Optimization in Energy Systems

Optimal Power Flow

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Repetition: Power Flow Equations

- Equations depending on bus type: Two variables per bus => we need two equation per bus
  - PQ bus
    $$-P_{L_k} = \sum_{m \in \Omega_k} P_{km}(\cdot) = U_k \sum_{m \in K} U_m (G_{km} cos\theta_{km} + B_{km} sin\theta_{km})$$
    $$-Q_{L_k} = -Q_{sh_k}(U_k) + \sum_{m \in \Omega_k} Q_{km}(\cdot) = U_k \sum_{m \in K} U_m (G_{km} sin\theta_{km} - B_{km} cos\theta_{km})$$
  - PU bus
    $$P_{G_k} - P_{L_k} = \sum_{m \in \Omega_k} P_{km}(\cdot) = U_k \sum_{m \in K} U_m (G_{km} cos\theta_{km} + B_{km} sin\theta_{km})$$
    $$U_k = U_{G_k}$$
  - slack bus
    $$U_k = U_{G_k}$$
    $$\theta_k = 0$$

Repetition: DC Power Flow

- Line Model
  - neglect loss terms (set $g_{km} = 0$)
  - set $U_k \approx U_m \approx 1 \text{ p.u.}$ => need to work with p.u. values!

$$\sin\theta_{km} \approx \theta_{km}$$

$$\Longrightarrow \quad P_{km} = \frac{\theta_{km}}{x_{km}} = \frac{\theta_k - \theta_m}{x_{km}}$$

models active power flows but cannot be used to model reactive power

- System Equations
  - Power balance at all buses except slack bus

$$P_k = P_{G_k} - P_{L_k} = \sum_{m \in \Omega_k} P_{km} \quad \Longrightarrow \quad P = B\theta$$

P: vector of net injections (except slack)

θ: vector of angles (except slack)

$$B_{km} = -\frac{1}{x_{km}}$$

$$B_{kk} = \sum_{m \in \Omega_k} \frac{1}{x_{km}}$$

  - Slack bus

$$\theta_k = 0$$

Learning Objectives

- After this lecture, you should be able
  - to set up and solve/implement an Optimal Power Flow problem using AC as well as DC power flow constraints
  - to set up and solve/implement a security constrained optimization problem using AC as well as DC power flow constraints

Optimization Problems

- Economic Dispatch
  - Find the generation dispatch which minimizes overall generation cost while supplying all loads

  $\Rightarrow$ Neglect grid

- Optimal Power Flow
  - Find the settings for the controllable variables, e.g. generation output, which minimizes the objective function, e.g. overall supply cost, taking into account the power flow equations and operational constraints.

  $\Rightarrow$ Take grid into account

Optimal Power Flow

- Optimal Power Flow (OPF):
  is a means to determine the optimal operational settings in an electric power system with respect to a given objective
- Power flow equations:
  are part of the constraint set
- Objective function:
  depends on objective to be achieved
- Control variables:
  depend on variables for which a setting is sought

Optimal Power Flow

- Optimization applied to Power Systems

$$
\begin{aligned}
&\min_{x,u} f(x,u) \\
&s.t.\;\; h(x,u) = 0 \\
&\qquad\; g(x,u) \le 0
\end{aligned}
$$

- State Variables x:
  - voltage magnitudes and angles
- Control Variables u:
  - depends on the problem, e.g.:
    - generator outputs
    - device settings
    - generator voltages, etc.
- Equality Constraints $h(x,u)$:
  - power flow equations

Optimal Power Flow

- Optimization applied to Power Systems (cont.)

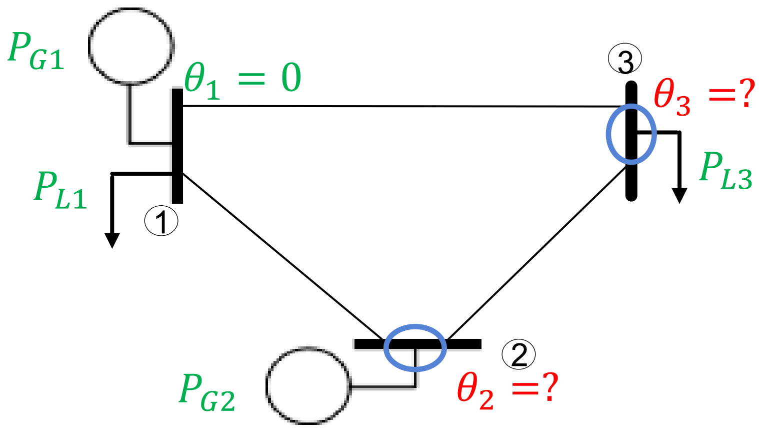
$$\min_{x,u} f(x,u)$$
$$s.t.\ \ h(x,u) = 0$$
$$g(x,u) \leq 0$$

- Inequality Constraints $g(x,u)$:
  - depends on considered problem, e.g.
    - Limits on line flows and/or voltages
    - Limits on control equipment settings, etc.

