ELECTRICAL ENGINEERING DEPARTMENT

ANTENNA AZIMUTH POSITION CONTROL PROJECT

SIGNALS & SYSTEMS

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SEC : 3

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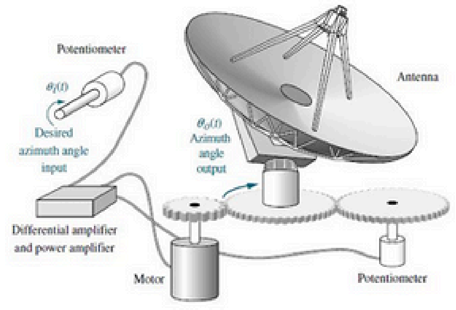
- FOR EACH OF THE RESULTING DISCRETE SYSTEMS, USE THE SIMULINK TO DISPLAY THE DISCRETE UNIT STEP RESPONSE. COMPARE YOUR RESULTS WITH PART E-2. COMMENT ON YOUR RESULTS.

Antenna Azimuth position control project

Note: All figures and data in this report description are taken from Control systems engineering by N. Nise.



Project description:

For the antenna Azimuth position control application (shown in figure 1 and Table 1), please do the following parts:

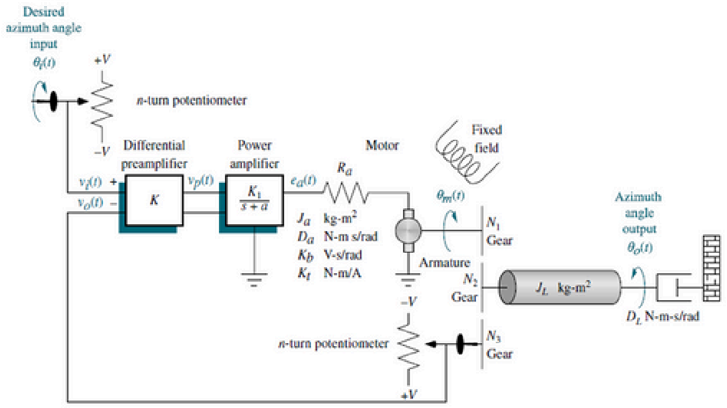


Figure 1

• PART A

Part A:

Find the transfer function for each subsystem of the antenna azimuth position control system schematic shown in figure 1. **Use Configuration 2**. The required subsystems are listed in the table below.

Subsystem	Input	Output
Input potentiometer	Angular rotation from user, $\theta_i(t)$	Voltage to preamp, $v_i(t)$
Preamp	Voltage from potentiometers, $v_e(t) = v_i(t) - v_0(t)$	Voltage to power amp, $v_p(t)$
Power amp	Voltage from preamp, $v_p(t)$	Voltage to motor, $e_a(t)$
Motor	Voltage from power amp, $e_a(t)$	Angular rotation to load, $\theta_0(t)$
Output potentiometer	Angular rotation from load, $\theta_0(t)$	Voltage to preamp, $v_0(t)$

Table 1: schematic parameters

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	—	—	—
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_t	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250
J_L	1	5	5
D_L	1	3	3

SOLUTION

① Part "A"

Input Potentiometer:-

∵ Input → $\theta_i(s)$
Output → $V_i(s)$

∴ $V_i(s) = (V^+ - V^-) * \frac{\theta_i(s)}{2\pi n} = [10 - (-10)] * \frac{\theta_i(s)}{2\pi \times 1}$

∴ transfer function:- $\frac{V_i(s)}{\theta_i(s)} = \frac{10}{\pi}$ * By using Configuration "2"

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②
PreAmplifier:-
∵ input:- $V_e(s)$
output:- $V_P(s)$

∴ transfer function:- $\frac{V_P(s)}{V_e(s)} = K$ * By using Configuration "2"

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③ Power Amplifier:-

input:- $V_P(s)$

output:- $e_a(s)$

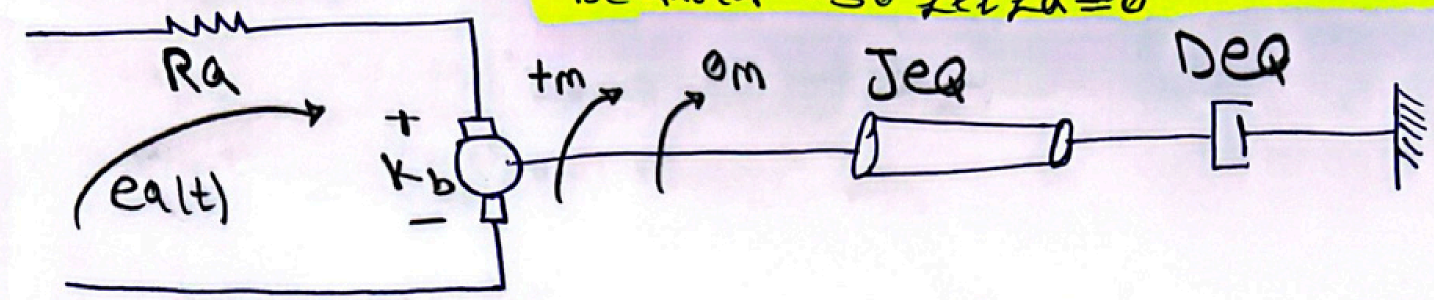
transfer function:- $\frac{e_a(s)}{V_P(s)} = \frac{K_1}{s+a}$ * By using Configuration "2"

∴ $\frac{e_a(s)}{V_P(s)} = \frac{150}{s+150}$

④ Moter & Load

*Note:- Assume Armature inductance L_a is small compared to Armature resistance which is usual for DC-motor so let $L_a = 0$

Schematic:-



- $J_{eq} = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 0.05 + 5 \times \left[\frac{50}{250}\right]^2 = 0.25$
- $D_{eq} = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 0.01 + 3\times\left(\frac{50}{250}\right)^2 = 0.13$
- $V_b(S) = K_b \cdot \theta_m(S) \cdot S$
- $T = I_a K_t$

For:-

$R_a = 5$	$J_{eq} = 0.25$
$\frac{N_1}{N_2} = 1/50$	$D_{eq} = 0.13$
$K_b = 1$	$K_t = 1$

$\therefore E_a(S) = I_a(S) \cdot R_a + V_b(S)$

$I_a(S) = \frac{T(S)}{K_t}$

$E_a(S) = \frac{T(S)}{K_t} \cdot R_a + K_b \theta_m(S) \times S$

- $\left[J_{eq} S^2 + D_{eq} S\right] \theta_m(S) = t(S)$

$\therefore E_a(S) = \theta_m(S) \cdot \left[\frac{R_a}{K_t}\left(J_{eq} S^2 + D_{eq} S\right) + K_b \cdot S\right]$

- $\frac{E_a(S)}{\theta_m(S)} = \frac{R_a J_{eq}}{K_t} \cdot S^2 + \left(\frac{R_a D_{eq}}{K_t} + K_b\right) \cdot S$

$\therefore \frac{\theta_m(S)}{E_a(S)} = \frac{1}{\frac{R_a J_{eq}}{K_t} \cdot S^2 + \left(\frac{R_a D_{eq}}{K_t} + K_b\right) S}$

$\frac{\theta_o}{\theta_m} = \frac{N_1}{N_2} \Longrightarrow \theta_o = \theta_m \times \frac{N_1}{N_2}$

$\therefore \frac{\theta_o(S)}{E_a(S)} = \frac{1 \times \left(\frac{N_1}{N_2}\right)}{\frac{R_a J_{eq}}{K_t} S^2 + \left(\frac{R_a D_{eq}}{K_t} + K_b\right) S}$

transfer function (moter)

$\frac{\theta_o(S)}{E_a(S)} = \frac{0.16}{S^2 + 1.32 S} \Longrightarrow \frac{\theta_o(S)}{E_a(S)}$

⑤ Output Potentiometer:-

input :- $\theta_o(s)$

output:- $V_o(s)$

$$\therefore V_o(s) = \frac{(V^+ - V^-) \times \theta_o(s) \times \frac{N_3}{N_2}}{2\pi n} = \frac{(10-(-10)) = \theta_o(s) \times \left(\frac{250}{250}\right)}{2\pi \times 1}$$

Transfer Function:-

$$\therefore \frac{V_o(s)}{\theta_o(s)} = \frac{10}{\pi}$$

- PART B

Part B:

For the schematic of the azimuth position control system shown in figure 1, **Configuration 2**, assume an open-loop system (feedback path disconnected).

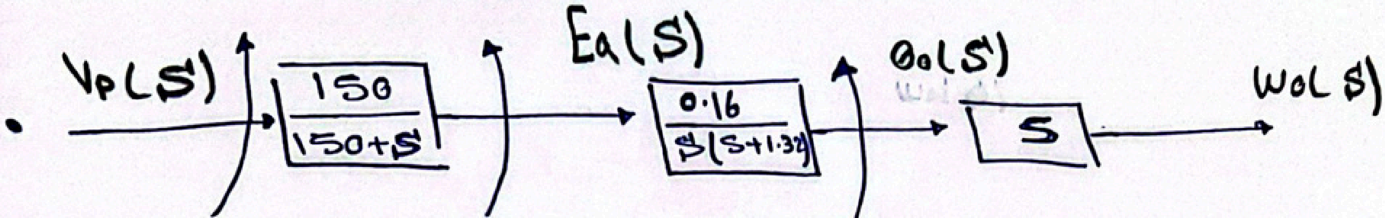
a. Predict, by inspection, the form of the open-loop angular velocity response of the load to a step-voltage input to the power amplifier.
b. Find the damping ratio and natural frequency of the open-loop system.
c. Derive the complete analytical expression for the open-loop angular velocity response of the load to a step-voltage input to the power amplifier, using transfer functions.
d. Use MATLAB to obtain a plot of the open-loop angular velocity response to a step-voltage input.

SOLUTION

- **A** a. Predict, by inspection, the form of the open-loop angular velocity response of the load to a step-voltage input to the power amplifier.

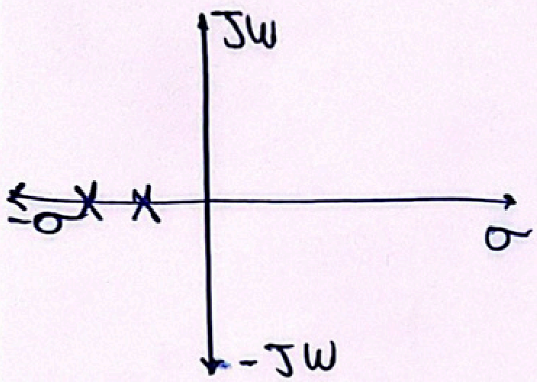
Part "B"

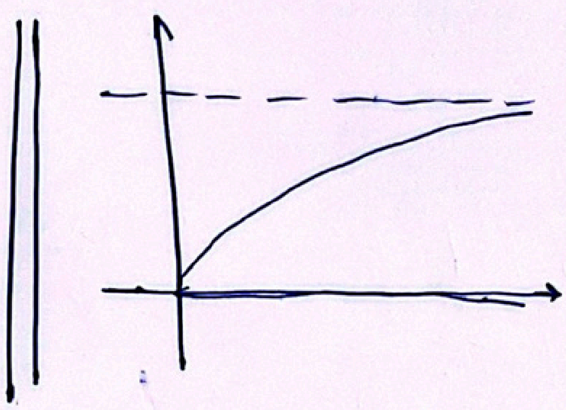
@



* $\omega_o(t) = A + Be^{-150t} + Ce^{-1.32t}$

By inspection the open loop system is 2nd order over damped system with distinct 2 real negative poles resulting in a non-oscillatory exponential response rise to a steady state value.





- **B** b. Find the damping ratio and natural frequency of the open-loop system.

(b)

$$\longrightarrow \boxed{\frac{24}{(S+150)(S+1.32)}} \longrightarrow$$

* solve transfer function

$$G(S) \Longrightarrow \frac{24}{S^2 + 151.32\,S + 198}$$

* the order of time Response is 2nd order time Response As we have 2 poles

* general Form → $\dfrac{\text{gain} \times \omega_n^2}{S^2 + 2\zeta\omega_n \cdot S + \omega_n^2}$

$\therefore \omega_n \rightarrow \sqrt{198} \longrightarrow 14.07124728$

* $2\zeta \times 14.07124728 = 151.32$

$\therefore \zeta \Longrightarrow 5.376922066 \simeq 5.38$ (over damped)

- **C** c. Derive the complete analytical expression for the open-loop angular velocity response of the load to a step-voltage input to the power amplifier, using transfer functions.

SOLUTION

(c)

input $\longrightarrow$ unit step

$\therefore$ output $\longrightarrow W_o(s)$

$$\therefore W_o(s) \Longrightarrow \frac{1}{s} \times \frac{24}{(s+150)(s+1.32)}$$

$$W_o(s) \longrightarrow \frac{24}{s(s+150)(s+1.32)}$$

$$\therefore \frac{A}{s} + \frac{B}{s+150} + \frac{C}{s+1.32} = \frac{24}{s(s+150)(s+1.32)}$$

*solve:- $A(s+150)(s+1.32) + Bs(s+1.32) + Cs(s+150) = 24$

@ $s=0$

$\therefore 198A + 0 + 0 = 24$

$\boxed{\therefore A \rightarrow 4/33}$

@ $s=-150 \rightarrow 0A + 22302B + 0C = 24$

$\therefore$

$\boxed{\therefore B \longrightarrow 1.076 \times 10^{-3}}$

@ $s = -1.32$

$0A + 0B - 196.2576C = 24$

$\boxed{C \rightarrow -0.1222}$

$$W_o(t) \rightarrow 4/33\, u(t) + 1.076 \times 10^{-3} e^{-150t} - 0.1222 e^{-1.32t}$$

$$\simeq 0.121 + 1.08 \times 10^{-3} \cdot e^{-150t} - 0.1222 e^{-1.32t}$$

- D

d. Use MATLAB to obtain a plot of the open-loop angular velocity response to a step-voltage input.

