* zeros → values of z for which $X(z) = 0$
* Poles → values of z for which $X(z) = \infty$

Sheet Five

3.1_a: Determine the z-transform of the following Signal:

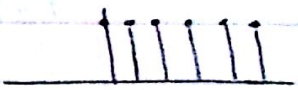
$$x(n) = \{\overset{-5}{3}, \overset{-4}{0}, \overset{-3}{0}, \overset{-2}{0}, \overset{-1}{0}, \underset{\uparrow}{\overset{0}{6}}, \overset{1}{1}, \overset{2}{-4}\}$$

$$X(z) = \sum_n x(n).z^{-n}$$

$$= 3z^5 + 6 + z^{-1} - 4z^{-2} \rightarrow \text{Roc}: 0 < |z| < \infty$$

3.2_a: Determine the z-transforms of the following Signals and sketch the corresponding pole-zero pattern

$\rightarrow x(n) = (1+n).u(n)$



$$u[n] = \frac{1}{1-z^{-1}}$$

$$X(z) = \sum_n x(n).z^{-n}$$

$$= \sum_{n=0}^{\infty} (1+n).z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n} + \sum_{n=0}^{\infty} n.z^{-n}$$

$$* \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \text{------} = \frac{1}{1-z^{-1}} \rightarrow \text{ROC}: z > 1$$

$$* \; x(n) = n.a^n.u(n)$$

$$x_1(n) = a^n.u(n) \rightarrow X_1(z) = \frac{1}{1-az^{-1}}$$

$$X(z) = -z.\frac{dX_1(z)}{dz} = \frac{az^{-1}}{(1-az^{-1})^2}$$

Scanned by CamScanner

So, $\sum_{n=0}^{\infty} n.z^{-n} \xrightarrow{u[n]} x(n)$

$$x_1(n) = \sum_{n=0}^{\infty} z^{-n}.u[n]$$

$$X_1(z) = \frac{1}{1-z^{-1}}$$

$$X(z) = -z.\frac{dX_1(z)}{dz} = -z\left[\frac{-(z^{-2})}{(1-z^{-1})^2}\right]$$

$$= \frac{z^{-1}}{(1-z^{-1})^2} \rightarrow ROC: |z|>1$$

Therefore: $X(z) = \frac{1-z^{-1}}{(1-z^{-1})^2}{}^{*} + \frac{z^{-1}}{(1-z^{-1})^2}$

$$= \frac{1}{(1-z^{-1})^2} = \frac{1}{1-2z^{-1}+z^{-2}} = \frac{z^2}{z^2-2z+1} = \frac{z^2}{(z-1)(z-1)}$$

zero $\rightarrow z=0$ And Double Pole at $z=1$

3.2.c: Determine z-transforms of the following signals and sketch corresponding pole-zero pattern

$x(n) = (-1)^n.2^{-n}.u(n)$

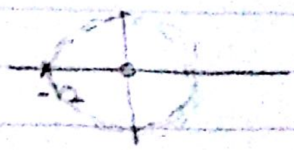
$$X(z) = \sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n.z^{-n}$$

Sequence $= \alpha^n.u[n] \rightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$

So, $X(z) = \frac{1}{1+1/2\, z^{-1}} = \frac{z}{z+1/2} \rightarrow ROC: |z|>1/2$

Pole $\rightarrow z=-1/2$

zero $\rightarrow z=0$



3.4.b: Determine the z-transform of the following Signal:

$$x(n) = n^2 . u(n)$$

Differentiation: $n.g[n] = -z \cdot \frac{dG(z)}{dz}$

$$X(z) = \sum_{n=0}^{\infty} n^2 . z^{-n}$$

$$= (-z)^2 . \frac{d^2}{dz^2} [\sum_{n=0}^{\infty} z^{-n}]$$

$$= z^2 . \frac{d^2}{dz^2} [\frac{1}{1-z^{-1}}]$$

$$= z^2 . [\frac{d}{dz} [\frac{-z^{-2}}{(1-z^{-1})^2}]]$$

$$= z^2 . [\frac{(2z^{-3})(1-z^{-1})^2 - (-z^{-2})(2)(1-z^{-1})(z^{-2})}{(1-z^{-1})^4}]$$

