Operations Research

Chapter 3

Part-1

Linear Programming

Graphical and Algebraic Solution

Linear Programming Graphical Solution

- Report Strategy Problem
- Mathematical Formula for Solution
- Graphical Solution
- Binding and non binding Constraint
- Finding the optimal solution
- Terminology
- The Windsor Glass Company Problem
- Reddy Mikks Problem

Report Strategy Problem

- *Suppose that Team members A and C have decided to work together in their tasks. They have decided that their skills are complementary and by working together in both of the tasks assigned to them they can do a better job than if they work independently. A has very good writing speed, he can write 4 pages per hour but his rate of grammatical errors is very high (3 errors per page). On the other hand, C is slower (2.5 pages per hour) but his error rate is very low (1 error per page). They want to write a report that will earn them the highest grade.*

  They have discovered that the grade given by the professor is very much influenced by the number of pages of the written assignments (the more pages the higher the grade assigned). However, they have also discovered that for every five errors in grammar, he deducts the equivalent of one written page, and also that the maximum number of mistakes the instructor will tolerate in an assignment is 80. Help this team of students to determine the best strategy (you need to define the meaning of this) given the total time constraint available to work in the project. The total time allocated to both of them is 8 hours. Also because of previous commitments C cannot work more than 6 hours in the project

Mathematical Formula for Solution

Determination of the objective function

Maximize $z = 4X_A + 2.5X_C – (4*(3/5)X_A + 2.5*(1/5)X_C)$

Maximize $z = 4X_A + 2.5X_C – (2.4X_A + 0.5X_C)$

Maximize $z = 4X_A + 2.5X_C – 2.4X_A - 0.5X_C$

Maximize $z = 1.6X_A + 2.0X_C$

The LP formulation of the problem is:

Maximize $z = 1.6X_A + 2.0X_C$

Subject to:

$12 X_A + 2.5X_C \leq 80$

$X_A + X_C \leq 8$

$X_C \leq 6$

$X_A, X_C \geq 0$

Graphical Solution

- The LP formulation of the problem is:

  Maximize $z = 1.6X_A + 2.0X_C$

  Subject to:

