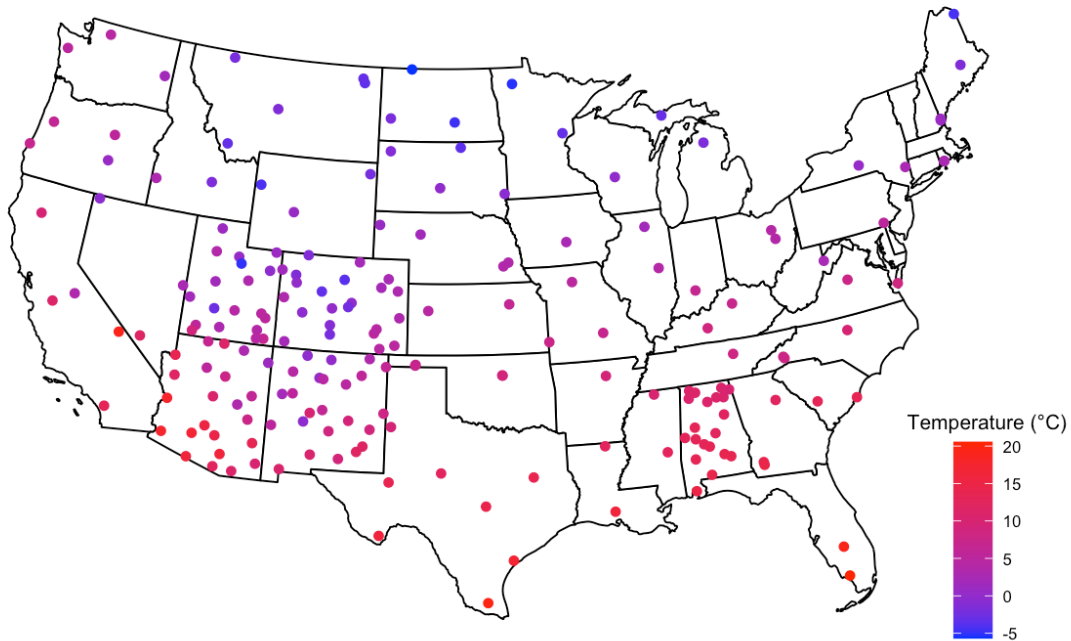



```
ggplot2::theme(legend.title = ggplot2::element_text(hjust=0.5),
               legend.position.inside=c(0.87,0.001),
               plot.title=ggplot2::element_text(color="black", hjust=0.5,
                                                size=15))
```

Average Station Temperatures (°C) March 2024



Objective-2: Interpolate Average Temperatures for March 2024

make station data on contiguous USA

```
HI_inds <- which(station_data$state == "HI") # Hawaii indexes
```

```
AK_inds <- which(station_data$state == "AK") # Alaska indexes
```

```
joint_inds <- c(AK_inds, HI_inds)
```

```
contig_data <- station_data[-joint_inds,]
```

extract station data

```
stat_ids <- contig_data$station_id
```

```
stat_lons <- contig_data$LONGITUDE
```

```
stat_lats <- contig_data$LATITUDE
```

```
n <- length(stat_ids)
```

