Operations Research

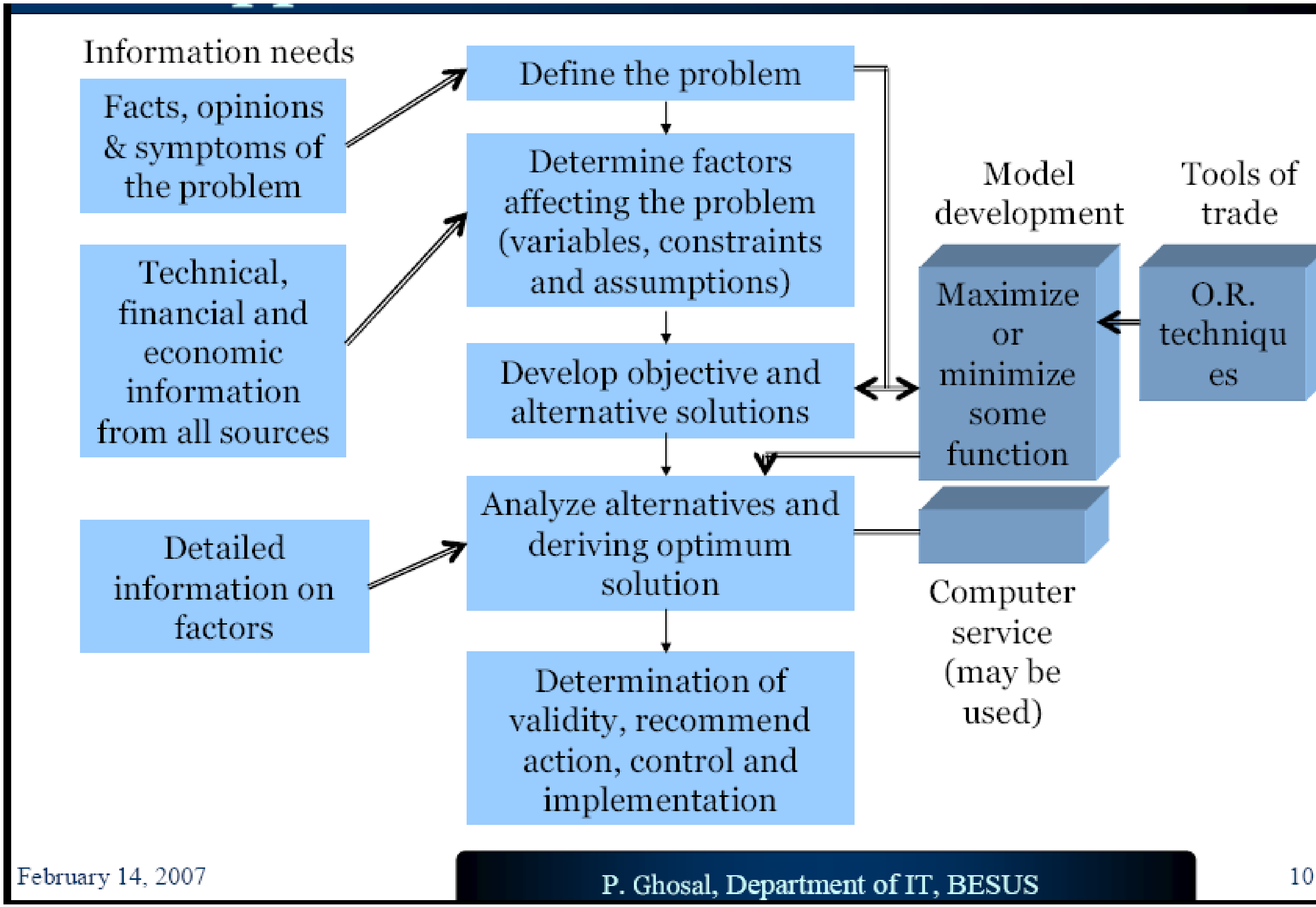
Chapter 2

Linear and non Linear Programming In general

Linear and non Linear Programming In general

- **OR Approach**
- **Linear Programming**
- **Non Linear Programming**
- **Standard Form of Linear Programming**
- **Solution of a Linear Programming Problem**
- **OR Problems Tree**
- **Report Strategy Problem**

OR Approach


Information needs
Facts, opinions & symptoms of the problem
Technical, financial and economic information from all sources
Detailed information on factors
Define the problem
Determine factors affecting the problem (variables, constraints and assumptions)
Develop objective and alternative solutions
Analyze alternatives and deriving optimum solution
Determination of validity, recommend action, control and implementation
Model development
Tools of trade
Maximize or minimize some function
O.R. techniqu es
Computer service (may be used)
February 14, 2007
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Linear Programming

The development of linear programming has been ranked among the most important scientific advances of the mid-20th century, and we must agree with this assessment. Its impact since just 1950 has been extraordinary. Today it is a standard tool that has saved many thousands or millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world; and its use in other sectors of society has been spreading rapidly. A major proportion of all scientific computation on computers is devoted to the use of linear programming..