  $12\,X_A + 2.5X_C \leq 80$

  $X_A + X_C \leq 8$

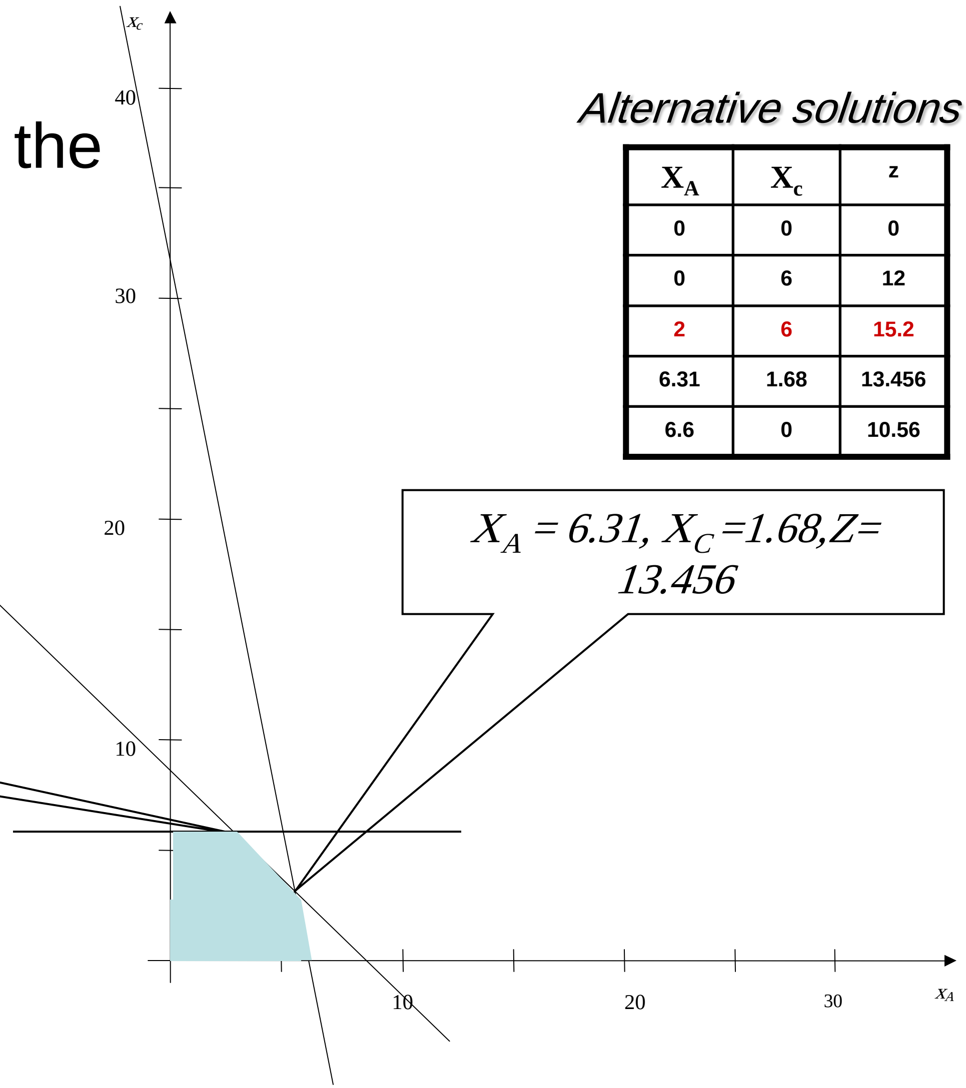
  $X_C \leq 6$

  $X_A, X_C \geq 0$

Optimal solution

$X_A = 2,\ X_C = 6, Z = 15.2$

X_A	X_c
0	32
6.6	0
0	8
8	0



$X_A = 6.31,\ X_C = 1.68, Z = 13.456$

Alternative solutions

X_A	X_c	z
0	0	0
0	6	12
2	6	15.2
6.31	1.68	13.456
6.6	0	10.56

Binding and non binding Constraint

To answer the second question if we increase the number of errors to 90 the first constraint would become:

$$12\ X_A + 2.5X_C <= 90$$

Then to find the point where the two constraints intersect we would have to solve the set of equations

$$12\ X_A + 2.5X_C = 90$$

$$X_A + X_C = 8$$

Which would give us: X_A=7.368 X_C = 0.6315. The objective function evaluated at this point would be 13.05 (No increase of the optimal solution →non-binding constraint)

Doing the same with the other constraint we get

$$12\ X_A + 2.5X_C = 80$$

$$X_A + X_C = 9$$

Which would give us : X_A=6.05 X_C = 2.94. The objective function evaluated at this point would be 15.57→Increase of the optimal solution→Binding constraint

We choose to increase the number of hours instead of the number of errors allowed

Finding the Optimal Solution

- Unfortunately we can get the graphical solution of the LP problem only for problems with 2 and 3 decision variables
- Therefore we need to find an alternative method to get the optimal solution of LP problems
- Some observations:
  - We know that the optimal solution must be in a corner point (why?)
  - We also know that a corner point is defined by the intersection of two constraints (for the 2 decision variable case we solve a different subset of two linear equations for each corner point). What about when we have three or more decision variables? Can you identify all the corner points?
  - So what we could do is solve the subsets of linear equations corresponding to all the corner points. Then we evaluate the objective function at those corner points and the one yielding the best objective value is our optimal solution.
  - However, since there is a large number of corner points (how many?) this is impractical. We need a smart way to search for the optimal solution. This is what the the simplex algorithm does for us.

Terminology

- A **solution** is any specification of values for the decision variables.
- A **feasible Solution** is a solution for which all the constraints are satisfied.
- The **feasible region** is the set of all feasible solutions.
- An **Optimal Solution** is a feasible solution that has the most favorable value of the objective function.
- A **Corner-Point Solution** corresponds to a solution of the subset of equations corresponding to the constraints meeting at that corner point (vertex)
- A **Corner-point feasible** (CPF) solution is a solution that lies at a corner of the feasible region .
- A **Basic Solution** is a corner-point solution
- A **Basic Feasible Solution** is a CPF solution for which all the variables are greater than or equal to zero.
- An **Optimal Solution** is a feasible solution that has the most favorable value of the objective function.

The Windsor Glass Company Problem (Hillier and Liberman)

The Windsor Glass Company is planning to launch two new products. Product 1 is an 8-foot glass door with aluminum framing and Product 2 a 4x6 foot double-hung wood-framed window

Aluminum frames are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glass and assembles the products. Product 1 requires some of the production capacity in Plants 1 and 3, but none in Plant 2. Product 2 needs only Plants 2 and 3. The marketing division has concluded that the company could sell as much of either product as could be processed by these plants. The management of the company wants to determine what mixture of both products would be the most profitable. The following table provides the information available.

Plant	Production time per batch, hours		Production time available per week, hours
	Product		
	1	2	
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	$3,000	$5,000	

The Windsor Glass Company
Problem Formulation (Hillier and Liberman)

Formulation as a linear programming problem

Decision variables:

x_1 = Number of batches of product 1 produced per week

x_2 = Number of batches of product 2 produced per week

Objective function:

Maximize z = $3X_1 + 5X_2$ (in thousands of dollars)

Subject to:

x_1	$\leq$	4	(Production Available in Plant 1)
$2x_2$	$\leq$	12	(Production Available in Plant 2)
$3x_1 + 2x_2$	$\leq$	18	(Production Available in Plant 3)

$x_1, x_2 \geq 0$

Reddy Mikks Problem (Taha)

- The Reddy Mikks company owns a small paint factory that produces both interior and exterior house paints. Two basic raw materials, A and B, are used to manufacture the paints.
- The maximum availability of A is 6 tons a day; that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table.

