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Homework #1

7. A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if (a) there are no restrictions? (b) there must be six men and six women? (c) there must be an even number of women? (d) there must be more women than men? (e) there must be at least eight men?

a) If there is no restriction, the number of ways to choose 12 people from 20 is:

$$C(20,12) = \frac{20!}{12!\,(20-12)!} = \frac{20!}{12!\,8!} = 125970$$

Therefore, there are **125970 way** to choose 12 from 20.

b) Since we need to select 6 men out of 10, and 6 women out of 10. We could use the product rule to get the total number of ways to select them.

$$= \frac{10!}{6!\,4!} \times \frac{10!}{6!\,4!} = 44100$$

Therefore, there are **44100 ways** to choose 6 men and women.

c) The number of women in committee can be 2,4,6,8 or 10 and corresponding to that men will be 10,8,6,4 and 2.

= C(10,2) × C(10,10) + C(10,4) × C(10,8) + C(10,6) × C(10,6) + C(10,8) × C(10,4) + C(10,10) × C(10,2)

= 63290

Therefore, there are **63290 chances** that the number of women would be even.

d) Number of women can be 7,8,9 or 10 and number of men will be 5,4,3,2 respectively.

= C(10,7) × C(10,5) + C(10,8) × C(10,4) + C(10,9) × C(10,3) + C(10,10) × C(10,2) = 40935

Therefore, there are 40935 chances that the number of women is more than men.

e) In this case, number of men can be 8,9 or 10 and respectively number of women can be 4, 3 and 2.

= C(10,8) × C(10,4) + C(10,9) × C(10,3) + C(10,10) × C(10,2) = 10695

Therefore, there are **10695 chances** to that there are at least 8 men within the committee of 12 people.

11. A student is to answer seven out of 10 questions on an examination.

In how many ways can he make his selection if (a) there are no restrictions? (b) he must answer the first two questions? (c) he must answer at least four of the first six questions?

a) Since there is no restrictions, the possible ways to answer 7 questions out of 10 can be found using the formula of Combination where $\frac{n!}{(r)!(n-r)!}$ which equals to = $\frac{10!}{(7)!(3)!}$ = C(10,7) = **120 ways.**

b) If he must make the first two answers, thus only 5 questions are to be selected from 8 questions. Using the same formula $\frac{n!}{(r)!(n-r)!}$, we get $\frac{8!}{5!3!}$ = C(8,5) = **56 ways.**

c) If the student must at least answer four out of the first six questions, then the possible number of questions to be answerd is 4, 5 and 6. Therefore, the number of ways 7 questions can be answered our of the 10 questions, where four of the six need to be answered can be found using the Rule of Sum.
= C(6,4) × C(4,3) + C(6,5) × C(4,2) + C(6,6) × C(4,1) = **100 ways.**

20. In the three parts of Fig. 1.8, eight points are equally spaced

and marked on the circumference of a given circle.

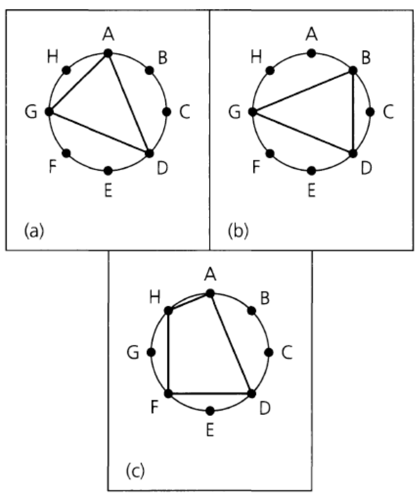

Figure 1.8

a) For parts (a) and (b) of Fig. 1.8 we have two different (though congruent) triangles. These two triangles (distinguished by their vertices) result from two selections of size 3 from the vertices A, B, C, D, E, F, G, H. How many different (whether congruent or not) triangles can we inscribe in the circle in this way?
b) How many different quadrilaterals can we inscribe in the circle, using the marked vertices? [One such quadrilateral appears in part (c) of Fig. 1.8.]

c) How many different polygons of three or more sides can we inscribe in the given circle by using three or more of the marked vertices?

a) To find the number of different triangles that can be formed from the eight points A, B, C, D, E, F, G, H using any three points, we know that from n objects, r objects can be selected in $\frac{n!}{(r)!\,(n-r)!}$ = C(n,r) ways. Therefore, the number of ways are equal to C(8,3). Using the formula, we get $\frac{8!}{3!5!}$ **= 56.**

b) In Figure (c) we see that to form a quadrilateral, any 4 points can be selected from the 8 points. Therefore, we will use combination as well, where C(8,4) = $\frac{8!}{4!4!}$ **= 70.**

c) Since there are 8 points given on the circumference of the circle, the maximum number of sides a polygon can be is 8. Also, we know that the minimum is 3 sides. Therefore,

Possible number of polygons of sides 3 are C(8,3)

