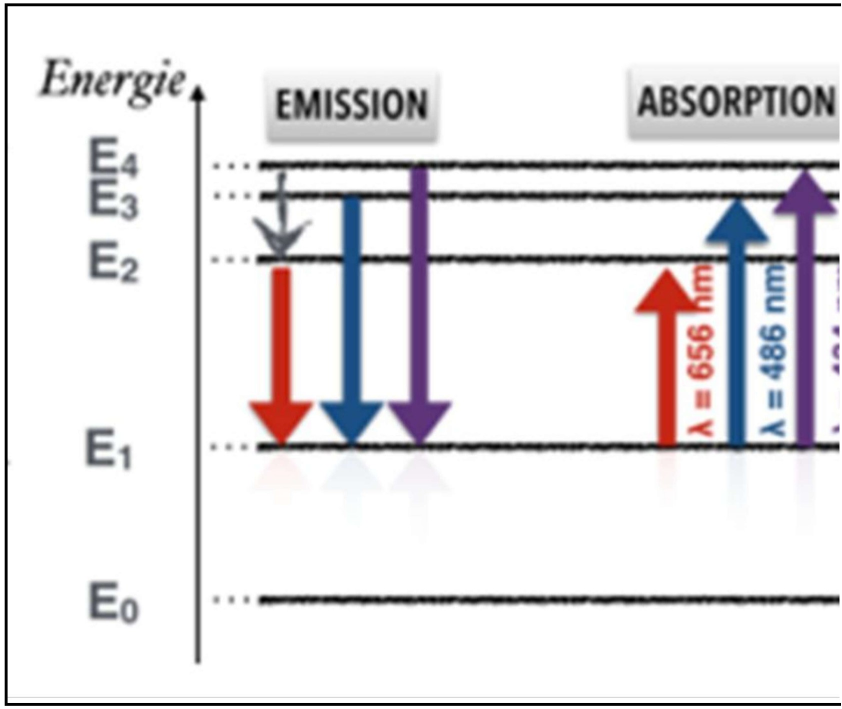
$$R_H = 1,096775\ 10^7\ m^{-1}$$

$$\boxed{\frac{1}{\lambda} = R_h Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

$n_2 > n_1$

Emission : $n_1 = n_f$ n_2

Absorption : $n_1 = n_i$



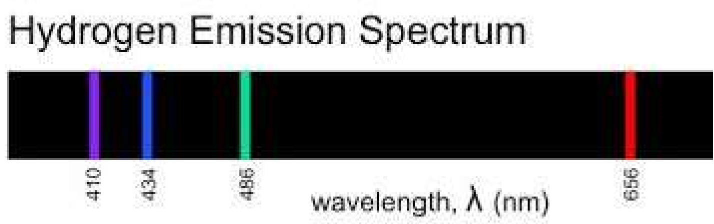
IV/ The emission spectrum of the hydrogen atom

Each absorbed or emitted wave corresponds to a spectral line, therefore:

Spectral line → wave → electronic transition

First spectral line: $n_2=n_1+1$

Limit line: $n_2=\infty$



IV.1/ The Lyman series

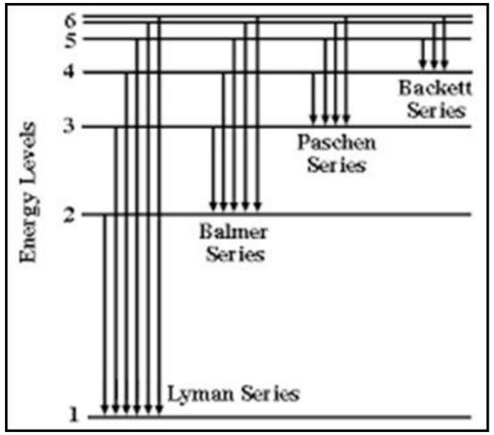
$n_1 = 1$; $n_2 > 1$

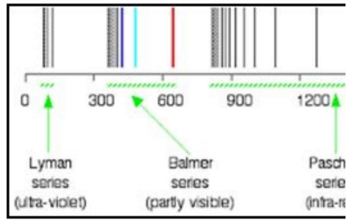
The first spectral line

 $n_1 = 1$; $n_2 = 2$ $\lambda_{1\to2} = \mathbf{121.6\ nm}$

The limit line

➢ $n_1 = 1$; $n_2 = +\infty$ $\lambda_{1\to\infty} = \mathbf{91.2\ nm}$





IV.2/ The Balmer series $n_1 = 2 ; n_2 > 2$

The first spectral line

 $n_1 = 2 ; n_2 = 3$ $\lambda_{2\rightarrow3}$ **= 656.3 nm**

The limit line

➢ $n_1 = 2 ; n_2 = +\infty$ $\lambda_{2\rightarrow\infty}$ **= 364.7 nm**

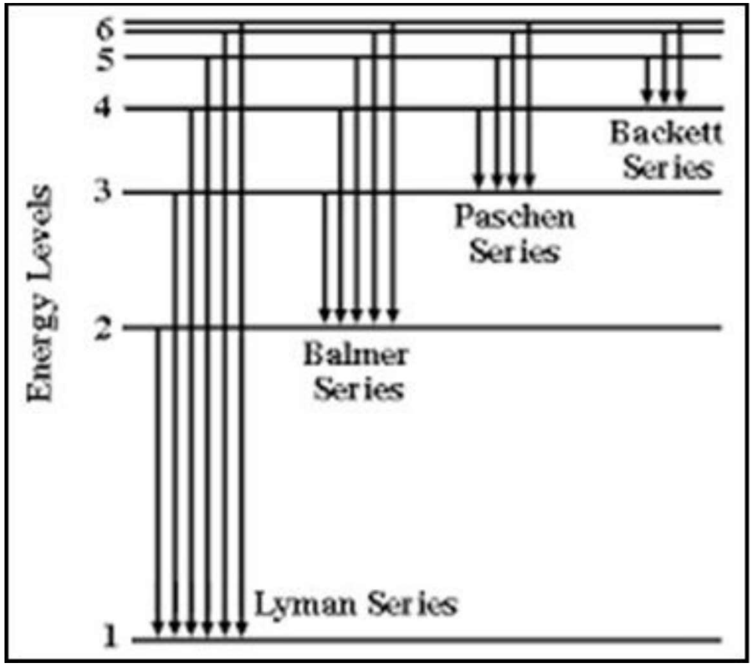
IV.3/ The Paschen series $n_1 = 3 ; n_2$

