ETH zürich

# Optimization in Energy Systems

## Multi-time Step Optimization & Unit Commitment

Lecturer: Dr. Mengshuo Jia

# Outline

- Multi-time Step Optimization
  - Motivation, constraints, example
- Unit Commitment Overview
  - Aim, application, challenge
- Deterministic Unit Commitment
  - Integer variables, constraints, objective, example
- Stochastic Unit Commitment
  - Scenario-based approach
- Solutions Method for Unit Commitment
  - Approximation, solutions, validation & reinforcement

# Learning Objectives

- After this lecture, you should be able
  - to formulate and solve a multi-time step optimization problem (with solvers) and discuss the advantage of multi-time step versus single time step
  - to formulate the deterministic unit commitment problems
  - to solve the DC-based deterministic unit commitment problem by programming
  - to understand the idea of stochastic unit commitment problems using the scenario-based approach
  - to understand the idea of solving unit commitment problems in reality

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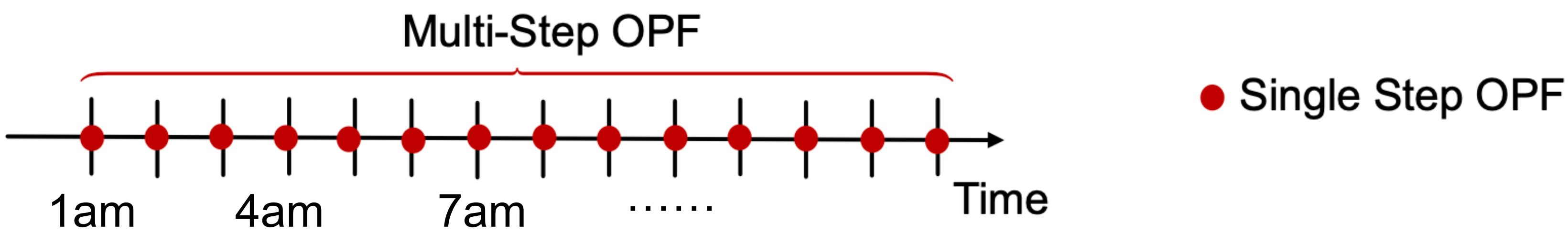
# Multi-time Step Optimization




- Motivation: to consider intertemporal constraints
  - $x(k+1) = f[x(k)]$, e.g., $x(k+1) = Ax(k)$
  - Storage charging/discharging behavior
  - Ramp limitations for units

# Multi-time Step Optimization

- Storage constraints

$$E(k+1) = E(k) + \eta P_{in}(k) - \frac{1}{\eta} P_{out}(k)$$

$\eta$, efficiency rate < 1

$$0 \leq P_{in}(k) \leq P_{in}^{max}, \quad 0 \leq P_{out}(k) \leq P_{out}^{max},$$

$$P_{in}(k) \cdot P_{out}(k) = 0,$$

Charging or discharging

$$E^{min} \leq E(k) \leq E^{max},$$

$$E(K) = E(0)$$

Restore the energy level

- Ramp constraints

$$-P_G^{rmax} \leq P_G(k+1) - P_G(k) \leq P_G^{rmax}$$

A unit cannot significantly change its output immediately

# Multi-time Step Optimization

- Multi-time DC OPF with storage

$$\min_{P_G} \sum_{k=1}^{K} \sum_{i=1}^{N_G} (a_i P_{G_i}^2(k) + b_i P_{G_i}(k) + c_i)$$

$$
\begin{aligned}
s.t.\ & P(k) = B\theta(k), && k = 1, \dots, K \\
& E(k+1) = E(k) + \eta P_{in}(k) - \frac{1}{\eta} P_{out}(k), && k = 0, \dots, K-1 \\
& E^{min} \le E(k) \le E^{max}, && k = 1, \dots, K \\
& P_G^{min} \le P_G(k) \le P_G^{max}, && k = 1, \dots, K \\
& 0 \le P_{in}(k) \le P_{in}^{max}, \quad 0 \le P_{out}(k) \le P_{out}^{max}, && k = 1, \dots, K \\
& P_{in}(k) \cdot P_{out}(k) = 0, && k = 1, \dots, K \\
& -P_{ij}^{max} \le P_{ij}(k) \le P_{ij}^{max}, && k = 1, \dots, K \\
& -P_G^{rmax} \le P_G(k+1) - P_G(k) \le P_G^{rmax}, && k = 0, \dots, K-1 \\
& E(K) = E(0)
\end{aligned}
$$

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# Unit Commitment – Overview

- Aim

  **Determine an optimal schedule for each generating unit so that**
  - the demand for electricity is met,
  - minimum cost for the system as a whole,

  **subject to**
  - ramping constraints,
  - network constraints,
  - minimum uptime/downtime constraints, etc.

  **with the decision variables, i.e.,**
  - generators' outputs
  - generators' on/off status

# Unit Commitment – Overview

- Aim

**Determine an optimal schedule for each generating unit so that**

- the demand for electricity is met,
- minimum cost for the system as a whole,

**subject to**

- ramping constraints,
- network constraints,
- minimum uptime/downtime constraints, etc.

