Examination Aid Sheet
Faculty of Applied Science & Engineering
Both sides of the sheet may be used;
must be printed on 8.5” x 11” paper.

Subject:______________________________

Candidate’s name:______________________

Candidate’s signature:___________________

Kinematics:

$v_f = v_i + a\Delta t$

$v_f^2 = v_i^2 + 2a\Delta x$

$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$

Circular Motion:

$v = \frac{2\pi r}{T} = \omega r$

$a = \alpha r$

$F_c = \frac{mv^2}{r}$

$a_c = \frac{v^2}{r} = \omega^2 r$

Drag:

$F_D = \frac{1}{2}\rho v^2 C_D A(-\hat{v})$

ρ: pressure; A: cross-sectional area; $(-\hat{v})$: opp dir.; C_D: Coefficient of drag, 0.25 — 0.5

Chapter 2:

$\frac{d\vec{p}}{d_t} = \vec{F}_{net} \rightarrow \Delta\vec{p} = \vec{F}_{net}\Delta t$

$\vec{J} = \int \vec{F}_{net} d_t$

$\Delta\vec{g} = -2R_E \frac{GM}{r^3}$

$F_g = G\frac{m_1 m_2}{r^2}$

$r_f = r_i + \frac{p_f}{m}\Delta t$

Energy:

Potential $\rightarrow$ mgh (gravitational), $\frac{1}{2}k\Delta x^2$ (elastic)

$K_{trans} = \frac{1}{2}mv^2$

$K_{rot} = \frac{1}{2}I\omega^2$

$c = \frac{\Delta E_{thermal}}{m\Delta T}$

Efficiency $= \frac{E_{useful}}{E_{input}}$

power: $P = \frac{dE}{d_t} = \frac{dW}{d_t} = \vec{F}\cdot\vec{v}$

constant power:

$dE = Pd_t$

Vibrational energy = Thermal

$Q = mc\Delta T \quad Q = -Q_{surr.}$

$\Delta U = Q - W$

Chapter 10:

$\vec{L} = \vec{r}\times\vec{p}$

Curl fingers toward $\vec{p}$

$\odot$ out of page

$\otimes$ into page

$\vec{L}_{rot} = I\vec{\omega} \rightarrow \vec{p} = m\vec{v}$

$\vec{\omega} \rightarrow \vec{v}$

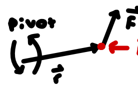
$I \rightarrow m$

$\vec{K}_{rot} = \frac{L_{rot}^2}{2I} \rightarrow K = \frac{p^2}{2m}$

$\vec{\alpha} \rightarrow \vec{a}$

$\vec{\tau} \rightarrow \vec{F}$

$\vec{\tau} = I\vec{\alpha} \rightarrow \vec{F} = m\vec{a}$



pivot

$\vec{F}$ applied here

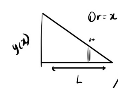
$\vec{\tau} = \vec{r}\times\vec{F}$

$= |\vec{r}||\vec{F}|\sin\theta$

$\vec{L} = \vec{L}_{trans} + \vec{L}_{rot}$

$[\vec{r}_{cm}\times\vec{p}_{tot}] + [\vec{r}_{1,cm}\times\vec{p}_1 + \ldots]$

Moment of inertia, triangle, uniform density



$y(x)$, L, dx

① $r = x$

$I = \Sigma mr^2 = \int_0^L x^2 \cdot dm$

$\cdot A = \frac{1}{2}Lh$

$\cdot M = \sigma A_{total}$ (σ: mass per area)

$\sigma = \frac{M}{\frac{1}{2}Lh} = \frac{2M}{Lh}$

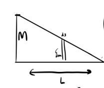
$y(x) = h\left(1 - \frac{x}{L}\right)$

$dA = dx \cdot y(x)$

$dm = \sigma dA = \frac{2M}{Lh}h\left(1 - \frac{x}{L}\right)dx$

$\int_0^L x^2\left[\frac{2M}{L\cancel{h}}\cancel{h}\left(1 - \frac{x}{L}\right)\right]dx$

Centre of mass, uniform density triangle



M, dx, $\int dm$, L

① $y = -\frac{h}{L}x + h$ ($\frac{0-h}{L-0} = -\frac{h}{L}$; h: yint when $x=0$)