- Objective Function $f(x,u)$:
  - depends on considered problem, e.g.
    - Improve voltage profile
    - Minimize overall cost
    - Minimize power losses, etc.

Power Flow Equations in OPF

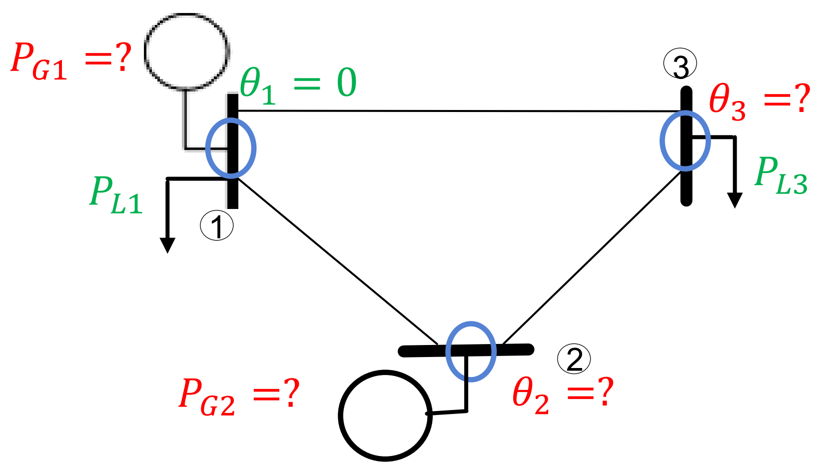
- Power Flow Computations



$$\theta_1 = 0$$

$$\begin{bmatrix} P_{G2} \\ -P_{L3} \end{bmatrix} = \begin{bmatrix} \frac{1}{x_{12}} + \frac{1}{x_{23}} & -\frac{1}{x_{23}} \\ -\frac{1}{x_{23}} & \frac{1}{x_{13}} + \frac{1}{x_{23}} \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}$$

- Optimal Power Flow



$$\theta_1 = 0$$

$$\begin{bmatrix} P_{G1} - P_{L1} \\ P_{G2} \\ -P_{L3} \end{bmatrix} = \begin{bmatrix} -\frac{1}{x_{12}} & -\frac{1}{x_{13}} \\ \frac{1}{x_{12}} + \frac{1}{x_{23}} & -\frac{1}{x_{23}} \\ -\frac{1}{x_{23}} & \frac{1}{x_{13}} + \frac{1}{x_{23}} \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}$$

Need to include power balance at slack bus in equation system

DC Optimal Power Flow (for Economic Dispatch)

- Objective

$$\min_{P_G} \sum_{i=1}^{N_G} C_{G_i}(P_{G_i}) = \sum_{i=1}^{N_G} (a_i P_{G_i}^2 + b_i P_{G_i} + c_i)$$

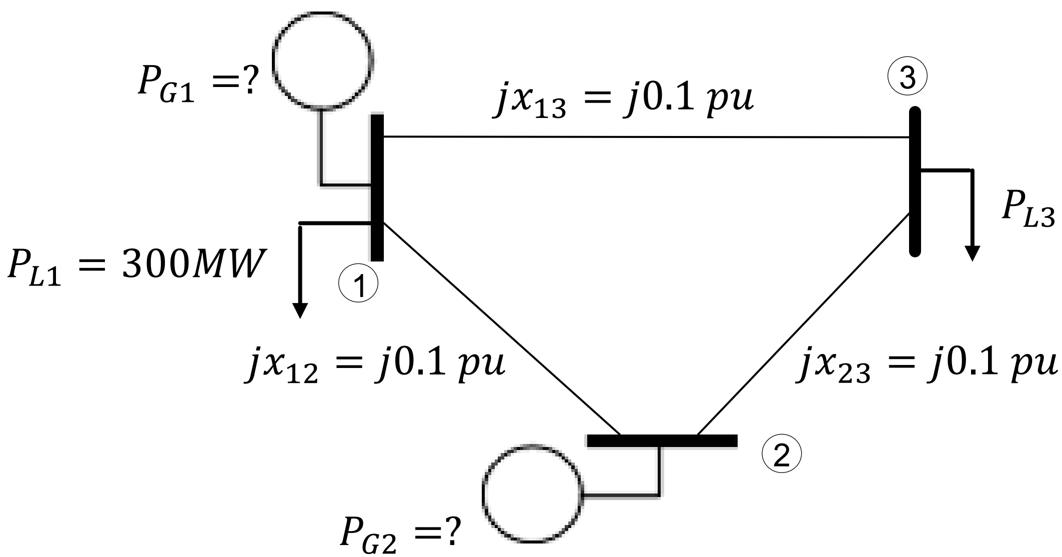
- Variables: P_G, θ (excluding angle at slack bus, i.e. $\theta_1 = 0$)
- Equality Constraints
  - Power balance at all nodes: $P = B \cdot \theta$
- Inequality Constraints
  - Limits on power generation: $P_G^{min} \leq P_G \leq P_G^{max}$
  - Limits on line flows: $-P_{ij}^{max} \leq P_{ij} \leq P_{ij}^{max}$