make average temperature vector & setup dates

```
avg_temps <- rep(NA, n)
```

```
start_d <- as.Date("2024-03-01") |> lubridate::yday()
```

```
end_d <- as.Date("2024-03-31") |> lubridate::yday()
```

```
for(i in 1:n){
```

```
  wbanno <- stat_ids[i]
```

```
  station <- estimate_yearly_cycle(weather_data, wbanno)
```

```
  avg_temps[i] <- mean(station[,2][start_d:end_d], na.rm=T)
```

```
}
```

```

# make data frame for temperature + spatial data
WD <- cbind.data.frame(lon=stat_lons, lat=stat_lats, avg_temp=avg_temps)
WD <- na.omit(WD) # remove NA

# make spatial data frame
spat.df <- data.frame(x=WD$lon, y=WD$lat)

# make response vector
Y <- WD$avg_temp

# get elevation data for model
prj_code <- 4326 # projection code
el.mod <- elevatr::get_elev_point(spat.df, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
spat.df$elev <- el.mod$elevation

# make design matrix for model
X_mod <- model.matrix(~1+x+y+elev, data=spat.df)

# get grid points in contiguous USA + elevation data
g <- create_grid_points(resolution=100)
colnames(g) <- c("x", "y") # change lon to x and lat to y for elevatr
el.interp <- elevatr::get_elev_point(g, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
g$elev <- el.interp$elevation

# make design matrix for interpolations
X_interp <- model.matrix(~1+x+y+elev, data=g)

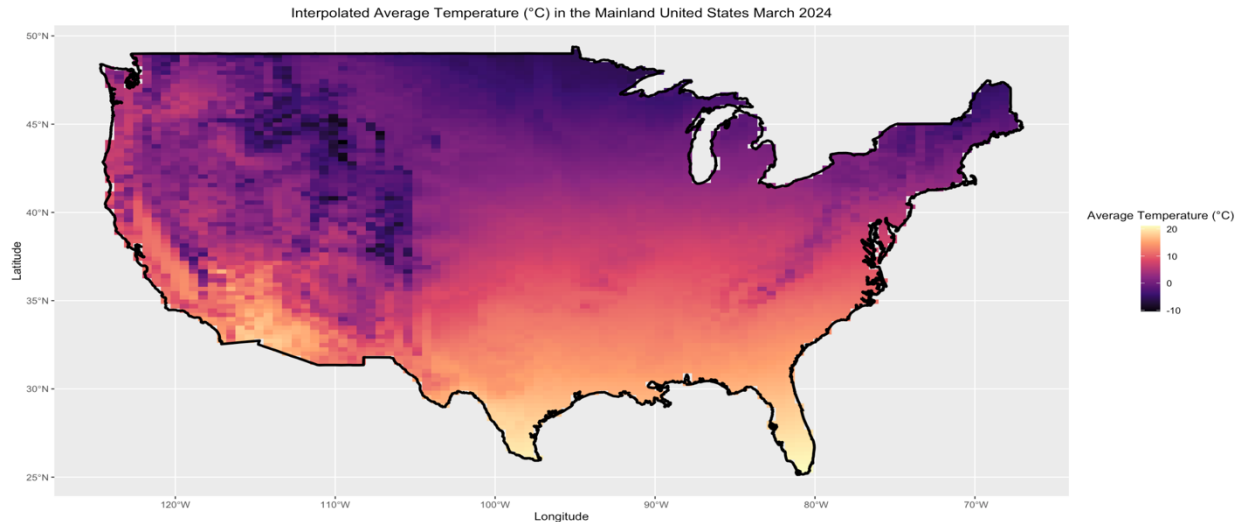
# make interpolations
g <- g[,1:2]
interps <- interpolate_points(WD, Y, X_mod, g, X_interp)
#> Assuming columns 1 and 2 of locs are (Longitude, Latidue) in degrees

# make title for plot & plot
t <- paste0("Interpolated Average Temperature (°C)",
  " in the Mainland United States March 2024")

int.plot <- plot_gridded_interpolations(interps, g, t=t)

# view plot
int.plot

```



Objective-3: Warmest and Coldest Days for USCRN Stations

The model being assumed is depicted below:

$$T_DAILY_AVG_i = \beta_0 + \beta_1 \cos(2\pi day_i / 365.25) + \beta_2 \sin(2\pi day_i / 365.25) + \beta_3 \cos^2(2\pi day_i / 365.25) + \beta_4 \cos((2\pi yday_i / 365.25) + 1.27) + \epsilon_i$$

Where $\epsilon_i \sim Normal(0, \sigma^2)$, day_i is the number of days from the epoch of 2000-01-01 and $yday_i$ is the day number (1-366). The null hypothesis is $H_0: \beta_i = 0$, i.e., the days do not have any effect on the average temperature, and it will be assessed at a significance level of $\alpha = 0.05$.

```
# make station data on contiguous USA
HI_inds <- which(station_data$state == "HI") # Hawaii indexes
AK_inds <- which(station_data$state == "AK") # Alaska indexes
joint_inds <- c(AK_inds, HI_inds)
contig_data <- station_data[-joint_inds,]

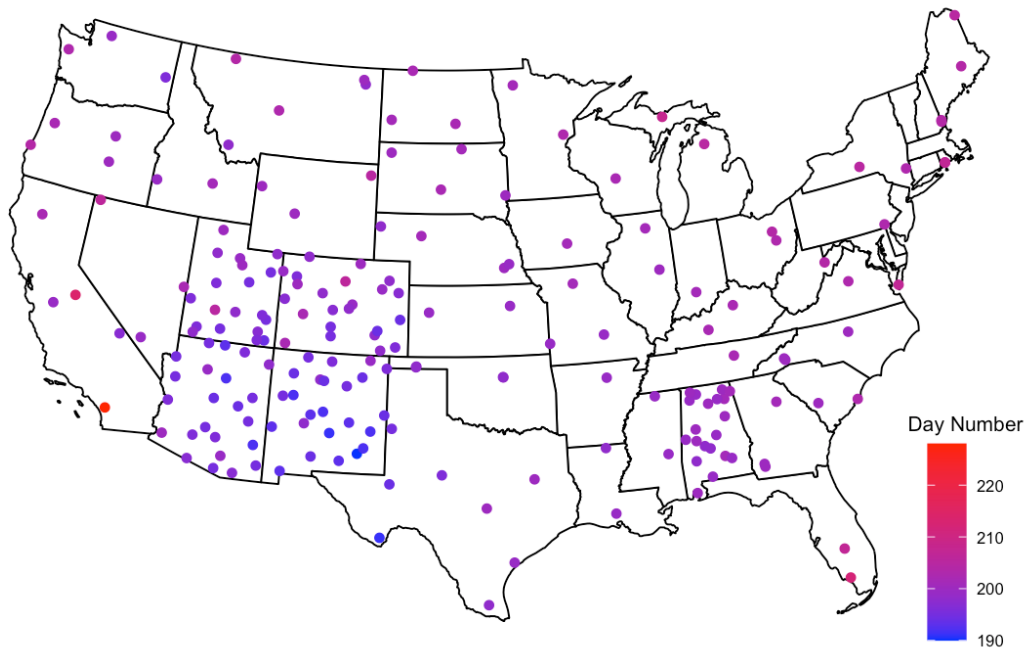
# extract station data
stat_ids <- contig_data$station_id
stat_lons <- contig_data$LONGITUDE
stat_lats <- contig_data$LATITUDE
n <- length(stat_ids)

# make warm and cold day vectors
warm <- rep(NA, n)
cold <- rep(NA, n)

# get yearly cycles & population warm and cold vectors
for(i in 1:n){
```