Non Linear Programming

A key assumption of linear programming is that *all its functions* (objective function and constraint functions) are linear. Although this assumption essentially holds for numerous practical problems, it frequently does not hold. In fact, many economists have found that some degree of nonlinearity is the rule and not the exception in economic planning problems. Therefore, it often is necessary to deal directly with nonlinear programming problems.

Standard Form of Linear Programming

- We define the standard form of a linear programming problem as:
  - One whose objective function is maximization
  - One whose constraints are expressed as equations and whose right hand side is non-negative (after the introduction of slack and surplus variables)
  - One in which all the decision variables are non-negative

Solution of a Linear Programming Problem

- Once a problem can be expressed as a linear programming problem the next step is to find its optimal solution
- For trivial 2 and 3 decision variable problems their optimal solution can be found graphically
- For more complex problems the simplex algorithm is traditionally used.
- In order to use the simplex algorithm it is necessary to:
  - Express the problem in a standard form
  - Represent the objective function as an equation

$$z - \sum_j c_j \chi_j = 0;$$

where z is the value of the objective function and the x_j's the decision variables

Non Linear Programming

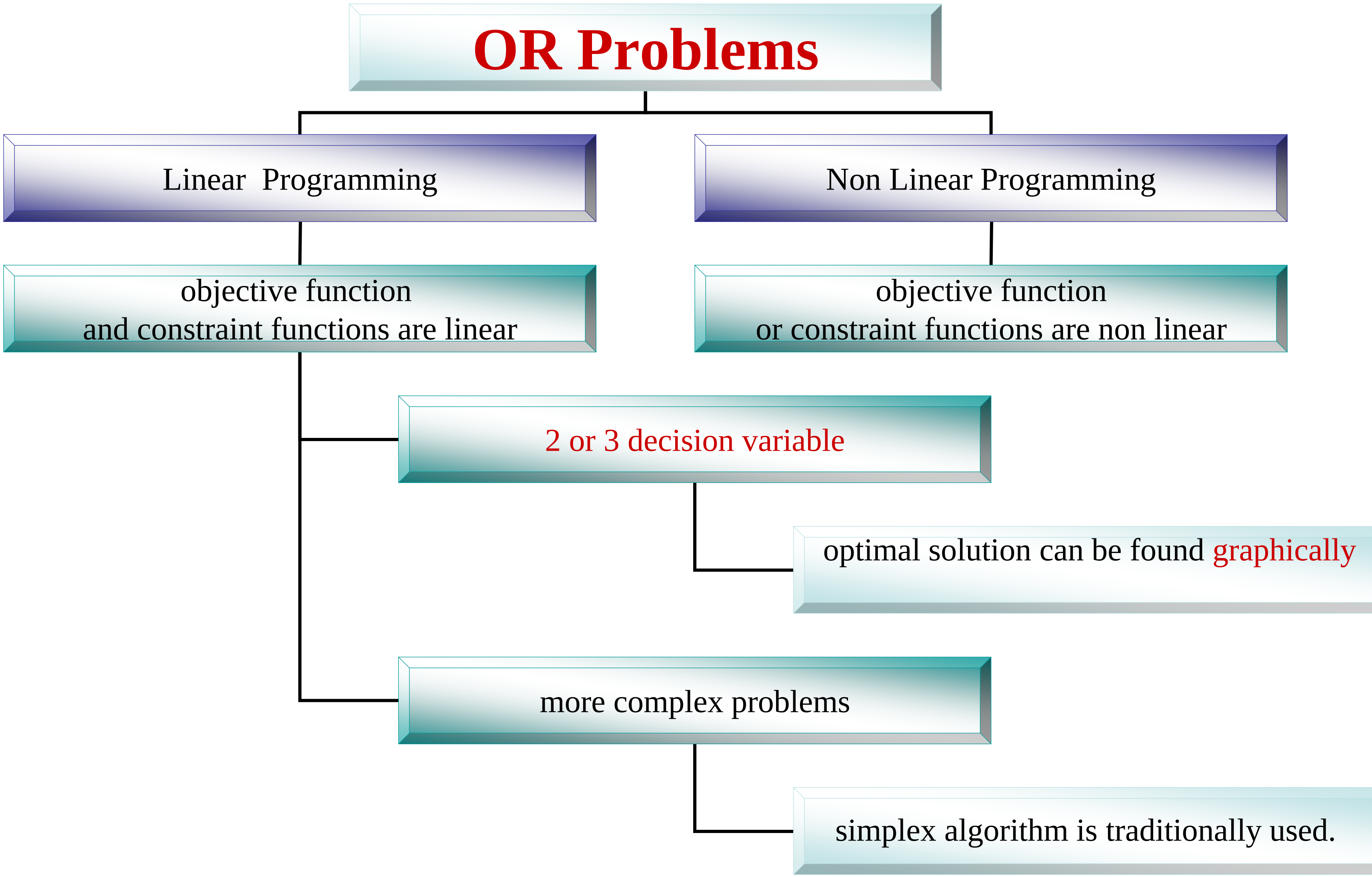
In one general form, the *nonlinear programming problem* is to find **x** $(x_1, x_2, \ldots, x_n)$ so as to Maximize $f(\mathbf{x})$, subject to $g_i(\mathbf{x})$ b_i, for i 1, 2, . . . , m, and $\mathbf{x} >= \mathbf{0}$, where $f(\mathbf{x})$ and the $g_i(\mathbf{x})$ are given functions of the n decision variables.