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \le P_{G1} \le 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \le P_{G2} \le 700MW$$



$$S_{base} = 100MW$$
$$U_{base} = U_n$$

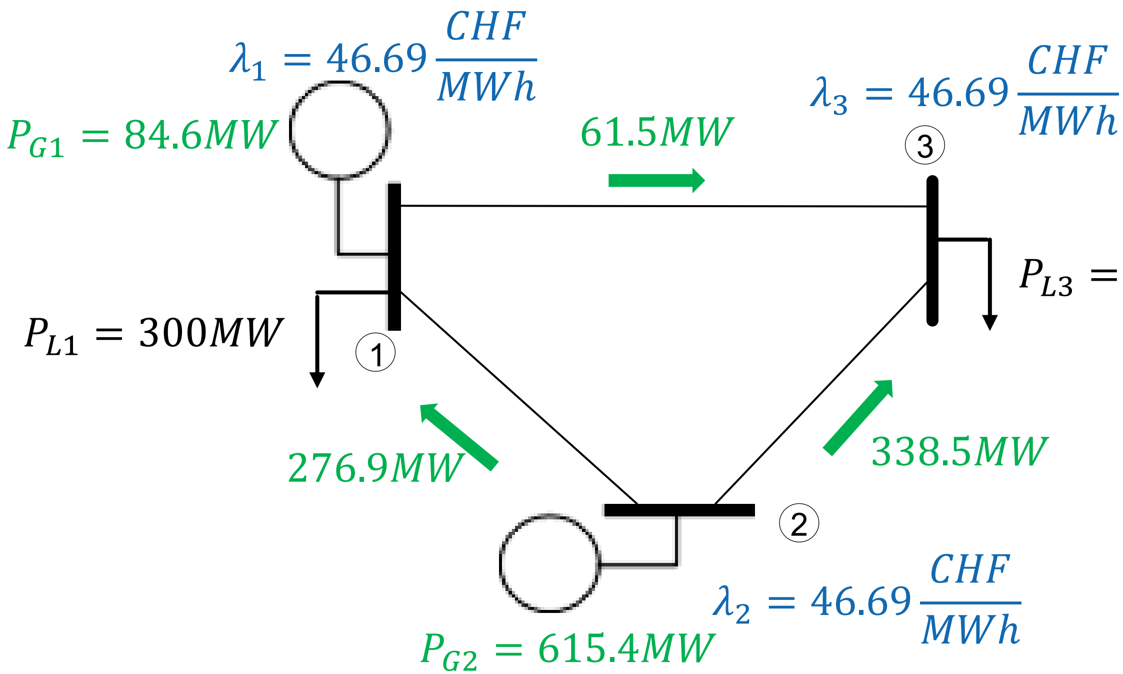
$$-300MW \le P_{12} \le 300MW$$
$$-200MW \le P_{13} \le 200MW$$
$$-400MW \le P_{23} \le 400MW$$

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq 700MW$$



$$S_{base} = 100MW$$
$$U_{base} = U_n$$

$$-300MW \leq P_{12} \leq 300MW$$
$$-200MW \leq P_{13} \leq 200MW$$
$$-400MW \leq P_{23} \leq 400MW$$

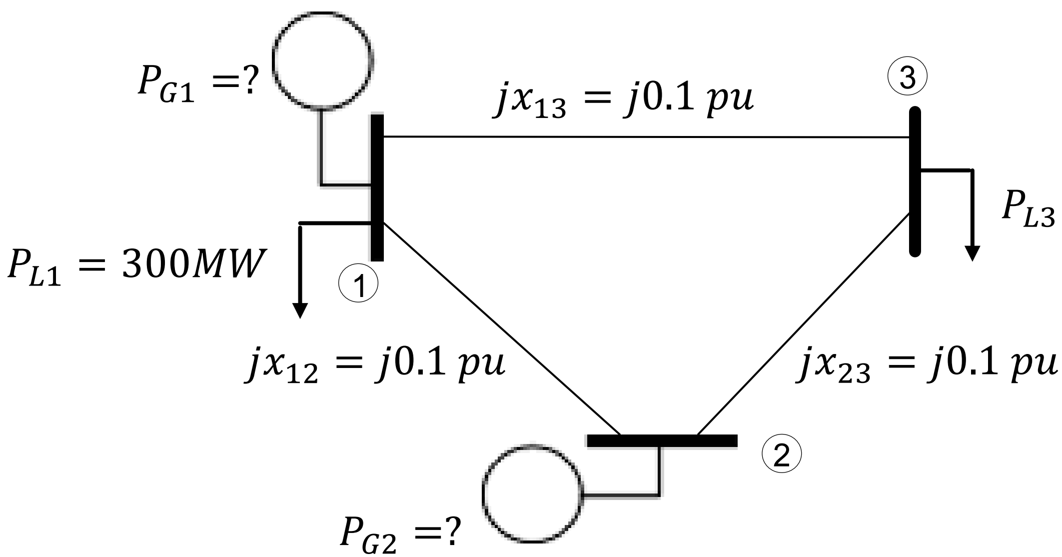
Line constraints non-binding => OPF solution = economic dispatch solution

