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***Inverted Pendulum by Fuzzy Control Inference System***

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## Abstract

An inverted pendulum is a classic control problem where the goal is to stabilize an upright pendulum in its inverted position. Fuzzy control is one approach used to address this problem.

The challenge in using fuzzy control for an inverted pendulum lies in designing the fuzzy sets, rules, and membership functions to adequately capture the system's dynamics and respond appropriately to changes in the pendulum's state.

Moreover, implementing fuzzy control for an inverted pendulum often involves simulation or physical setup with sensors, actuators, and a control system that can interpret fuzzy logic rules and apply the necessary control actions.

## Introduction

Controlling an inverted pendulum using fuzzy logic involves employing a fuzzy control system to stabilize the pendulum in an upright position. An inverted pendulum is an unstable system where the challenge lies in maintaining balance by applying appropriate forces.

Understanding the Inverted Pendulum System:

System Dynamics:

An inverted pendulum consists of a rod or a stick with a mass (the pendulum) that needs to be balanced in the upright position.

The system is inherently unstable; without control, it will fall due to gravity.

Fuzzy logic provides a way to deal with uncertain or imprecise information by using linguistic variables.

It allows for the creation of rules based on human-like reasoning rather than precise mathematical models.

Components of Fuzzy Control:

Fuzzification, fuzzy rules, inference engine, defuzzication and fuzzy rules

Real-world Implementation: Translating the fuzzy logic control into a physical system with sensors and actuators can pose challenges.

## MATHEMATICAL MODELING OF INVERTED PENDULUM

The inverted pendulum system is a classic control problem that is used in universities around the world. It is a suitable process to test prototype controllers due to its high nonlinearities and lack of stability.

In this section, the model of the single inverted pendulum is

established and the dynamical equations of the system will be

derived. Figure 1 shows the free body diagram of the system.

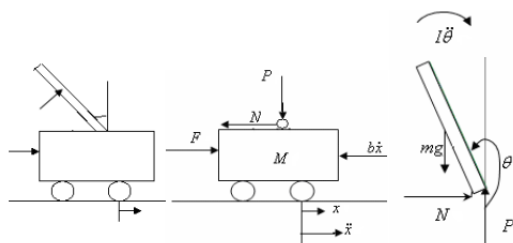


Figure 1

The single inverted pendulum system is made of a cart on top of which pendulum is pivoted. The cart is constrained to move only in the horizontal x direction, while the pendulum can only rotate in the x-y plane. The single inverted pendulum system has two degrees of freedom and can therefore be fully represented using two generalized coordinates: horizontal displacement of the cart, and rotational displacement of pendulum. The physical properties of the system are fixed in Table 1.

M	Mass of cart	0.5kg
m	Mass of pendulum	0.2kg
b	Friction of cart	0.1N/m/sec
l	Length to pendulum centre of mass	0.3m
I	inertia of the pendulum	0.006kg*m*m
F	Force applied to cart	
x	Cart position coordinate	
$\phi$	Pendulum angle from the vertical	
$\theta$	Pendulum angle from the vertical downwards	

## Angular Acceleration

$$x_{dd} = \left(\frac{1}{m}\right) * (u(1) - u(2) - b * u(3))$$

## Normal Acceleration

$$t_{dd} = \left(\frac{1}{l}\right) * (-u(1) * l * \cos(u(3)) - u(2) * l * \sin(u(3)))$$

