CSc22000 - Algorithms
Spring 2021 Final Exam
Instructor: Ahmet C. Yuksel
24 May'20
Time Limit: Submit before 05/25 11:59pm

Name (Print): ____________________

This exam contains 9 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

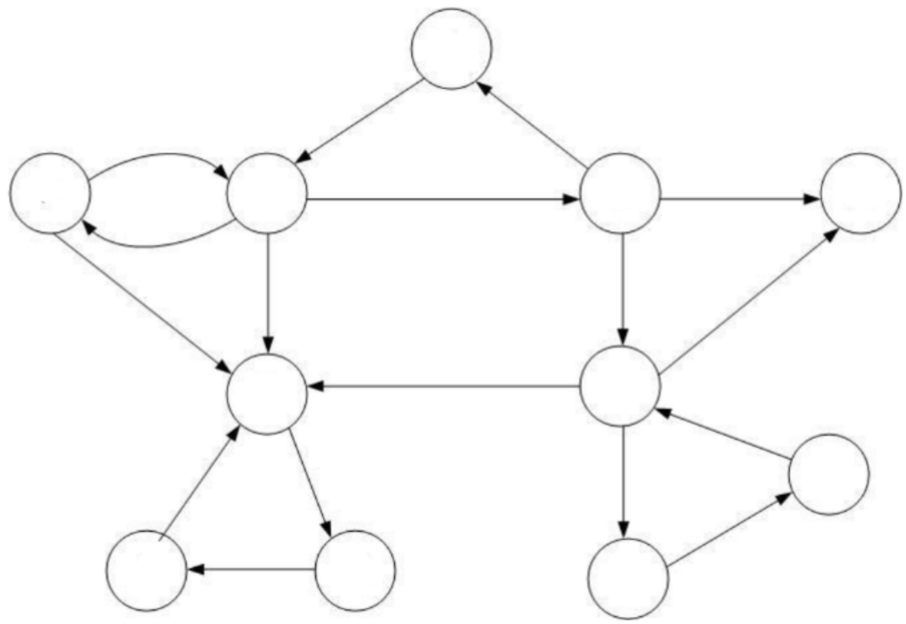
- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
  Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
  Abbreviations, shortcuts and the handwriting that the instructor cannot read will receive no credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	7	
2	10	
3	5	
4	10	
5	10	
6	5	
7	3	
8	10	
9	6	
10	12	
11	10	
12	12	
Total:	100	

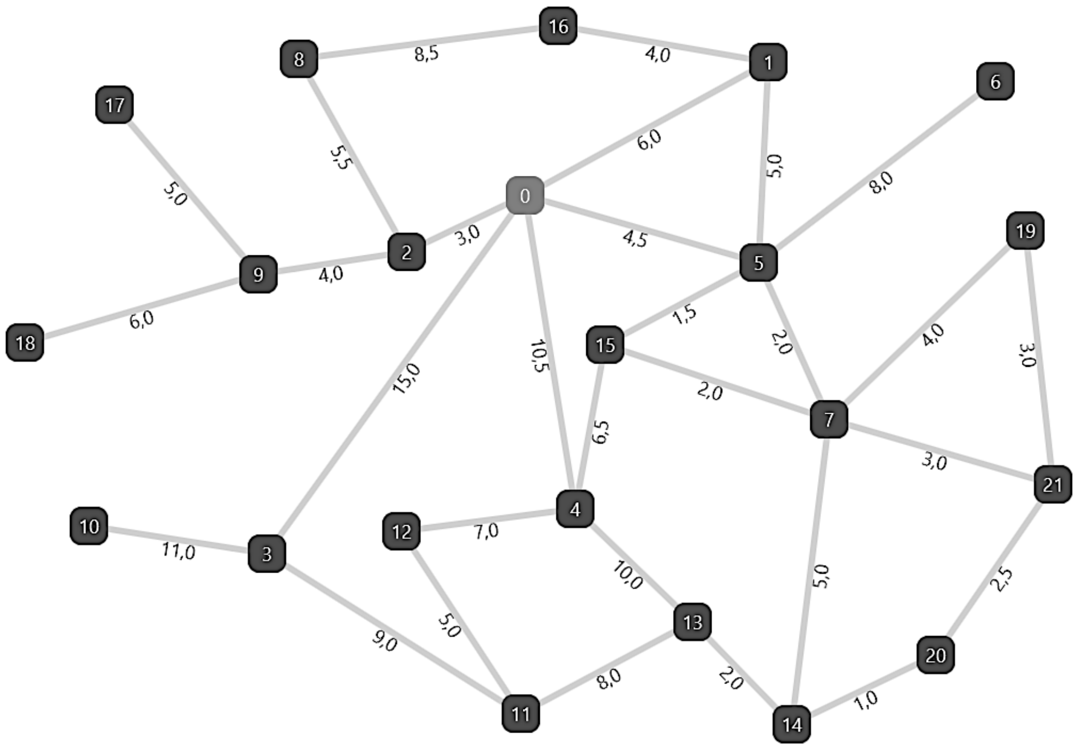
1. Define each of the following terms.
   (a) (1 point) Why the sum of the degrees of every vertex in a graph is $2 * |E|$
   (b) (1 point) A graph G={V,E,W}
   (c) (1 point) A walk, trail and path.
   (d) (1 point) Adjacency list, Adjacency matrix.
   (e) (1 point) A graph G'={V',E',W'} where G={V,E,W} and $V' \subseteq V$
   (f) (1 point) A spanning tree
   (g) (1 point) A minimum spanning tree

2. (10 points) Use Depth First Search(DFS) algorithm on the graph given below. Do the necessary labeling. Show all steps including the resulting tree(s). What is the worst case running time of DFS algorithm?