MATLAB CODE

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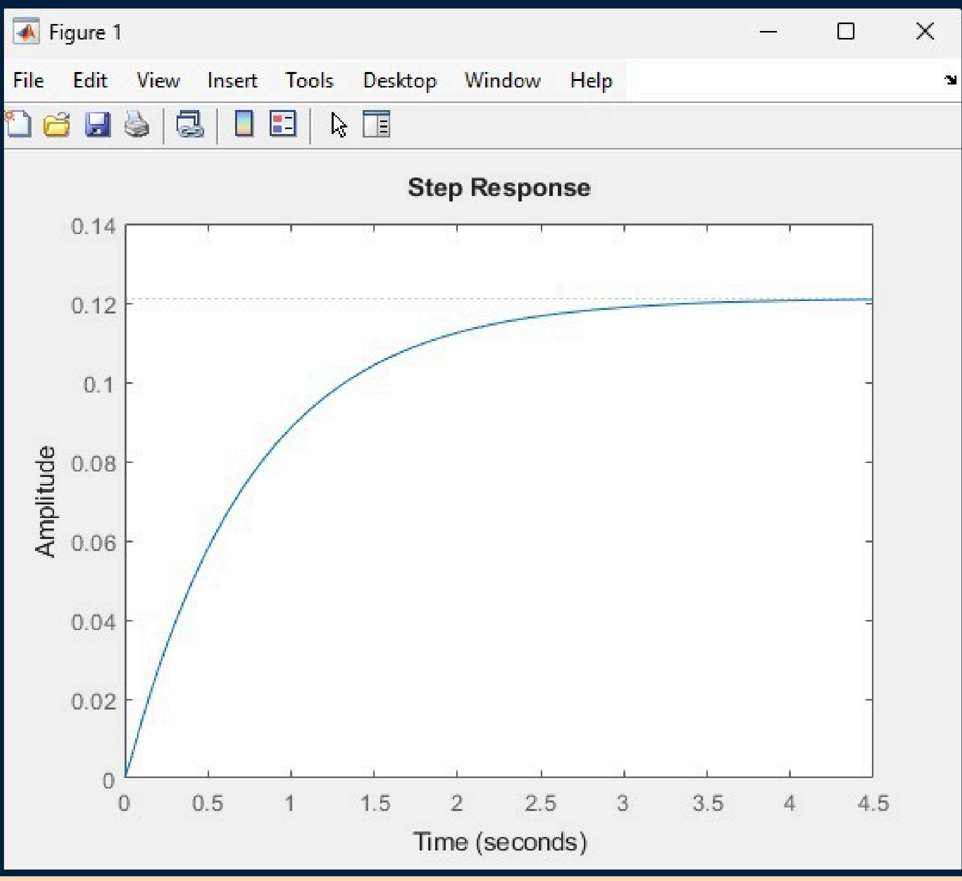
untitled.m

```
num=24;
den=poly([-150 -1.32]);
G=tf(num,den);
step(G)
```

PLOT RESPONSE



• PART C

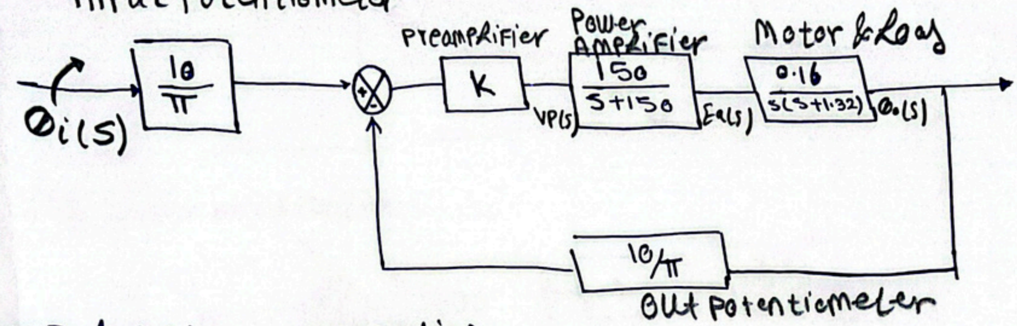
• A

a. Find the closed-loop transfer function using block diagram reduction.

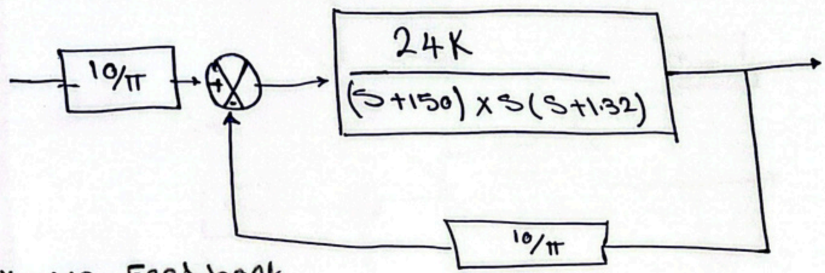
SOLUTION

Part "c"

@



* Block diagram reduction

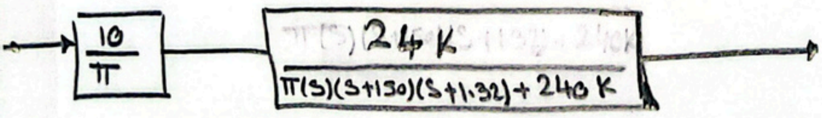


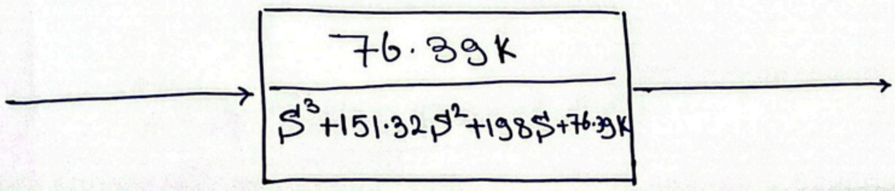
* -ve - Feedback

* STEPS →

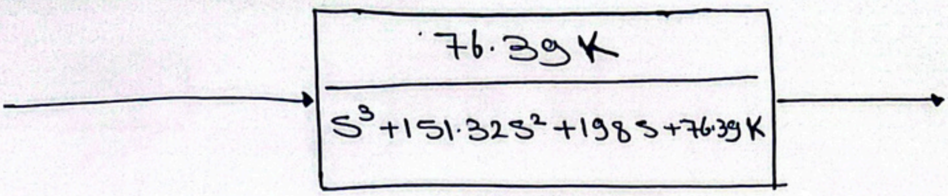
$$\frac{\frac{24K}{s(s+150)\cdot(s+1.32)}}{\frac{1}{1}+\frac{24K\cdot 10}{\pi\cdot(s)(s+150)(s+1.32)}}$$

$$\frac{\pi(s)(s+150)(s+1.32)+24K\times 10}{\pi(s)(s+150)(s+1.32)}$$





@ Final Answer:-

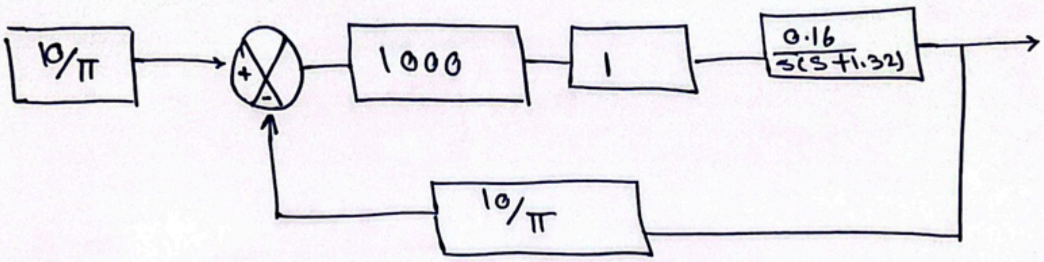


• B

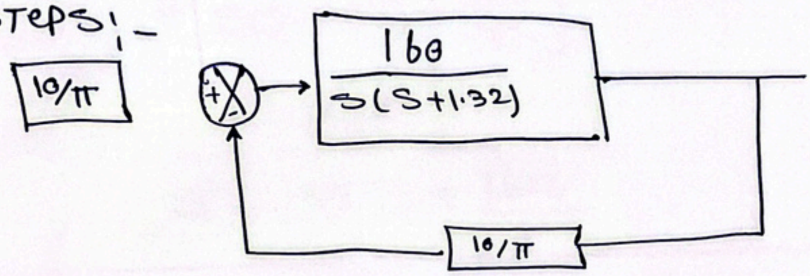
b. Replace the power amplifier with a transfer function of unity and evaluate the closed-loop peak time, percent overshoot, and settling time for K = 1000.

SOLUTION

(b) After modification



Steps:-



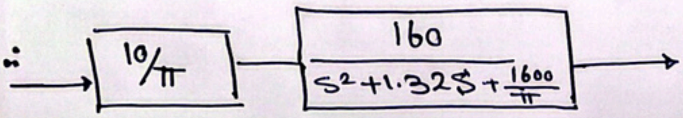
* -ve Feed Back

$$\frac{\dfrac{160}{\cancel{s^2+1.32s}}}{1+\dfrac{160}{s^2+1.32s}\times\dfrac{10}{\pi}}$$

$$\frac{s^2+1.32s+\dfrac{1600}{\pi}}{\cancel{s^2+1.32s}}$$

-ve feed Back $\longrightarrow$ $$\frac{160}{s^2+1.32s+\dfrac{1600}{\pi}}$$

∴



transfer Function:- $$\frac{1600/\pi}{s^2+1.32s+\dfrac{1600}{\pi}}$$

- B

* General Form:- $\dfrac{gain \times wn^2}{s^2 + 2\zeta wn\, s + wn^2}$ $\boxed{\cdot gain \rightarrow 1}$

$\therefore wn^2 = \dfrac{1600}{\pi}$ $\therefore wn \rightarrow 22.56758334$

* $2\zeta wn = 1.32$ $\therefore \zeta \rightarrow 0.0292454854$

· Peak time $= \dfrac{\pi}{wn\sqrt{1-\zeta^2}} \rightarrow 0.1392677704 \simeq 0.14$ sec

· $O.S\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} = 91.21811051\%$

· $T.s = \dfrac{4}{wn\zeta} = 6.060606061 \simeq 6.061$ sec.

- C c. For the system of "b", derive the expression for the closed-loop step response of the system.

SOLUTION

(c)

input: - unit step

* transfer Function: - $\dfrac{1600/\pi}{s^2+1.32s+\frac{1600}{\pi}}$

• output $\longrightarrow \dfrac{1}{s} \cdot \dfrac{1600/\pi}{s^2+1.32s+\frac{1600}{\pi}}$

. solve

$$\frac{A}{s} + \frac{Bs+D}{s^2+1.32s+\frac{1600}{\pi}} = \frac{1600/\pi}{s\left(s^2+1.32s+\frac{1600}{\pi}\right)}$$

$$\therefore A\left(s^2+1.32s+\frac{1600}{\pi}\right)+[Bs+D]\,s = \frac{1600}{\pi}$$

$$@\ s=0 \quad \therefore \quad A \times \frac{1600}{\pi} + 0 = \frac{1600}{\pi}$$

$$\therefore \underline{\underline{A=1}}$$

@ s=1

$$A\left[1+1.32+\frac{1600}{\pi}\right]+[B+D] = \frac{1600}{\pi}$$

$$\boxed{B+D=-2.32} \longrightarrow (1)$$

@ s=-1

$$A\left(1-1.32+\frac{1600}{\cancel{\pi}}\right)+[B-D] = \frac{1600}{\cancel{\pi}}$$

$$\boxed{B-D=0.32} \longrightarrow (2)$$

From (1) & (2)

$B=-1$

$D=-1.32$

- C

$$\therefore \frac{A}{S} + \frac{BS + D}{S^2 + 1.32S + \frac{1600}{\pi}} = \frac{1600/\pi}{S\left(S^2 + 1.32S + \frac{1600}{\pi}\right)}$$

$$\frac{1}{S} + \frac{-1S - 1.32}{S^2 + 1.32S + \frac{1600}{\pi}}$$

$$\frac{1}{S} + \frac{-1\left[S + 1.32\right]}{(S + 0.66)^2 + (22.55793027)^2}$$

$$\therefore \frac{1}{S} + \left[-1 \times \left[\frac{S + 0.66 \quad + \quad 0.66}{(S + 0.66)^2 + (22.55793027)^2}\right]\right]$$

$$\frac{1}{S} + \left[-1 * \left[\frac{S + 0.66}{(S + 0.66)^2 + (22.55793027)^2} + \frac{0.66}{(S + 0.66)^2 + (22.55793027)^2}\right]\right]$$

$$1\,u(t) + \left[-1 \times \left[e^{-0.66t} \cos(22.55793027t) + 0.02925800338\, e^{-0.66t} \times \sin(22.557\right.\right.$$

$\therefore$ Final Answer:-

$$1u(t) + \left[-1 * \left[e^{-0.66t} \cos(22.55793027t) + 0.02925800338\, e^{-0.66t} \sin(22.5793027t)\right]\right]$$

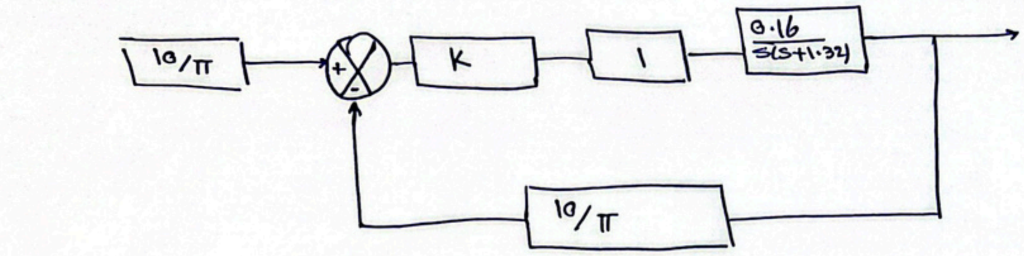
* with Approximation:-

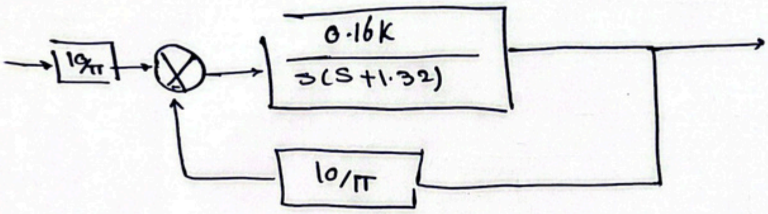
$$1u(t) + \left[-1 * \left[e^{-0.66t} \cos(22.56t) + 0.0293\, e^{-0.66t} \sin(22.56t)\right]\right]$$

• D d. For the simplified model of "b", find the value of K that yields a 10% overshoot.