$y(x) = -h\left(1 - \frac{x}{L}\right)$

Areas of each rectangle:

$dx \times y(x)$

$dA = h\left(1 - \frac{x}{L}\right)dx$

② $x_{cm} = \frac{\int x\,dA}{\int dA}$

$= \frac{\int_0^L x\cancel{h}\left(1 - \frac{x}{L}\right)dx}{\int_0^L \cancel{h}\left(1 - \frac{x}{L}\right)dx}$

$= \frac{\int_0^L x - \frac{x^2}{L}}{\int_0^L 1 - \frac{x}{L}}$

$= \frac{\int_0^L \frac{x^2}{2} - \frac{x^3}{3L}}{\int_0^L x - \frac{x^2}{2L}}$

Plug in L's, get $L/3$

$\bar{x}_{cm} = \frac{\int x(\sigma y(x)dx)}{\int \sigma y(x)dx}$ (here it cancels out)

Chapter 3:

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$F_{grav\,2\,by\,1} = -\frac{Gm_1 m_2}{r^2}\hat{r}$ } 1 by 2 = opp. dir.

Binomial Approximation:

$(1+x^2) \sim (1+nx) \quad x \ll 1$

How grav. changes for small heights:

$|\vec{F}_g| \simeq mg\left(1 - \frac{2y}{R_E}\right)$

$F_{A\,on\,B} = -F_{B\,on\,A}$

Centre of mass:

$\vec{r}_{cm} = \frac{\Sigma m_i r_i}{\Sigma m_i} \quad \vec{v}_{cm} = \frac{\Sigma m_i v_i}{\Sigma m_i}$

$\vec{p}_{System} = m_{System}\vec{v}_{System}$

$\vec{v} = \frac{1}{T}\int v(t)d_t$ or $\frac{1}{T}\Delta x$

$\vec{a} = \frac{d|\vec{v}|}{d_t}\hat{v} + |\vec{v}|\frac{d\hat{v}}{d_t}$

Uniform Circular Motion:

$\vec{a} = \frac{v^2}{R} \quad \hat{r} = \left(\frac{r}{R}\right)^3$

Work:

$W_{net} = \Delta K$

$W = Fd\cos\theta$

$\frac{dQ}{d_t} = \vec{F}\cdot\vec{v} = P$

$W_{friction} = -$heat energy

$W = \int F d_x$

$\Delta U = -W = -\int_0^h mg(-\hat{y})d_y$

Work = motion energy + spring or stored

Time?

$\hookrightarrow$ Impulse!

$\int F d_t = \Delta p$

Distance

$\hookrightarrow$ work

$\int F d_x = \frac{1}{2}mv^2$

Chapter 11:

System = both particles

Elastic: kinetic energy conserved

$\Sigma K_i = \Sigma K_f$

Inelastic: kinetic energy changes

Before $\rightarrow$ After

Totally inelastic: Stick together

$\rightarrow$ loses maximum possible kinetic energy

One at rest, one not (equal masses)

$2P_{1f}P_{2f} = 0$

Unequal:

$\circ \rightarrow$ ② $\quad \vec{P}_{1i} \quad \vec{P}_{2i} = 0$

$v_{2f} \sim 0$

$v_{2f} \ll v_{1f}$

$\vec{P}_{1f} = -\vec{P}_{1i}$

$\vec{P}_{2f} = \vec{P}_{2f}$

Reference Frames:

$A \rightarrow v$

② $\rightarrow v \quad 0$

② $\overset{v}{\leftarrow} 0$

② $\quad 0 \rightarrow v$ relative to mass 2

② $\overset{v}{\rightarrow} \quad 0 \overset{2v}{\rightarrow}$ relative to earth

Chapter 4:

$F_k = \mu_k F_N$ (kinetic friction) $\quad F_s \le \mu_s F_N$ (static friction)

Size of interatomic bonds (d):

ρ = density $= \frac{m}{d^3}$

$d = \sqrt[3]{\frac{m}{\rho}}$

$k_T = k_i \frac{N_B}{N_L}$ (k_T: Total; k_i: micro; N_B: # of atoms in area; N_L: # in row)

