Operations Research

Chapter 5

Part-2

Transportation Problem
Find optimal Solution

Stepping Stone Method

- *Once we have obtained an initial basic feasible solution using any of the methods previously discussed, the next step is to get the the optimal solution. A method to do this is the stepping stone method.*
- *To introduce this method, we first recognize that*
  - *A solution is optimal if the relative profit for every non-basic variable is greater than or equal to zero.*
  - *The relative profit can be computed by increasing by one unit the current assignment to each of the non-basic variables and computing the total change in z.*

- *For instance, in the following problem find the relative profit ($\bar{c}_{14}$) of increasing the value of the non-basic variable X_{14} by one unit.*

Relative Profit

Reduce this cell by one unit

	Demand				
Source	A	B	C	D	Avail
1	10	15 − 0	20	+ 11	15
2	0 12	0 + 7	15 0 9	10 − 20	25
3	5 0	14	16	18	5
Demand	5	15	15	10	

We want to increase the current assignment of this cell by +1

To keep the current solution balanced, we need to find a path using the current basic variables (cells) from which we are going to reduce and increase the current allocation. The path will be constructed by taking 90 degree turns at the current basic cells

Increase this cell by one unit

We need to reduce the assignment to this cell by one unit

Relative profit

$$\bar{c}_{14} = 11(1)+20(-1)+7(1)+0(-1)=-2$$

u-v Method (Modified Distribution)

- *Using the previous method, we could find the relative profits for all the non-basic cells to identify the one that would give us the most savings. However, for larger problems, this would be too cumbersome. There is a better way to do this.*
- ***u-v Method (Modified distribution)***
- *For any basic variable, find numbers $\mathbf{u}_i$ for the source (row) and $\mathbf{v}_j$ for the destination (column) such that:*

$$c_{ij} = u_i + v_j.,$$

Where c_{ij} is the original cost associated with the cell ij.

- *Since we have m+n-1 basic variables and we have m $\mathbf{u}_i$'s and n $\mathbf{v}_j$'s, we will have more unknowns than linear equations. Thus, we set one of the unknowns to zero and solve for the rest of the unknowns.*
- *Once we have solved for the $\mathbf{u}_i$'s and $\mathbf{v}_j$'s, we compute the relative profit for each of the non basic variables.*

$$\bar{c}_{ij} = c_{ij} - (u_i + v_j)$$

Example on How to Use the u-v Method

	Demand				
Source	A	B	C	D	Avail
1	10	15 / 0	20	11	15
2	0 / 12	0 / 7	15 0 / 9	10 / 20	25
3	5 / 0	14	16	18	5
Demand	5	15	15	10	

The basic variables are $\mathbf{X}_{12}$, $\mathbf{X}_{21}$, $\mathbf{X}_{22}$, $\mathbf{X}_{23}$, $\mathbf{X}_{24}$, $\mathbf{X}_{31}$ *and the u-v equations are:*

$\mathbf{u}_1 + \mathbf{v}_2 = 0$ $\mathbf{u}_2 + \mathbf{v}_1 = 12$ $\mathbf{u}_2 + \mathbf{v}_2 = 7$

$\mathbf{u}_2 + \mathbf{v}_3 = 9$ $\mathbf{u}_2 + \mathbf{v}_4 = 20$ $\mathbf{u}_3 + \mathbf{v}_1 = 0$

Since we have 6 equations and 7 unknowns, there exists an infinite number of solutions. Arbitrarily set one of the variables to zero ($\mathbf{u}_1 = 0$*) and solve for the rest. (*$\mathbf{v}_2=0$, $\mathbf{v}_1=5$, $\mathbf{u}_2 = 7$, $\mathbf{v}_4=13$, $\mathbf{u}_3 = -5$, $\mathbf{v}_3=2$.

