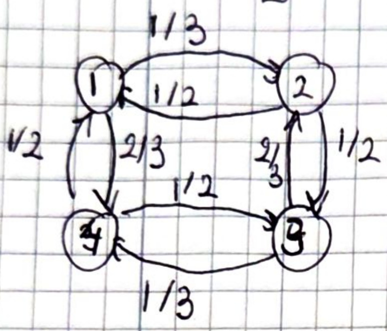
Markov chain

$S = \{1, 2, 3, 4\}$

$$[P_{ij}] = \begin{bmatrix} 0 & 1/3 & 0 & 2/3 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}$$

From state j to i



a) $X_0 = 1 \rightarrow p_0 = [1 \quad 0 \quad 0 \quad 0]^T$

$$p_1 = [P_{ij}] \cdot p_0 = \begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$$p_2 = [P_{ij}]^2 \cdot p_0$$

$$= \begin{bmatrix} 0 & 1/3 & 0 & 2/3 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$p_2 = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix}$$

b) Convergence of X_n as $n \rightarrow \infty$

Key properties of $[P_{ij}]$

1. All states communicate
2. Periodic with period 2 $\rightarrow$ cycles of even lengths dominate

$$p_n = \begin{cases} [0 \quad 1/2 \quad 0 \quad 1/2]^T & \text{for } n \text{ ~~even~~ odd} \\ [1/2 \quad 0 \quad 1/2 \quad 0]^T & \text{for } n \text{ even} \end{cases}$$

$$p_{n+1}(1) + p_{n+1}(3) = p_{n+1}(2) + p_{n+1}(4)$$

$$\Rightarrow p_0(1) + p_0(3) \neq p_0(2) + p_0(4)$$

The distribution of X_n does **not** converge to a single limit.