[illegible]

Warmest Days for Stations



```
# plot cold
```

```
# make title for plot
```

```
t <- "Coldest Days for Stations"
```

```
# make plotting data frame
```

```
g <- cbind.data.frame(stat_lons, stat_lats)
```

```
colnames(g) <- c("lon", "lat")
```

```
contig_usa <- maps::map("usa", "main", exact=T, plot=F)
```

```
pts_poly <- sp::point.in.polygon(g$lon, g$lat, contig_usa$x, contig_usa$y)
```

```
keep <- which(pts_poly == 1)
```

```
img.mat <- cbind.data.frame(g, cold)
```

```
img.mat <- img.mat[keep, ]
```

```
img.mat <- na.omit(img.mat)
```

```
colnames(img.mat) <- c("lon", "lat", "cold")
```

```
# get United States data
```

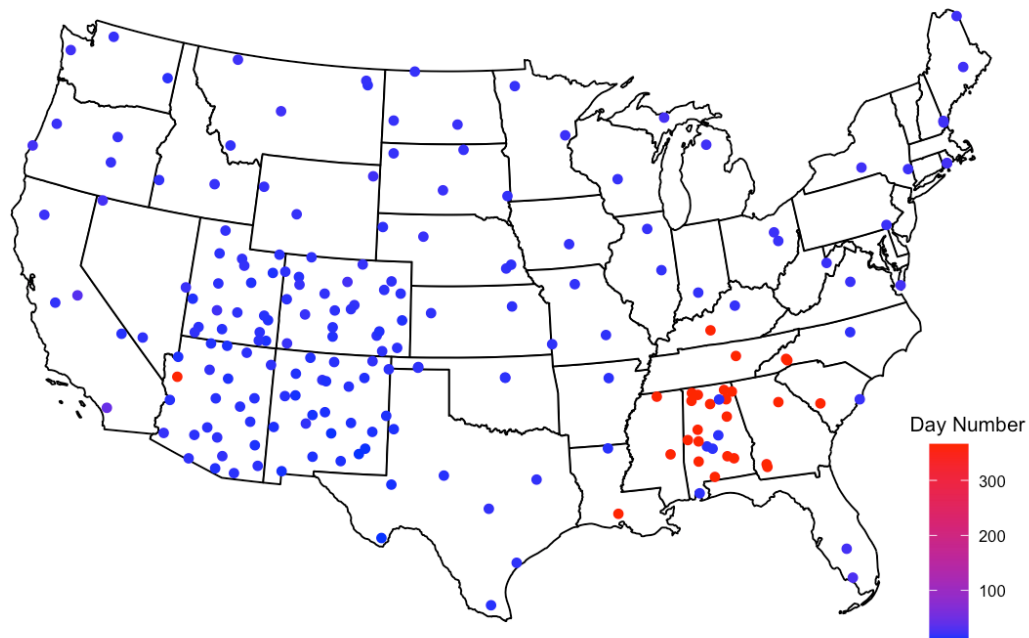
```
usm.img.mat <- usmap::usmap_transform(img.mat)
```

```
# plot data
```

```
usmap::plot_usmap("states", exclude=c("AK", "HI")) +  
  ggplot2::geom_sf(data=usm.img.mat, ggplot2::aes(colour=cold)) +  
  ggplot2::coord_sf() + ggplot2::ggtitle(t)+  
  ggplot2::scale_color_gradient(low="blue", high="red",  
                                name="Day Number") +  
  ggplot2::theme(legend.title = ggplot2::element_text(hjust=0.5),
```

```
legend.position.inside=c(0.87,0.001),
plot.title=ggplot2::element_text(color="black", hjust=0.5,
size=15))
```

