Discrete Math (CSC 10400)- Homework 3 **Name:** **Mohamed Kharma**

1. (25 points) On the first day of new year, Bob deposits \$1000 in an account that pays 6% annual interest compounded monthly. At the beginning of each month, he adds \$200 to his account. If he continues to do so for the next four years, making 47 additional deposits of \$200, how much will his account be worth exactly four years after he opened it? What is the general formula for $P(n)$, the amount after n months?

1) $a_0 = \$1000$

→ rate per month $= \frac{6}{12} = 0.5\%$, therefore, $P_1 = 1000 + \frac{0.5(1000)}{100} + 200$

$= 1000(1.005) + 200$

∴ In the n^{th} month, he would have $a_n = 1.005a_{n-1} + 200$, where $0 \leq n \leq 46$

∵ The homogeneous relation is $a_n^{(h)} = C(1.005)^n$

∴ The particular relation is $a_n^{(p)} = A$

Therefore, $A = 1.005A + 200$, we have $a_n = C(1.005)^n - 40000$

$A = -40000$

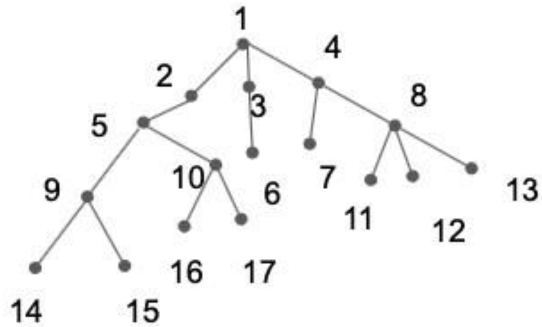
let $n = 0$ → $1000 = C - 40000$, Thus $a_n = 41000(1.005)^n - 40000$

$C = 41000$

For $n = 47$ → $a_{47} = 41000(1.005)^{47} - 40000$

$\boxed{= \$11830.40}$

2. (25 points) List the vertices in the tree shown below, when they are visited in a pre-order and in a post-order traversal.



2) Pre-order Traversal

(1, 2, 5, 9, 14, 15, 10, 16, 17, 3, 6, 4, 7, 8, 11, 12, 13)

Post-order traversal

(14, 15, 9, 16, 17, 10, 5, 2, 6, 3, 7, 11, 12, 13, 8, 4, 1)

3. (25 points) 1. Let $T = (V,E)$ be a binary tree. If the number of vertices is equal to n, what is the maximum height h that tree can attain?

   2. If T is a complete binary tree with n vertices, what is the maximum height it can reach in this case?

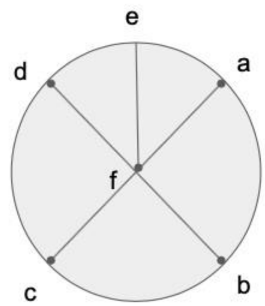
3)

1. A binary tree is a tree with nodes such that each node can have at most 2. Therefore, the maximum hight for the tree would be $\boxed{n-1}$ because each node must only one to get the max height.

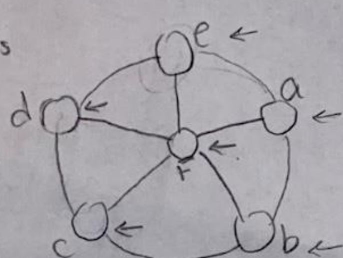
2. Let binary tree hight = (h) and has (i) leaves
Since at most we have 2^h leaves in a binary tree, so $i \leq 2^h$
Therefore, by (log) both sides, we get → $\log_2 i \leq h$
Also, since $(h-1) < \log_2 i \leq h$
This means, the maximum heigh for a complete binary tree is $\boxed{h = \lceil \log_2 i \rceil}$

4. (25 points) For the graph shown below, find the breath first and depth first spanning trees with the vertices ordered as

$$a,b,c,d,e,f$$

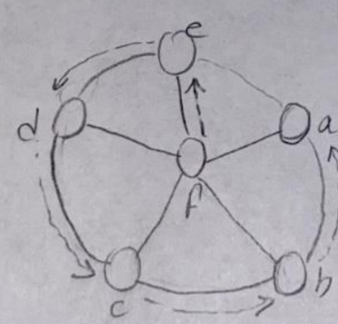


4) Breath first spanning tree is



⇒ (a, b, e, f, c, d)

Depth First spanning tree is



⇒ (f, e, d, c, b, a)