**with the decision variables, i.e.,**

- generators' outputs --- continuous variable
- generators' on/off status --- (integer) binary variable, 1: on, 0: off.

# Unit Commitment – Overview

- Application

**For those with markets**:

- Standard tool for clearing electricity markets
- Particularly day-ahead markets

**For those without markets**:

- Determine the day-ahead commitments and dispatches

- Challenge

- Mixed-integer nonlinear optimization problem
- Generally large-scale and nonconvex

**Important but challenging**

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# Deterministic Unit Commitment

- Deterministic

  No uncertainty is considered here

- Variables (Unit $j$ at Time $t$)
  - Generators' outputs, **continuous**
    - $p_j(t)$, power generated
  - Generators' status, **binary**
    - $v_j(t)$, on/off status
    - $y_j(t)$, when starts up
    - $z_j(t)$, when shuts down

# Deterministic Unit Commitment

- Binary Decision Variables (Unit $j$ at Time $t$)
  - $v_j(t)$, on/off status
  - $y_j(t)$, when starts up
  - $z_j(t)$, when shuts down

# Deterministic Unit Commitment

- Binary Decision Variables (Unit $j$ at Time $t$)
  - $v_j(t)$, is 1 if unit j is on in time period t, o/w 0
  - $y_j(t)$, is 1 if unit j starts up at the beginning of time period t, o/w 0
  - $z_j(t)$, is 1 if unit j shuts down at the beginning of time period t, o/w 0

  **Easier to**
  - ensure that unit j cannot start up and shut down at the same time
  - describe the ramping constraints
  - describe uptime and downtime constraints
  - describe the cost for starting up or shutting down
  - …

# Deterministic Unit Commitment

- Binary Decision Variables (Unit $j$ at Time $t$)
  - $v_j(t)$, is 1 if unit j is on in time period t, o/w 0
  - $y_j(t)$, is 1 if unit j starts up at the beginning of time period t, o/w 0
  - $z_j(t)$, is 1 if unit j shuts down at the beginning of time period t, o/w 0
- Logical Coherence between Binary Variables

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0 \qquad y_j(t) + z_j(t) \leq 1$$

  - Ensure that $y_j(t)$ and $z_j(t)$ take appropriate values when unit j starts up or shuts down
  - E.g., $v_j(0) = 1$ and $v_j(1) = 0$
    - $y_j(1) - z_j(1) = -1$, only if $y_j(1) = 0$ and $z_j(1) = 1$ (Hint: binary)

# Deterministic Unit Commitment

- Logical Coherence between Binary Variables

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0 \qquad y_j(t) + z_j(t) \leq 1$$

  - Ensure that $y_j(t)$ and $z_j(t)$ take appropriate values when unit j starts up or shuts down
  - E.g., for a longer period

| | t=0 | t=1 | t=2 | t=3 | t=4 | t=5 |
|---|---|---|---|---|---|---|
| Status ($v_j$) | 0 | 0 | 1 | 1 | 0 | 0 |
| Startup ($y_j$) | 0 | 0 | 1 | 0 | 0 | 0 |
| Shutdown ($z_j$) | 0 | 0 | 0 | 0 | 1 | 0 |

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# Deterministic Unit Commitment

- Ramping Constraints
  - In the Continuous OPF Problem:
    - Ramp-up: $p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}}$
    - Ramp-down: $p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}}$
    - $R_j^U$: maximum ramp-up rate of unit j
    - $R_j^D$: maximum ramp-down rate of unit j
  - In the UC Problem:
    - Ramp-up: $p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}} \, v_j(t-1) + S_j^{\mathrm{U}} \, y_j(t)$
    - Ramp-down: $p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}} \, v_j(t) + S_j^{\mathrm{D}} \, z_j(t)$
    - $S_j^U$: maximum start-up rate of unit j
    - $S_j^D$: maximum shut-down rate of unit j

# Deterministic Unit Commitment

- Ramping Constraints
  - Ramp-up Constraint in UC

$$p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t)$$

  - If $v_j(t-1) = 1$
    - Unit j is on at $(t-1)$, i.e., $y_j(t) = 0$ must hold
    - Because unit j cannot be turned on again
    - Hence: $p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}}$
  - If $y_j(t) = 1$
    - Unit j starts up at $t$, then $v_j(t-1) = 0$ must hold
    - Because unit j cannot be turned on again
    - Hence: $p_j(t) - p_j(t-1) \leq S_j^U$

# Deterministic Unit Commitment

- Ramping Constraints
  - Ramp-down Constraint in UC

$$p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t)$$

- If $v_j(t) = 1$
  - Unit j is on at $t$, i.e., $z_j(t) = 0$ must hold
  - Because unit j is still on at $t$
  - Hence: $p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}}$
- If $z_j(t) = 1$
  - Unit j shuts down at $t$, then $v_j(t) = 0$ must hold
  - Because unit j is already off at $t$
  - Hence: $p_j(t-1) - p_j(t) \leq S_j^{D}$