Example on How to Use the u-v Method

	Demand				
Source	A	B	C	D	Avail
1	10	15 / 0	20	11	15
2	**0** / 12	**0** / 7	15 / 9	10 / 20	25
3	5 / 0	14	16	18	5
Demand	5	15	15	10	

u_1 **=0**, u_2 **= 7**, u_3 **= -5**

v_1 **=5** v_2 **=0** v_3 **=2** v_4 **=13**

If all the relative profits ($\bar{c}_{ij} = c_{ij} - (u_i + v_j)$ *) are positive, the solution is optimal; otherwise, at least another iteration of the transportation algorithm is needed.*

Example on How to Use the u-v Method

	Demand				
Source	A	B	C	D	Avail
1	5 / 10	15 / 0	18 / 20	-2 / 11	15
2	**0** / 12	**0** / 7	15 / 9	10 / 20	25
3	5 / 0	19 / 14	19 / 16	10 / 18	5
Demand	5	15	15	10	

$u_1 = 0$, $u_2 = 7$, $u_3 = -5$

$v_1 = 5$, $v_2 = 0$, $v_3 = 2$, $v_4 = 13$

$$\bar{c}_{11} = c_{11} - (u_1 + v_1) = 10 - (0 + 5) = 5$$

$$\bar{c}_{14} = 11 - (0 + 13) = -2$$

Since there is at least a cell with a negative relative profit, then the current solution is not optimal and at least another iteration is needed.

An Iteration of the Transportation Method

- *Step 1- Determination of the entering variable: The candidate variables are the ones having negative relative profits ($\bar{c}_{ij}$). Among the candidate variables, select the most negative value of ($\bar{c}_{ij}$) as the entering variable.*
- *Step 2- Determination of the leaving variable: Increasing the value of the entering variable will cause the reduction in value of some of the basic variables (donors) and an increase in value of some others(recipients). A path of horizontal and vertical lines can always be formed to identify the donors and recipients. The first basic variable to be decreased to zero becomes the leaving variable.*
- *Step 3- The new basic feasible solution is identified by adding the value of the leaving basic variable to the allocation for each recipient cell and subtracting this amount from the allocation for each donor cell.*

Example of an Iteration

	Demand				
Source	A	B	C	D	Avail
1	5 10	15 − 0	18 20	+ -2 11	15
2	**0** 12	**0** + 7	15 9	10 − 20	25
3	5 0	19 14	19 16	10 18	5
Demand	5	15	15	10	

We increase this cell until one of the donor cells becomes zero

New Basic Solution

	Demand					
Source	A	B	C	D	Avail	
1	5 10	5 0	18 20	10 11	15	u_1 **=0**
2	**0** 12	10 7	15 9	2 20	25	u_2 **= 7**
3	5 0	19 14	19 16	13 18	5	u_3 **= -5**
Demand	5	15	15	10		
	v_1 **=5**	v_2 **=0**	v_3 **=2**	v_4**=11**		

Since all of the relative profits are positive, we stop. The solution is optimal. Total cost?

Quiz in next Lecture

- *Faced with a court order to desegregate its schools, a county school board decides to redistribute its minority students through busing. The plan calls for busing 50 students from each of the three cities- A, B and C , to four schools- E, W, N and S. For a perfect desegregation, the schools need 20, 40, 30, and 60 minority students respectively. The dollar cost for busing each student is given as follows:*

		Schools			
City		E	W	N	S
	A	7	6	5	4
	B	9	7	3	6
	C	8	8	7	3

Set up the transportation table for this problem

- *Find an initial basic feasible solution using the VAM method*
- *Determine the optimal solution using the u-v method*

Use of Big M in the Transportation Tableau

- *When the assignment of one source to one destination is not allowed, we use a cost of M for the corresponding cell*
- *This M represents a very large positive penalty for making that assignment*
- *The transportation algorithm is used in a regular way, just considering M as having a very large penalty*

Example (Ravindran, Phillips and Solberg)

- *Consider the problem of scheduling the weekly production of a certain item for the next four weeks. The production cost for the item is $10 for the first two weeks and $15 for the last two weeks. The weekly demands are 300, 700, 900, and 800, which must be met. The plant can produce a maximum of 700 units each week. In addition, the company can employ overtime during the second and third week. This increases the daily production by an additional 200 units, but the production cost increases by $5 per item. Excess production can be stored at a unit cost of $3/week.*
- *How should production be scheduled so as to minimize the total costs?*

Example (Ravindran, Phillips and Solberg)

	Demands				
Supplies	Week 1	Week 2	Week 3	Week 4	
Week 1-n	10	13	16	19	700
Week 2-n	M	10	13	16	700
Week 2-o	M	15	18	21	200
Week 3-n	M	M	15	18	700
Week 3-o	M	M	20	23	200
Week 4-n	M	M	M	15	700
	300	700	900	800	