# ITGD4207 Operations Research

## Chapter 5

## Linear Programming

## Transportation Model

# Linear Programming
# Transportation Model

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# The Transportation Model

The transportation model is a special class of LPPs that deals with transporting(=shipping) a commodity from **sources** (e.g. factories) to **destinations** (e.g. warehouses). The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. We assume that the shipping cost is proportional to the number of units shipped on a given route.

We assume that there are m sources 1,2, …, m and n destinations 1, 2, …, n. The cost of shipping one unit from Source i to Destination j is $c_{ij}$.

We assume that the availability at source i is $a_i$ (i=1, 2, …, m) and the demand at the destination j is $b_j$ (j=1, 2, …, n). We make an important assumption: the problem is a **balanced** one. That is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

That is, total availability equals total demand.

We can always meet this condition by introducing a dummy source (if the total demand is more than the total supply) or a dummy destination (if the total supply is more than the total demand).

Let $x_{ij}$ be the amount of commodity to be shipped from the source i to the destination j.

Though we can solve the above LPP by Simplex method, we solve it by a special algorithm called the transportation algorithm. We present the data in an m×n tableau as explained below.

Thus the problem becomes the LPP

Minimize $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1,2,\ldots,n)$$

$$x_{ij} \geq 0$$

Destination

| | | 1 | 2 | . | . | n | Supply |
|---|---|---|---|---|---|---|---|
| Source | 1 | $c_{11}$ | $c_{12}$ | | | $c_{1n}$ | $a_1$ |
| | 2 | $c_{21}$ | $c_{22}$ | | | $c_{2n}$ | $a_2$ |
| | . | | | | | | |
| | . | | | | | | |
| | m | $c_{m1}$ | $c_{m2}$ | | | $c_{mn}$ | $a_m$ |
| Demand | | $b_1$ | $b_2$ | | | $b_n$ | |

# Formulation of Transportation Models

## Example 1

MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The capacities of the three plants during the next quarter are 1000, 1300 and 1200 cars. The quarterly demands at the two distribution centers are 2300 and 1400 cars. The transportation cost per car from Los Angeles to Denver and Miami are $80 and $215 respectively. The corresponding figures from Detroit and New Orleans are 100, 108 and 102, 68 respectively.

# Formulate the transportation Model.

Since the total demand = 3700 > 3500 (Total supply) we introduce a dummy supply with availability 3700-3500=200 units to make the problem a balanced one. If a destination receives $u$ units from the dummy source, it means that that destination gets $u$ units less than what it demanded. We usually put the cost per unit of transporting from a dummy source as zero (unless some restrictions are there). Thus we get the transportation tableau

Destination

| Source | | Denver | Miami | Supply |
|---|---|---|---|---|
| S | Los Angeles | 80 | 215 | 1000 |
| o | Detroit | 100 | 108 | 1300 |
| u r | New Orleans | 102 | 68 | 1200 |
| c | Dummy | 0 | 0 | 200 |
| e | Demand | 2300 | 1400 | |

We write inside the (i,j) cell the amount to be shipped from source i to destination j. A blank inside a cell indicates no amount was shipped.

# Example 2

In the previous problem, penalty costs are levied at the rate of $200 and $300 for each undelivered car at Denver and Miami respectively. Additionally no deliveries are made from the Los Angeles plant to the Miami distribution center. Set up the transportation model.

The above imply that the "cost" of transporting a car from the dummy source to Denver and Miami are respectively 200 and 300. The second condition means we put a "high" transportation cost from Los Angeles to Miami. We thus get the tableau

Destination

| Source | Denver | Miami | Supply |
| --- | --- | --- | --- |
| Los Angeles | 80 | M | 1000 |
| Detroit | 100 | 108 | 1300 |
| New Orleans | 102 | 68 | 1200 |
| Dummy | 200 | 300 | 200 |
| Demand | 2300 | 1400 | |

Note: M indicates a very "big" positive number. It is denoted by "infinity".

## Determination of the starting

## Basic Feasible Solution BFS

In any transportation model we determine a starting BFS and then iteratively move towards the optimal solution which has the least shipping cost.