Possible number of polygons of sides 4 are C(8,4)

Possible number of polygons of sides 5 are C(8,5)

Possible number of polygons of sides 6 are C(8,6)

Possible number of polygons of sides 7 are C(8,7)

Possible number of polygons of sides 8 are C(8,8)

Thus, the possible number of polygons of 3 or more sides that are built by 8 points are C(8,3) + C(8,4) + C(8,5) + C(8,6) + C(8,7) + C(8,8) = $\frac{8!}{3!5!}+\frac{8!}{4!4!}+\frac{8!}{5!3!}+\frac{8!}{6!2!}+\frac{8!}{7!1!}+\frac{8!}{8!0!}$ = 56+70+56+28+8+1 **= 219.**

15. Define the connective "Nand" or "Not . . . and . . ." by $(p \uparrow q) \Longleftrightarrow \neg(p \wedge q)$, for any statements p, q. Represent the following using only this connective.

b) $p \vee q$ **c)** $p \wedge q$

B) To represent $p \vee q$ into the connectives by $(p \uparrow q) \Leftrightarrow \neg\,(p \wedge q)$.

We start by the Double Negation law, we get $p \vee q \Leftrightarrow \neg\neg\, p \vee \neg\neg\, q$.

Then, applying DeMorgan's law, we get $\neg\, p \vee \neg\, q \Leftrightarrow \neg\,(p \wedge q)$

Therefore, $p \vee q \Leftrightarrow \neg\,(\neg\, p \wedge \neg\, q)$

Since, $(p \uparrow q) \Leftrightarrow \neg (p \wedge q)$, then $\neg (\neg p \wedge \neg q) \Leftrightarrow (\neg p \uparrow \neg q)$

Thus, $p \vee q \Leftrightarrow (\neg p \uparrow \neg q)$

Also, we have $\neg p \Leftrightarrow (p \uparrow q)$, then $\neg p \uparrow \neg q \Leftrightarrow (p \uparrow p) \uparrow (q \uparrow q)$

This means that the statement $\boldsymbol{p \vee q}$ is represented by $\boldsymbol{(p \uparrow p) \uparrow (q \uparrow q)}$

C) To represent $p \wedge q$ into the connectives by $(p \uparrow q) \Leftrightarrow \neg (p \wedge q)$.

We start by using the Double Negation law, which is $\neg\neg p \Leftrightarrow p$.

Therefore, $p \wedge q$ can be rewritten as $p \wedge q \Leftrightarrow \neg\neg (p \wedge q)$

Since we have $\neg(p \wedge q) \Leftrightarrow (p \uparrow q)$, then $p \wedge q \Leftrightarrow \neg (p \uparrow q)$

This means $\neg (p \uparrow q) \Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$

Thus $(p \wedge q) \Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$

Which means the statement $\boldsymbol{p \wedge q}$ is represented by $\boldsymbol{(p \uparrow q) \uparrow (p \uparrow q)}$

10. Establish the validity of the following arguments.

a) $[(p \wedge \neg q) \wedge r] \rightarrow [(p \wedge r) \vee q]$

b) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

A) Using the Inference Rules:

Step	**Statement**	**Explanation**
1	$(p \wedge \neg q)$	Premise
2	p	Step 1 and Rule of Conjunctive Simplification $\frac{p \wedge q}{p}$
3	r	Premise
4	$(p \wedge r)$	Step 2, 3 and Rule of Conjunction. $\frac{p, q}{p \wedge q}$
5	$\therefore (p \wedge r) \vee q$	Step 4 and Disjunctive Amplification Rule $\frac{p}{p \vee q}$

Therefore, $[(p \wedge \neg q) \wedge r] \rightarrow [(p \wedge r) \vee q]$ **is valid.**

B) Using the Inference Rules:

Step	**Statement**	**Explanation**
1	$(p, p \to q)$	Premise
2	q	Step 1 and Rule of Detachment
3	$\neg q \vee r$	Premise
4	$q \to r$	Step 3 and $(\neg q \vee r) \Leftrightarrow (q \to r)$
5	$\therefore r$	Step 2,4 and Rule of Detachment

Therefore, $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$ **is valid.**

11. Show that each of the following arguments is invalid by providing a counterexample — that is, an assignment of truth values for the given primitive statements /?, q , r, and s such that all premises are true (have the truth value 1) while the conclusion is false (has the truth value 0).

a) $[(p \wedge \neg q) \wedge [p \to (q \to r)]] \to \neg r$

Answer: The only time the conclusion $(\neg r)$ is false when r is true meaning $(\neg r)$: 0 and r: 1. Since all premises are 1, therefore, for the premise $(P \wedge \neg q)$ to be 1, both p and $\neg q$ must be 1. Which gives the truth value of p=1 and of q=0. Thus, we have $(p \to (p \to r)) = 1$, which mean that $p: 1, q: 0, r: 1$