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq \cancel{700MW}\ {\color{red}600MW}$$



$$S_{base} = 100MW$$
$$U_{base} = U_n$$

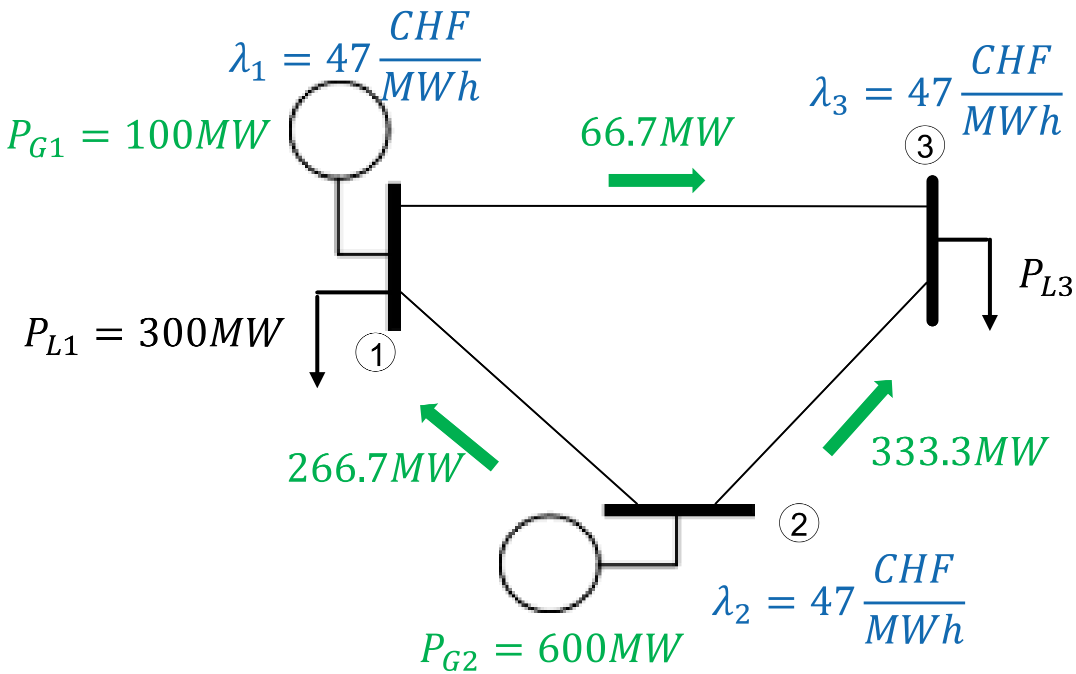
$$-300MW \leq P_{12} \leq 300MW$$
$$-200MW \leq P_{13} \leq 200MW$$
$$-400MW \leq P_{23} \leq 400MW$$

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq \cancel{700MW}\ 600MW$$



$S_{base} = 100MW$

$U_{base} = U_n$

$-300MW \leq P_{12} \leq 300MW$

$-200MW \leq P_{13} \leq 200MW$

$-400MW \leq P_{23} \leq 400MW$

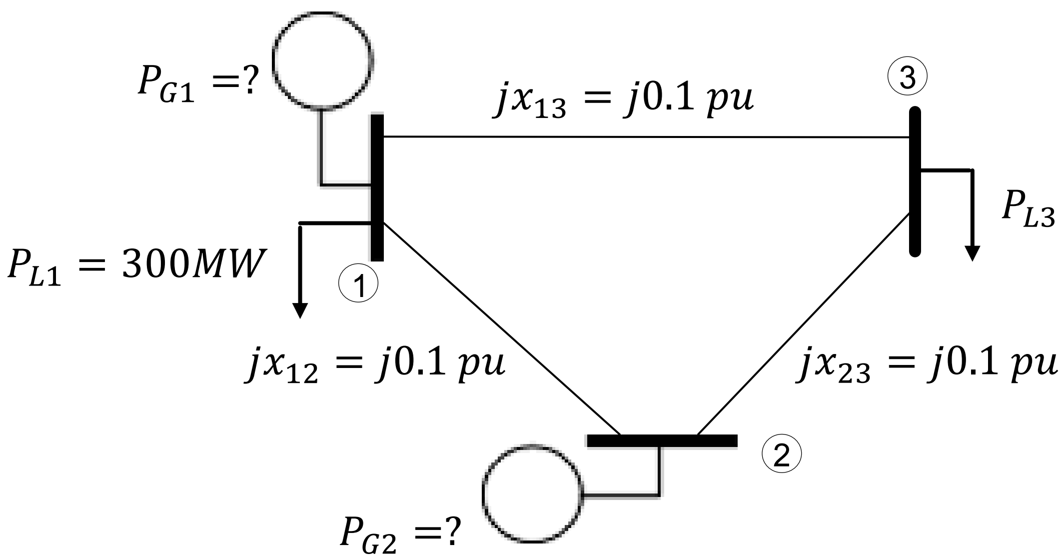
Line constraints non-binding => OPF solution = economic dispatch solution