$k_i = \frac{mg}{\Delta L}\frac{N_L}{N_A}$

$k_T = \frac{mg}{\Delta L}$

Stress: $\sigma = \frac{F}{A} \sim \frac{ks}{d^2}$

Strain: $\varepsilon = \frac{\Delta L}{L} = \frac{s}{d}$

$E = \frac{\sigma}{\varepsilon} = \frac{k}{d} = \frac{kL}{A}$

$\Delta L = \frac{FL}{EA}$

$\frac{dF}{d_t} = m\frac{d^2x}{d_t^2} = -kx$

$\frac{d^2x}{d_t^2} = -\omega^2 x \rightarrow \omega^2 = \frac{k}{m} \rightarrow \omega = \sqrt{\frac{k}{m}}$ (angular freq.)

$\omega = \frac{2\pi}{T} \quad f = \frac{1}{T}$

$T = 2\pi\sqrt{\frac{m}{k}}$ Young's modulus?

Speed of Sound:

$\vec{v}_{sound} = \sqrt{\frac{E}{\rho}}$ (ρ: density)

$a = -\omega^2 x$

$f = \frac{\omega}{2\pi} = \frac{\sqrt{k/m}}{2\pi} \quad Pa = \frac{N}{m^2}$

dist. from equilibrium $= \frac{2A}{\pi}$

Chapter 8 and 12:

h: Planck's constant

$\Delta E \sim \hbar\omega = \hbar\sqrt{\frac{k}{m}}$

$\cdot\cdot|\cdot|\cdot$ } microstate $\quad \begin{matrix} q=4 \\ N=3 \end{matrix}$ } macrostate

$(N-1) = |$

$q = \cdot$

ways: $\Omega = \frac{(q+N-1)!}{q!(N-1)!}$

$k_B T = \frac{q}{\Delta(\ln\Omega)}$

$\Delta E = q = \hbar\omega \quad \omega = \sqrt{\frac{k}{m}}$

$T = \frac{q}{k_B\Delta(\ln\Omega)} \quad c = \frac{\Delta E/N}{\Delta T}$ (N: Num atoms)

Hold the string from a yo-yo and let it unwind, how long will it take to fully unwind **M**

Assume
- thin string (r ~ constant)
- disk: $I = \frac{1}{2}mR^2$

$T = \frac{2\pi}{\omega}$

$\Sigma F = T - mg$

$T - mg = ma$

$T = ma + mg$

$\tau = T \times r$

$(ma + mg) \times r = \alpha I_{cm}$

$I = \frac{1}{2}mR^2$ The big disk

$\alpha = \frac{a}{r}$

$r(ma + mg) = I \frac{a}{r}$

$rT = \frac{1}{2}mR^2 \frac{a}{r}$

$r(m(a+g)) = \frac{1}{2}mR^2 \frac{a}{r}$

$a = \frac{2gr^2}{R^2 - 2r^2}$

Then use kinematics:

$L = \frac{1}{2}at^2$

$t = \sqrt{\frac{L(R^2 - 2r^2)}{gr^2}}$

assume all

Modeling: Max Speed of a tennis ball, initially at rest, after being hit by a racket?

$V_{\text{ball rel. to racket}} = V_{Racket} - V_{Ball}$
$= -V_R$

Collision becomes "ball hits an unmoving wall"
just reverses velocity: $-V_R = +V_R$

Switch back to Earth's frame:
add rackets speed back

$V_f = 2V_R$

Modelling: Time for gyroscope to precess one full circle:

How to find?

$T = \frac{2\pi}{\Omega}$

$\tau = mgd$

$\tau = |L| \frac{dL}{dt}$

$\tau = |L| \Omega$

$mgd = I\omega\Omega$

$\cancel{m}gd = \cancel{m}R^2\omega\Omega$

$\Omega = \frac{gd}{R^2\omega}$

$T = \frac{2\pi R^2\omega}{gd}$

Assume
- d ~ 10cm
- R ~ 30cm
- m ~ 3kg
- hoop ~ $I \sim mR^2$
- ω ~ 10 rad/s

Angular acc. of hoop rolling down a hill

$\tau = mgr\sin\theta$

$\alpha = \frac{mgr\sin\theta}{2mr^2}$

$\alpha = \frac{g\sin\theta}{2mr}$

Child, merry ground, Spin speed: ? jumps

Before After

$L_i = mv(R+r)$ $L_f = I\omega$

$mv(R+r) = (I_{disk} + I_{kid})\omega$

simplify, point was so close to disk edge

$mv(\cancel{R+r}) = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$ (R)