Coldest Days for Stations



```
# interpolate & plot warm
```

```
# make data frame for day + spatial data
```

```
WD <- cbind.data.frame(lon=stat_lons, lat=stat_lats, warm=warm)
```

```
WD <- na.omit(WD) # remove NA
```

```
# make spatial data frame
```

```
spat.df <- data.frame(x=WD$lon, y=WD$lat)
```

```
# make response vector
```

```
Y <- WD$warm
```

```
# get elevation data for model
```

```
prj_code <- 4326 # projection code
```

```
el.mod <- elevatr::get_elev_point(spat.df, prj=prj_code, src="aws")
```

```
#> Mosaicing & Projecting
```

```
#> Note: Elevation units are in meters
```

```
spat.df$elev <- el.mod$elevation
```

```
# make design matrix for model
```

```
X_mod <- model.matrix(~1+x+y+elev, data=spat.df)
```

```

# get grid points in contiguous USA + elevation data
g <- create_grid_points(resolution=100)
colnames(g) <- c("x", "y") # change Lon to x and Lat to y for elevatr
el.interp <- elevatr::get_elev_point(g, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
g$elev <- el.interp$elevation

# make design matrix for interpolations
X_interp <- model.matrix(~1+x+y+elev, data=g)

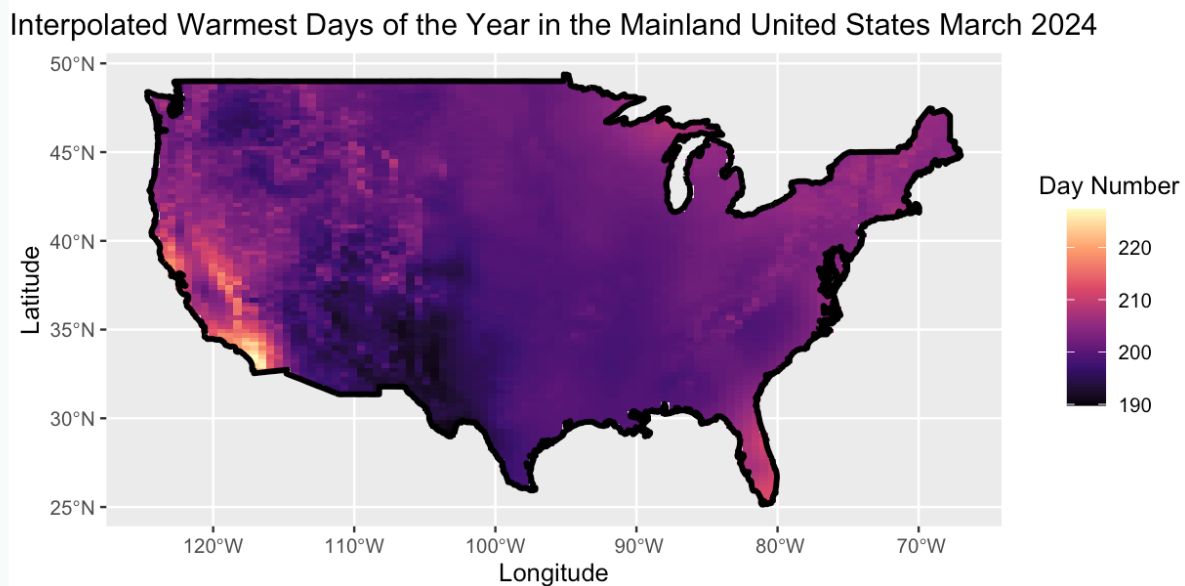
# make interpolations
g <- g[,1:2]
interps <- interpolate_points(WD, Y, X_mod, g, X_interp)
#> Assuming columns 1 and 2 of locs are (Longitude,latitude) in degrees

# make title for plot & plot
t <- paste0("Interpolated Warmest Days of the Year",
  " in the Mainland United States March 2024")
f <- "Day Number"

int.plot <- plot_gridded_interpolations(interps, g, t, f)

# view plot
int.plot