## Horizontal Direction

$$N = m * (u(1) - l * u(3)^2 * \sin(u(4)) + l * u(2) * \cos(u(4)))$$

## Vertical Direction

$$P = m * (l * (u(2))^2 * \cos(u(3)) + l * u(1) * \sin(u(3)) + g$$

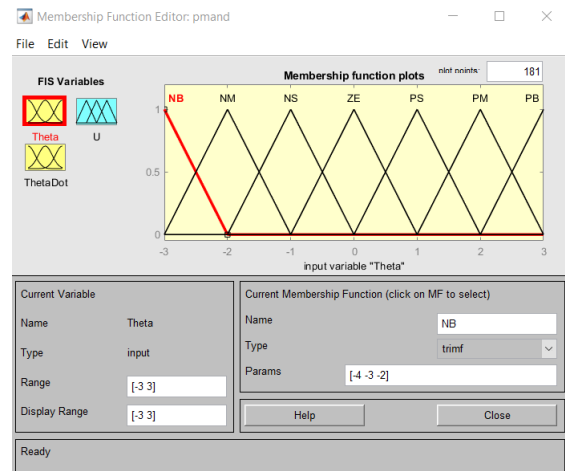
# FUZZY CONTROLLER DESIGN

Designing a fuzzy control system for an inverted pendulum involves creating a controller that uses fuzzy logic to stabilize the pendulum in its upright position. Here's a step-by-step guide:

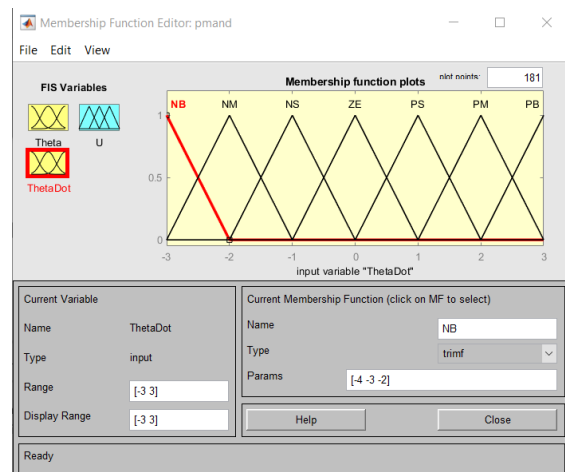
### 1. Define Inputs and Outputs:

Inputs: Angle of the pendulum, angular velocity.

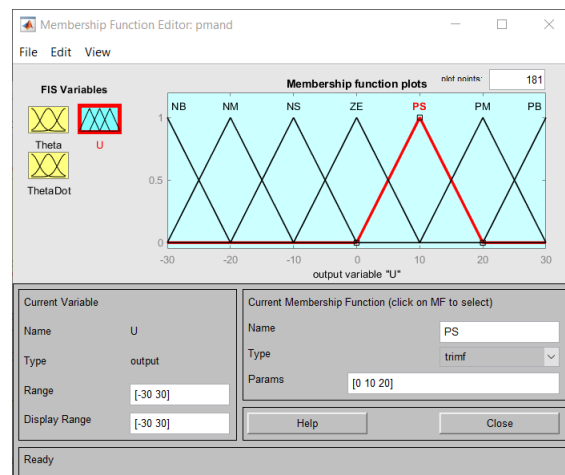
Output: Control action (force or torque applied).



### Theta Membership Function



### ThetaDot Membership Function



### U Membership Function

2. Fuzzification:

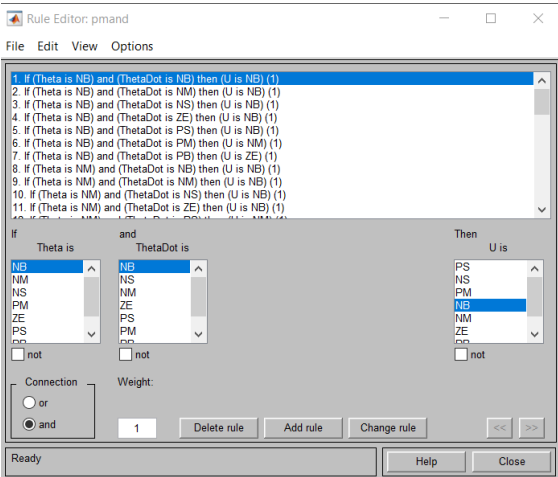
Convert crisp inputs (angle, velocity) into fuzzy sets using linguistic variables ("tilted forward," "tilted backward," "falling rapidly," etc.).

Define membership functions for these linguistic variables.

