Problem 2:- given Π_0 [initial distribution]

$$\Pi_n = P^n \Pi_0 \quad , \quad \hat{\Pi}_n = \underbrace{(S_M P)^n}_{\text{Sampling of } P} \underbrace{S_M \Pi_0}_{\text{sample of } \Pi_0}$$

I) Identify Π_{n+1} & $\hat{\Pi}_{n+1}$

$$\Pi_{n+1} = P \cdot \Pi_n \quad , \quad \hat{\Pi}_{n+1} = S_M \cdot P \cdot \hat{\Pi}_n$$

II) The distance $d(\Pi_{n+1}, \hat{\Pi}_{n+1})$ Can be identified as:-

$$d(\Pi_{n+1}, \hat{\Pi}_{n+1}) = d(P\Pi_n, S_M \cdot P\hat{\Pi}_n)$$

using triangular Inquality with $P\hat{\Pi}_n$ as intermidiate.

$$d(\Pi_{n+1}, \hat{\Pi}_{n+1}) = d(P\Pi_n, S_M P\hat{\Pi}_n) \leqslant d(P\Pi_n, P\hat{\Pi}_n) + d(P\hat{\Pi}_n, S_M P\hat{\Pi}_n)$$

III) break down of R.H.S of Inquality:-

1) $d(P\Pi_n, P\hat{\Pi}_n) \leqslant d(\Pi_n, \hat{\Pi}_n)$ Property of $\underline{P}$

2) $d(P\hat{\Pi}_n, S_M P\hat{\Pi}_n) \leqslant \frac{1}{M}$ Property of S_M Sampling

\+ Subsitute in the inquality:-

$$\therefore d(\Pi_{n+1}, \hat{\Pi}_{n+1}) \leqslant d(P\Pi_n, P\hat{\Pi}_n) + d(P\hat{\Pi}_n, S_M P \cdot \hat{\Pi}_n)$$

$$\leqslant d(\Pi_n, \hat{\Pi}_n) + \frac{1}{M} \quad \#$$