```




```

# interpolate & plot cold

# make data frame for day + spatial data
WD <- cbind.data.frame(lon=stat_lons, lat=stat_lats, cold=cold)
WD <- na.omit(WD) # remove NA

# make spatial data frame
spat.df <- data.frame(x=WD$lon, y=WD$lat)

# make response vector
Y <- WD$cold

# get elevation data for model
prj_code <- 4326 # projection code
el.mod <- elevatr::get_elev_point(spat.df, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
spat.df$elev <- el.mod$elevation

# make design matrix for model
X_mod <- model.matrix(~1+x+y+elev, data=spat.df)

# get grid points in contiguous USA + elevation data
g <- create_grid_points(resolution=100)
colnames(g) <- c("x", "y") # change lon to x and lat to y for elevatr
el.interp <- elevatr::get_elev_point(g, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
g$elev <- el.interp$elevation

# make design matrix for interpolations
X_interp <- model.matrix(~1+x+y+elev, data=g)

# make interpolations
g <- g[,1:2]
interps <- interpolate_points(WD, Y, X_mod, g, X_interp)
#> Assuming columns 1 and 2 of locs are (Longitude, Latitude) in degrees

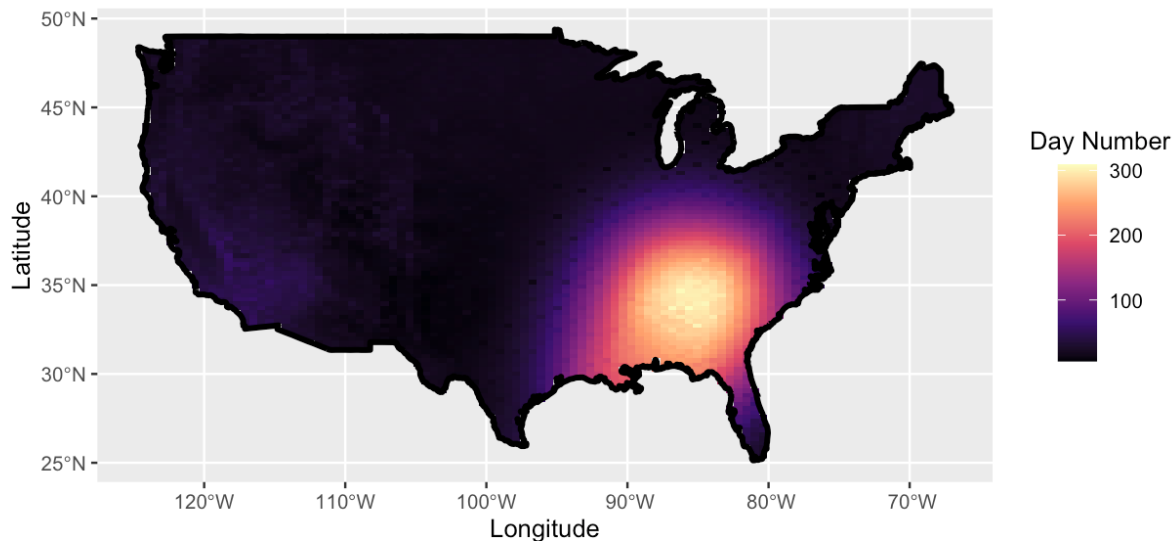
# make title for plot & plot
t <- paste0("Interpolated Coldest Days of the Year",
  " in the Mainland United States March 2024")
f <- "Day Number" # Legend title

int.plot <- plot_gridded_interpolations(interps, g, t, f)

# view plot
int.plot

```

Interpolated Coldest Days of the Year in the Mainland United States March 2024



Objective-4: Yearly Cycles for 10 USCRN Stations

get estimates for 10 diverse stations

```
GA_Newton_8_W <- estimate_yearly_cycle(weather_data, 63828)
RI_Kingston_1_W <- estimate_yearly_cycle(weather_data, 54797)
AZ_Tucson_11_W <- estimate_yearly_cycle(weather_data, 53131)
MT_Wolf_Point_34_NE <- estimate_yearly_cycle(weather_data, 94059)
AL_Northport_2_S <- estimate_yearly_cycle(weather_data, 73801)
CO_Grand_Junction_9_W <- estimate_yearly_cycle(weather_data, 3076)
NM_Reserve_1_W <- estimate_yearly_cycle(weather_data, 3080)
CA_Yosemite_Village_12_W <- estimate_yearly_cycle(weather_data, 53150)
FL_Everglades_City_5_NE <- estimate_yearly_cycle(weather_data, 92826)
SD_Pierre_24_S <- estimate_yearly_cycle(weather_data, 94085)
```