# Deterministic Unit Commitment

- Ramping Constraints
  - Assumption
    - ramp-up rate = start-up rate;
    - ramp-down rate = shut-down rate

$$p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t)$$

$$p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t)$$

$$R_j^U = S_j^U \qquad \Downarrow \qquad R_j^D = S_j^D$$

$$p_j(t) - p_j(t-1) \leq R_j^U\left[v_j(t-1) + y_j(t)\right]$$

$$p_j(t-1) - p_j(t) \leq R_j^D\left[v_j + z(t)\right]$$

# Deterministic Unit Commitment

- Ramping Constraints
  - Special Case for $t = 1$
    - Up : $p_j(t) - p_j(t-1) \leq R_j^U[v_j(t-1) + y_j(t)]$
    - Down: $p_j(t-1) - p_j(t) \leq R_j^D[v_j + z(t)]$
    - $p_j(0) = p_j^{ini}$ and $v_j(0) = v_j^{ini}$ are parameters i/o variables
    - To distinguish:

**Up**

- $p_j(t) - p_j^{ini} \leq R_j^U[v_j^{ini} + y_j(t)], \quad \forall j, \quad t = 1$
- $p_j(t) - p_j(t-1) \leq R_j^U[v_j(t-1) + y_j(t)], \quad \forall j, \quad \forall t > 1$

**Down**

- $p_j^{ini} - p_j(t) \leq R_j^D[v_j + z(t)], \quad \forall j, \quad t = 1$
- $p_j(t-1) - p_j(t) \leq R_j^D[v_j + z(t)], \quad \forall j, \quad \forall t > 1$

# Deterministic Unit Commitment

- Uptime/downtime Constraints
  - Description: a unit cannot be turned on or off arbitrarily
    - If unit $j$ starts up at time period $k$
      - Then it must stay "on" for $T_j^U$ time periods (including the time period $k$ itself) => uptime constraint
    - If unit $j$ shuts down at time period $k$
      - then it must stay "off" for $T_j^D$ time periods (including the time period $k$ itself) => downtime constraint

# Deterministic Unit Commitment

- Uptime constraint

$$\sum_{k=t-T_j^{\mathrm{U}}+1,\ k\geq 1}^{t} y_j(k) \leq v_j(t)$$

  - If unit $j$ starts up at time period $k$
    - Then it must stay "on" for $T_j^U$ time periods (including the time period $k$ itself)
  - How it works?

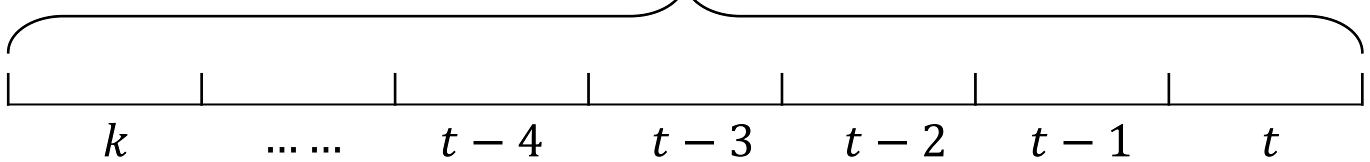
# Deterministic Unit Commitment

- Uptime constraint

$$\sum_{k=t-T_j^{\mathrm{U}}+1,\ k\geq 1}^{t} y_j(k) \leq v_j(t)$$

$$\Downarrow$$

$T_j^U$ time periods
(because $k = t - T_j^U + 1$)



# Deterministic Unit Commitment

- Uptime constraint

$$\sum_{k=t-T_j^{\mathrm{U}}+1,\ k\geq 1}^{t} y_j(k) \leq v_j(t)$$

$$\downarrow$$

$$\Sigma_j^U \leq v_j(t)$$

| $k$ | ⋯ ⋯ | $t-4$ | $t-3$ | $t-2$ | $t-1$ | $t$ |
|---|---|---|---|---|---|---|
| $y_j$=1 | $y_j$=0 | $y_j$=0 | $y_j$=0 | $y_j$=0 | $y_j$=0 | $y_j$=0 |
| $v_j$=1 | $v_j$=1 | $v_j$=1 | $v_j$=1 | $v_j$=1 | $v_j$=1 | $v_j$=1 |

- If unit $j$ starts up at time period $k$, it must stay "on" for $T_j^U$ time periods (including the time period $k$ itself)