SOLUTION

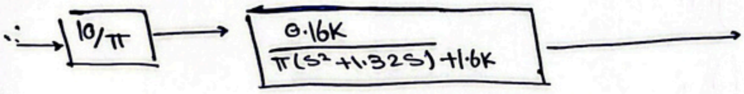
(d) simplified model of "b"





Negative-Feedback:-

$$\frac{\frac{0.16K}{s^2+1.32s}}{1+\frac{1.6K}{\pi(s^2+1.32s)}} \rightarrow \boxed{\frac{0.16K}{\pi(s^2+1.32s)+1.6K}}$$



$$\rightarrow \boxed{\frac{\frac{1.6K}{\pi}}{\pi(s^2+1.32s)+1.6K}} \rightarrow \qquad \frac{1.6K/\pi^2}{s^2+1.32s+\frac{1.6}{\pi}K}$$

↳ transfer function

$0.50\% \rightarrow 10\% \quad \therefore e^{-\pi\zeta/(\sqrt{1-\zeta^2})}$

$\zeta \rightarrow 0.591155 0338 \simeq \underline{0.5912}$

$\omega_n^2 \rightarrow \frac{1.6K}{\pi} \quad \therefore \omega_n = \sqrt{\frac{1.6K}{\pi}}$

$\therefore 1.32 = 2\zeta\omega_n$

$$\frac{1.32}{2\times 0.5912} = \sqrt{\frac{1.6K}{\pi}}$$

$\therefore K = 2.447084185 \simeq \underline{2.447}$

• PART D

For the antenna azimuth position control system shown in figure 1, **Configuration 2**. Find the range of preamplifier gain required to keep the closed-loop system stable.

SOLUTION

Part "D"

* Transfer Function:-

$$\frac{76.39K}{S^3+151.32S^2+198S+76.39K}$$

Routh table:-

S^3	1	198
S^2	151.32	76.39K
S^1	$\frac{29961.36-76.39K}{151.32}$	0
S^0	76.39K	0

$\therefore \frac{29961.36-76.39K}{151.32}=\frac{0}{1}$

$\therefore 29961.36-76.39K=0$

$\therefore K=392.215735$

$\therefore 76.39K=0$

$\therefore K=0$

$\therefore$ the range of K for stability

$$\boxed{0<K<392.215735}$$

$\therefore K\in\,]0,392.215735[$

• PART E

Consider the transfer function derived in **Part C-b**, use a Simulink simulation to

1- Check the following properties: a) linearity b) time invariance c) causality (Your report should include the responses of the system)

transfer function:- $\dfrac{1600/\pi}{s^2+1.32s+\frac{1600}{\pi}}$

HOW TO PROOF LINEARITY

4. Linearity

How can we mathematically proof the Linearity property?

Step 1 calculate the system output $y_1(t)$, assuming that its input is $x_1(t)$.

Step 2 calculate the system output $y_2(t)$, assuming that its input is $x_2(t)$.

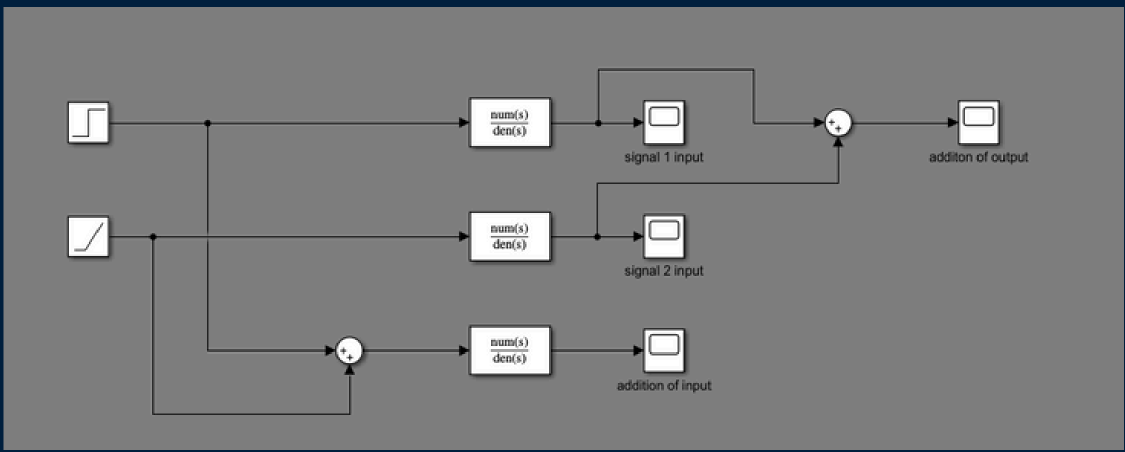
Step 3 calculate the system output $y_3(t)$, assuming that its input is $x_3(t)$.

Step 4 calculate $a\,y_1(t) + b\,y_2(t)$.

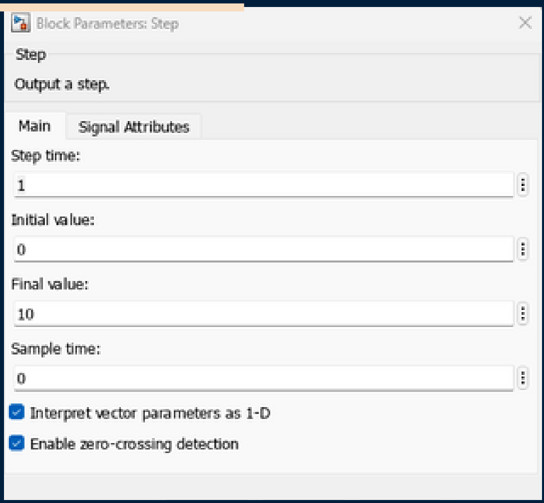
Step 5 Find $y_3(t)$ when $x_3(t) = a\,x_1(t) + b\,x_2(t)$.

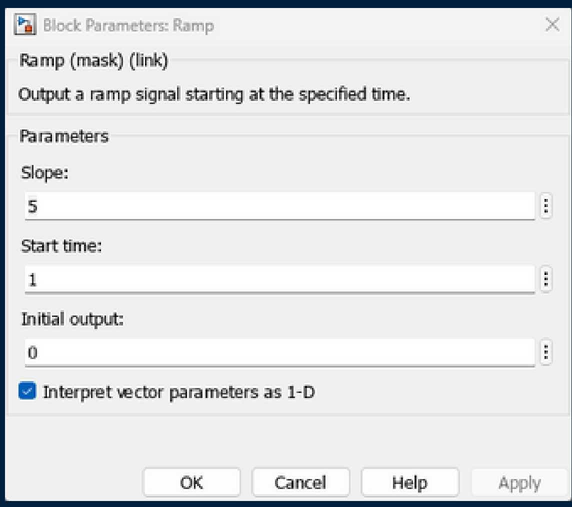
If the results of steps 4 and 5 are equal, then the system is linear. Otherwise, it is nonlinear.

SIMULINK

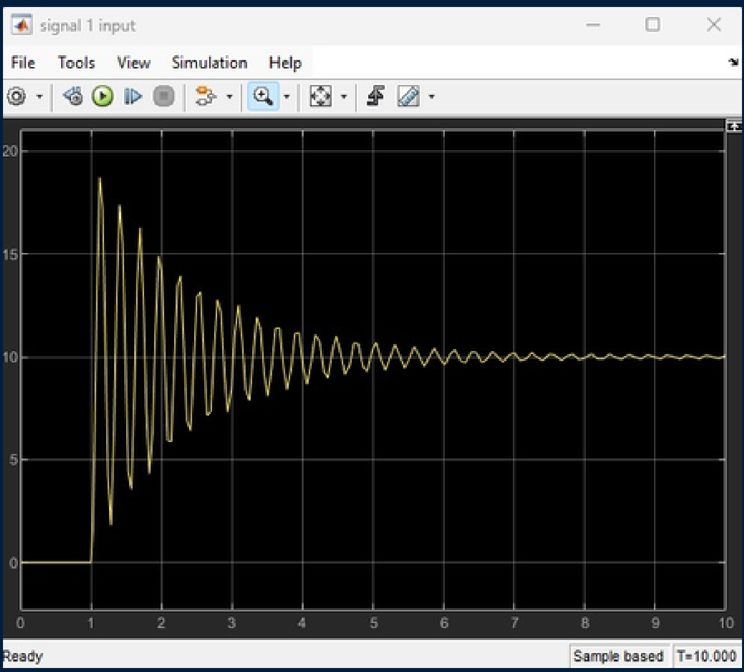


CONFIGURATIONS

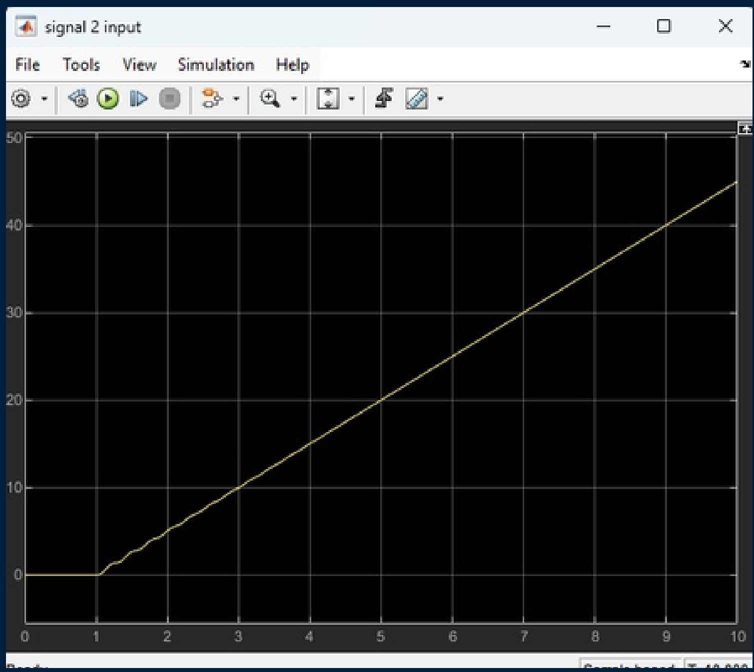




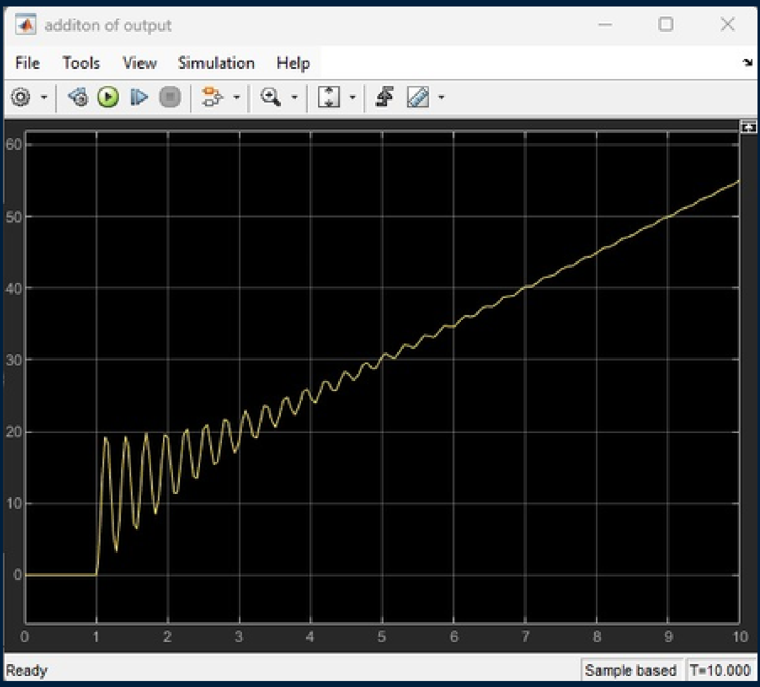
GRAPHS



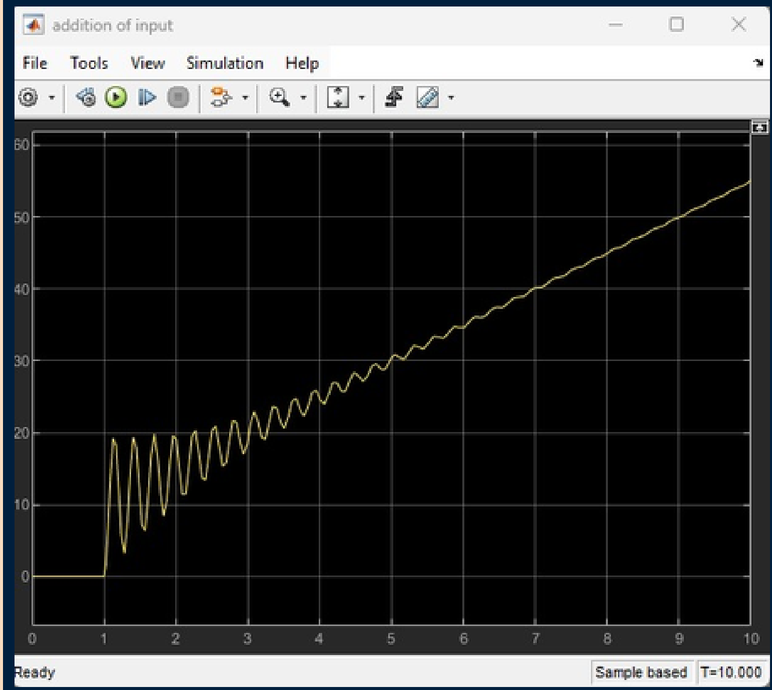
STEP 1



STEP 2



STEP 4



STEP 3

FROM THE PREVIOUS STEPS WE CAN CONCLUDE THAT OUR SYSTEM IS LINEAR

HOW TO PROOF TI

3. Time Invariance

How can we mathematically proof the TI property?