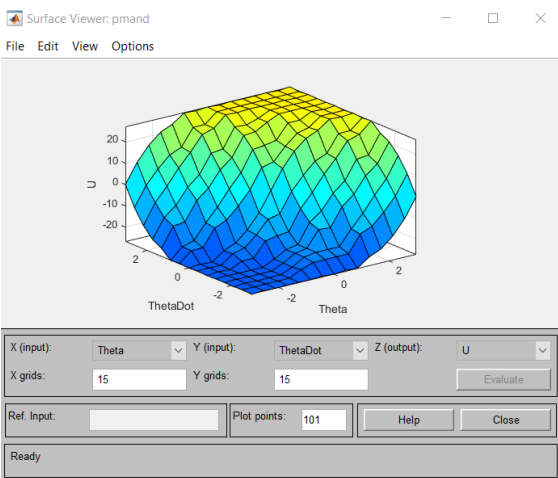
3. Rule Base Formation:

Create a set of IF-THEN rules based on expert knowledge or system behavior.

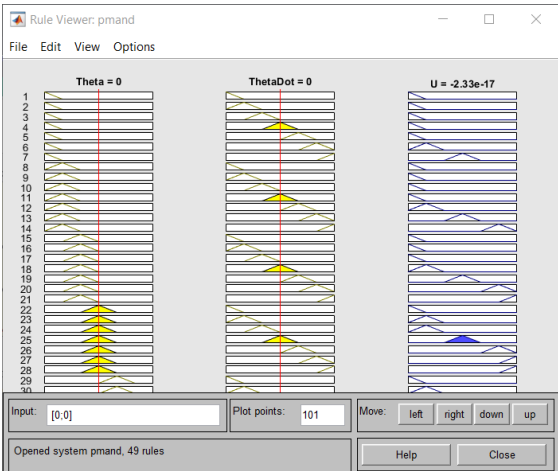
Rules could be of the form: "IF angle is A AND velocity is B, THEN apply control action C."



Rule Table



Rule Surface



Rules

4. Fuzzy Inference:

Apply fuzzy logic to determine appropriate control actions based on the rules and current fuzzy input values.

Use inference methods like Mamdani or Sugeno to process the rules.

$\begin{matrix} \text{cc} \\ \text{e} \end{matrix}$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NM	ZE
NM	NB	NB	NB	NB	NM	ZE	PM
NS	NB	NB	NB	NM	ZE	PM	PB
ZE	NB	NB	NM	ZE	PM	PB	PB
PS	NB	NM	ZE	PM	PB	PB	PB
PM	NM	ZE	PM	PB	PB	PB	PB
PB	ZE	PM	PB	PB	PB	PB	PB

"NB" to represent "negative big"

"NM" to represent "negative medium"

"ZE" to represent "zero"

"PS" to represent "positive small".

"PM" to represent "positive medium"

"PB" to represent "positive big"

5. Defuzzification:

Convert the fuzzy output into a crisp control signal (force or torque) using methods like centroid or weighted average.

## 6. Membership Function and Rule Tuning:

Fine-tune membership functions and rules to improve controller performance.

Use simulation or experimental data to adjust these parameters.

## IV. SIMULATION AND EXPERIMENTAL TESTS

The structure of the inverted pendulum system by Simulink is shown in Figure 6. Applying the dynamic model of inverted pendulum obtained in section 2, Figures 7 show control results of the inverted pendulum system by Simulink simulation.