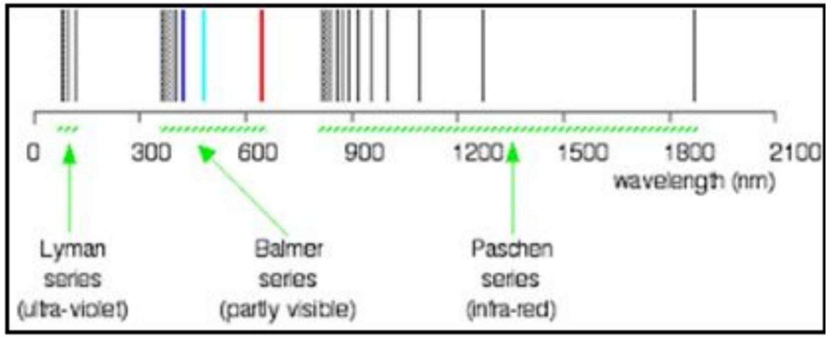
The first spectral line

 $n_1 = 3 ; n_2 = 4$ $\lambda_{3\rightarrow4}$ **= 1876 nm**

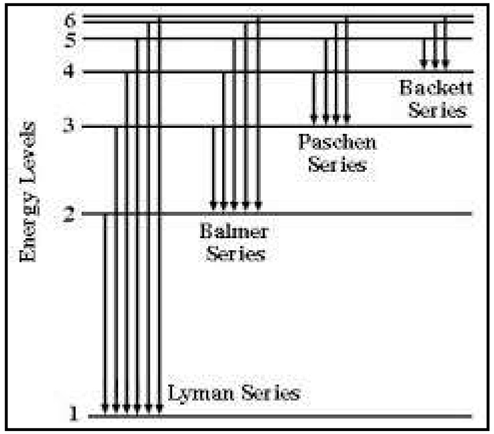
The limit line

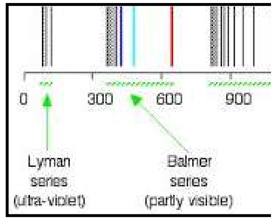
➢ $n_1 = 3 ; n_2 = +\infty$ $\lambda_{3\rightarrow\infty} =$





IV.4/ The Brackett series $n_1 = 4 \; ; n_2 > 4$





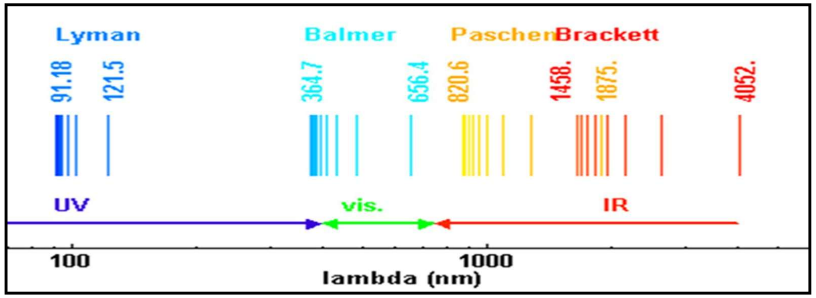
The first spectral line

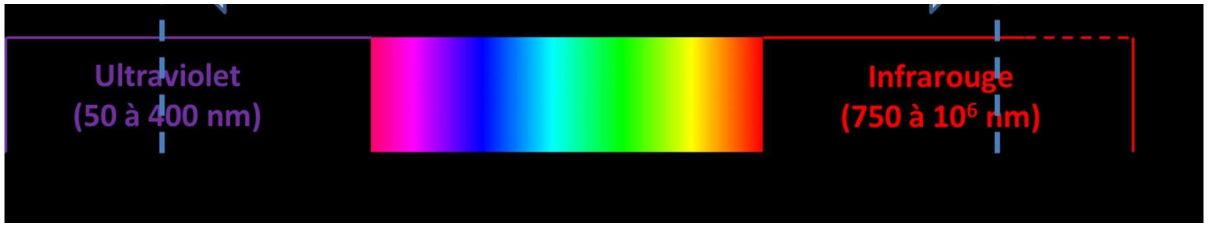
 $n_1 = 4 \; ; n_2 = 5$ $\lambda_{4\to5} = \mathbf{4052\ nm.}$

The limit line

➢ $n_1 = 4 \; ; n_2 = +\infty$ $\lambda_{4\to\infty} = \mathbf{1458\ nm}$

V/ The electromagnetic spectrum





V/ Limitations of the Bohr Model

The Bohr model correctly predicts the energy levels of the hydrogen atom and other single-electron systems.

However, the depiction of an electron moving in well-defined orbits, like in a "miniat solar system," is incompatible with the wave-like nature of the electron.

In fact, when an electron is confined within a space comparable in size to its waveler **wave character** becomes evident.

It is this **wave nature of matter** that leads to the concept of **atomic orbitals**, which e the **geometric structure** of the atom.