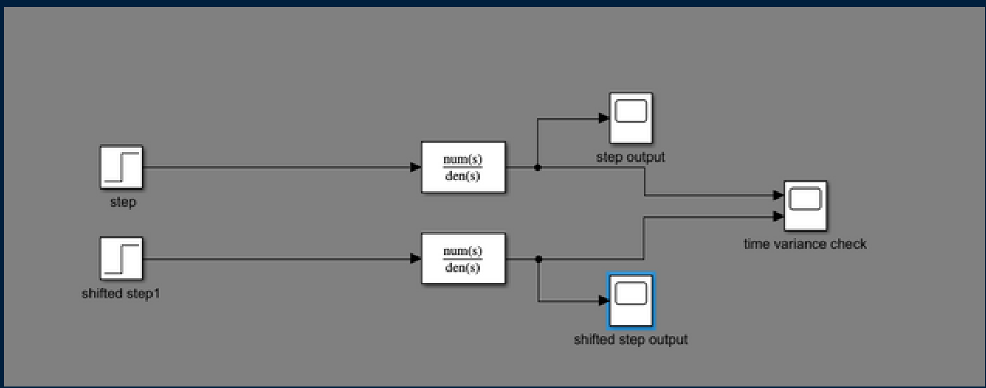
Step 1 calculate the system output $y_1(t)$, assuming that its input is $x_1(t)$.

Step 2 calculate the system output $y_2(t)$, assuming that its input is $x_2(t) = x_1(t - t_0)$.

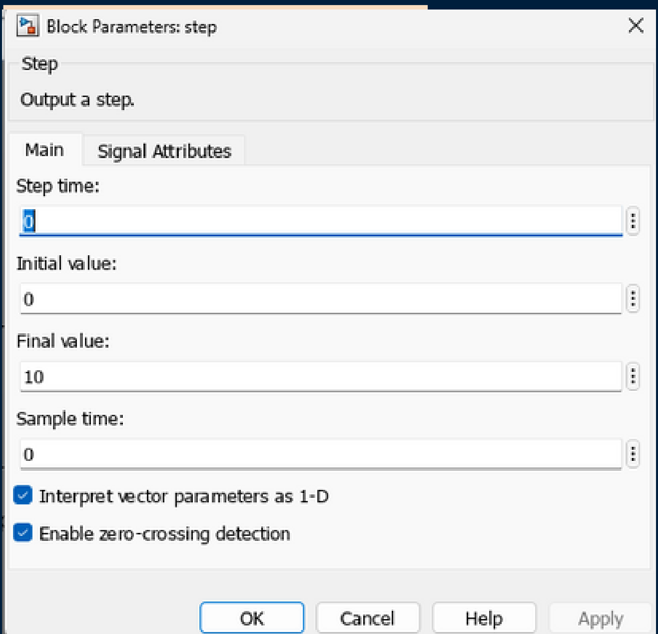
Step 3 calculate $y_1(t - t_0)$.

If $y_1(t - t_0) = y_2(t)$, then the system is TI. Otherwise, it is TV.

SIMULINK



CONFIGURATIONS


Block Parameters: step

Step

Output a step.

Main | Signal Attributes

Step time:

0

Initial value:

0

Final value:

10

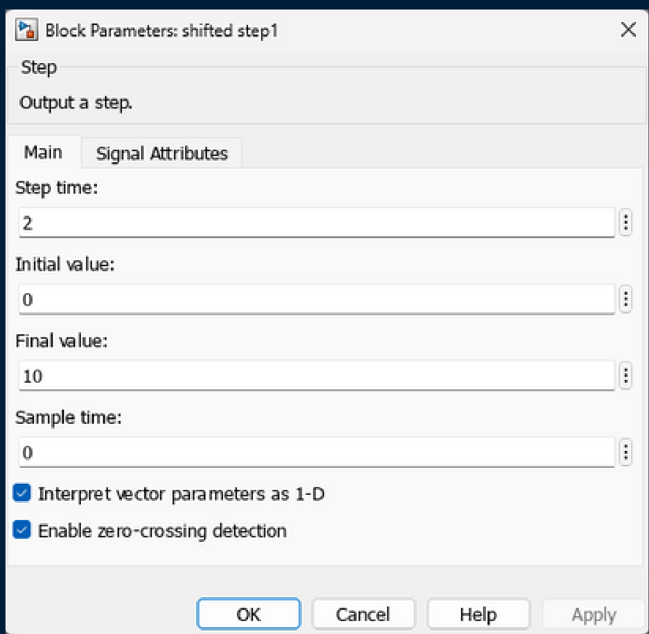
Sample time:

0

☑ Interpret vector parameters as 1-D

☑ Enable zero-crossing detection

OK | Cancel | Help | Apply


Block Parameters: shifted step1

Step

Output a step.

Main | Signal Attributes

Step time:

2

Initial value:

0

Final value:

10

Sample time:

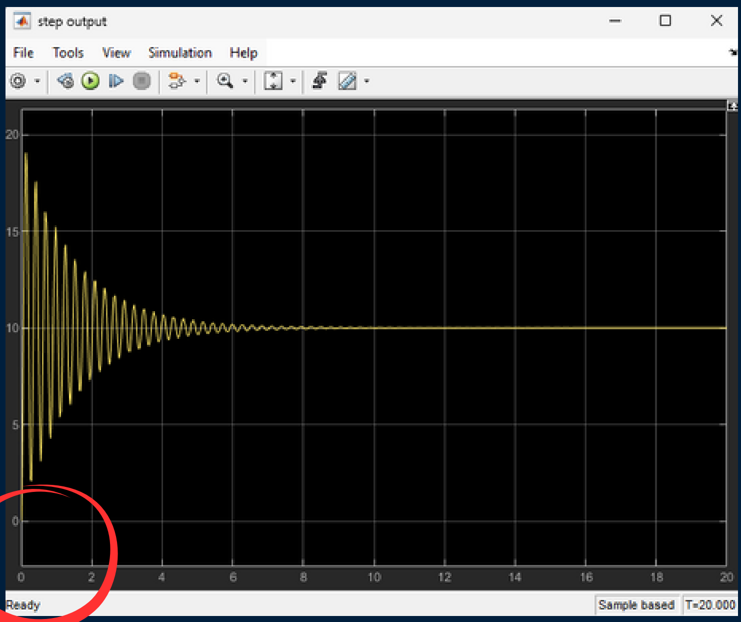
0

☑ Interpret vector parameters as 1-D

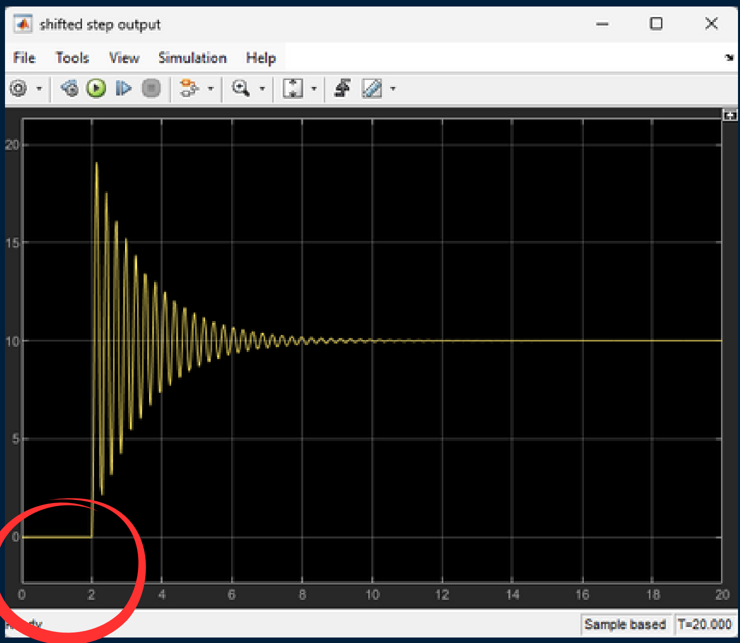
☑ Enable zero-crossing detection

OK | Cancel | Help | Apply

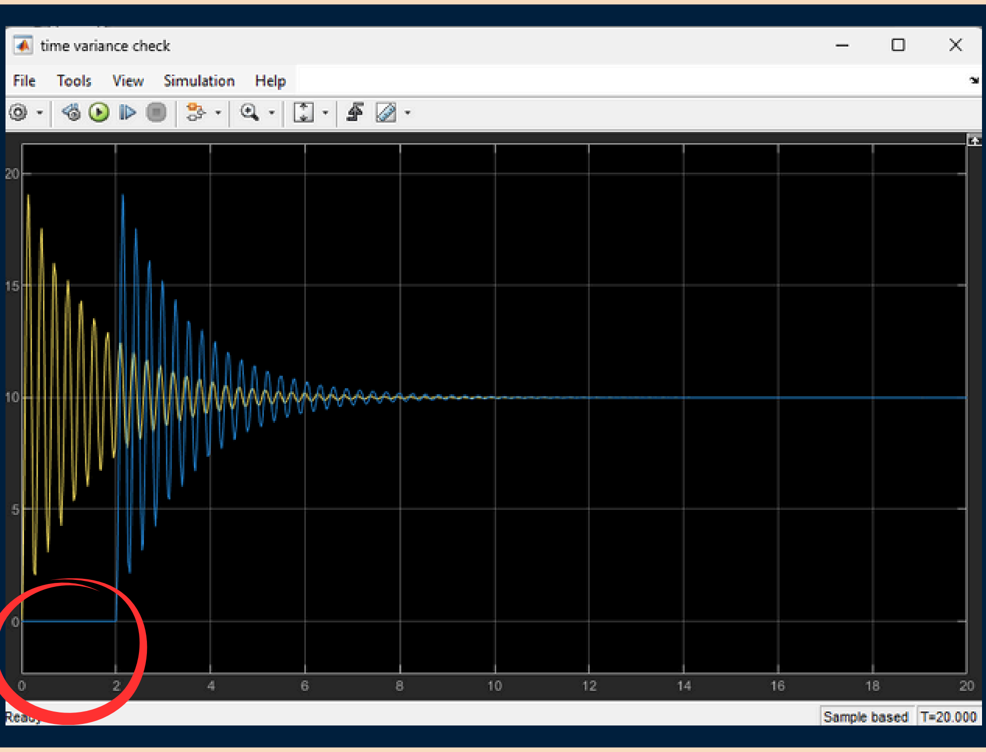
GRAPHS



STEP 1



STEP 2



STEP 3

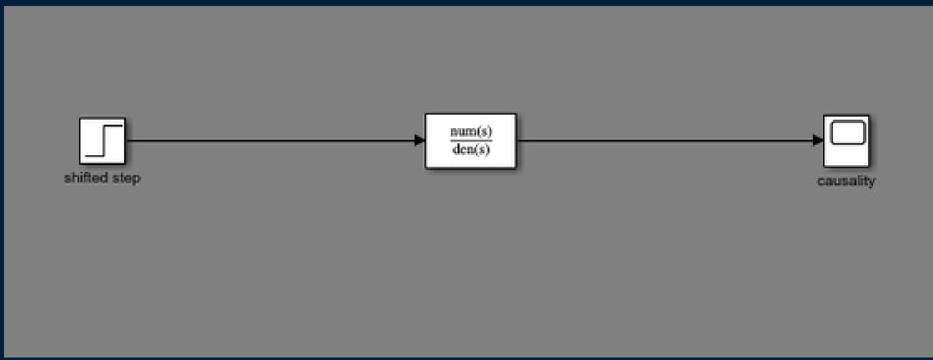
FROM THE PREVIOUS STEPS WE CAN CONCLUDE THAT OUR SYSTEM IS TIME INVARIANT

HOW TO PROOF CAUSALITY

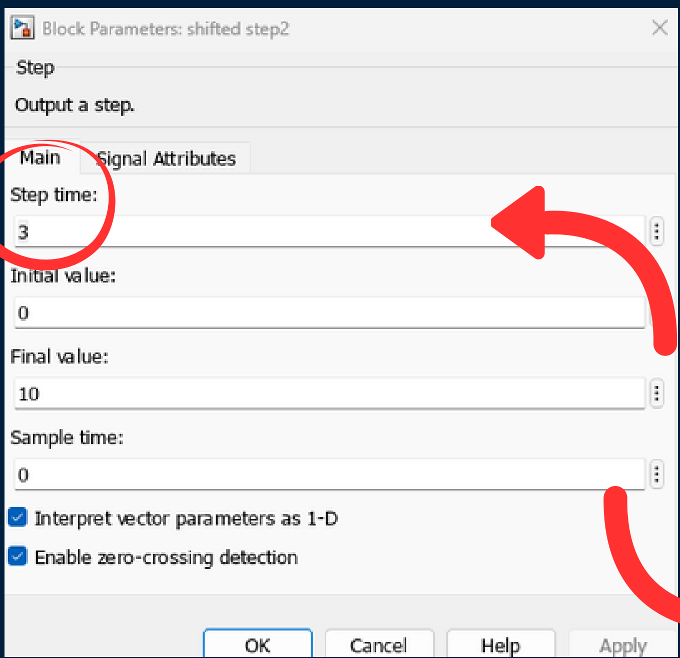
Causality

A system is **causal** if the output at any time depends only on values of input at present time and its past.