	Tons of Raw Material per Ton of Paint		
	Exterior	Interior	Maximum Availability (tons)
Raw Material A	1	2	6
Raw Material B	2	1	8

- A market survey has established that the daily demand for the interior paint cannot exceed that of exterior paint by more than 1 ton. The survey also showed that the maximum demand for the interior paint is limited to 2 tons daily.
- The wholesale price per ton is $3000 for exterior paint and $2000 per interior paint. How much interior and exterior paint should the company produce daily to maximize gross income?

Reddy Mikks Problem Formulation

Define:

X_E = Tons of exterior paint to be produced

X_I = Tons of interior paint to be produced

Thus, the LP formulation of the Reddy-Mikks Company is as follows:

Maximize $z = 3X_E + 2X_I$

Subject to:

$X_E + 2X_I \leq 6$ (1) (availability of raw material A)

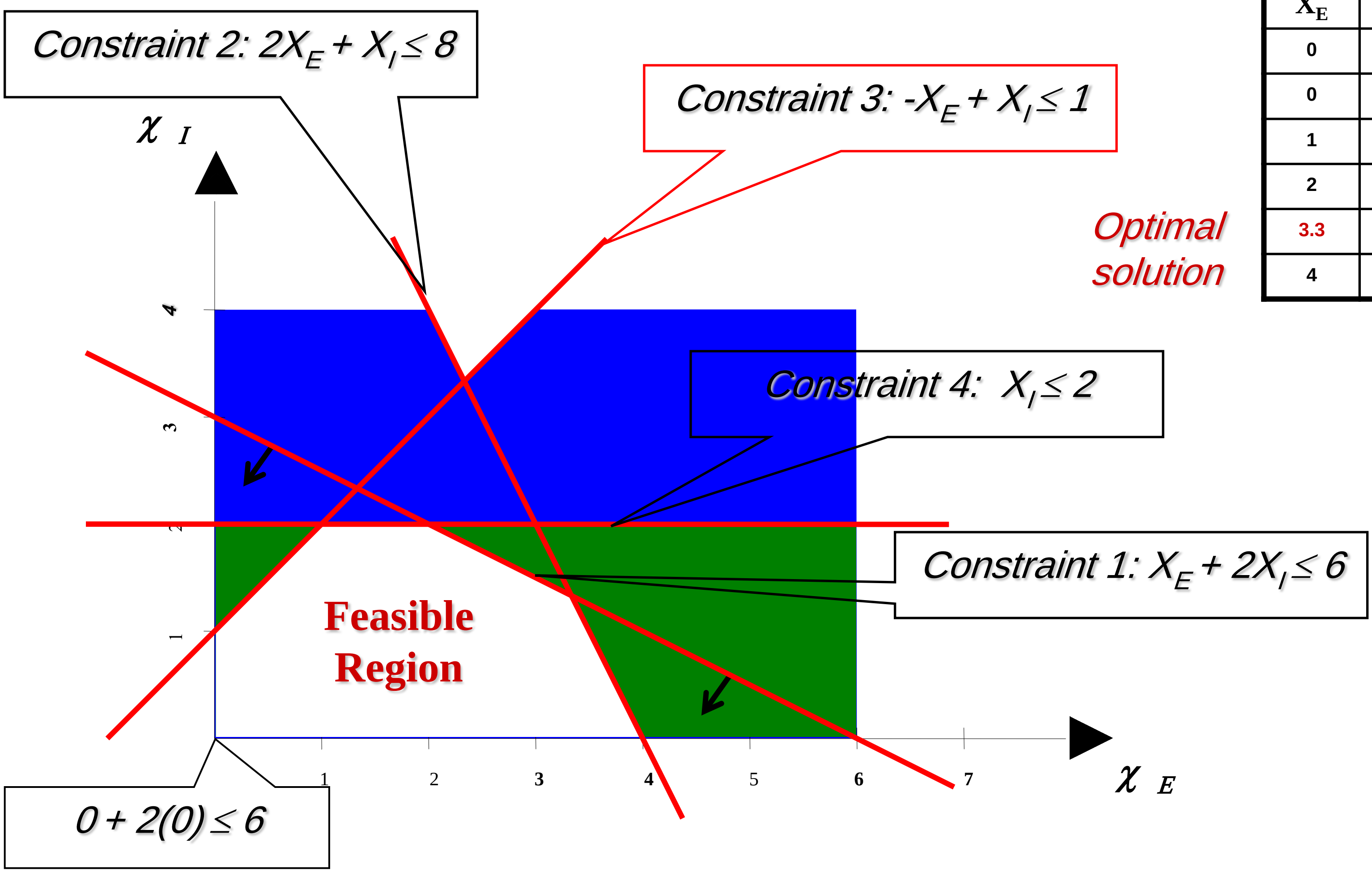
$2X_E + X_I \leq 8$ (2) (availability of raw material B)

$-X_E + X_I \leq 1$ (3) (Restriction in production)

$X_I \leq 2$ (4) (Demand Restriction)

$X_E , X_I \geq 0$

Graphical Solution
of the Ready Mikks Problem



Alternative solutions

X_E	X_I	z
0	0	0
0	1	2
1	2	7
2	2	10
3.3	1.35	12.6
4	0	12

Optimal solution