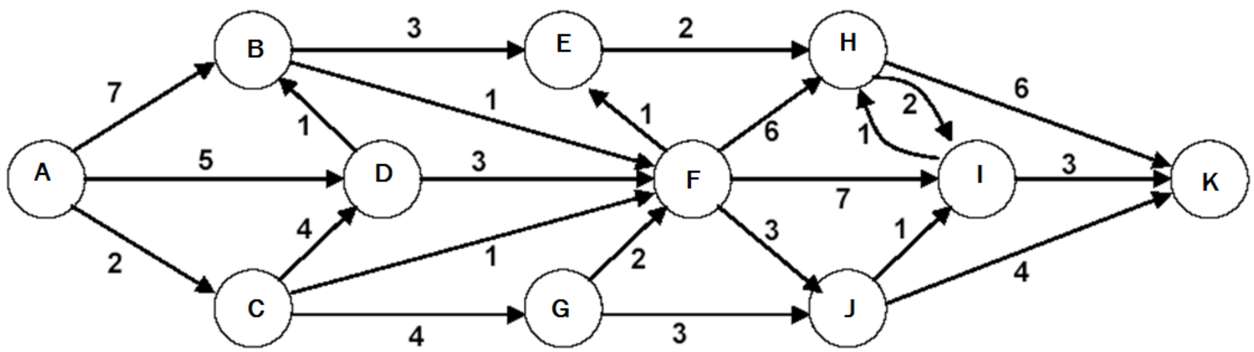


3. (5 points) How can we modify the DFS algorithm to detect cycles in a graph?

4. Given the graph below, find the minimum spanning tree by using;
   (a) (5 points) Kruskal's Algorithm (Also write its running time)
   (b) (5 points) Prim's Algorithm (Starting from 'vertex 0') (Also write its running time)



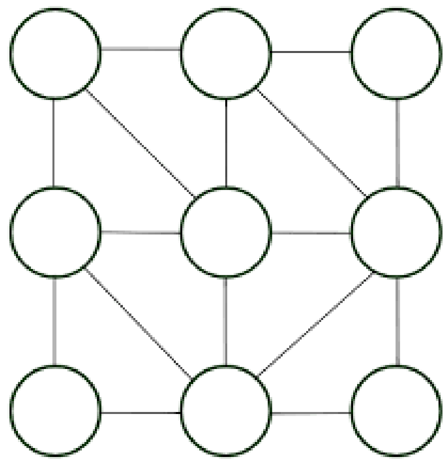
5. (10 points) Use Dijkstra's algorithm to find the shortest path between the vertices A and K. Use the similar data structures, we used in class. Show all steps and the shortest path by using the proper notation (You can cross out the previous values if there is an update). What are the worst case time and space complexity of Dijkstra's algorithm?



6. (5 points) Can we use Dijkstra's shortest path algorithm on the graphs with negative weights? Why? Why not?

7. (3 points) Explain the optimal substructure in Dynamic Programming and Greedy Algorithms. Why do we need an optimal substructure to solve problems by using Dynamic Programming and Greedy Algorithms?

8. (10 points) Using Breadth First Search(BFS) algorithm traverse the given graph below. Show the resulting tree. What is the worst case time complexity of BFS algorithm?



9. (6 points) Find the longest common subsequence between

$$X = \langle A\beta\Theta\beta\Delta A\beta \rangle$$

$$Y = \langle \beta\Delta\Theta A\beta A \rangle$$

by using dynamic programming.

10. (12 points) Suppose we are given a set of rectangular 3-D objects, where the i^{th} object has width $w(i)$, depth $d(i)$, height $h(i)$ (all real numbers). We want to create a stack of objects which is as tall as possible, trying to maximize the total height, but we can only stack an object on top of another object if the dimensions of the 2-D base of the lower object are each larger than or equal to those of the 2-D base of the higher object. This rule applies to any object that you put on the first object. Design an algorithm with the dynamic programming approach to solve the problem described above and apply your algorithm to the set of objects below;

$$S = \{(2,4,1),(4,7,10),(5,7,2),(6,6,3),(1,4,3),(2,2,12),(1,5,10),(3,3,2)\}$$

where each tuple represents (width,depth,height) of an object respectively.
Hint: We may rotate the objects.

11. (10 points) Consider the matrices below. Use dynamic programming approach to find the minimum cost of multiplying $\{A1, A2, A3, A4, A5\}$

$$A1 : 2X3 \qquad A2 : 3X6 \qquad A3 : 6X4 \qquad A4 : 4X5 \qquad A5 : 5X2$$

Reminder:

$$m[i,j] = \begin{cases} 0 & \textbf{if } i = j\ , \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j\} & \textbf{if } i < j \end{cases}$$

12. (12 points) Using Floyd-Warshall algorithm(all pairs shortest path), find the shortest path in the graph below. Also compare this algorithm to the Dijkstra's algorithm by its time and space complexity.