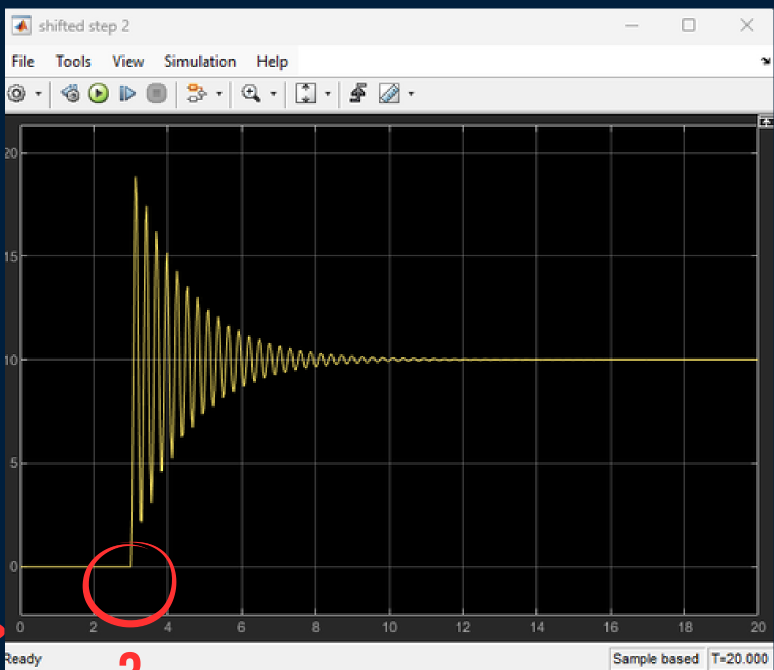
SIMULINK



CONFIGURATIONS



GRAPH



3

THE OUTPUT IS GENERATED AT THE SAME TIME AS THE INPUT

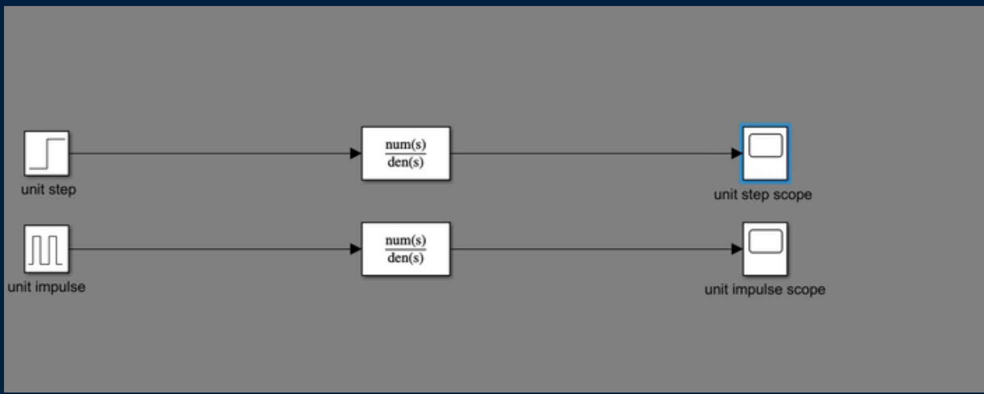
FROM THE PREVIOUS STEPS WE CAN CONCLUDE THAT OUR SYSTEM IS CAUSAL

PART E

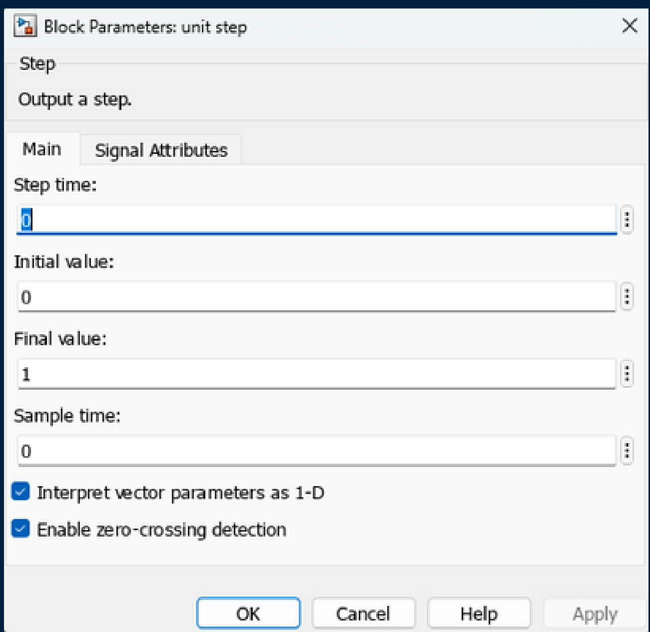
2- Plot the unit-step and unit-impulse response of the transfer function in **Part C-b**.

transfer function :- $\dfrac{1600/\pi}{s^2+1.32\,s+\frac{1600}{\pi}}$

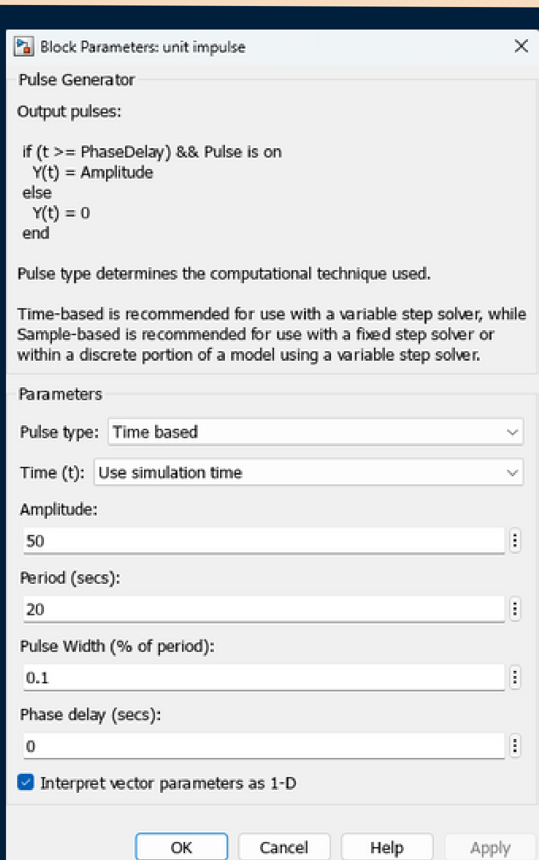
SIMULINK



CONFIGURATIONS

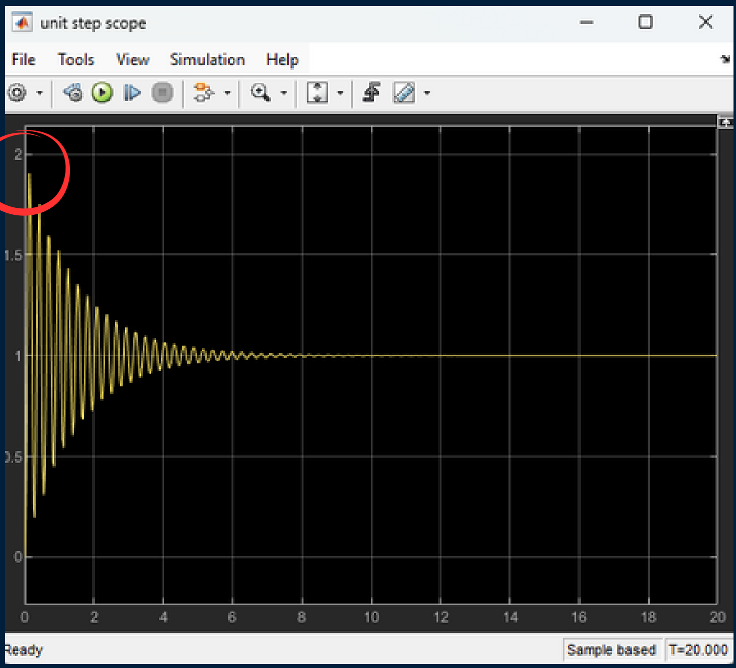


UNIT STEP PROPERTIES

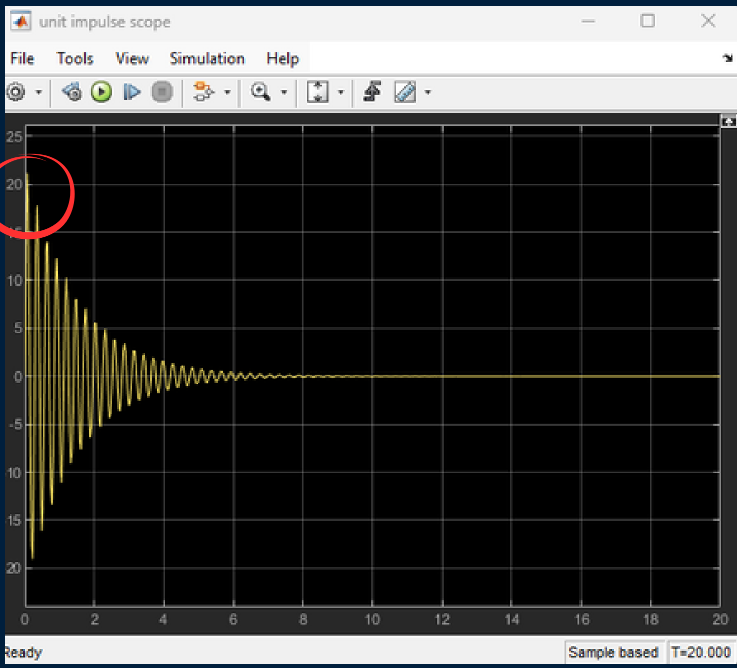


UNIT IMPULSE PROPERTIES

GRAPHS



UNIT STEP SCOPE



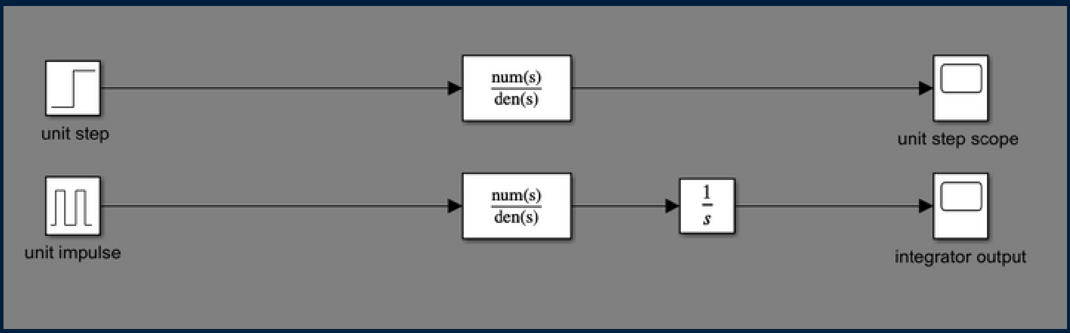
UNIT IMPULSE SCOPE

• PART E

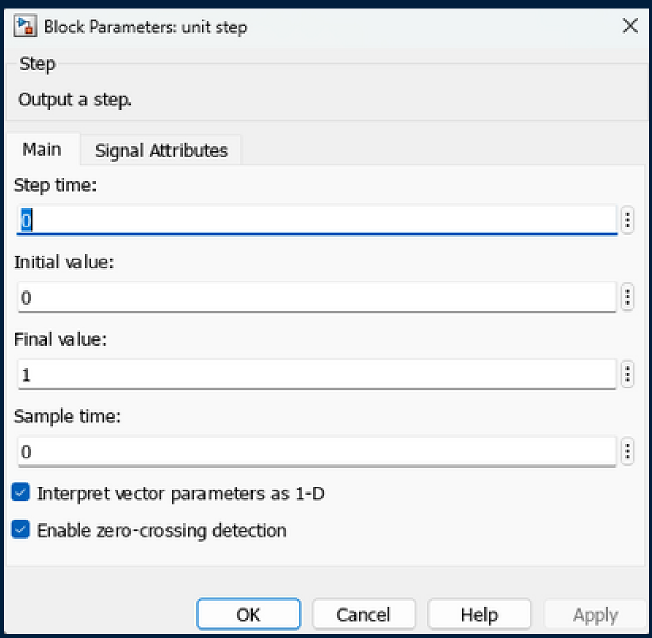
• C

3- Integrate the unit-impulse response in **Part E-2** and compare the result with its unit-step response in **Part E-2**. Comment on your results.

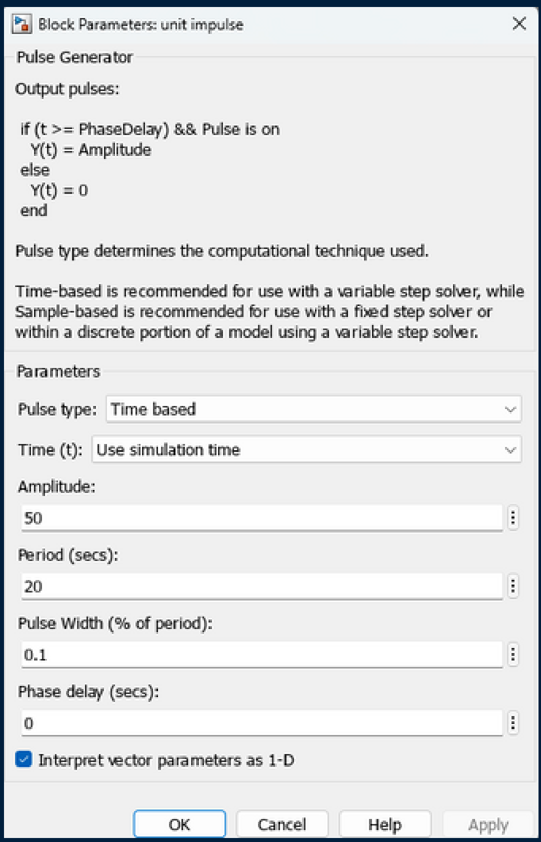
SIMULINK



CONFIGURATIONS

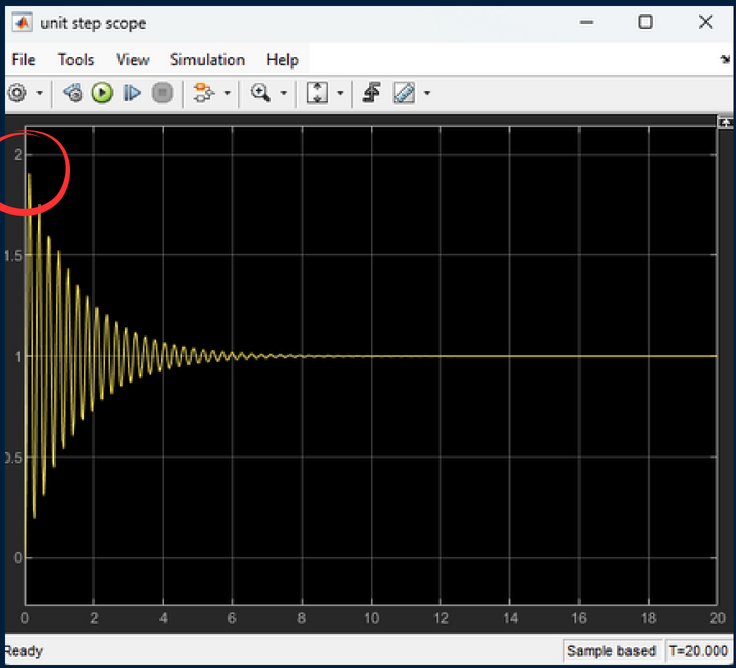


UNIT STEP PROPERTIES

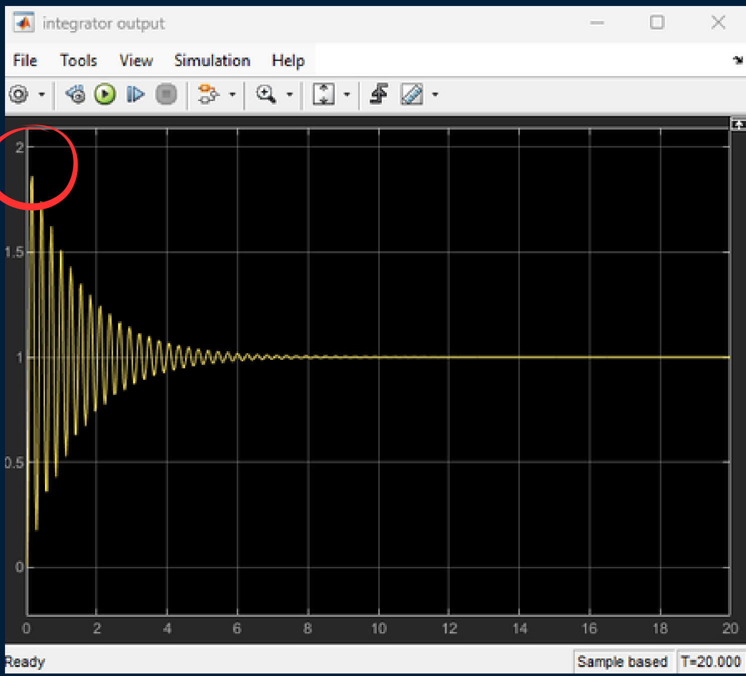


UNIT IMPULSE PROPERTIES

GRAPHS



UNIT STEP SCOPE



UNIT IMPULSE INTEGRATOR SCOPE

COMMENT

IF WE MAKE INTEGRATION FOR THE IMPULSE INPUT WE WILL GET THE SAME OUTPUT OF UNIT STEP AND THIS IS BECAUSE :