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq 700MW$$



$$S_{base} = 100MW$$

$$U_{base} = U_n$$

$$-250MW \leq P_{12} \leq 250MW$$

$$-200MW \leq P_{13} \leq 200MW$$

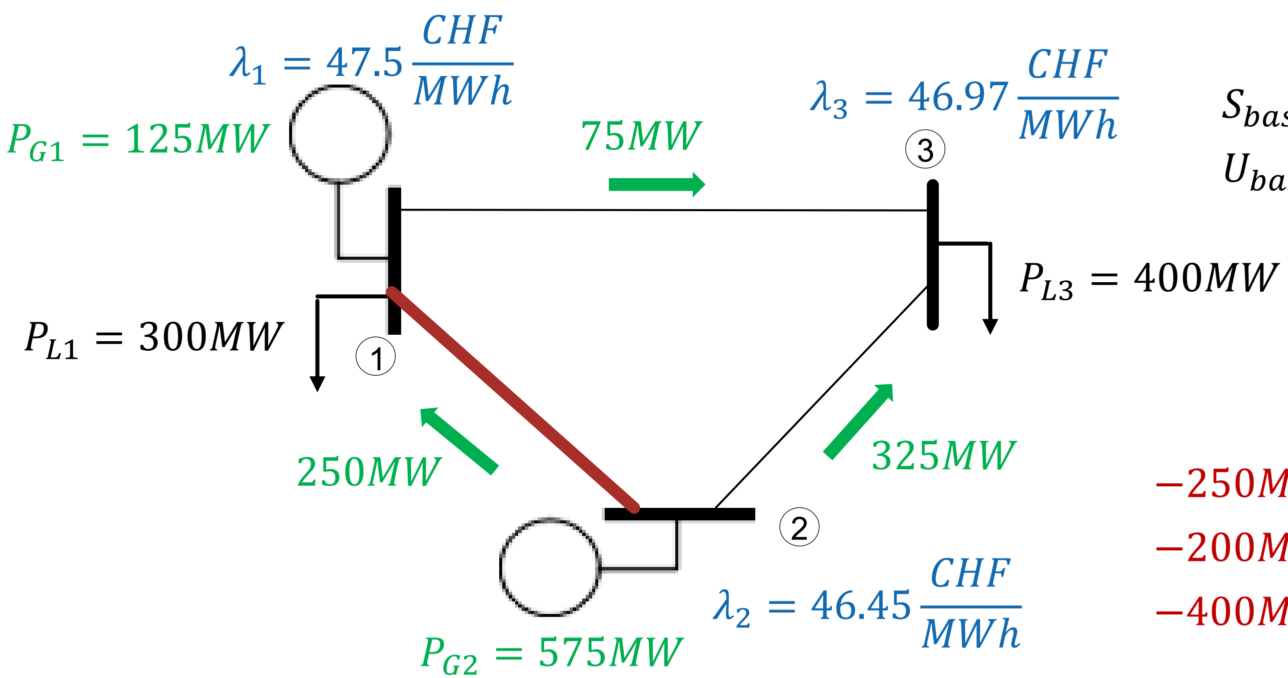
$$-400MW \leq P_{23} \leq 400MW$$

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq 700MW$$



$$S_{base} = 100MW$$

$$U_{base} = U_n$$

$$-250MW \leq P_{12} \leq 250MW$$

$$-200MW \leq P_{13} \leq 200MW$$

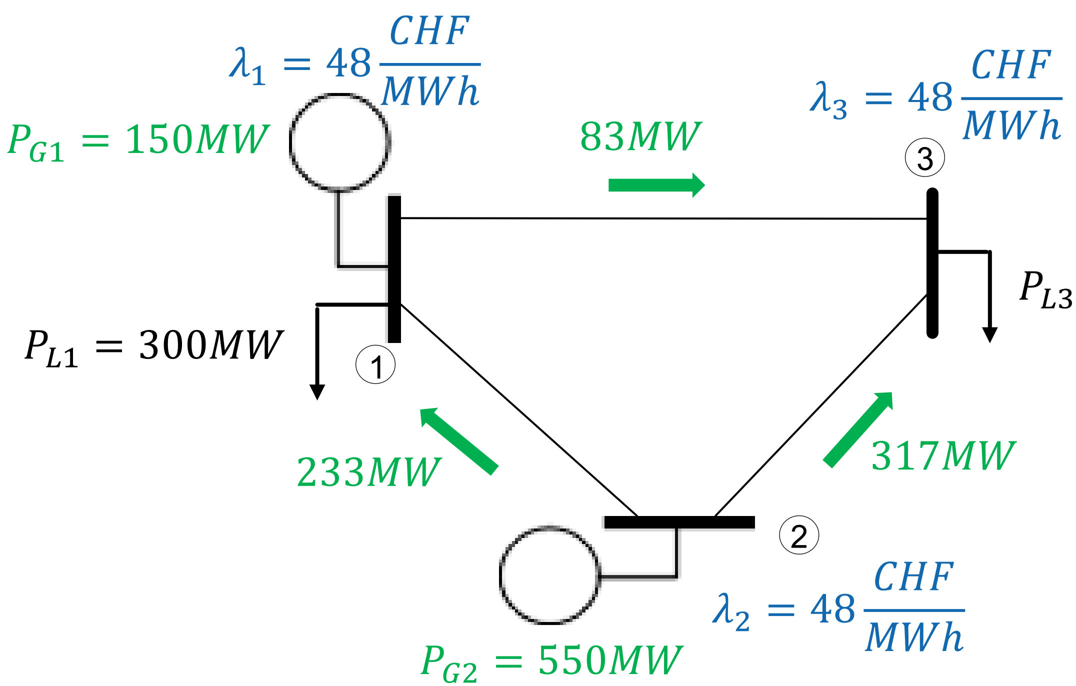
$$-400MW \leq P_{23} \leq 400MW$$

DC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq 550MW$$



$S_{base} = 100MW$

$U_{base} = U_n$

$-250MW \leq P_{12} \leq 250MW$

$-200MW \leq P_{13} \leq 200MW$

$-400MW \leq P_{23} \leq 400MW$

DC Optimal Power Flow

- Conclusions
  - Case 1: line flows do NOT reach their limits
    - Results equal to economic dispatch
    - Lagrange Multiplier for power balance constraints all equal and equal to price in economic dispatch
  - Case 2: at least one line limit is reached
    - Results deviate from pure economic dispatch
