1. Compute the DFT for the following sequence

$$x(n) = \{ 0, 1 , 2, 3\}$$

Sketch the relation frequency versus amplitude and phase respectively, knowing that the sampling frequency is 4 kHz.

- Convert from time domain to frequency domain

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-J 2\pi k n}{N}}$$

$$e^{-J\theta} = \cos\theta - J\sin\theta$$

- 1st harmonic $\underset{K}{X(0)} = \underset{\substack{n=0\\K=0}}{0} + \underset{\substack{n=1\\K=0}}{e^0} + \underset{\substack{n=2\\K=0}}{2e^0} + \underset{\substack{n=3\\K=0}}{3e^0} = 6$

- 2nd harmonic $\underset{K}{X(1)} = \underset{\substack{n=0\\K=1}}{0} + \underset{\substack{n=1\\K=1}}{e^{\frac{-J\cancel{2}\pi(1)(1)}{\cancel{4}}}} + \underset{\substack{n=2\\K=1}}{2e^{\frac{-J\cancel{2}\pi(\cancel{2})(1)}{\cancel{4}}}} + \underset{\substack{n=3\\K=1}}{3e^{\frac{-J\cancel{2}\pi(3)(1)}{\cancel{4}}}}$

$$= 0 + e^{-J\frac{\pi}{2}} + 2e^{-J\pi} + 3e^{\frac{-J3\pi}{2}}$$

- $e^{-J\frac{\pi}{2}} = \overset{0}{\cos\frac{\pi}{2}} - J\overset{1}{\sin\frac{\pi}{2}} = -J$
- $e^{-J\pi} = \overset{-1}{\cos\pi} - J\overset{0}{\sin\pi} = -1$
- $e^{-J\frac{3\pi}{2}} = \overset{0}{\cos\frac{3\pi}{2}} - J\overset{-1}{\sin\frac{3\pi}{2}} = J$

$$= 0 + (-J) + (2)(-1) + (3)(J)$$

$$= -2 + 2J$$

- 3rd harmonic $X(\overset{K}{2}) = \underset{k=2}{\overset{n=0}{0}} + \overset{n=1,\,k=2}{e^{\frac{-j\cancel{2}\pi(1)(\cancel{2})}{\cancel{4}}}} + \overset{n=2,\,k=2}{2e^{\frac{-j\cancel{2}\pi(\cancel{2})(2)}{\cancel{4}}}} + \overset{n=3,\,k=2}{3e^{\frac{-j\cancel{2}\pi(\cancel{2})(3)}{\cancel{4}}}}$

- $e^{-j\pi} = \overset{-1}{\cos\pi} - j\overset{0}{\sin\pi} = -1$
- $e^{-j2\pi} = \overset{1}{\cos 2\pi} - j\overset{0}{\sin 2\pi} = 1$
- $e^{-j3\pi} = \overset{-1}{\cos 3\pi} - j\overset{0}{\sin 3\pi} = -1$

$= 0 + (-1) + (2)(1) + (3)(-1) = -2$

- 4th harmonic $X(\overset{k}{3}) = \underset{k=3}{\overset{n=0}{0}} + \overset{n=1,\,k=3}{e^{\frac{-j\cancel{2}\pi(1)(3)}{\cancel{4}}}} + \overset{n=2,\,k=3}{2e^{\frac{-j\cancel{2}\pi(\cancel{2})(3)}{\cancel{4}}}} + \overset{n=3,\,k=3}{3e^{\frac{-j\cancel{2}\pi(3)(3)}{\cancel{4}}}}$

- $e^{-j\frac{3\pi}{2}} = \overset{0}{\cos\frac{3\pi}{2}} - j\overset{-1}{\sin\frac{3\pi}{2}} = j$
- $e^{-j3\pi} = \overset{-1}{\cos 3\pi} - j\overset{0}{\sin 3\pi} = -1$
- $e^{-j\frac{9\pi}{2}} = \overset{0}{\cos\frac{9\pi}{2}} - j\overset{1}{\sin\frac{9\pi}{2}} = -j$