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

THE FINAL OUTPUT IS THE SAME

• PART E

• D

4- Draw the system response when its input is sinusoidal with fixed magnitude and different frequencies (Use three frequencies). Manually calculate the magnitude and phase angle of the system in each of the previous cases and compare them to the Simulink results. Comment on your results.

HAND ANALYSIS

Part E

(d) hand Analysis

Transfer Function :- $\dfrac{1600/\pi}{s^2+1.32s+\frac{1600}{\pi}} \longrightarrow G(s)$

input sine wave.

$\therefore$ let input $\rightarrow 1 \times \sin(\omega t) \rightarrow \boxed{G(t)} \rightarrow A\sin(\omega t+\phi)$

As our system is LTI so we conclude that the magnitude and phase angle will change.

* 1st assumption

$$G(s) = \frac{1600/\pi}{s^2+1.32s+\frac{1600}{\pi}}$$

• let $s = j\omega \longrightarrow H(j\omega) \rightarrow \dfrac{1600/\pi}{-\omega^2+1.32j\omega+\frac{1600}{\pi}}$

$$\therefore |H(j\omega)| = \frac{1600/\pi}{\sqrt{(-\omega^2+\frac{1600}{\pi})^2+(1.32\omega)^2}}$$

$\longrightarrow$ Formula for magnitude

• Angle $\angle H(j\omega)$:- $0 - \tan^{-1}\left(\dfrac{1.32\omega}{\frac{1600}{\pi}-\omega^2}\right)$

$\longrightarrow$ Formula for phase angle

Note our $\omega_n \rightarrow \sqrt{\frac{1600}{\pi}} \rightarrow 22.56758334$

so let $\omega_1 = 5 \therefore f_1 \rightarrow \dfrac{5}{2\pi}$

$\omega_2 = 22.56 \therefore f_2 = \dfrac{22.56}{2\pi}$

$\omega_3 = 40 \therefore f_3 \rightarrow \dfrac{40}{2\pi}$

* Rule $\omega = 2\pi f$

$f = \dfrac{\omega}{2\pi}$

• PART E
HAND ANALYSIS

Solve for $w_1 = 5$ & $f_1 = \frac{5}{2\pi}$

Magnitude:-

$$|H(jw)| = \frac{A}{1} = \frac{1600/\pi}{\sqrt{\left(\frac{1600}{\pi} - 5^2\right)^2 + (1.32 \times 5)^2}}$$

$\therefore A \longrightarrow 1.0515237 \longrightarrow$ 1st magnitude.

first phase shift $\longrightarrow -0.7807 \simeq \underline{\underline{-0.8^\circ}}$

Solve for $w_2 = 22.56$ & $f_2 = \frac{22.56}{2\pi}$

$$\therefore |H(jw)| = \frac{A}{1} = \frac{1600/\pi}{\sqrt{\left(\frac{1600}{\pi} - 22.56^2\right)^2 + (1.32 \times 22.56)^2}}$$

$A \longrightarrow 17.10127175 \longrightarrow$ 2nd Magnitude

second phase shift $\longrightarrow -89.34159487 \simeq -90^\circ$

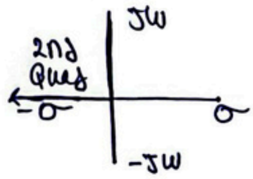
Solve for $w_3 = 40$ & $f_3 = 40/2\pi$

$$\therefore |H(jw)| = \frac{A}{1} = \frac{1600/\pi}{\sqrt{\left(\frac{1600}{\pi} - 40^2\right)^2 + (1.32 \times 40)^2}}$$

$A \longrightarrow 0.46639604 11 \simeq 0.47 \longrightarrow$ 3rd Magnitude

third phase shift $\longrightarrow -177.2285269$

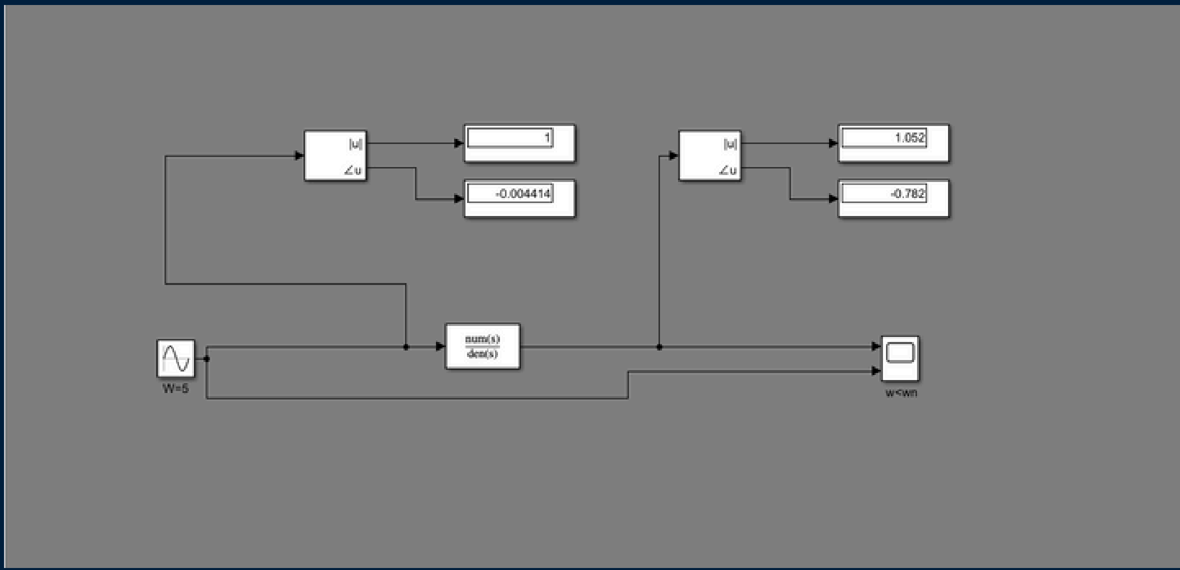
$\simeq \underline{\underline{-177^\circ}}$



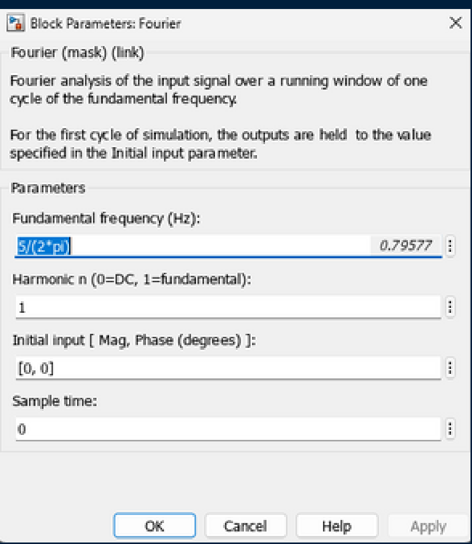
FOURIER ANALYSIS

@ W=5

SIMULINK



CONFIGURATIONS


Block Parameters: Fourier

Fourier (mask) (link)

Fourier analysis of the input signal over a running window of one cycle of the fundamental frequency.

For the first cycle of simulation, the outputs are held to the value specified in the Initial input parameter.

Parameters

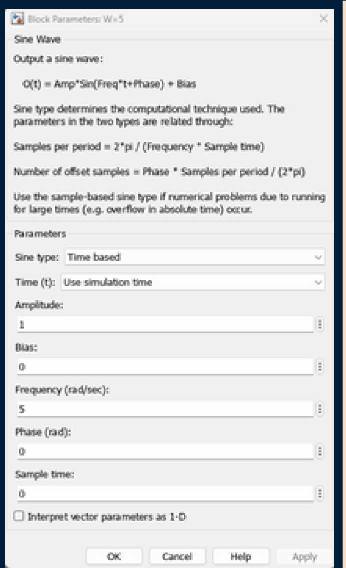
Fundamental frequency (Hz): 5/(2*pi) 0.79577

Harmonic n (0=DC, 1=fundamental): 1

Initial input [Mag, Phase (degrees)]: [0, 0]

Sample time: 0

OK Cancel Help Apply


Block Parameters: W=5

Sine Wave

Output a sine wave:

O(t) = Amp*Sin(Freq*t+Phase) + Bias

Sine type determines the computational technique used. The parameters in the two types are related through:

Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude: 1

Bias: 0

Frequency (rad/sec): 5

Phase (rad): 0

Sample time: 0

Interpret vector parameters as 1-D

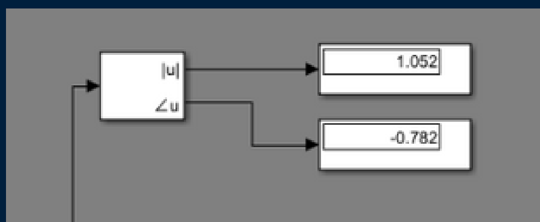
OK Cancel Help Apply

HAND ANALYSIS

∴ A ⟶ 1.0515237 ⟹ 1st magnitude.

first phase shift ⟶ −0.7807 ≃ $\underline{-0.8°}$

SIMULINK OUTPUT



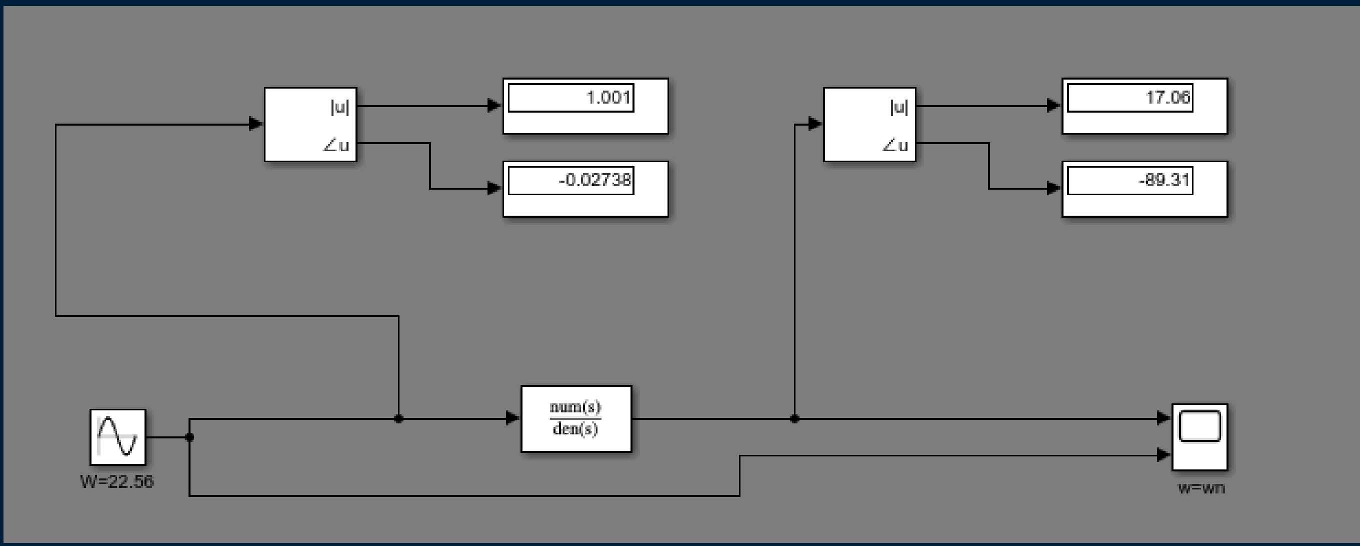
COMMENT

- THE ANALYTICAL AND THE SIMULINK OUTPUT ARE THE SAME
- AT ω=5 RAD/S, THE OUTPUT CLOSELY FOLLOWS THE INPUT WITH NEARLY THE SAME AMPLITUDE AND A VERY SMALL PHASE SHIFT, INDICATING MINIMAL DISTORTION AT LOW FREQUENCIES.

