Invariant Reduction for Symmetric TMSV Gaussian Channels

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For a TMSV in standard form,

$$A = B = a\mathbb{I}_2, \quad C = \text{diag}(c, -c), \quad a = \cosh(2r), \ c = \sinh(2r).$$

A symmetric phase-insensitive channel preserves this structure:

$$A' = B' = a'\mathbb{I}_2, \qquad C' = \text{diag}(c', -c').$$

The invariants are

$$\tilde{\Delta} = 2\big((a')^2 + (c')^2\big), \qquad \det V' = \big((a')^2 - (c')^2\big)^2.$$

Using the expression for $\tilde{\nu}_-$ in [1],

$$\tilde{\nu}_- = a' - c'.$$

Thus $\tilde{\nu}_- < 1 \iff a' - c' < 1$.

If we now consider thermol loss, with:

$$a' = \eta a + (1 - \eta)(2N_{\text{th}} + 1), \qquad c' = \eta c,$$

we would be able to get the expression

$$\eta e^{-2r} + (1 - \eta)(2N_{\text{th}} + 1) < 1.$$

Afterwards considering quantum-limited amplification:

$$a' = ga + (g - 1), \qquad c' = gc,$$

one reaches the conclusion that:

$$\tilde{\nu}_- = ge^{-2r} + (g - 1).$$

For our above expression, there is only two possibilities:

- $g \geq 2$: since $\tilde{\nu}_- = ge^{-2r} + (g - 1) \geq 1$ for all r, entanglement is impossible.

- $1 < g < 2$: This would require satisfying the expression:

$$r > \frac{1}{2} \ln\left(\frac{g}{2 - g}\right).$$

At $g = 2$ the added quantum-limited noise equals one shot-noise unit per mode, giving a hard noise budget that no amount of squeezing can overcome.

References

[1] A. Serafini, F. Illuminati, G. Adesso, and S. De Siena, *Phys. Rev. A* **70**, 022318 (2004).