

$$f(x) = \frac{1}{1 + e^{-x}}$$

$$\begin{aligned} \text{out}_{h_1} &= f(w_1 \times i_1 + w_2 \times i_2 + b_1 \times 1) \\ &= f(0.15 \times 0.05 + 0.20 \times 0.10 + 0.35 \times 1) \\ &= f(0.3775) \\ &= \frac{1}{1 + e^{-0.3775}} \end{aligned}$$

$$= 0.59327$$

$$\begin{aligned} \text{out}_{h_2} &= f(w_3 \times i_1 + w_4 \times i_2 + b_2 \times 1) \\ &= f(0.25 \times 0.05 + 0.30 \times 0.10 + 0.35 \times 1) \\ &= f(0.3925) \\ &= \frac{1}{1 + e^{-0.3925}} \end{aligned}$$

$$= 0.59689$$

$$\begin{aligned} \text{out}_{o_1} &= f(w_5 \times \text{out}_{h_1} + w_6 \times \text{out}_{h_2} + b_3 \times 1) \\ &= f(0.40 \times 0.59327 + 0.45 \times 0.59689 + 0.60 \times 1) \\ &= f(1.10591) \\ &= \frac{1}{1 + e^{-1.10591}} \end{aligned}$$

$$= 0.75137$$

$$\begin{aligned} \text{out}_{o_2} &= f(w_7 \times \text{out}_{h_1} + w_8 \times \text{out}_{h_2} + b_4 \times 1) \\ &= f(0.50 \times 0.59327 + 0.55 \times 0.59689 + 0.60 \times 1) \\ &= f(1.22492) \\ &= \frac{1}{1 + e^{-1.22492}} \end{aligned}$$

$$= 0.77293$$

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$$\begin{aligned} E &= \frac{1}{2} (T_1 - \text{out}_{o1})^2 + \frac{1}{2} (T_2 - \text{out}_{o2})^2 \\ &= (0.01 - 0.75137)^2 + (0.99 - 0.77293)^2 \\ &= 0.298371 \end{aligned}$$

$$\begin{aligned} \delta_{o1} &= (T_1 - \text{out}_{o1}) * \text{out}_{o1} * (1 - \text{out}_{o1}) \\ &= (0.01 - 0.75137) * 0.75137 * (1 - 0.75137) \\ &= -0.13850 \end{aligned}$$

$$\begin{aligned} w_5^+ &= w_5 + \eta * \delta_{o1} * \text{out}_{n1} \\ &= 0.40 + 0.5 * (-0.13850) * 0.59327 \\ &= 0.35892648 \approx 0.35892 \end{aligned}$$

$$\begin{aligned} w_6^+ &= w_6 + \eta * \delta_{o1} * \text{out}_{n2} \\ &= 0.45 + 0.5 * (-0.13850) * 0.59689 \\ &= 0.408666186 \approx 0.40867 \end{aligned}$$

$$\begin{aligned} b_3^+ &= b_3 + \eta * \delta_{o1} * 1 \\ &= 0.60 + 0.5 * (-0.13850) * 1 \\ &= 0.53075 \end{aligned}$$

$$\begin{aligned} \delta_{o2} &= (T_2 - \text{out}_{o2}) * \text{out}_{o2} * (1 - \text{out}_{o2}) \\ &= (0.99 - 0.77293) * 0.77293 * (1 - 0.77293) \\ &= 0.03810 \end{aligned}$$

$$\begin{aligned} w_7^+ &= w_7 + \eta * \delta_{o2} * \text{out}_{n1} \\ &= 0.50 + 0.5 * 0.03810 * 0.59327 \\ &= 0.511301270 \approx 0.51130 \end{aligned}$$

$$\begin{aligned}
 w_8^+ &= w_8 + \eta * \delta_{o2} * out_{h2} \\
 &= 0.55 + 0.5 * 0.03810 * 0.59689 \\
 &= 0.561370121 \approx 0.56137
 \end{aligned}$$

$$\begin{aligned}
 b_4^+ &= b_4 + \eta * \delta_{o2} * 1 \\
 &= 0.60 + 0.5 * 0.03810 * 1 \\
 &= 0.61905
 \end{aligned}$$

$$\begin{aligned}
 \delta_{h1} &= (\delta_{o1} * w_5 + \delta_{o2} * w_7) * out_{h1} * (1 - out_{h1}) \\
 &= (-0.13850 * 0.40 + 0.03810 * 0.50) * 0.59327 * (1 - 0.59327) \\
 &= -0.00877
 \end{aligned}$$

$$\begin{aligned}
 w_1^+ &= w_1 + \eta * \delta_{h1} * i_1 \\
 &= 0.15 + 0.5 * (-0.00877) * 0.05 \\
 &= 0.149780716 \approx 0.14978
 \end{aligned}$$

$$\begin{aligned}
 w_2^+ &= w_2 + \eta * \delta_{h1} * i_2 \\
 &= 0.20 + 0.5 * (-0.00877) * 0.10 \\
 &= 0.19956143 \approx 0.19956
 \end{aligned}$$

$$\begin{aligned}
 b_1^+ &= b_1 + \eta * \delta_{h1} * 1 \\
 &= 0.35 + 0.5 * (-0.00877) * 1 \\
 &= 0.34562
 \end{aligned}$$