There are three methods to determine a starting BFS. As mentioned earlier, any BFS will have only m+n-1 basic variables (which may assume non-zero =positive values) and the remaining variables will all be non-basic and so have zero values. In any transportation tableau, we only indicate the values of basic variables. The cells corresponding to non-basic variables will be blank.

# 1. NORTH-WEST Corner Method

# for determining a starting BFS

The method starts at the north-west corner cell (i.e. cell (1,1)).

Step 1. We allocate as much as possible to the selected cell and adjust the associated amounts of supply and demand by subtracting the allocated amount.

Step 2. Cross out the row (column) with zero supply (zero demand) to indicate that no further assignments can be made to that row(column).

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column).

Step 3. If no further allocation is to be made, stop. Else move to the cell to the right (if a column has just been crossed out) or to the cell below if a row has just been crossed out. Go to Step 1.

Consider the transportation tableau:

Destination

| Source | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| 1 | 3<br>**3** | 7<br>**2** | 6 | 4 | ~~5~~ ~~2~~ |
| 2 | 2 | 4<br>**1** | 3<br>**1** | 2 | ~~2~~ ~~1~~ |
| 3 | 4 | 3 | 8<br>**1** | 5<br>**2** | ~~3~~ ~~2~~ |
| Demand | ~~3~~ | ~~3~~ ~~1~~ | ~~2~~ ~~1~~ | ~~2~~ | |

Total shipping cost = 48

# 2. LEAST COST Method

# of determining the starting BFS.

In this method we start assigning as much as possible to the cell with the least unit transportation cost (ties are broken arbitrarily) and the associated amounts of supply and demand are adjusted by subtracting the allocated amount.

Cross out the row (column) with zero supply (zero demand) to indicate that no further assignments can be made to that row(column).

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column).

Next look for the uncrossed out cell with the smallest unit cost and repeat the process until no further allocations are to be made.

# Consider the transportation tableau:

| | | Destination 1 | Destination 2 | Destination 3 | Destination 4 | Supply |
|---|---|---|---|---|---|---|
| Source | 1 | 3 **1** | 7 **0** | 6 **2** | 4 **2** | ~~5~~ ~~4~~ ~~2~~ |
| | 2 | 2 **2** | 4 | 3 | 2 | ~~2~~ |
| | 3 | 4 | 3 **3** | 8 | 5 | ~~3~~ |
| Demand | | ~~3~~ ~~1~~ | ~~3~~ ~~0~~ | ~~2~~ | ~~2~~ | |

Total shipping cost = 36

## 3. Vogel's approximation method (VAM)

Step 1. For each row (column) remaining under consideration, determine a penalty by subtracting the smallest unit cost in the row (column) from the next smallest unit cost in the same row(column). ( If two unit costs tie for being the smallest unit cost, **then the penalty is 0**).

Step2. Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the cell with the least unit cost in the selected row or column.(Again break the ties arbitrarily.) Adjust the supply and demand and cross out the satisfied row or column.

If both a row and a column are simultaneously satisfied then

if exactly one row or column is left uncrossed make the obvious allocations and stop. Else cross out one only (either the row or the column) and leave a zero supply(demand) in the uncrossed out row(column). (**<u>But omit that row or column for calculating future penalties</u>**).

Step 3. If all allocations are made, stop. Else go to step 1.

| Source | Destination 1 | Destination 2 | Destination 3 | Destination 4 | Supply | Row Penalties |
|---|---|---|---|---|---|---|
| 1 | 3 **3** | 7 **0** | 6 **0** | 4 **2** | $\cancel{5}$ | $\cancel{1}$ 1 |
| 2 | 2 | 4 | 3 **2** | 2 | $\cancel{2}$ | $\cancel{0}$ - |
| 3 | 4 | 3 **3** | 8 | 5 | $\cancel{3}$ | $\cancel{1}$ 1 |
| Demand | $\cancel{3}$ | $\cancel{3}$ $\cancel{0}$ | $\cancel{2}$ $\cancel{0}$ | $\cancel{2}$ | | |
| Column Penalties | $\cancel{1}$ 1 | $\cancel{1}$ 4 | $\cancel{3}$ - | $\cancel{2}$ 1 | | |

Total shipping cost = 32