$\omega = \frac{mvR}{\frac{1}{2}MR^2 + mR^2}$

Assume:
- conserve ΔL
- child lands on edge and pi at edge
- child is point mass

Numberless

Max Speed of a racecar.

for 1st second $V_2 = 30$ m/s, $T \sim 1$ s

$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}mv_2^2}{T}$

top speed (V_m)

$P = F \cdot v = \left(\frac{1}{2}\rho V_m^2 A C_D\right) \cdot V_m = \frac{mv_2^2}{2T}$

$V_m^3 = \frac{mv_2^2}{T\rho A C_D}$

$V_m = \sqrt[3]{\frac{10^6}{1}}$

~ 100 m/s

assume
- 1st second
- ignore air
- $\rho \sim 1$ kg/m³
- $A \sim (1 \times 2 \text{m}^2)$
- $C_D \sim 0.1$-0.3
- $m \sim 1000$ kg

Max Speed of tennis ball initially at rest

Max speed: elastic

$V_1' = \left(\frac{2(9)}{1+9}\right)(30)$

$V_f = \frac{18}{10}(30) = 54$ m/s ✓

Assume:
elastic, 1D,
$m_B / m_1 = 1/9$

2024

(N): Runner starts at rest, runs d in T. Constant Power. Top speed?

$P = \frac{dE}{dt} = \frac{\frac{1}{2}mv^2}{dt}$

$\int P\,dt = \frac{1}{2}mv^2$

$PT = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2PT}{m}}$

$d = \int \sqrt{\frac{2PT}{m}}$

isolate & plug back

(M): Throw a ball ($r = 0.01$) at wall. Max force between?
Not constant force.

$\cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}kx^2$

$mv^2 = kx^2$

$mv^2 = \frac{F^2}{k}$

$F_{max} = kx_{max}$

$F_{max} = \frac{EA}{L} \cdot mv^2$

$kmv^2 = F^2$

$\frac{EA}{L}mv^2 = F^2$

$\frac{E(\pi R^2)mv}{2\cancel{R}} = F^2$

$\sqrt{\frac{EmvR}{2}\pi} = F$

(N): Period of rod swinging as a pendulum

$T = \frac{2\pi}{\omega}$ $\omega^2 x = \frac{d^2x}{dt}$

SHM

$\tau = mg\left(\frac{L}{2}\right)\theta$

$I = \frac{mL^2}{3}$

$\alpha = \frac{g\theta}{2} \times \frac{3}{L}$

$\alpha = \frac{3g\theta}{2L}$

$\frac{3g}{2L} \cdot \theta$

$\omega = \sqrt{\frac{3g}{2L}}$

$T = 2\pi \cdot \sqrt{\frac{2L}{3g}}$

(N): Two solid spheres in space. One at rest, other moving w/ speed v. Neither rotating. Stick together & collide. Neither deforms. KE lost in this process? m, R, v.

Perfectly inelastic

$\cancel{m}v_i = 2\cancel{m}v_f$

$v_f = \frac{v_i}{2}$

$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2$

$\frac{1}{2}mv_i^2 = \frac{1}{2}m\left(\frac{v_i}{2}\right)^2 + \frac{1}{2}m\left(\frac{v_i}{2}\right)^2 + \frac{1}{2}I\omega^2 \times 2$

Find from:
$I\omega = mvR$

$\underline{V \neq \omega R \text{ (in space)}}$

(M): A ball moves to person. Ball's moving N [10], athlete kicks and it goes East. Ball v after kick?

$\dot{P}_y = 0$

$10v_2\cos 60° = 1(10)$

$v_2 = 2$

$v_f = 10(2)\sin 60°$

Hoop, Pivot, acts like Pendulum, T: ?