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$$\begin{aligned}\delta_{hz} &= (\delta_{01} * w_6 + \delta_{02} * w_8) * out_{hz} * (1 - out_{hz}) \\ &= ((-0.13850) * 0.45 + 0.03810 * 0.55) * 0.59689 * \\ &\quad (1 - 0.59689) \\ &= -0.00995\end{aligned}$$

$$\begin{aligned}w_3^+ &= w_3 + \eta * \delta_{hz} * i_1 \\ &= 0.25 + 0.5 * (-0.00995) * 0.05 \\ &= 0.24975114 \approx 0.24975\end{aligned}$$

$$\begin{aligned}w_4^+ &= w_4 + \eta * \delta_{hz} * i_2 \\ &= 0.30 + 0.5 * (-0.00995) * 0.10 \\ &= 0.29950229 \approx 0.29950\end{aligned}$$

$$\begin{aligned}b_2^+ &= b_2 + \eta * \delta_{hz} * 1 \\ &= 0.35 + 0.5 * (-0.00995) * 1 \\ &= 0.34503\end{aligned}$$

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In [1]: def sigmoid(x):
        return 1 / (1 + (2.71828 ** -x))

        def sigmoid_derivative(x):
            return x * (1 - x)

        inputs = [0.05, 0.10]
        weights_input_hidden = [0.15, 0.20, 0.25, 0.30]
        bias_hidden = [0.35, 0.35]
        weights_hidden_output = [0.40, 0.45, 0.50, 0.55]
        bias_output = [0.60, 0.60]
        targets = [0.1, 0.99]
        learning_rate = 0.5

        h1_input = inputs[0] * weights_input_hidden[0] + inputs[1] * weights_input_hidden[2] + bias_hidden[0]
        h2_input = inputs[0] * weights_input_hidden[1] + inputs[1] * weights_input_hidden[3] + bias_hidden[1]
        h1_output, h2_output = sigmoid(h1_input), sigmoid(h2_input)

        o1_input = h1_output * weights_hidden_output[0] + h2_output * weights_hidden_output[1] + bias_output[0]
        o2_input = h1_output * weights_hidden_output[2] + h2_output * weights_hidden_output[3] + bias_output[1]
        o1_output, o2_output = sigmoid(o1_input), sigmoid(o2_input)

        error_o1, error_o2 = targets[0] - o1_output, targets[1] - o2_output
        grad_o1, grad_o2 = error_o1 * sigmoid_derivative(o1_output), error_o2 * sigmoid_derivative(o2_output)

        grad_h1 = (grad_o1 * weights_hidden_output[0] + grad_o2 * weights_hidden_output[2]) * sigmoid_derivative(h1_output)
        grad_h2 = (grad_o1 * weights_hidden_output[1] + grad_o2 * weights_hidden_output[3]) * sigmoid_derivative(h2_output)

        weights_hidden_output[0] += learning_rate * h1_output * grad_o1 # w5
        weights_hidden_output[1] += learning_rate * h2_output * grad_o1 # w6
        weights_hidden_output[2] += learning_rate * h1_output * grad_o2 # w7
        weights_hidden_output[3] += learning_rate * h2_output * grad_o2 # w8

        weights_input_hidden[0] += learning_rate * inputs[0] * grad_h1 # w1
        weights_input_hidden[1] += learning_rate * inputs[0] * grad_h2 # w2
        weights_input_hidden[2] += learning_rate * inputs[1] * grad_h1 # w3
        weights_input_hidden[3] += learning_rate * inputs[1] * grad_h2 # w4

        bias_output[0] += learning_rate * grad_o1
        bias_output[1] += learning_rate * grad_o2
        bias_hidden[0] += learning_rate * grad_h1
        bias_hidden[1] += learning_rate * grad_h2

        print("Outputs:", o1_output, o2_output)
        print("Updated Weights:")
        print("w1, w2, w3, w4:", weights_input_hidden)
        print("w5, w6, w7, w8:", weights_hidden_output)
```

Outputs: 0.7514044524275139 0.7729760524059158

Updated Weights:

w1, w2, w3, w4: [0.14982142587210845, 0.19979652631822406, 0.2496428517442169, 0.2995930526364481]
w5, w6, w7, w8: [0.3638322020897263, 0.41372227870602696, 0.511320082807667, 0.5613544874957562]