- Unit $j$ starts up at time period $k$
  - $\Rightarrow y_j\ (k) = 1$
  - $\Rightarrow v_j(k) = 1$
  - $\Rightarrow \Sigma_j^U \geq 1$
  - $\Sigma_j^U > 1$ or $\Sigma_j^U = 1$?
- If $\Sigma_j^U > 1$
  - Maximal $v_j(t)$ is 1
  - $\Sigma_j^U > 1 = v_j(t)_{max}$, contradictory!
  - $\Rightarrow \Sigma_j^U = 1$
- $\Sigma_j^U = 1$
  - $y_j(k) = 1$ and $\Sigma_j^U = 1$
  - $\Rightarrow y_j(t), y_j(t-1), \ldots, y_j(k+1) = 0$
  - $\Sigma_j^U \leq v_j(t)$ and $\Sigma_j^U = 1$
  - $\Rightarrow v_j(t) = 1$
- If $v_j(m) = 0,\ m \in [k+1, t-1]$?
  - $\Rightarrow y_j(n) = 1,\ \exists n \in [m+1, t]$
  - But $y_j(t), y_j(t-1), \ldots, y_j(k+1) = 0$
  - $\Rightarrow v_j(t-1), \ldots, v_j(k+1) = 1$

# Deterministic Unit Commitment

- Downtime constraint

$$v_j(t) + \sum_{k=t-T_j^{\mathrm{D}}+1,\ k\geq 1}^{t} z_j(k) \leq 1$$

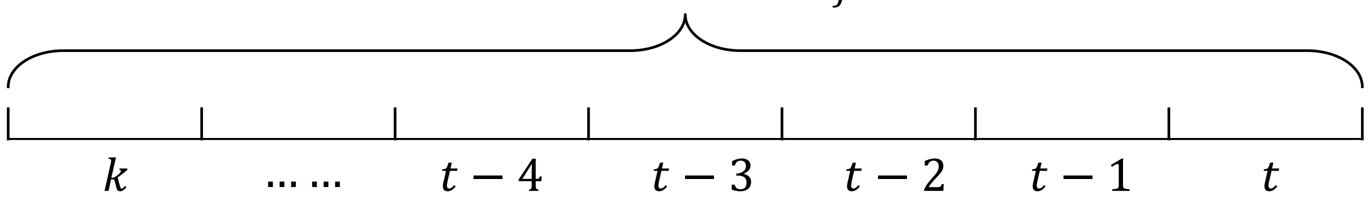
  - If unit $j$ shuts down at time period $k$
    - then it must stay “off” for $T_j^D$ time periods (including the time period $k$ itself)
  - How it works?

# Deterministic Unit Commitment

- Downtime constraint

$$v_j(t) + \sum_{k=t-T_j^{\mathrm{D}}+1,\ k\geq 1}^{t} z_j(k) \leq 1$$

$T_j^D$ time periods
(because $k = t - T_j^D + 1$)



# Deterministic Unit Commitment

- Downtime constraint

$$v_j(t) + \sum_{k=t-T_j^{\mathrm{D}}+1,\ k\geq 1}^{t} z_j(k) \leq 1$$

$$\downarrow$$

$$v_j(t) + \Sigma_j^D \leq 1$$

| $k$ | …… | $t-4$ | $t-3$ | $t-2$ | $t-1$ | $t$ |
|---|---|---|---|---|---|---|
| $z_j$=1 | $z_j$=0 | $z_j$=0 | $z_j$=0 | $z_j$=0 | $z_j$=0 | $z_j$=0 |
| $v_j$=0 | $v_j$=0 | $v_j$=0 | $v_j$=0 | $v_j$=0 | $v_j$=0 | $v_j$=0 |

- If unit $j$ shuts down at time period $k$, it must stay "off" for $T_j^D$ time periods (including the time period $k$ itself)

- Unit $j$ shuts down at time period $k$
  - $\Rightarrow z_j\ (k) = 1$
  - $\Rightarrow v_j(k) = 0$
  - $\Rightarrow \Sigma_j^D \geq 1$
  - $\Sigma_j^D > 1$ or $\Sigma_j^D = 1$?
- If $\Sigma_j^D > 1$
  - Minimal $v_j(t)$ is 0
  - $v_j(t)_{min} + \Sigma_j^D > 1$, contradictory!
  - $\Rightarrow \Sigma_j^D = 1$
- $\Sigma_j^D = 1$
  - $z_j(k) = 1$ and $\Sigma_j^D = 1$
  - $\Rightarrow z_j(t), z_j(t-1), \dots, z_j(k+1) = 0$
  - $v_j(t) + \Sigma_j^D \leq 1$ and $\Sigma_j^D = 1$
  - $\Rightarrow v_j(t) = 0$
- If $v_j(m) = 1$, $m \in [k+1, t-1]$?
  - $\Rightarrow z_j(n) = 1$, $\exists n \in [m+1, t]$
  - But $z_j(t), z_j(t-1), \dots, z_j(k+1) = 0$!
  - $\Rightarrow v_j(t-1), \dots, v_j(k+1) = 0$

# Deterministic Unit Commitment

- Generation Constraints
  - In the Continuous OPF Problem:
    - Upper limit: $p_j(t) \leq \overline{p}_j$
    - Lower limit: $\underline{p}_j \leq p_j(t)$
  - In the UC Problem:
    - Upper limit: $p_j(t) \leq \overline{p}_j v_j(t)$
    - Lower limit: $\underline{p}_j v_j(t) \leq p_j(t)$
    - Still linear