No algorithm that will solve *every* specific problem fitting this format is available.

However, substantial progress has been made for some important special cases of this problem by making various assumptions about these functions, and research is continuing very actively. This area is a large one, and we do not have the space to survey it completely.

However, we do present a few sample applications and then introduce some of the basic ideas for solving certain important types of nonlinear programming problems.

OR Problems Tree



Report Strategy Problem

- *Suppose that Team members A and C have decided to work together in their tasks. They have decided that their skills are complementary and by working together in both of the tasks assigned to them they can do a better job than if they work independently. A has very good writing speed, he can write 4 pages per hour but his rate of grammatical errors is very high (3 errors per page). On the other hand, C is slower (2.5 pages per hour) but his error rate is very low (1 error per page). They want to write a report that will earn them the highest grade.*

  They have discovered that the grade given by the professor is very much influenced by the number of pages of the written assignments (the more pages the higher the grade assigned). However, they have also discovered that for every five errors in grammar, he deducts the equivalent of one written page, and also that the maximum number of mistakes the instructor will tolerate in an assignment is 80. Help this team of students to determine the best strategy (you need to define the meaning of this) given the total time constraint available to work in the project. The total time allocated to both of them is 8 hours. Also because of previous commitments C cannot work more than 6 hours in the project

Formula for Solution

From last lecture the LP formulation of the problem is:

Maximize $z = 1.6X_A + 2.0X_C$

Subject to:

$12\ X_A + 2.5X_C \leq 80$

$X_A + X_C \leq 8$

$X_C \leq 6$

$X_A,\ X_C \geq 0$

The Windsor Glass Company Problem (Hillier and Liberman)

The Windsor Glass Company is planning to launch two new products. Product 1 is an 8-foot glass door with aluminum framing and Product 2 a 4x6 foot double-hung wood-framed window

Aluminum frames are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glass and assembles the products. Product 1 requires some of the production capacity in Plants 1 and 3, but none in Plant 2. Product 2 needs only Plants 2 and 3. The marketing division has concluded that the company could sell as much of either product as could be processed by these plants. The management of the company wants to determine what mixture of both products would be the most profitable. The following table provides the information available.

Plant	Production time per batch, hours		Production time available per week, hours
	Product		
	1	2	
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	$3,000	$5,000	

The Windsor Glass Company
Problem Formulation (Hillier and Liberman)

Formulation as a linear programming problem

Decision variables:

x_1 = Number of batches of product 1 produced per week

x_2 = Number of batches of product 2 produced per week

Objective function:

Maximize z = $3\,X_1 + 5X_2$ (in thousands of dollars)

Subject to:

x_1	$\leq$	4	(Production Available in Plant 1)
$2x_2$	$\leq$	12	(Production Available in Plant 2)
$3x_1 + 2x_2$	$\leq$	18	(Production Available in Plant 3)

$x_1, x_2 \geq 0$

Reddy Mikks Problem (Taha)

- The Reddy Mikks company owns a small paint factory that produces both interior and exterior house paints. Two basic raw materials, A and B, are used to manufacture the paints.
- The maximum availability of A is 6 tons a day; that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table.

	Tons of Raw Material per Ton of Paint		
	Exterior	Interior	Maximum Availability (tons)
Raw Material A	1	2	6
Raw Material B	2	1	8

- A market survey has established that the daily demand for the interior paint cannot exceed that of exterior paint by more than 1 ton. The survey also showed that the maximum demand for the interior paint is limited to 2 tons daily.
- The wholesale price per ton is $3000 for exterior paint and $2000 per interior paint. How much interior and exterior paint should the company produce daily to maximize gross income?

Reddy Mikks Problem Formulation

Define:

X_E = Tons of exterior paint to be produced

X_I = Tons of interior paint to be produced

Thus, the LP formulation of the Reddy-Mikks Company is as follows:

Maximize $z = 3X_E + 2X_I$

Subject to:

$X_E + 2X_I \leq 6$ (1) (availability of raw material A)

$2X_E + X_I \leq 8$ (2) (availability of raw material B)

$-X_E + X_I \leq 1$ (3) (Restriction in production)

$X_I \leq 2$ (4) (Demand Restriction)

$X_E, X_I \geq 0$