$$= z^2 [\frac{(2z^{-3})(1-z^{-1}) + (2z^{-4})}{(1-z^{-1})^3}]$$

$$= \frac{2z^{-1}(1-z^{-1}) + 2z^{-2}}{(1-z^{-1})^3}$$

$$= \frac{2z^{-1} - 2z^{-2} + 2z^{-2}}{(1-z^{-1})^3}$$

$$= \frac{2z^{-1}}{(1-z^{-1})^3} \rightarrow \text{ROC: } |z|>1$$

?!

* Find z-transform for $x(n) = u(-n+4)$

$\boxed{g[-n] = G(1/z)}$ and $\boxed{g[n+n_o] = z^{n_o} \cdot G(z)}$

$$u[n] = \frac{1}{1-z^{-1}}$$

$$u[-n] = \frac{1}{1-(1/z)^{-1}} = \frac{1}{1-z}$$

$$u[-n+4] = \frac{z^4}{1-z}$$

3.11-a: Using long division, determine the inverse z-transform of

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

$$\begin{array}{r|l}
 & 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \cdots \\
\hline
1-2z^{-1}+z^{-2} & 1 + 2z^{-1} \\
 & \ominus\; 1 - 2z^{-1} + z^{-2} \\
\hline
 & 4z^{-1} - z^{-2} \\
 & \ominus\; 4z^{-1} - 8z^{-2} + 4z^{-3} \\
\hline
 & 7z^{-2} - 4z^{-3} \\
 & \ominus\; 7z^{-2} - 14z^{-3} + 7z^{-4} \\
\hline
 & 10z^{-3} - 7z^{-4}
\end{array}$$

Therefore: $X(n) = \{\underset{\uparrow}{1}, 4, 7, 10, \ldots, 3n+1, \ldots\}$

$\quad 0 \quad 1 \quad 2 \quad 3, \ldots$

3.14_a: Determine the casual signal $x(n)$ if the z-transform $X(z)$ given by:

$$X(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$$

$$\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}} = \frac{1+3z^{-1}}{(1+z^{-1})(1+2z^{-1})}$$ "Partial Fraction"

$$\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}} = \frac{A}{(1+z^{-1})} + \frac{B}{(1+2z^{-1})}$$ → Mult. "$1+3z^{-1}+2z^{-2}$"

$$1+3z^{-1} = A(1+2z^{-1}) + B(1+z^{-1})$$

$$1+3z^{-1} = A + 2Az^{-1} + B + Bz^{-1}$$

$$1+3z^{-1} = (A+B) + z^{-1}(2A+B)$$

$A+B=1 \rightarrow ①$

$2A+B=3 \rightarrow ②$ "Solving 2 equations"

$-A=-2 \rightarrow A=2$ and $B=-1$

So, $X(z) = \left(\frac{2}{1+z^{-1}} + \frac{-1}{1+2z^{-1}}\right)$

$$\alpha^n . u[n] = \frac{1}{1-\alpha z^{-1}}$$

$$x(n) = [2[-1]^n - 1[-2]^n] . u(n)$$

3.19: Determine the casual Signal $x(n)$ having the z-transform

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2} \quad \rightarrow \text{Mult. } (1-2z^{-1})(1-z^{-1})^2$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})$$

$$1 = A[1-2z^{-1}+z^{-2}] + B[1-3z^{-1}+2z^{-2}] + Cz^{-1}[1-2z^{-1}]$$

$$1 = (A+B) + z^{-1}(-2A-3B+C) + z^{-2}(A+2B-2C)$$

$A+B=1 \rightarrow ①$

$-2A-3B+C=0 \rightarrow ②$

$A+2B-2C=0 \rightarrow ③$

Solving 3 equations together:

$\boxed{A=4}$ $\boxed{B=-3}$ $\boxed{C=-1}$

So, $x(n) = [4(2)^n - 3 - n].u(n)$

$$\boxed{n\,g[n] = -z\frac{dG(z)}{dz}}$$