    - Lagrange Multipliers for power balance constraints deviate from each other => they correspond to locational marginal prices

- Locational Marginal Pricing
  - Lagrange Multiplier = Locational Marginal Price (LMP)
  - Indicates the cost of covering the next additional MWh of load at that particular node

AC Optimal Power Flow (for Economic Dispatch)

- Objective

$$\min_{P_G} \sum_{i=1}^{N_G} C_{G_i}(P_{G_i}) = \sum_{i=1}^{N_G} (a_i P_{G_i}^2 + b_i P_{G_i} + c_i)$$

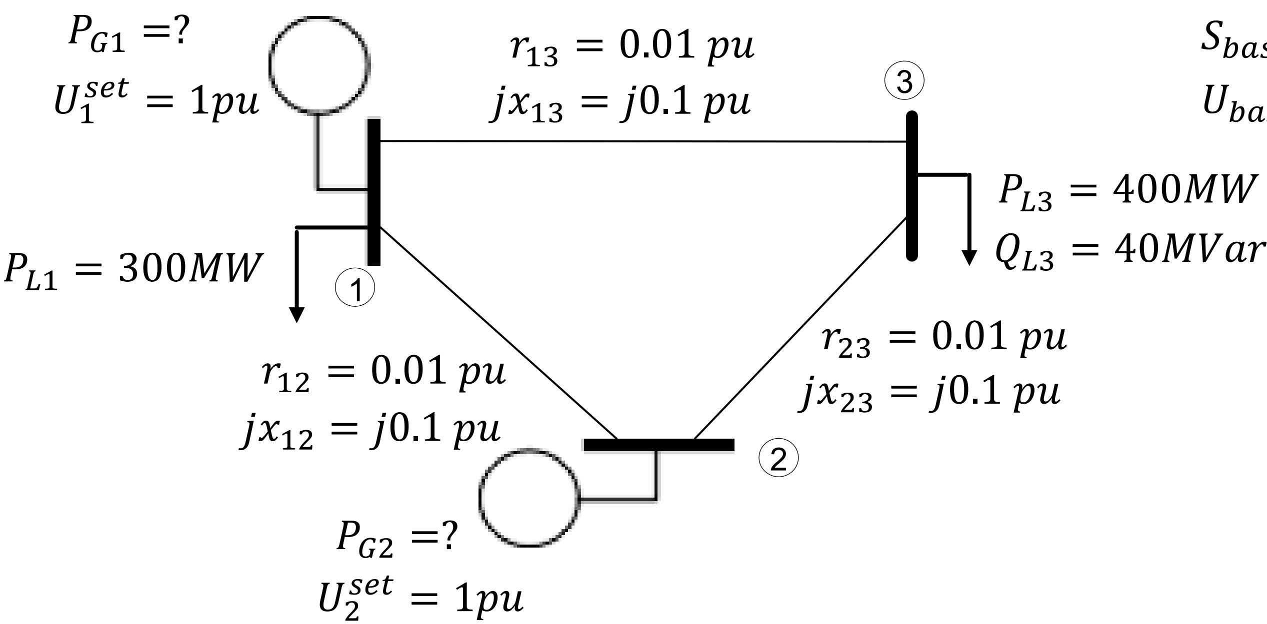
- Variables: P_G, U, θ
- Equality Constraints
  - Active Power balance at all nodes: $P_i = \sum_{ij\in\Omega_i} P_{ij}(U_i, U_j, \theta_i, \theta_j)$
  - Reactive Power balance at PQ nodes: $Q_i = \sum_{ij\in\Omega_i} Q_{ij}(U_i, U_j, \theta_i, \theta_j)$
  - Voltage Control at PU nodes: $U_i = U_i^{set}$
- Inequality Constraints
  - Limits on power generation: $P_G^{min} \leq P_G \leq P_G^{max}$
  - Limits on line flows: $-S_{ij}^{max} \leq S_{ij} \leq S_{ij}^{max}$ or $-I_{ij}^{max} \leq I_{ij} \leq I_{ij}^{max}$
  - Limits on voltage magnitudes of PQ buses: $U_i^{min} \leq U_i \leq U_i^{max}$