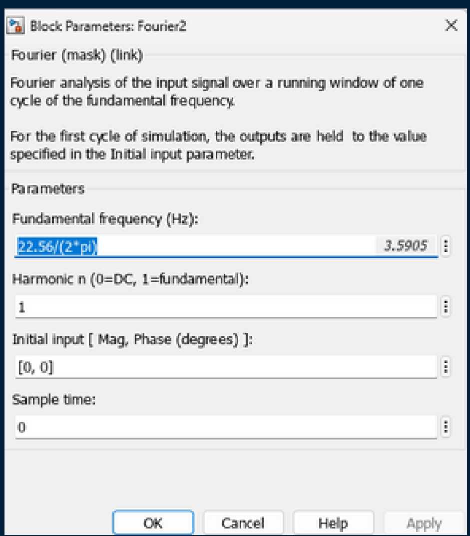
FOURIER ANALYSIS

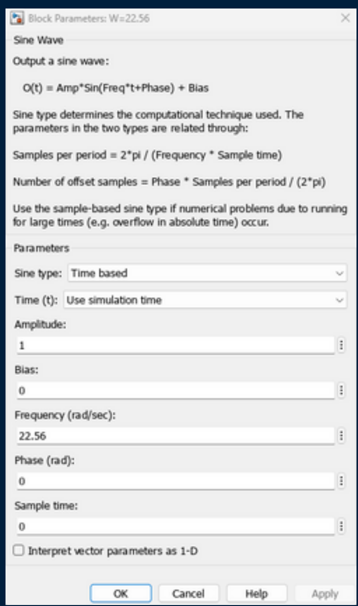
@ W=22.56

SIMULINK



CONFIGURATIONS



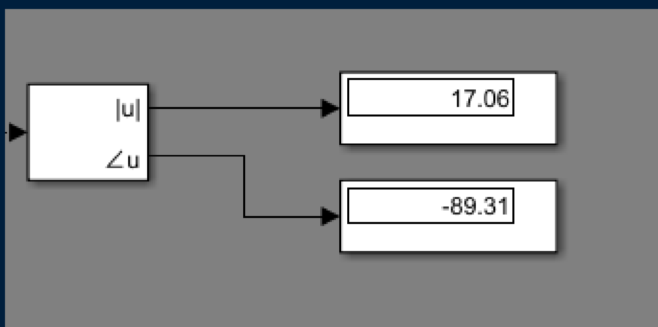


HAND ANALYSIS

A → 17.1012717.5 → 2nd Magnitude

Second Phase shift → −89.34159487 ≃ −90°

SIMULINK OUTPUT



COMMENT

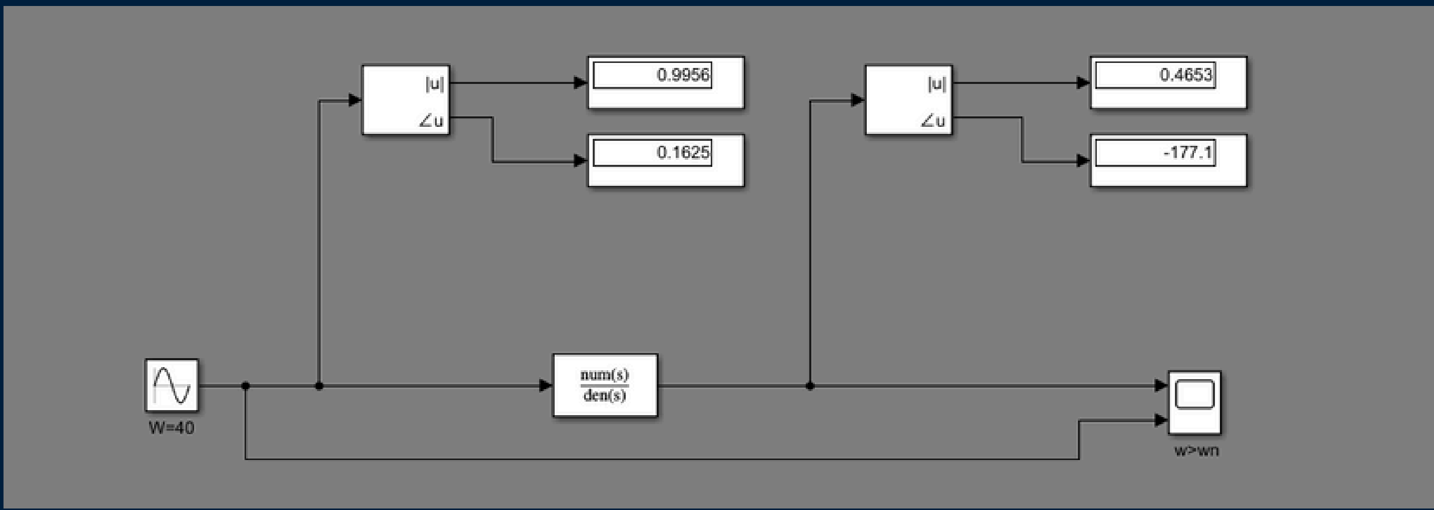
- THE ANALYTICAL AND THE SIMULINK OUTPUT ARE THE SAME

- AT ω=22.56 RAD/S, WHICH IS EQUAL TO THE NATURAL FREQUENCY OF THE SYSTEM, THE OUTPUT AMPLITUDE BECOMES MUCH LARGER THAN THE INPUT DUE TO RESONANCE, AND THE PHASE SHIFT IS APPROXIMATELY −90°.

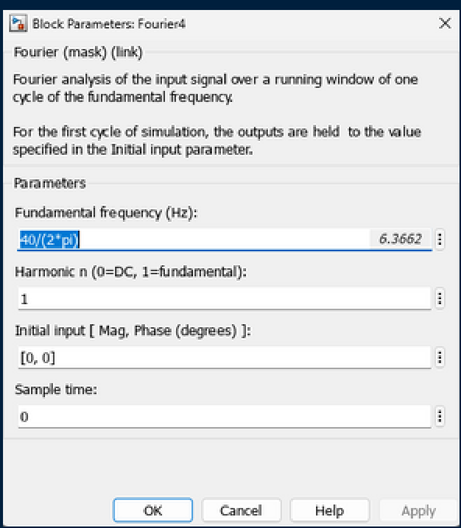
FOURIER ANALYSIS

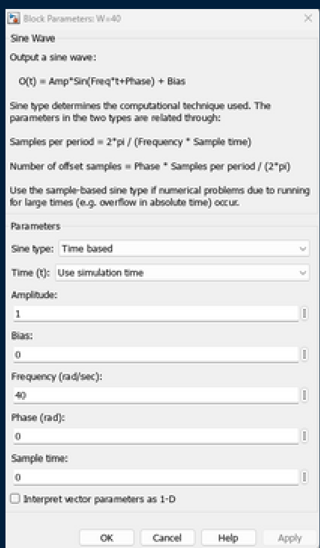
@ W=40

SIMULINK

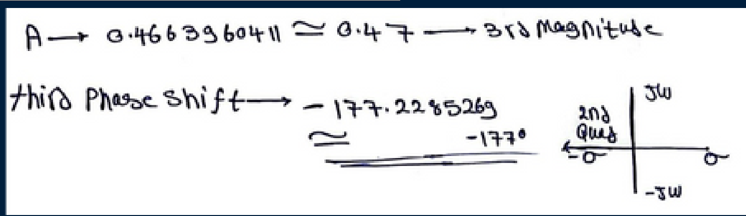


CONFIGURATIONS

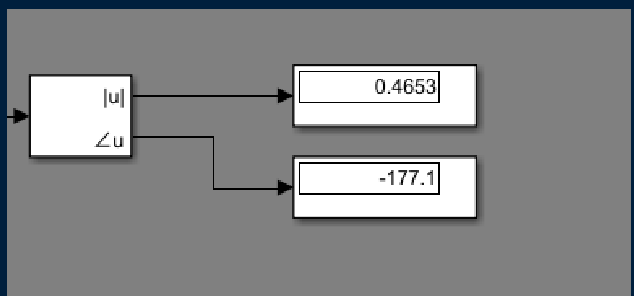




HAND ANALYSIS



SIMULINK OUTPUT



COMMENT

- THE ANALYTICAL AND THE SIMULINK OUTPUT ARE THE SAME

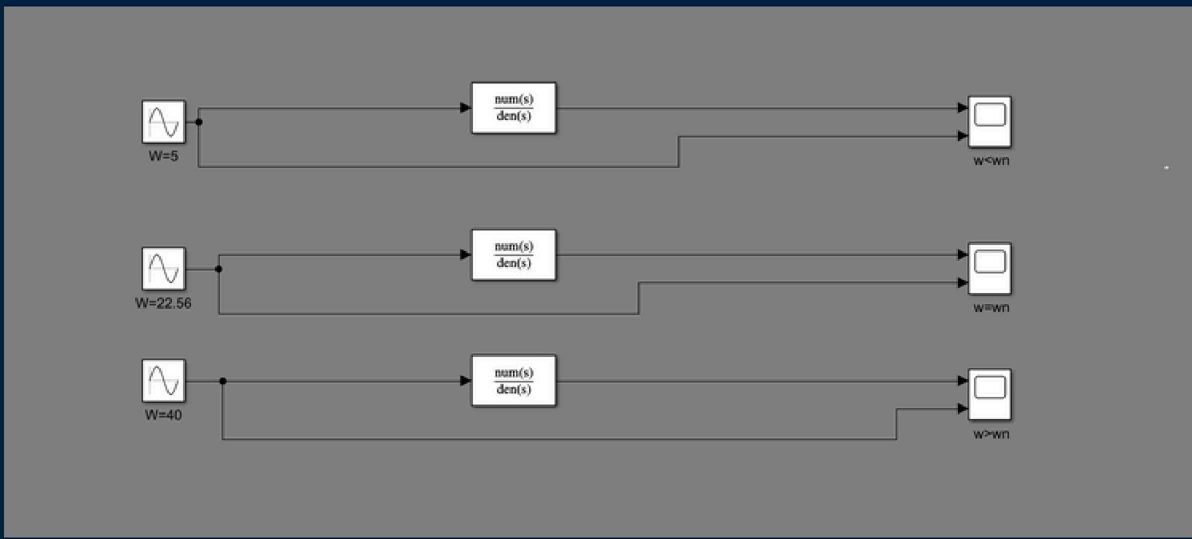
- AT ω=40 RAD/S, THE OUTPUT AMPLITUDE IS SIGNIFICANTLY SMALLER THAN THE INPUT, INDICATING STRONG ATTENUATION, AND THE PHASE SHIFT APPROACHES −180°, MEANING THAT THE OUTPUT IS ALMOST OUT OF PHASE WITH THE INPUT.

