

**Numerical Analysis**  
**Tutorial : Numerical solution of linear systems**

Level : 3A & 3 B

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**Exercice 1**

We consider the system of linear equations  $(S) : AX = b$ , with

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 4 & -1 \\ -1 & 1 & 3 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \text{et } b = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}.$$

**Partie I :**

1. (a) Show that  $(S)$  has a unique solution in  $\mathbb{R}^3$ .  
(b) Solve  $(S)$  using the Gauss elimination method.
2. (a) Justify the convergence of the Jacobi method and the Gauss-Seidel method for solving the system  $(S)$ .  
(b) Write the iterative methods of Jacobi and Gauss-Seidel for solving the system  $(S)$ .
- (c) For the initial vector  $X^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ , Give the results of the first two iterations using
  - i. Jacobi's method.
  - ii. Gauss-Seidel's method.

**Partie II :**

3. By considering the error  $E = \|X - X^{(k)}\|_2$ , with  $X$  the exact solution,  $X^{(k)}$  ( $k \in \{1, 2\}$ ) an approximate solution by one of the two methods and  $\|\cdot\|_2$  the Euclidean norm defined as :

$$\|X\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2}, \quad \forall X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3,$$

Calculate the errors made by the Jacobi and Gauss-Seidel methods for the first two iterations.

4. Compare the Jacobi and Gauss-Seidel methods in terms of accuracy for the first two iterations for solving the  $(S)$  system.