AC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \le P_{G1} \le 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \le P_{G2} \le 700MW$$



$S_{base} = 100MW$

$U_{base} = U_n$

$|S_{12}| \le 250MVA$

$|S_{13}| \le 200MVA$

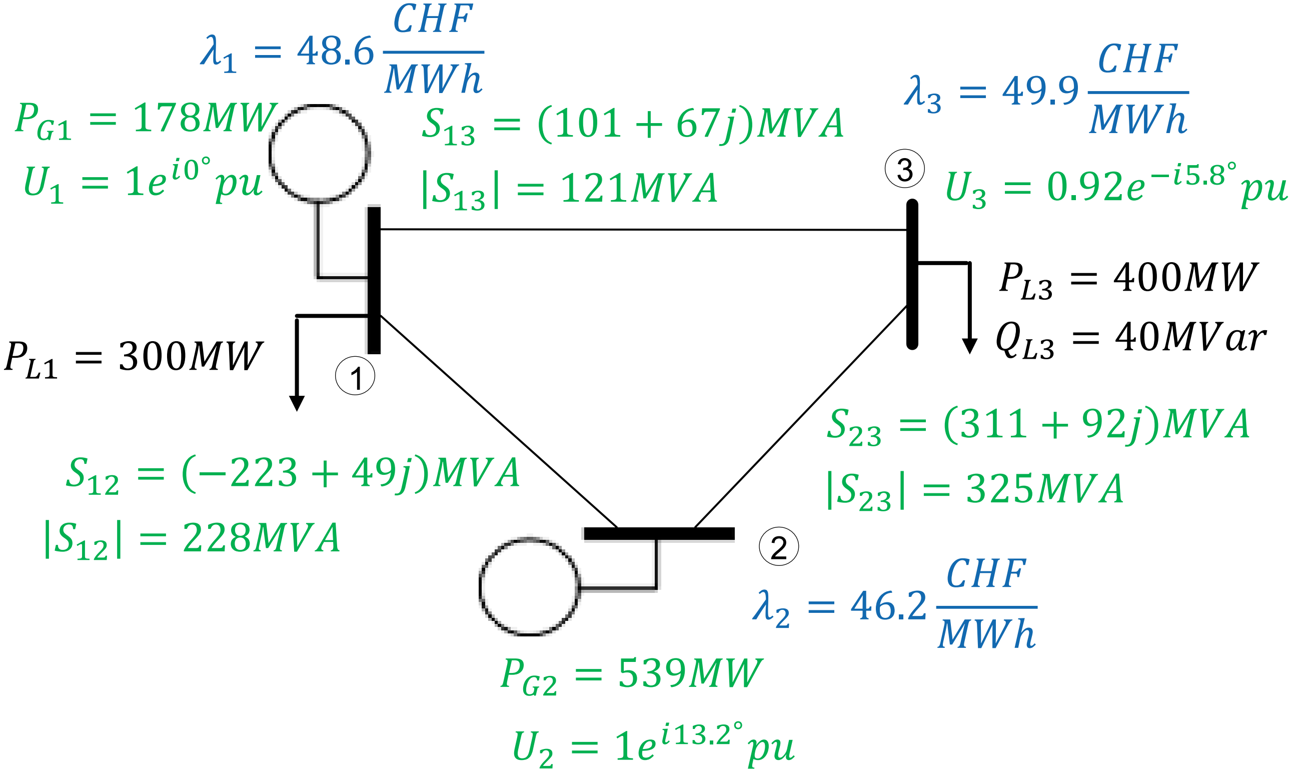
$|S_{23}| \le 400MVA$

AC Optimal Power Flow

- Example
  - Two Generators:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \qquad 0MW \leq P_{G1} \leq 200MW$$

$$C_2(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \qquad 0MW \leq P_{G2} \leq 700MW$$