plotting

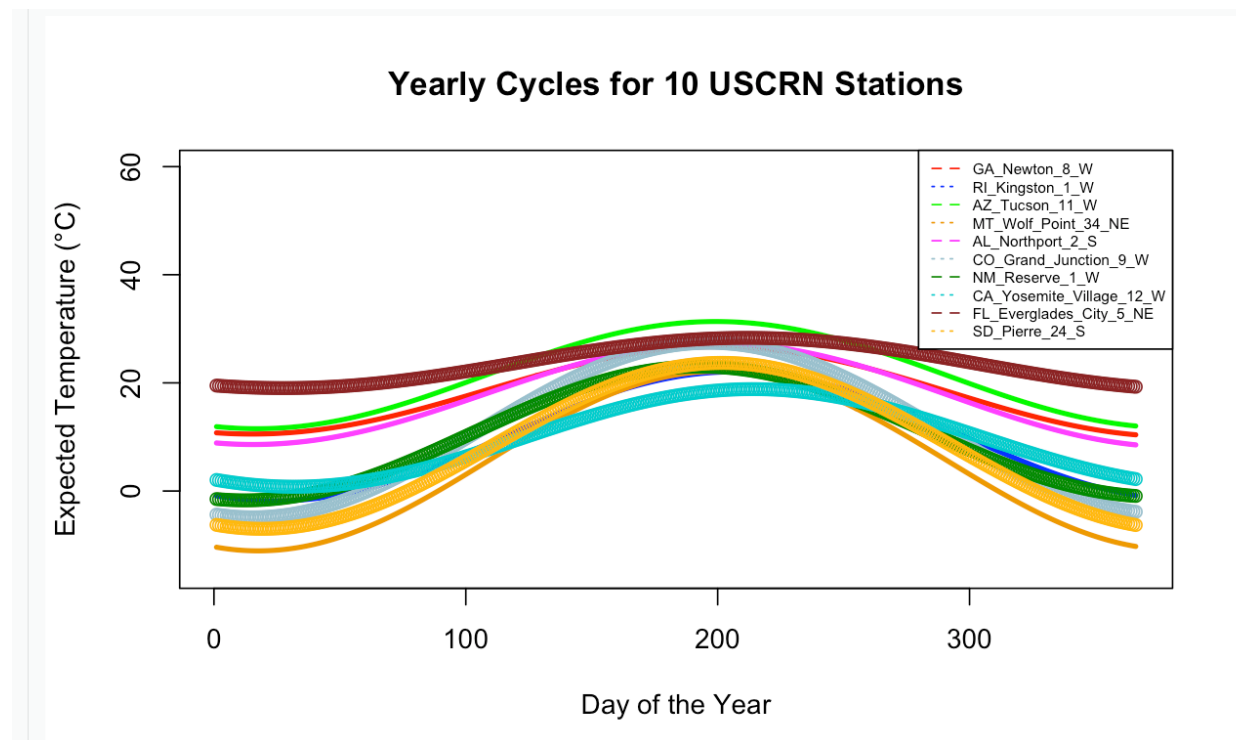
```
x <- 1:366
```

```
plot(x, GA_Newton_8_W$EXPECTED_TEMP, type = "n",
     xlab="Day of the Year",
     ylab="Expected Temperature (°C)",
     main = "Yearly Cycles for 10 USCRN Stations",
     ylim=c(-15,60))
lines(x, GA_Newton_8_W$EXPECTED_TEMP, type = "l", col = "red", lw=3)
lines(x, RI_Kingston_1_W$EXPECTED_TEMP, type = "l", col = "blue", lw=3)
lines(x, AZ_Tucson_11_W$EXPECTED_TEMP, type = "l", col = "green", lw=3)
lines(x, MT_Wolf_Point_34_NE$EXPECTED_TEMP, type = "l", col = "orange2",
      lw=3)
lines(x, AL_Northport_2_S$EXPECTED_TEMP, type = "l", col = "magenta", lw=3)
lines(x, CO_Grand_Junction_9_W$EXPECTED_TEMP, type = "b", col = "lightblue3")
```

```

lines(x, NM_Reserve_1_W$EXPECTED_TEMP, type = "b", col = "green4")
lines(x, CA_Yosemite_Village_12_W$EXPECTED_TEMP, type = "b", col = "cyan3")
lines(x, FL_Everglades_City_5_NE$EXPECTED_TEMP, type = "b", col = "brown4")
lines(x, SD_Pierre_24_S$EXPECTED_TEMP, type = "b", col = "darkgoldenrod1")
par(xpd=TRUE)
legend("topright", legend = c("GA_Newton_8_W", "RI_Kingston_1_W",
                              "AZ_Tucson_11_W", "MT_Wolf_Point_34_NE",
                              "AL_Northport_2_S", "CO_Grand_Junction_9_W",
                              "NM_Reserve_1_W", "CA_Yosemite_Village_12_W",
                              "FL_Everglades_City_5_NE", "SD_Pierre_24_S"),
      col = c("red", "blue", "green", "orange2", "magenta",
              "lightblue3", "green4", "cyan3", "brown4", "darkgoldenrod1"),
      lty=c(2,3), cex=0.55)

```



Objective-5: Estimating and Interpolating Station Trends

The model being assumed is depicted below: $T_DAILY_AVG_i = \beta_0 + \beta_1 day_i + \beta_2 \cos(2\pi day_i/365.25) + \beta_3 \sin(2\pi day_i/365.25) + \beta_4 \cos^2(2\pi day_i/365.25) + \beta_5 \cos((2\pi yday_i/365.25) + 1.27) + \beta_6 \sin(2\pi yday_i/365.25) + \epsilon_i$ Where $\epsilon_i \sim Normal(0, \sigma^2)$, and day_i is the number of days from the epoch of 2000-01-01, and $yday_i$ is the day number (1-366). The null hypothesis is $H_0: \beta_i = 0$, i.e., the days do not have any effect on the average temperature, and it will be assessed at a significance level of $\alpha = 0.05$.

```

# make station data on contiguous USA
HI_inds <- which(station_data$state == "HI") # Hawaii indexes
AK_inds <- which(station_data$state == "AK") # Alaska indexes
joint_inds <- c(AK_inds, HI_inds)

