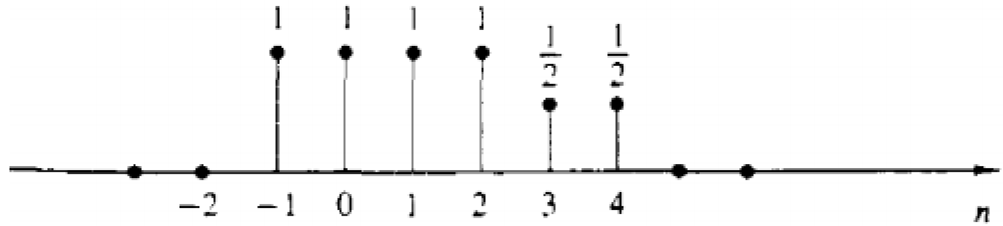
1. Sketch the output sequence in each case for the following input sequence X



$X(n) = \{---, \underset{-2}{0}, \underset{-1}{1}, \underset{0}{1}, \underset{1}{1}, \underset{2}{1}, \underset{3}{\tfrac{1}{2}}, \underset{4}{\tfrac{1}{2}}, 0, ---\}$

a. X(n-2)

→ shift Right by 2

$\{\dots, 0, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \dots\}$

$1+K$, $-1-K$ } shift left

$-1+k$, $1-K$ } shift Right

b. X (-n+4)

① fold → $\{, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, 1, 1, 0, ---\}$

② shift Right by 4

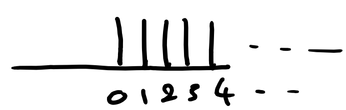
$\{---, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, 1, 1, 0, ---\}$

c. X(n+2)

$\{---, 0, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, ---\}$

d. X(n) * u (2-n)

unit step u(n) 1 on $n \geq 0$



① fold u then shift Right by 2

$u(n) = \{---, 0, 1, 1, 1, 1, \dots 1\}$

$u(-n) = \{1, --, 1, 1, 1, 0, ---\}$

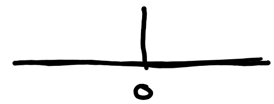
$u(2-n) = \{1, --, 1, 1, 1, 1, 0, ---\}$

② multiply x(n) by u(2-n)

$X(n) = \{---, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, --\}$

→ $\{---, 1, 1, 1, 1, 0, ---\}$

Unit sample $\delta(n)$ 1 on $n=0$



e. $X(n-1) * \delta(n-3)$

① Shift δ Right by 3

$$\delta(n) = \{\ldots, 0, \underset{\uparrow}{1}, 0, \ldots\}$$

$$\delta(n-3) = \{\ldots, \underset{\uparrow}{0}, 0, 0, 1, 0, \ldots\}$$

② Shift x Right 1

$$x(n-1) = \{\ldots, \underset{\uparrow}{1}, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, \ldots\}$$

③ multiply

$$\{\ldots, \underset{\uparrow}{0}, 0, 0, 1, 0, 0, \ldots\}$$

f. $X(n^2)$

$y(n) = x(n^2)$

$y(0) = x(0)$ $\quad$ $y(-1) = x(1)$

$y(1) = x(1)$ $\quad$ $y(-2) = x(4)$

$y(2) = x(4)$ $\quad$ $y(-3) = x(9)$

$y(3) = x(9)$

$$y(n) = \{\ldots, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, \ldots\}$$

2. Find the sequence of Y(n)

a. $Y(n) = u(n) - u(n-3) - 2\,\delta(n-1)$

$y(0) = u(0) - u(-3) - 2\,\delta(-1) = 1 - 0 - 0 = 1$

$y(1) = u(1) - u(-2) - 2\,\delta(0) = 1 - 0 - 2 = -1$

$y(2) = u(2) - u(-1) - 2\,\delta(1) = 1 - 0 - 0 = 1$

$y(3) = u(3) - u(0) - 2\,\delta(2) = 1 - 1 - 0 = 0$

$y(n) = \{ \cdots, \underset{\uparrow}{1}, -1, 1, 0, \cdots \}$

---

b. $Y(n) = u(n+2) - u(n) + 3\,\delta(n+1)$

$y(-2) = u(0) - u(-2) + 3\,\delta(-1) = 1 - 0 + 0 = 1$

$y(-1) = u(1) - u(-1) + 3\,\delta(0) = 1 - 0 + 3(1) = 4$

$y(0) = u(2) - u(0) + 3\,\delta(1) = 1 - 1 + 0 = 0$

$y(n) = \{ \cdots, 1, 4, \underset{\uparrow}{0}, \cdots \}$

1. A discrete system can 1) static or dynamic 2) time invariant or time variant
   Examine the following systems with respect to the properties above.

① static: Depends at most on input sample at same time but no past or future samples
present

② Time invariant: iff the input, output characteristics don't change with time

① output shift

② input shift

Compare ① & ② → there is a change "variant"
→ " " no " "in variant"

a. Y(n) = cos(x[n])

① static depends on present

② $\left.\begin{matrix} \cos(x(n-k)) \\ \cos(x(n-k)) \end{matrix}\right\} \to$ no change in variant

b. Y(n) = x(-n + 2)

① $y(0) = x(2)$ "future" → dynamic

② $\left.\begin{matrix} x(-(n-k)+2) \\ x(-n-k+2) \end{matrix}\right\} \to$ Change "variant"

c. y(n)=x(2n)

① $y(1) = x(2)$ "future" → dynamic

② $\left.\begin{matrix} x(2(n-k)) \\ x(2n-k) \end{matrix}\right\} \to$ variant

d. y(n) = |x(n)|

① static

② $|x(n-k)|$

$|x(n-k)|$ → invariant

e. y(n) =Round[x(n)] integer part of x(n)

① static

② Round[x(n-k)]

Round[x(n-k)] → invariant

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