DSP Lab 2

SHEET 1 SOLUTIONS

(1st Problem) 1.8 in book

An analog Electrocardiogram “ECG” signal contains useful frequencies up to 100HZ.

a) What is the Nyquist rate for this signal?

b) Suppose that we sample this signal at a rate of 250 samples. What is the highest frequency that can be represented uniquely at this sampling rate?

a) Nyquist $\rightarrow F_s \geq 2 F_{max}$

since $F_{max} = 100\ Hz$ "given"

$F_s \geq 2(100) \rightarrow F_s = 200\ Hz$

b) highest frequency that can be represented uniquely "folding frequency"

$$\circledast\ F_{Fold} = \frac{F_s}{2} = \frac{250}{2} = 125\ Hz$$

(2nd Problem) 1.9 in book

An analog signal X_a(t)= Sin(480 πt)+3 Sin(720πt) is sampled 600 times per second.

a) Determine the Nyquist sampling rate at X_a(t)
b) Determine the Folding frequency
c) What are the frequencies in radian, in the resulting discrerte time signal X(n)?
d) can we reconstruct this signal again correctly from the sampled signal using the previously mentioned sampling frequency?

a) $F_s \geq 2F_{max}$

$$f_1 = \overbrace{480}^{2f}/2 = 240$$

$$f_2 = 720/2 = \underline{360} \quad \text{max}$$

So $F_s = 2(360) = 720\ Hz$

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b) $F_{fold} = \dfrac{F_s}{2} = \dfrac{600}{2} = 300\ Hz$

$600 \rightarrow$ sampling freq. given

c) $x_d(t) = \sin\left(\frac{480}{600}\pi n\right) + 3\sin\left(\frac{720}{600}\pi n\right)$

$= \sin\left(\frac{4\pi}{5} n\right) + 3\sin\left(\frac{6\pi n}{5}\right)$

7 π

$\downarrow$

$= \sin\left(\frac{4\pi n}{5}\right) + 3\sin\left(2\pi - \frac{4\pi}{5}\right)$

$= \sin\left(\frac{4\pi n}{5}\right) + 3\sin\left(-\frac{4\pi n}{5}\right)$

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$= -2\sin\left(\frac{4\pi n}{5}\right)$

$\omega = \frac{4\pi}{5}$

$F_s = \frac{1}{T}$

discrete

$t = nT = \frac{n}{F_s}$

$x_d(t) = A\cos\left(\frac{2\pi F n}{F_s} + \theta\right)$

analog

$x_a(t) = A\cos(\Omega t + \theta)$

$\Omega = 2\pi f$

$x_a(t) = A\cos(2\pi f t + \theta)$

d) No we can't due to the aliasing happend to the 2^{nd} component.

3^{rd} Problem

$Y(t) = 5\cos(250\pi t) + 10\sin(80\pi t)$ if $y(t)$ is sampled at $F_s = 100$

a) find the digital signal $y(n)$.

b) Can we retrieve back the original signal correctly from $y(n)$ or not ?

a) $y_d(t) = 5\cos\left(\frac{250\pi n}{100}\right) + 10\sin\left(\frac{80\pi n}{100}\right)$

$= 5\cos(\underbrace{2.5\pi}_{>2\pi} n) + 10\sin(0.8\pi n)$

$= 2\pi + 0.5\pi$

$= 5\cos(0.5\pi n) + 10\sin(0.8\pi n)$

b) No due to aliasing.

F_s must be $2 * 125 = \boxed{250\text{ Hz}}$

$F_s = \frac{1}{T}$

discrete

$t = nT = \frac{n}{F_s}$

$x_d(t) = A\cos\left(\frac{2\pi F n}{F_s} + \theta\right)$

analog

$x_a(t) = A\cos(\Omega t + \theta)$

$\Omega = 2\pi f$

$x_a(t) = A\cos(2\pi f t + \theta)$

4th Problem

$Y(t) = 3\cos(300\,\pi t)$

a) What is the Nyquist frequency?
b) If the signal is sampled by 100 HZ, find y(n)
c) If y(n) is converted back to analog what will be the value of analog F, is it the same as original?

a) $f_1 = 300/2 = 150\,Hz \rightarrow max$

$F_s = 2 * 150 = 300\,Hz$

b) $F_s = 100\,Hz$

$y_d(t) = 3\cos\left(\frac{300}{100}\pi n\right) = 3\cos(3\pi n)$

$> 2\pi$

$2\pi + \pi$

$= 3\cos(\pi n)$

$2\pi f = 1$

$\neq$

c)

$f_d = \frac{1}{2} \Rightarrow \frac{1}{2} * 100 = 50\,Hz$

due of aliasing

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