```

```

contig_data <- station_data[-joint_inds,]

# extract station data
stat_ids <- contig_data$station_id
stat_lons <- contig_data$LONGITUDE
stat_lats <- contig_data$LATITUDE
n <- length(stat_ids)

# make trend vectors
trend <- rep(NA, n)
trend_pv <- rep(NA, n)
trend_se <- rep(NA, n)

# get yearly cycles & population trend vectors
for(i in 1:n){
  wbanno <- stat_ids[i]
  dat3 <- estimate_trends(weather_data, wbanno)
  trend[i] <- dat3[1]
  trend_pv[i] <- dat3[2]
  trend_se[i] <- dat3[3]
}

trend <- 365.25*trend # to make from day to year

# plot

# make title for plot
t <- "Trends(°C/yr) in for Stations"

# get statistical significance
trend_pv <- ifelse(trend_pv < 0.05, "Significant", "Insignificant")

# make plotting data frame
g <- cbind.data.frame(stat_lons, stat_lats)
colnames(g) <- c("lon", "lat")
contig_usa <- maps::map("usa", "main", exact=T, plot=F)
pts_poly <- sp::point.in.polygon(g$lon, g$lat, contig_usa$x, contig_usa$y)
keep <- which(pts_poly == 1)
img.mat <- cbind.data.frame(g, trend, trend_pv, trend_se)
img.mat <- img.mat[keep, ]
img.mat <- na.omit(img.mat)
colnames(img.mat) <- c("lon", "lat", "beta", "p.val", "se")

# get United States data
usm.img.mat <- usmap::usmap_transform(img.mat)

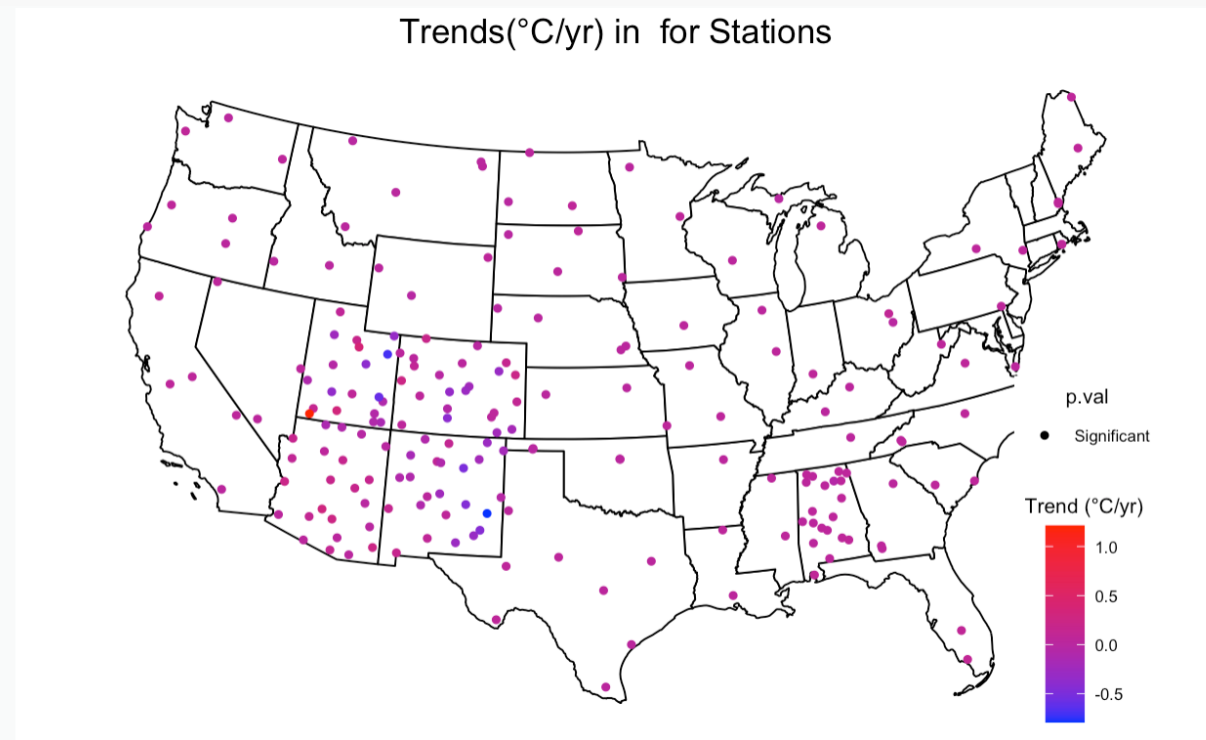
# plot data
usmap::plot_usmap("states", exclude=c("AK", "HI")) +

```

```

ggplot2::geom_sf(data=usm.img.mat, ggplot2::aes(colour=beta, shape=p.val)) +
ggplot2::coord_sf() + ggplot2::ggtitle(t)+
ggplot2::scale_color_gradient(low="blue", high="red",
                             name="Trend (°C/yr)") +
ggplot2::theme(legend.title = ggplot2::element_text(hjust=0.5),
               legend.position.inside=c(0.87,0.001),
               plot.title=ggplot2::element_text(color="black", hjust=0.5,
                                                  size=15))

