DSP Lab 3

SHEET 2 SOLUTIONS

(1st Problem) 1.10 in book

A digital communication link carries binary coded words representing samples of an input signal

$$X_a(t) = 3\cos(600\,\pi t) + 2\cos(1800\,\pi t)$$

The link is operated at 10000 bits/s and each input sample is quantized into 1024 different voltage levels.

a) What is the sampling frequency and folding frequency?
b) What is the Nyquist rate for the signal $X_a(\mathrm{t})$?
c) What are the frequencies in the resulting discrete time signal $X(\mathrm{n})$?
d) What is the resolution Δ ?

(a) $\#\text{bits/sample} = \log_2(1024)$

$= 10$

Since 10000 is number of bits/s

Then $F_s = \dfrac{\text{bits/s}}{\text{bits/sample}}$

$= \dfrac{\cancel{\text{bits}}}{s} * \dfrac{\text{sample}}{\cancel{\text{bits}}} \rightarrow \dfrac{\text{sample}}{s}$

$= \dfrac{1000\cancel{0}}{1\cancel{0}} = 1000 Hz$

- $\#\text{bits} = \log_2(\text{levels})$
- $\#\text{levels} = 2^{(\text{bits})}$

$F_s \rightarrow \dfrac{\text{Samples}}{\text{Second}}$

- Folding frequency $= \frac{F_s}{2} = \frac{1000}{2} = 500\ Hz$

(b) $f_1 = \frac{600}{2} = 300$

$f_2 = \frac{1800}{2} = 900$ max

Nyquist $= 2(900) = 1800 Hz$

c) using (Relative or Normalized frequency)

$f = \frac{F}{F_s} \rightarrow$ from properties of f $\boxed{-\frac{1}{2} < f < \frac{1}{2}}$

So,

$$f_1 = \frac{600}{2} * \frac{1}{1000} = 0.3$$ in range ✓

$$f_2 = \frac{1800}{2} * \frac{1}{1000} = 0.9 > 0.5$$ $2\pi(1) - 2\pi(0.1) = 2\pi(0.9)$

$$x(n) = 3\cos(2\pi(0.3)n) + 2\cos(2\pi(0.1)n)$$

d) $\Delta = \dfrac{X_{max} - X_{min}}{\# \text{ Levels}}$

X_{max} → max of analog cos 1

$3(1) + 2(1) = 5$

X_{min} → min of analog cos −1

$3(-1) + 2(-1) = -5$

$\Delta = \dfrac{5 - (-5)}{1024} = \dfrac{10}{1024}$

X_{max}: max amplitude

X_{min}: min amp.

Levels 1024 "given"

(2nd Problem)

Quantize and encode the following sampled signal x(n) using 4 quantization levels and minimum number of bits. Compute the average error power in both cases.

$$X(n) = \{-1.22, 1.5, 3.24, 3.94, 2.20, -1.10, -2.26, -1.88, -1.2\}$$

* Steps:

1. find min & max amp.
2. find (delta Δ)
3. make ranges which equals #levels $+\Delta$
4. calculate mid point for each range
5. Quantize
6. average power error $= \frac{1}{N} \sum_{i=1}^{N} eq_i^2$

#samples ← N

X (n) = {-1.22, 1.5, 3.24, 3.94, 2.20, -1.10, -2.26, -1.88, -1.2}

① min = −2.26 , max = 3.94

② $\Delta = \dfrac{max - min}{l} = \dfrac{3.94 - (-2.26)}{4} = 1.55$

4 → given

③ [min, min + Δ] , [Z , Z + Δ] ----- [, max]

Z

[−2.26, −0.71] , [−0.71, 0.84] , [0.84, 2.39] , [2.39, 3.94]

① ② ③ ④

④ midpoins = $\dfrac{start + end}{2}$ foreach range

$\left\{ \dfrac{-2.26 + (-0.71)}{2} , ----- \right\} \Rightarrow \{-1.485, 0.065, 1.615, 3.165\}$

(5)

n	$x(n)$	interval index	$x_q(n)$	$e_q(n) = x_q(n) - x(n)$	e_q^2
0	-1.22	1	-1.485	-0.265	$(\quad)^2$
1	1.5	3	1.615	0.115	$(\quad)^2$
2	3.24	4	3.165	-0.075	
3	3.94	4	//	-0.775	
4	2.2	3	1.615	-0.585	
5	-1.1	1	-1.485	-0.385	
6	-2.26	1	//	0.775	
7	-1.88	1	//	0.395	
8	-1.2	1	//	-0.285	

{-1.485, 0.065, 1.615, 3.165}

(1) (2) (3) (4)

6 average power error $= \frac{1}{N} \sum_{i=1}^{N} e_{q_i}^2$

#samples ← N

$N = 9 \Rightarrow \frac{1}{9}\Big[(-0.265)^2 + (0.115)^2 + (-0.075)^2 + (-0.755)^2 + (-0.585)^2 + (-0.385)^2 + (0.775)^2 + (0.395)^2 + (0.285)^2\Big]$

$= \frac{1}{9}[2.018025] = \boxed{0.224225}$

* $\#\text{bits} = \log_2(4) = 2$

$\{00, 01, 10, 11\} \rightarrow$ encoding for 4 levels

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