$= 0 + (j) + (2)(-1) + (3)(-j) = -2 - 2j$

So, $X(K) = \{6, -2+2j, -2, -2-2j\}$

• sketch:

$$A = \sqrt{r^2 + (img)^2} \qquad \Phi = \tan^{-1}\left(\frac{img}{r}\right)$$

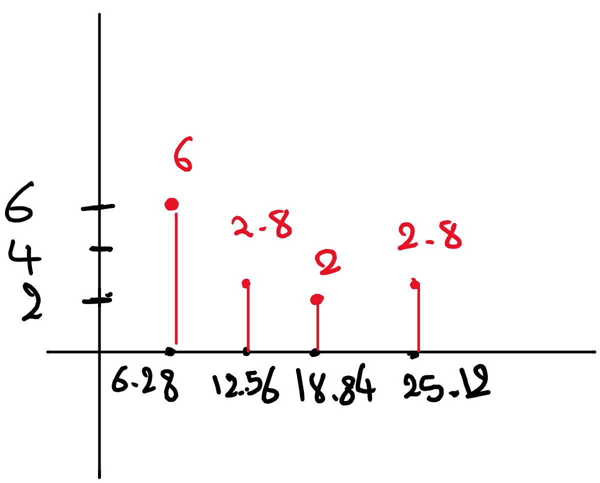
img: coffecient of imaginary

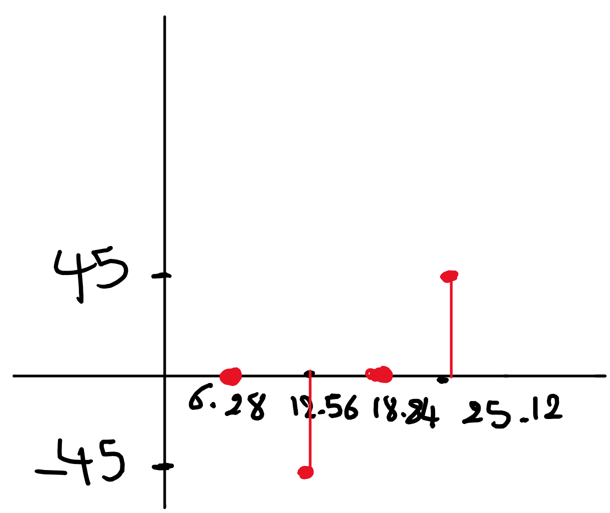
r: real

	A	Φ
$X(0)$	$\sqrt{6^2} = 6$	$\tan^{-1}(\frac{0}{6}) = 0$
$X(1)$	$\sqrt{(-2)^2+2^2} = \sqrt{8}$	$\tan^{-1}(\frac{2}{-2}) = -45$
$X(2)$	$\sqrt{(-2)^2} = 2$	$\tan^{-1}(\frac{0}{-2}) = 0$
$X(3)$	$\sqrt{(-2)^2+(-2)^2} = \sqrt{8}$	$\tan^{-1}(\frac{-2}{-2}) = 45$

$F_s = 4$ KHz

$$\boxed{\Omega = \frac{2\pi}{NT_s}} = \frac{2\pi}{4 * \frac{1}{4}} = 6.28$$ "fundamental freq."





2. Compute the continuous FT for the sequence

$$x(n) = \{ 0, 1 , 2, 3\}$$

from its DFT components knowing that the sampling frequency is 4kHZ.