# Deterministic Unit Commitment

- Objective Function
  - In the Continuous OPF Problem:

$$\min_{\Xi} \quad \sum_{t\in T}\sum_{j\in J} c_j(p_j(t))$$

    - $c_j(p_j(t))$ represents the generation cost of unit $j$ at time $t$
  - In the UC Problem:

$$\min_{\Xi} \quad \sum_{t\in T}\sum_{j\in J} \left(c_j(p_j(t)) + c_j^{\mathrm{U}} y_j(t) + c_j^D z_j(t)\right)$$

    - $c_j^U$ is a constant cost for starting up unit $j$
    - $c_j^D$ is a constant cost for shutting down unit $j$

# Deterministic Unit Commitment

- Example: DC-based UC

$$\underset{\Xi^{\mathrm{UC}}}{\text{Minimize}} \sum_t \sum_i (C_i p_{it} + C_i^{\mathrm{SU}} y_{it} + C_i^{\mathrm{SD}} z_{it}) \quad (1a)$$

**Objective**: minimize the cost of energy production and the start-up/shut-down cost of all units.

subject to

$$\sum_i \mathcal{M}_n^I p_{it} + \sum_j \mathcal{M}_n^J w_{jt} = L_{nt} + \sum_\ell A(\ell, n) f_{\ell t}, \quad \forall n, \forall t, \quad (1b)$$

Power balance at each node

$$y_{it} - z_{it} = u_{it} - u_i^{\mathrm{ini}}, \quad \forall i, t = 1, \quad (1c)$$

$$y_{it} - z_{it} = u_{it} - u_{i,t-1}, \quad \forall i, \forall t > 1 \quad (1d)$$

$$y_{it} + z_{it} \le 1, \quad \forall i, \forall t, \quad (1e)$$

Commitment status of generators
Correlation among status variables

Cannot start up and shut down at the same time

$$p_{it} \le \overline{p}_i u_{it}, \quad \forall i, \forall t, \quad (1f)$$

$$\underline{p}_i u_{it} \le p_{it}, \quad \forall i, \forall t, \quad (1g)$$

Upper & lower generation limits of units

$$p_{it} - P_i^{\mathrm{ini}} \le R_i^{\mathrm{U}}(u_i^{\mathrm{ini}} + y_{it}), \quad \forall i, t = 1, \quad (1h)$$

$$p_{it} - p_{i,t-1} \le R_i^{\mathrm{U}}(u_{i,t-1} + y_{it}), \quad \forall i, \forall t > 1 \quad (1i)$$

$$P_i^{\mathrm{ini}} - p_{i,t} \le R_i^{\mathrm{D}}(u_{it} + z_{it}), \quad \forall i, t = 1, \quad (1j)$$

$$p_{i,t-1} - p_{i,t} \le R_i^{\mathrm{D}}(u_{it} + z_{it}), \quad \forall i, \forall t > 1 \quad (1k)$$

Ramping limits
No uptime/downtime constraints

$$w_{jt} \le \hat{w}_j, \quad \forall j, \forall t, \quad (1l)$$

Wind power bounds (expected value)

$$f_{\ell t} = B_\ell \sum_n A(\ell, n) \delta_{nt}, \quad \forall \ell, \forall t, \quad (1m)$$

Power flows

$$-\overline{f}_\ell \le f_{\ell t} \le \overline{f}_\ell, \quad \forall \ell, \forall t, \quad (1n)$$

Transmission capacity limits

$$\delta_{n_1 t} = 0, \forall t \quad (1o)$$

Reference node

$$p_{it} \ge 0,\ \forall i, \forall t; w_{jt} \ge 0,\ \forall j, \forall t;\ f_{\ell t} \text{ free } \forall \ell, \forall t;\ \delta_{nt} \text{ free } \forall n, \forall t; \quad (1p)$$

$$u_{it} \in \{0,1\}; y_{it} \in \{0,1\}; z_{it} \in \{0,1\},\ \forall i, \forall t, \quad (1q)$$

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# Stochastic Unit Commitment

- Uncertainty
  - Stochastic sources
  - Most notably wind and solar generation
  - Demand, yet can be assumed to be deterministic (as in this lecture)
- Modelling
  - Stochastic (Scenario-based) optimization
  - Chance-constrained optimization
  - Robust optimization
  - Distributionally robust optimization

# Stochastic Unit Commitment

- Uncertainty
  - Stochastic sources
  - Most notably wind and solar generation
  - Demand, yet can be assumed to be deterministic (as in this lecture)
- Modelling
  - **Stochastic (Scenario-based) optimization for UC problems**
    - **Uncertainties from wind power generators**
  - Chance-constrained optimization
  - Robust optimization
  - Distributionally robust optimization

# Stochastic Unit Commitment

- Stochastic UC Formulation
  - **Two-stage** stochastic programming problem
  - First stage
    - Make decisions in advance, e.g., day-ahead UC
    - Uncertainties have not been realized
    - Assume renewable generators are dispatchable
      - Like conventional generators
    - Similar to a deterministic UC problem
      - Difference: introduce reserve variables $r_i^+(t)$, $r_i^-(t)$