```



```
# interpolate & plot
```

```
# keep small se trends
```

```
keep.i <- which(trend_se < 1)
trend <- trend[keep.i]
stat_lons <- stat_lons[keep.i]
stat_lats <- stat_lats[keep.i]
```

```
# make data frame for day + spatial data
```

```
WD <- cbind.data.frame(lon=stat_lons, lat=stat_lats, tr=trend)
WD <- na.omit(WD) # remove NA
```

```
# make spatial data frame
```

```
spat.df <- data.frame(x=WD$lon, y=WD$lat)
```

```
# make response vector
```

```
Y <- WD$tr
```

```

# get elevation data for model
prj_code <- 4326 # projection code
el.mod <- elevatr::get_elev_point(spat.df, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
spat.df$elev <- el.mod$elevation

# make design matrix for model
X_mod <- model.matrix(~1+x+y+elev, data=spat.df)

# get grid points in contiguous USA + elevation data
g <- create_grid_points(resolution=100)
colnames(g) <- c("x", "y") # change Lon to x and Lat to y for elavatr
el.interp <- elevatr::get_elev_point(g, prj=prj_code, src="aws")
#> Mosaicing & Projecting
#> Note: Elevation units are in meters
g$elev <- el.interp$elevation

# make design matrix for interpolations
X_interp <- model.matrix(~1+x+y+elev, data=g)

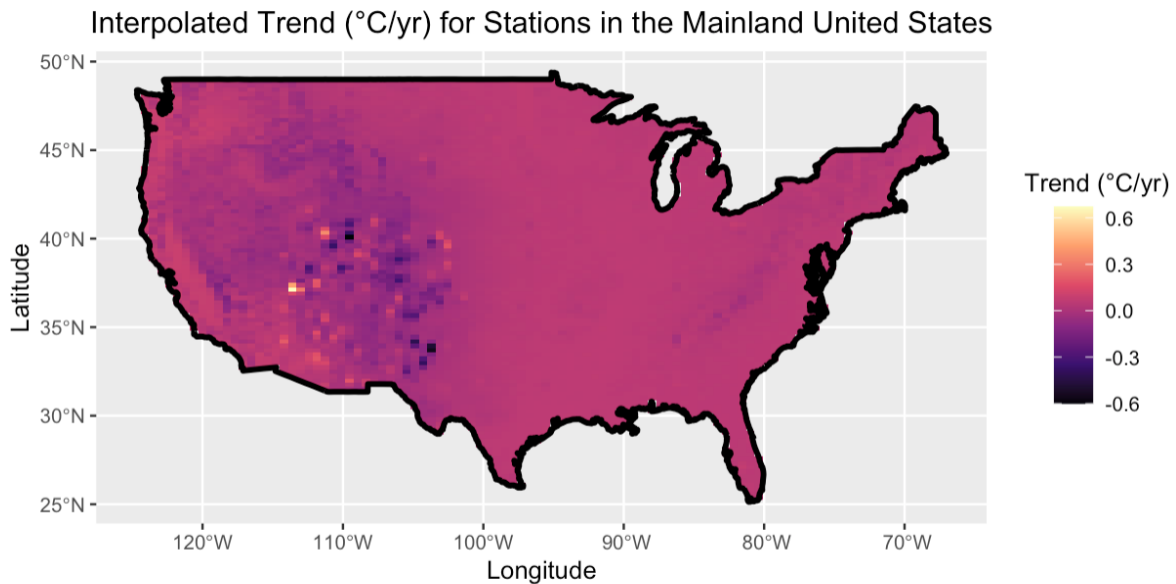
# make interpolations
g <- g[,1:2]
interps <- interpolate_points(WD, Y, X_mod, g, X_interp)
#> Assuming columns 1 and 2 of Locs are (Longitude, Latidue) in degrees

# make title for plot & plot
t <- paste0("Interpolated Trend (°C/yr) for Stations",
            " in the Mainland United States")
f <- "Trend (°C/yr)"

int.plot <- plot_gridded_interpolations(interps, g, t, f)

# view plot
int.plot

```



Objective-6: Validation with Reputable Sources

I found a research paper by Gil-Alana et al. 2022 which can be found at this <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC9616705/> link. The authors claim that the overall temperature trend in the United States is approximately $+1.60^{\circ}\text{F}/100$ year which is approximately $+0.71^{\circ}\text{C}/100$ year. I obtained answers similar to this range ranging from $-0.6^{\circ}\text{C}/\text{year}$ to $+0.6^{\circ}\text{C}/\text{year}$, however, the units are off since mine is estimated per year and theirs is the average estimated temperature trend over the past 100 years. My work is slightly off and is probably due to my models not accounting for the data correctly or being slightly insufficient.