① calculate DFT "See last problem"

② multiply each component by T_s

① $X(K) = \{ 6, -2+2J, -2, -2-2J \}$

② $F(j\omega) = \left\{ \frac{6}{4000}, \frac{-2+2J}{4000}, \frac{-2}{4000}, \frac{-2-2J}{4000} \right\}$

3. Compute the data sequence back from its DFT components {6, -2+2j, -2, -2-2j}.

Convert from frequency domain to time domain

- $x(n) = \frac{1}{N} \sum_{K=0}^{N-1} x(K) \cdot e^{\frac{J2\pi nK}{N}}$
- $e^{J\theta} = \cos\theta + J\sin\theta$

n	n=0, K=0	n=0, K=1	n=0, K=2	n=0, K=3

- $x(0) = 6e^{0} + (-2+2J)e^{0} + (-2)e^{0} + (-2-2J)e^{0}$

$= 6 - 2 + 2J - 2 - 2 - 2J = 0$

n	n=1, K=0	n=1, K=1	n=1, K=2	n=1, K=3

- $x(1) = 6e^{0} + (-2+2J)e^{\frac{J\cancel{2}\pi(1)(1)}{\cancel{4}}} + (-2)e^{\frac{J\cancel{2}\pi(1)(\cancel{2})}{\cancel{4}}} + (-2-2J)e^{\frac{J\cancel{2}\pi(1)(3)}{\cancel{4}}}$

→ $e^{\frac{J\pi}{2}} = \overset{0}{\cos\frac{\pi}{2}} + J\overset{1}{\sin\frac{\pi}{2}} = J$

→ $e^{J\pi} = \overset{-1}{\cos\pi} + J\overset{0}{\sin\pi} = -1$

→ $e^{J\frac{3}{2}\pi} = \overset{0}{\cos\frac{3\pi}{2}} + J\overset{-1}{\sin\frac{3\pi}{2}} = -J$

$J^2 = -1$

$= 6 + (-2+2J)(J) + (-2)(-1) + (-2-2J)(-J)$

$= 6 - 2J - 2 + 2 + 2J - 2 = \frac{4}{4} = 1$

$n \qquad n=2,\ k=0 \qquad n=2,\ k=1 \qquad n=2,\ k=2 \qquad n=2,\ k=3$

$$\bullet\ X(2) = 6e^{0} + (-2+2J)\,e^{\cancel{\frac{J2\pi(2)(1)}{4}}} + (-2)\,e^{\cancel{\frac{J2\pi(2)(2)}{4}}} + (-2-2J)\,e^{\cancel{\frac{J2\pi 6}{4}}}$$

$$\rightarrow e^{J\pi} = \overset{-1}{\cos}\,\pi + J\overset{0}{\sin}\,\pi = -1$$

$$\rightarrow e^{J2\pi} = \overset{1}{\cos}\,2\pi + J\overset{0}{\sin}\,2\pi = 1$$

$$\rightarrow e^{J3\pi} = \overset{-1}{\cos}\,3\pi + J\overset{0}{\sin}\,3\pi = -1$$

$$= 6 + (-2+2J)(-1) + (-2)(1) + (-2-2J)(-1)$$

$$= 6 + 2 - 2J - 2 + 2 + 2J = \frac{8}{4} = 2$$

$n \qquad n=3,\ k=0 \qquad n=3,\ k=1 \qquad n=3,\ k=2 \qquad n=3,\ k=3$

$$\bullet\ X(3) = 6e^{0} + (-2+2J)\,e^{\frac{J2\pi(3)(1)}{4}} + (-2)\,e^{\frac{J2\pi(3)(2)}{4}} + (-2-2J)\,e^{\frac{J2\pi(3)(3)}{4}}$$

$$\rightarrow e^{J\frac{3\pi}{2}} = \overset{0}{\cos}\,\frac{3\pi}{2} + J\overset{-1}{\sin}\,\frac{3\pi}{2} = -J$$

$$\rightarrow e^{J3\pi} = \overset{-1}{\cos}\,3\pi + J\overset{0}{\sin}\,3\pi = -1$$

$$\rightarrow e^{J\frac{9\pi}{2}} = \overset{0}{\cos}\,\frac{9\pi}{2} + J\overset{1}{\sin}\,\frac{9\pi}{2} = J$$

$$= 6 + (-2+2J)(-J) + (-2)(-1) + (-2-2J)(J)$$

$$= 6 + 2J + 2 + 2 - 2J + 2 = \frac{12}{4} = 3$$

So, $X(n) = \{0, 1, 2, 3\}$

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