$S_{base} = 100MW$

$U_{base} = U_n$

$|S_{12}| \leq 250MVA$

$|S_{13}| \leq 200MVA$

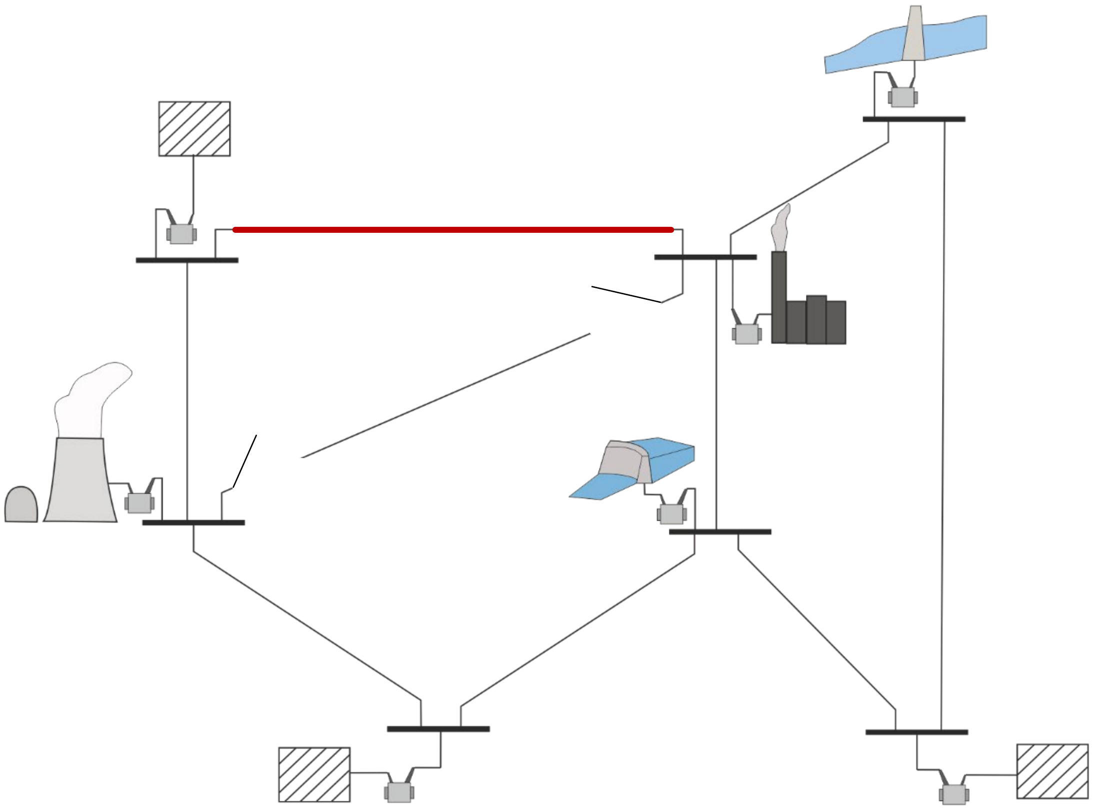
$|S_{23}| \leq 400MVA$

AC OPF vs. DC OPF

	DC OPF	AC OPF
$U_1\angle\theta_1$	$1\angle 0°\ pu$	$1\angle 0°\ pu$
$U_2\angle\theta_2$	$1\angle 15.87°\ pu$	$1\angle 13.2°\ pu$
$U_3\angle\theta_3$	$1\angle -3.53°\ pu$	$0.92\angle -5.8°\ pu$
P_{G1}	$84.6\ MW$	$178\ MW$
P_{G2}	$615.4\ MW$	$539\ MW$
P_{loss}	$0\ MW\ (21\ MW)$	$17\ MW$

Security Constrained Optimal Power Flow

- N-1 Security
  - System is operated such that no constraint is violated in normal operation or in case of a failure of one element in the system



Security Constrained Optimal Power Flow

- Optimization Problem
  - Same as AC OPF or as DC OPF
  - Additional power flow balances for considered failure cases
  - Additional operational limitations for considered cases
- Variables
  - Power Generation: P_G
  - Voltages in normal operation: U_0, θ_0
  - Voltages for each contingency case: $U_q, \theta_q, \quad q = 1, \ldots, N$
- Problem Formulation

Minimize Generation Cost

$$\sum_{i=1}^{N_G} C_i(P_{G_i})$$

$$s.t. \quad g_0(x_0, P_G) = 0$$

Normal Operation

$$h_0(x_0, P_G) \leq \overline{h}$$

$$g_q(x_q, P_G) = 0, \qquad q = 1, \ldots, N$$

Contingency Cases

$$h_q(x_q, P_G) \leq \overline{h}, \qquad q = 1, \ldots, N$$

Summary

- Economic Dispatch
  - Find generation dispatch that minimizes cost of supply
  - Constraints include generation limits
  - Lagrange Multiplier of power balance constraint equal to marginal price

- DC OPF and AC OPF
  - Find generation dispatch that minimizes cost of supply (or other objective)
  - Constraints include grid limitations (line flows, voltages, etc.)
  - Lagrange Multipliers of power balance constraints correspond to locational marginal prices (LMP) => congestions and losses make LMPs deviate from each other

Summary

- Security Constrained OPF
  - Same as DC or AC OPF but includes constraints for contingency cases
  - Additional set of power flow equations per considered contingency