$I = 2mR^2$ from pivot

$\tau = mgR\sin\theta$

$I\alpha = mgR\sin\theta$

$\alpha = -\frac{mgR\sin\theta}{2mR^2}$

$\sin\theta \approx \theta$

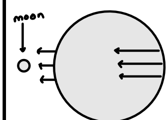
$\frac{d^2\theta}{dt^2} \approx -\frac{g}{2R}\theta$

OH! SHM: $\frac{d^2x}{dt^2} = -\omega^2 x$

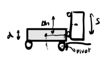
$\omega = \sqrt{\frac{g}{2R}}$

$T = \frac{2\pi}{\omega} \rightarrow T = 2\pi\sqrt{\frac{2R}{g}}$

Tidal waves:



Wagon, Curb, flips over, speed=?



Assume
- curb h ~ wheel diameter
- energy conserved after collision
- $\hat{L}$ cons.

Distance to pivot: $\frac{d}{2} + r_{wheel}$

$L_i = L_f$

$mv\left(\frac{d}{2} + r\right) = I\omega$

$I_{rod} + m\left(\frac{d}{2} + r\right)^2$

$\omega = \frac{mv\left(\frac{d}{2} + r\right)}{I}$

energy:

$mg\Delta h = I\omega^2$

$h_f = s/2$

$h_i = \frac{d}{2}$

$\Delta h = \frac{s}{2} - \frac{h}{2}$

plug in for ω

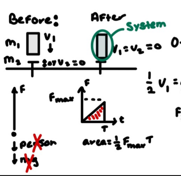
$\frac{1}{2} I_a \frac{m^2v^2d^2}{I_a^2} = mg\left(\frac{s}{2} - r\right)$

Isolate for v.

Numberless

Max force of hammer on nail.

Objects: Hammer, Nail



$\Delta P = \int F\,dt$

$0 - m_1v_1 = \frac{1}{2}F_{max}T$

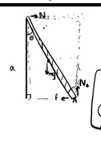
$\frac{1}{2}v_1 = \frac{\Delta y}{T} \longrightarrow T = \frac{2\Delta y}{v_1}$

$F_{max} = \frac{2m_1v_1}{2\Delta y / v_1} = \frac{m_1v_1^2}{\Delta y}$

$= 10$ kN

Chap 2

Assumptions:
- Inelastic collision
- Ignore gravity and person
- $F(t)$ is linear
- Assume T found by const. acceleration
- $m_1 \sim 1$ kg
- $v_1 \sim 10$ m/s
- $\Delta y \sim 1$ cm



Forces:

$\hat{x} = N_1 - f$

$\hat{x} = N_1 - \mu_k N_2$

① $N_1 = \mu_k N_2$

$\hat{y} = N_2 - mg$

② $N_2 = mg$

$N_1 = \mu_k mg$

Torques:

$\tau_A = mg\left(\frac{\cancel{a}}{2}\cos\theta\right) - N_1(\cancel{a}\sin\theta)$

$\tau_A = mg \cdot \frac{1}{2}\cos\theta - N_1\sin\theta$

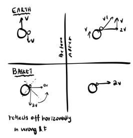
$N_1 = \frac{mg\frac{1}{2} \cdot \frac{\cos\theta}{\sin\theta}}{\cancel{\mu_k g}}$

$\mu_k = \frac{1}{2}\cot\theta$

ladder on wall
μ_k: ?

Modelling:

Tennis ball, ontop basketball, rolls off, 45° when hits ground, v



reflects off horizontally in wrong R.F

$V_f = \sqrt{(2v)^2 + v^2}$

$= \sqrt{5}\,v$

$= \sqrt{10gh}$

Midterm: Moment of inertia triangle

$\frac{dm}{dx} = Ax^2$

$dm = Ax^2\,dx$

$M = \int \frac{dm}{dx} = \int Ax^2$

$\int_0^L Ax^2 = A\frac{L^3}{3}$

$M = \frac{AL^3}{3}$

$3M = AL^3$

$A = \frac{3M}{L^3}$

$I = \int_0^L x^2\,dm$

$I = A\int_0^L x^4\,dx$

$I = \frac{3M}{L^3}\int_0^L x^4\,dx$

For small angles:

$\alpha\left(\frac{d^2\theta}{dt}\right) = -\frac{g}{2r} \cdot \theta$

For Small angles:
$\sin\theta = \theta$
$\cos\theta = 1$
$\tan\theta = \theta$