# Stochastic Unit Commitment

- Stochastic UC Formulation
  - **Two-stage** stochastic programming problem
  - Second stage
    - To make decisions in "real-time"
      - Uncertainties => realizations, e.g., wind power realizations
    - Adjust the first stage decisions after observing the realizations
      - Reserve deployment variables $p_{i\omega}^{+}(t)$, $p_{i\omega}^{-}(t)$
      - Wind power spillage variables $w_{j\omega}^{spil}(t)$
      - Load shedding variables $l_{\omega}^{shed}(t)$

# Stochastic Unit Commitment

- Stochastic UC Formulation
  - **Two-stage** stochastic programming problem
  - Second stage
    - To make decisions in ”real-time”
      - Uncertainties => realizations, e.g., wind power realizations
    - Adjust the first stage decisions after observing the realizations
    - In fact, no realizations have been seen => one day ahead
      - **Assume** a set of realizations of wind power generations
      - A realization => a scenario
    - Do adjustment for **each scenario**

# Stochastic Unit Commitment

- Stochastic UC Formulation
  - **Two-stage** stochastic programming problem
  - Combine the first and second stage together
    - The decisions made in the first stage can satisfy all possible scenarios in the second stage
      - Conventional generators' status
      - Conventional generators' output
      - Conventional generators' reserves provided

# Stochastic Unit Commitment

- First Stage
  - Variables
    - $w_i(\mathrm{t})$, power generated by wind power unit $i$ at time $t$
    - $r_i^+(t)$, upward reserve provided by conventional unit $i$ at time $t$
    - $r_i^-(t)$, downward reserve provided by conventional unit $i$ at time $t$
    - Reserve:
      - Amount of additional capacity available beyond the actual power output
      - Typically used to provide a buffer
        - In cases of, e.g., unexpected increases/decreases in renewable generations

# Stochastic Unit Commitment

- First Stage
  - Variables
    - $w_i(\mathrm{t})$, power generated by wind power unit $i$ at time $t$
    - $r_i^+(t)$, upward reserve provided by conventional unit $i$ at time $t$
    - $r_i^-(t)$, downward reserve provided by conventional unit $i$ at time $t$
    - $p_i(t)$, $v_i(t)$, $y_i(t)$, $z_i(t)$
      - Active power generation of unit $i$ at time $t$
      - Commitment variables of unit $i$ at time $t$

# Stochastic Unit Commitment

- First Stage
  - Main Constraints
    - $\sum_i p_i(t) + \sum_j w_j(t) = L_t, \ \forall t$ Power Balance
    - $y_i(t) - z_i(t) = v_i(t) - v_i(t-1), \forall i, t$ Commitment Status
    - $y_i(t) + z_i(t) \leq 1, \forall i, t$ Commitment Status
    - $p_i(t) + r_i^+(t) \leq \overline{p}_i v_i(t), \forall i, t$ Generation upper limit
    - $\underline{p}_i v_i(t) \leq p_i(t) - r_i^-(t), \forall i, t$ Generation lower limit

# Stochastic Unit Commitment

- First Stage
  - Other Constraints
    - $v_i(t), y_i(t), z_i(t) \in \{0,1\}, \forall i, t,$ Binary restriction
    - $0 \leq w_j(t) \leq \overline{w}_j,, \forall j, t$ Wind power bounds
    - Omit uptime/downtime, ramping, network constraints
  - Objective
    - $\sum_t \sum_i \{c_i\,[p_i(t)] + c_i^U y_j(t) + c_i^D z_j(t) + c_i^{RU} r_i^+(t) + c_i^{RD} r_i^-(t)\}$
    - Minimize the overall cost of
      - Generation, start up, shut down
      - Providing upward reserve and downward reserve

# Stochastic Unit Commitment

- Second Stage
  - Scenario
    - Use index $\omega$ to denote scenario $\omega$
    - Use $w^{*}_{j\omega}(t)$ to denote the realization of the output of wind power unit $j$ in scenario $\omega$ at time $t$ --- parameter
  - Introduce new Variables for adjustments
    - $p^{+}_{i\omega}(t)$, $p^{-}_{i\omega}(t)$, upward/downward reserve deployments from conventional unit $i$ at time $t$ under scenario $\omega$
    - $w^{spil}_{j\omega}(t)$, power spillage from wind power unit $j$ at time $t$ under scenario $\omega$
    - $l^{shed}_{\omega}(t)$, overall load shedding at time $t$ under scenario $\omega$

# Stochastic Unit Commitment

- Second Stage
  - Why there is a need for adjustment?
    - Scheduled output from the first stage: $w_j(t)$
    - Realized output from scenario $\omega$: $w^*_{j\omega}(t)$
    - Deviation: $\sum_j\left[w^*_{j\omega}(t) - w_j(t)\right]$ --- aim to eliminate the deviation
  - Adjustment Method
    - $\sum_i[p^+_{i\omega}(t) - p^-_{i\omega}(t)]$ , the regulation provided by all conv. units
    - $\sum_i\left[-w^{spil}_{j\omega}(t)\right]$, the spillage provided by all wind power units
    - $l^{shed}_{\omega}(t)$, the load shedding provided by the overall load
  - Aim
    - $\sum_j\left[w^*_{j\omega}(t) - w_j(t)\right] + \sum_i[p^+_{i\omega}(t) - p^-_{i\omega}(t)] + \sum_i\left[-w^{spil}_{j\omega}(t)\right] + l^{shed}_{\omega}(t) = 0$