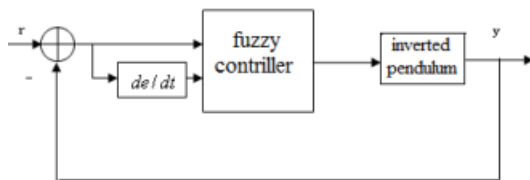


Figure 6

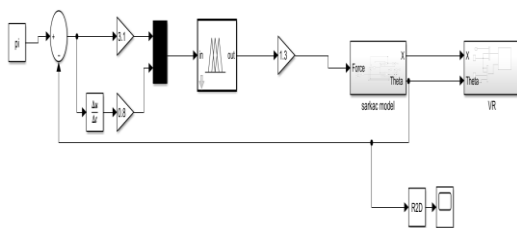
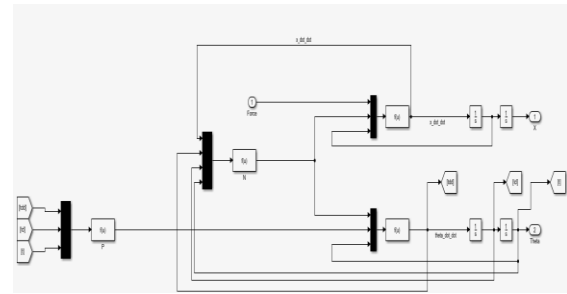
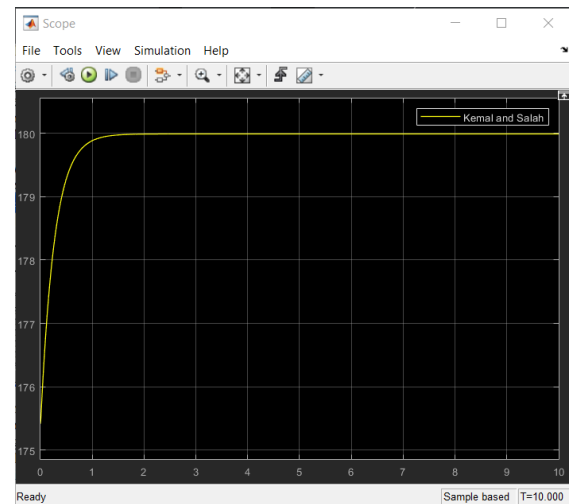


Figure 7

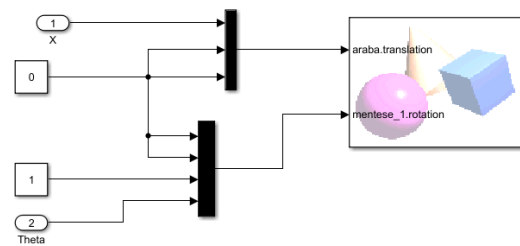


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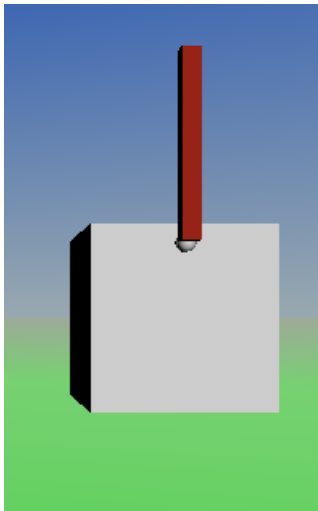


Scope

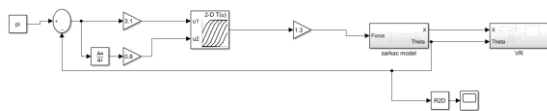
Figure 8



VR



VR Model



LOOKUP Table Diagram



LookUp Table Scope

Viewing "n-D Lookup Table" block data (T(:,:,)):

Breakpoints	Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Row		-3	-2	-1	0	1	2	3
(1)	-3	-3	-3	-3	-3	-3	-2	0
(2)	-2	-3	-3	-3	-3	-2	0	2
(3)	-1	-3	-3	-3	-2	0	2	3
(4)	0	-3	-3	-2	0	2	3	3
(5)	1	-3	-2	0	2	3	3	3
(6)	2	-2	0	2	3	3	3	3
(7)	3	0	2	3	3	3	3	3

Look Up Table Block Data

## V. CONCLUSION

**Fuzzy Logic Control:** Offers the advantage of handling imprecise information but might require fine-tuning of membership functions and rules for optimal performance.

**PID Control:** Provides a well-understood and widely used method but might struggle with tuning and adaptability to varying system conditions.

**Lookup Table Control:** Efficient for simple systems but might lack adaptability and struggle with scalability for complex systems.

Each method has its strengths and weaknesses. The choice of control method often depends on the specific characteristics of the system, the available resources, and the desired balance between precision and computational complexity.

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