# Stochastic Unit Commitment

- Second Stage
  - Constraints
    - $\sum_j \left[ w_{j\omega}^*(t) - w_j(t) - w_{j\omega}^{spil}(t) \right] + \sum_i [p_{i\omega}^+(t) - p_{i\omega}^-(t)] + l_\omega^{shed}(t) = 0\,,\ \forall t, \omega$
    - $0 \le p_{i\omega}^+(t) \le r_i^+(t),\ \forall i, t, \omega.$ Recall $r_i^+(t)$ is from the first stage
    - $0 \le p_{i\omega}^-(t) \le r_i^-(t),\ \forall i, t, \omega.$ Recall $r_i^-(t)$ is from the first stage
    - $0 \le l_\omega^{shed}(t) \le L_t,\ \forall t, \omega$
    - $0 \le w_{j\omega}^{spil}(t) \le w_{j\omega}^*(t),\ \forall j, t, \omega$

# Stochastic Unit Commitment

- Second Stage
  - Objective
    - $\mathbb{E}\{\sum_t(\sum_i[c_i(p^+_{i\omega}(t) - p^-_{i\omega}(t))] + c^{shed} l^{shed}_{\omega}(t)) \mid \forall\omega\}$
    - Minimize the expectation of the regulation cost of
      - Reserve deployments, load shedding, for each scenario
      - Wind power is for free, i.e., spillage has no cost
    - Assume $\pi_\omega$ is the probability of scenario $\omega$ --- known
    - $\mathbb{E}\{\cdot \mid \forall\omega\} = \sum_\omega \pi_\omega \sum_t(\sum_i[c_i(p^+_{i\omega}(t) - p^-_{i\omega}(t))] + c^{shed} l^{shed}_{\omega}(t))$

# Stochastic Unit Commitment

- Stochastic UC Problem
  - Objective
    - Objective of the 1st stage + Objective of the 2nd stage
  - Constraints
    - Union of the constraints in the 1st and 2nd stages
  - Variables
    - Union of the variables in the 1st and 2nd stages
    - However, only some variables in the 1st stage will be outputted
      - Conventional generators' status
      - Conventional generators' output
      - Conventional generators' reserves provided

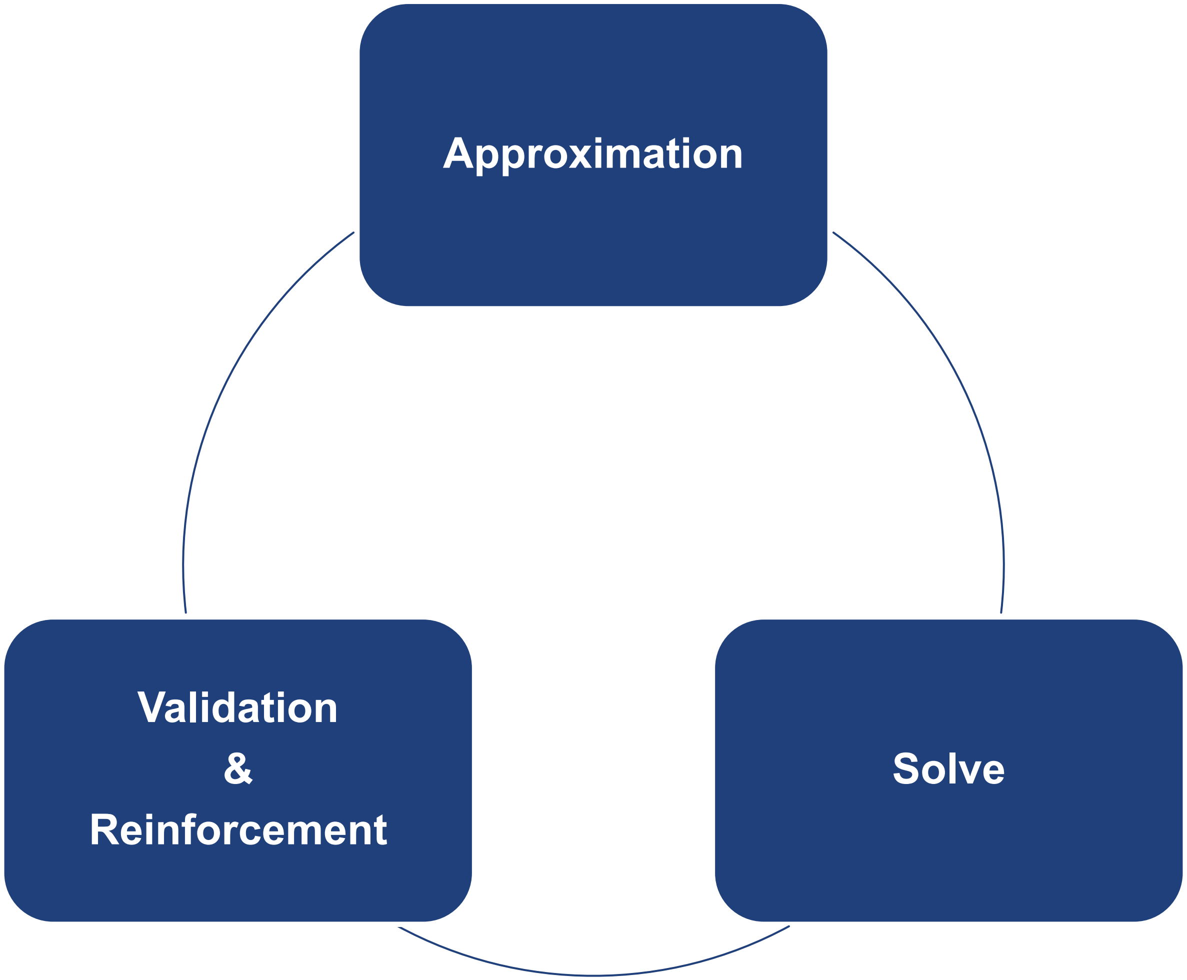
# Stochastic Unit Commitment

- Drawbacks
  - Assume explicit knowledge of the probability distribution of the uncertainties.
    - Estimated empirically
    - Data-driven
    - Simulation-driven
    - Impact the quality of the results
  - As the number of scenarios increases, the optimization problem become larger and more challenging
    - Large scale => Solvers may fail even for mixed integer linear programming

# Outline

- Multi-time Step Optimization
  - Motivation, constraints, example
- Unit Commitment Overview
  - Aim, application, challenge
- Deterministic Unit Commitment
  - Integer variables, constraints, objective, example
- Stochastic Unit Commitment
  - Scenario-based approach
- **Solutions Method for Unit Commitment**
  - Approximation, solutions, validation & reinforcement

# Solution Methods for Unit Commitment



# Solution Methods for Unit Commitment

- Approximation
  - AC-based UC
    - Mixed integer nonlinear programming (MINLP) --- challenging
  - DC approximation => DC-based UC
    - Mixed integer linear programming (MILP) --- Solvable
    - However, large scale => still challenging
      - Keep the number of scenarios in stochastic UC small
      - Research for scenario and/or scale reduction
    - Most common approximation method
  - Question => how to solve an MILP problem?

# Solution Methods for Unit Commitment

- Solve
  - State-of-the-art Solvers
    - E.g., Gurobi, CPLEX, AMPL, GAMS => Enough for this lecture
    - The common method to solve UC on a day-ahead basis
  - Algorithms Inside
    - General principles: divide-and-conquer
      - Recursively partitions the feasible region
      - => Find integer (binary) solutions
    - Typical algorithm: branch-and-bound (BnB)
      - Most (mixed) integer optimization solvers use some form of branch-and-bound algorithm

# Solution Methods for Unit Commitment

- Solve
  - Branch-and-bound algorithm: basic idea
    - Relaxation
      - General MILP: remove integer restrictions, i.e., $x \in \mathbb{R}$
      - 0/1 MILP: replace 0/1 restrictions with $0 \leq x \leq 1$
    - Branch/divide/add constraints with relaxation
      - General MILP: $x \leq N$ and $x \geq N+1 \Rightarrow$ two subproblems
        - E.g., $Min\ f(x),\ s.t.\ g(x) \leq 0$. If $x^* = 5.4$, then
        - Add $x \leq 5$ to $g(x) \leq 0$, resolve, see if $x^* = 5, or\ 4$, ...
        - Add $x \geq 6$ to $g(x) \leq 0$, resolve, see if $x^* = 6, or\ 7$, ...
      - 0/1 MILP: $x = 0$ and $x = 1 \Rightarrow$ two subproblems

# Solution Methods for Unit Commitment

- Validation & Reinforcement
  - DC-based UC Result
    - Optimal solution for the approximated problem
    - May not even feasible for the original AC-based problem
  - Check & Add Constraints
    - Input the DC-based UC solutions into AC power flow equations
      - Violations of transmission flows and/or bus voltages?
      - If so, add additional constraints to the DC-based UC
      - Re-solve & re-validation & re-reinforcement

# Summary

- Multi-time Step Optimization
  - Motivation, ramping constraints, storage constraints, receding horizon
- Unit Commitment Overview
  - Aim, application, challenge
- Deterministic Unit Commitment
  - Integer variables, logic constraints, ramping constraints, uptime/downtime constraints, generation Constraints, objectives
- Stochastic Unit Commitment
  - Scenarios, additional variables and constraints, two-stage formulations
- Solutions Method for Unit Commitment
  - Approximation, solutions (BnB), validation & reinforcement