PART E

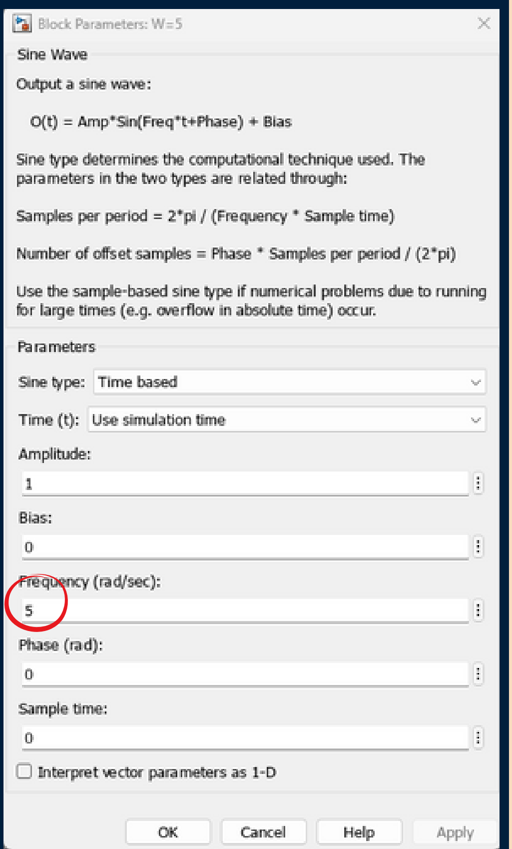
- 4

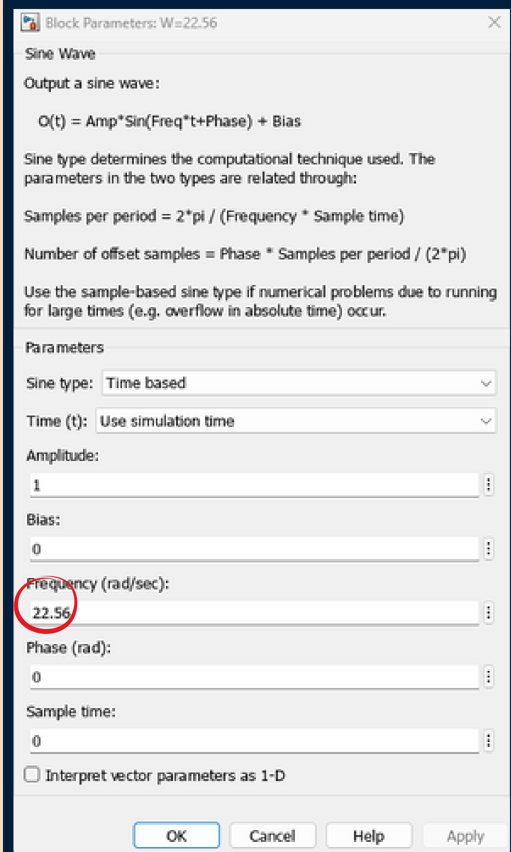
ANOTHER SOLUTION FROM THE GRAPHS

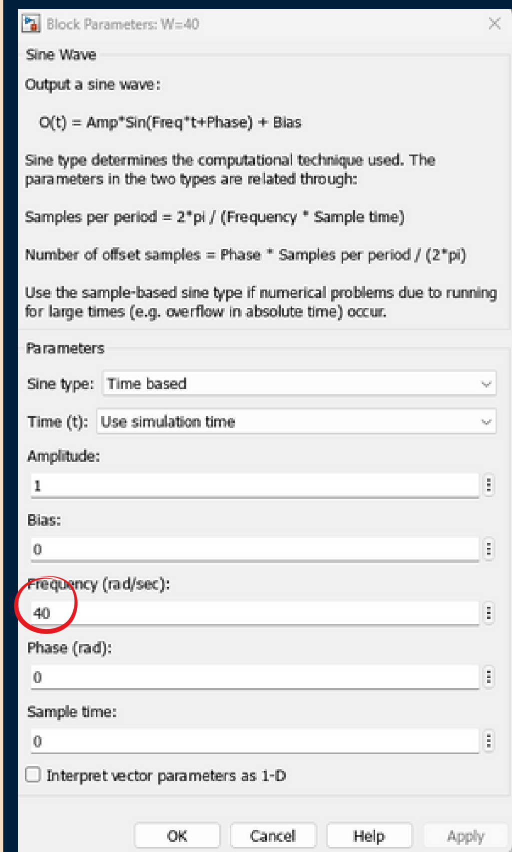
SIMULINK



CONFIGURATIONS



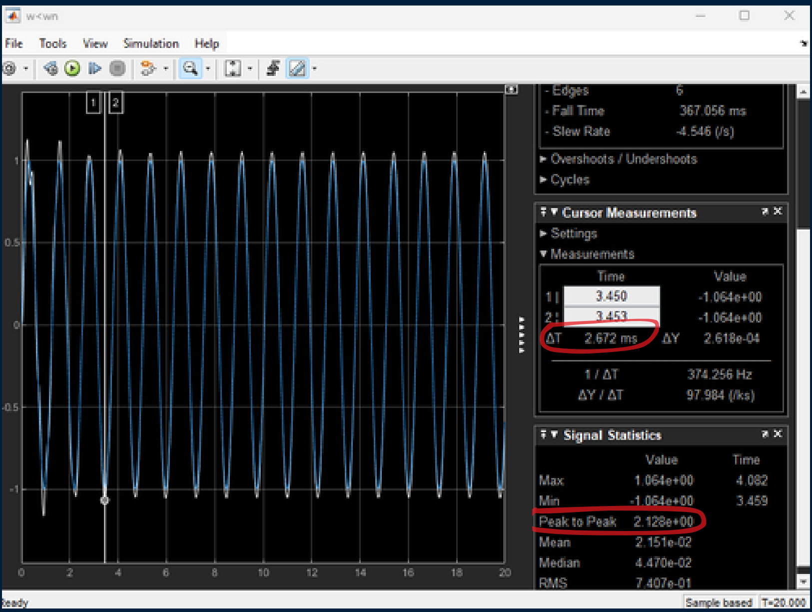




ANOTHER SOLUTION FROM THE GRAPHS

TO CALCULATE MAGNITUDE:
PEAK-TO-PEAK/2

TO GET THE PHASE SHIFT WE WILL GET THE ΔTIME AND CALCULATE THE PHASE SHIFT FROM IT .
THETA =(180/PI)*W*Δ TIME



∴ A ⟶ 1.0515237 ⟶ 1st magnitude.
first Phase shift ⟶ −0.7807 ≃ −0.8°

first phase shift ⟶ −0.7807° ≃ −0.8°

$\theta = \frac{180}{\pi} \times w \times \delta t$

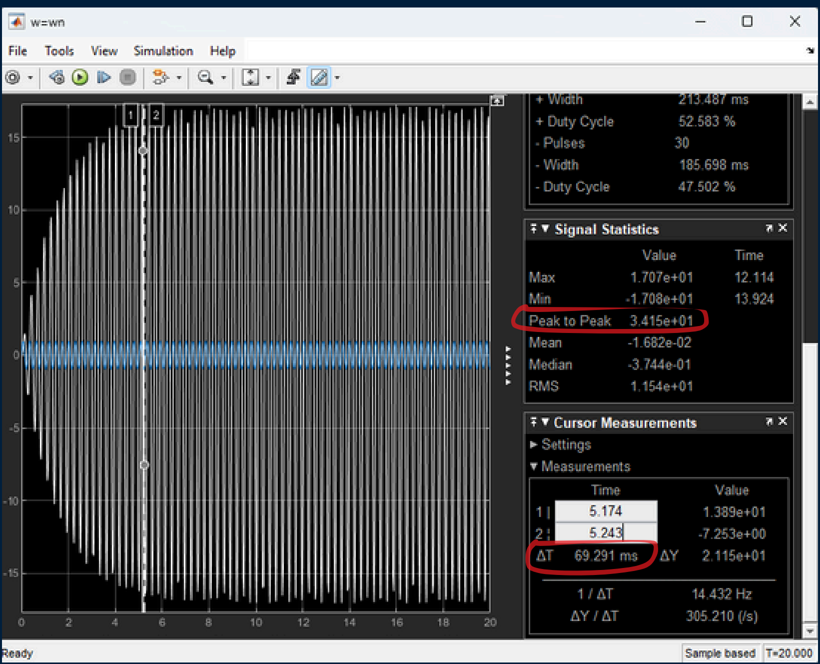
$\therefore -0.8 = \frac{180}{\pi} \times 5 \times \delta t$

∴ δt ⟶ Δt ⟶ 2.7925 ms

MAGNITUDE 2.128/2 1.064

θ (180/PI)*W*ΔT = (180/PI)*5*2.672*10^-3 0.7654

THE -VE SIGN IS JUST TO DEMONSTRATE THAT
THE PHASE SHIFT IS "DELAYED"



A ⟶ 17.1012717 5 ⟶ 2nd Magnitude
second Phase shift ⟶ −89.34159487 ≃ −90°

second Phase shift ⟶ −89.34159487 ≃ −90°

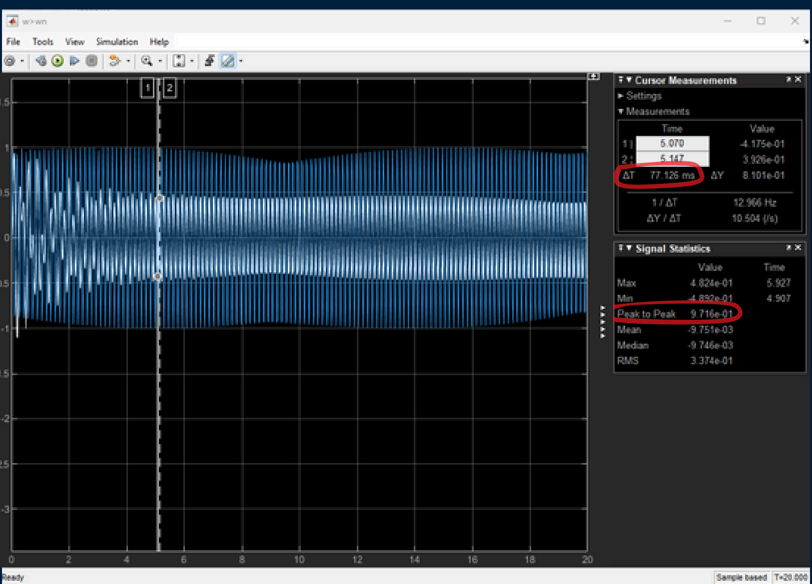
$\therefore -90 = \frac{180}{\pi} \times 22.56\ \delta t$

∴ δt ⟶ −0.069627 s ⟶ −69.627 ms

MAGNITUDE 34.15/2 17.075

θ (180/PI)*W*ΔT = (180/PI)*22.56*69.291*10^-3 89.565

THE -VE SIGN IS JUST TO DEMONSTRATE THAT
THE PHASE SHIFT IS "DELAYED"



A ⟶ 0.46639604 ≃ 0.47 ⟶ 3rd Magnitude
third Phase shift ⟶ −177.2285269 ≃ −177°
2nd Quad, 3w, −3w

third Phase shift ⟶ −177.2285269 ≃ −177°

$-177° = \frac{180}{\pi} \times 40 \times \delta t$

∴ δt ⟶ −0.07723 08194 ⟶ −77.23 ms

∗ −ve sign ⟶ is just to know that the phase shift is "delayed"

MAGNITUDE (9.716*(10^(-1)))/2 0.4858

θ (180/PI)*W*ΔT (180/PI)*40*77.125*10^-3 176.757

THE -VE SIGN IS JUST TO DEMONSTRATE THAT
THE PHASE SHIFT IS "DELAYED"

CONCLUSION

- At ω=5 rad/s, the output closely follows the input with nearly the same amplitude and a very small phase shift, indicating minimal distortion at low frequencies.

- At ω=22.56 rad/s, which is equal to the natural frequency of the system, the output amplitude becomes much larger than the input due to resonance, and the phase shift is approximately −90°.

- At ω=40 rad/s, the output amplitude is significantly smaller than the input, indicating strong attenuation, and the phase shift approaches −180°, meaning that the output is almost out of phase with the input.

- # PART E

- E

5- You can use the command c2d (continuous to discrete) to convert the continuous transfer function in **Part C-b** to discrete using sampling periods of 0.01, 0.1 and 1 second.

MATLAB CODE

transfer Function:- $\dfrac{1600/\pi}{S^2+1.32\,S+\frac{1600}{\pi}}$

```
% 1. Define the continuous-time transfer function
num = [509.2958];
den = [1, 1.32, 509.2958];
Gs = tf(num, den);

% 2. Define the sampling periods
Ts_list = [0.01, 0.1, 1];

% 3. Convert and display results for each Ts
for Ts = Ts_list
    % Convert using Zero-Order Hold (default method)
    Gz = c2d(Gs, Ts, 'zoh');

    fprintf('\n--- Discrete Transfer Function for Ts = %.2f s ---\n', Ts);
    display(Gz);
end
```

PART E

- E

OUTPUT MATLAB

```
--- Discrete Transfer Function for Ts = 0.01 s ---

Gz =

   0.02525 z + 0.02513
  ----------------------
  z^2 - 1.937 z + 0.9869

Sample time: 0.01 seconds
Discrete-time transfer function.
```

SAMPLING =0.01S

```
--- Discrete Transfer Function for Ts = 0.10 s ---

Gz =

      1.571 z + 1.49
  ----------------------
  z^2 + 1.185 z + 0.8763

Sample time: 0.1 seconds
Discrete-time transfer function.
```

SAMPLING =0.1S

```
--- Discrete Transfer Function for Ts = 1.00 s ---

Gz =

      1.444 z + 0.695
  -----------------------
  z^2 + 0.8721 z + 0.2671

Sample time: 1 seconds
Discrete-time transfer function.
```

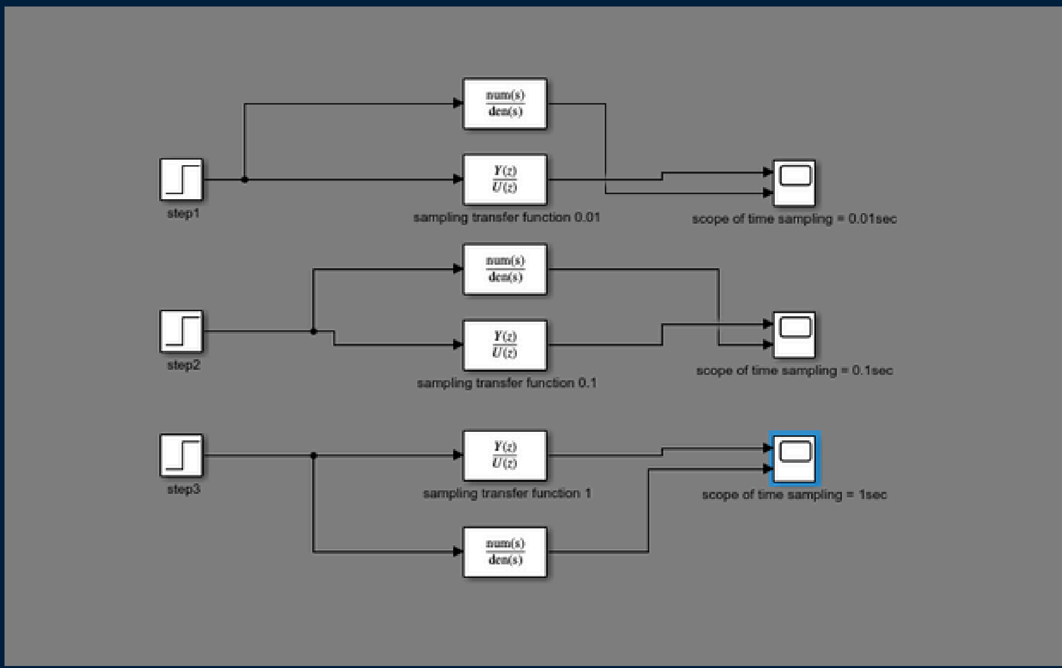
SAMPLING =1S

• PART E

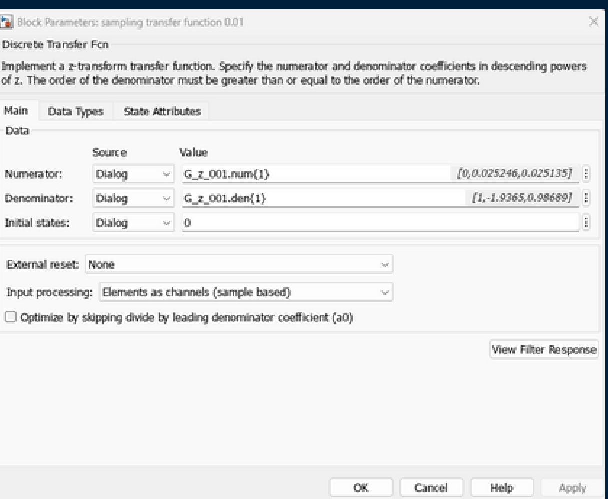
• F

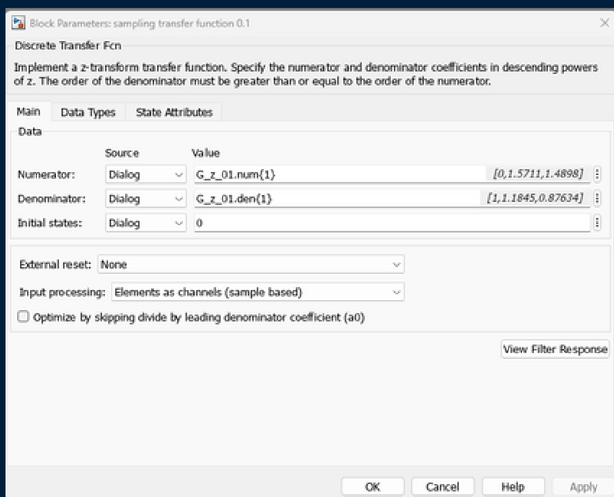
6- For each of the resulting discrete systems, use the Simulink to display the discrete unit step response. Compare your results with **Part E-2**. Comment on your results.

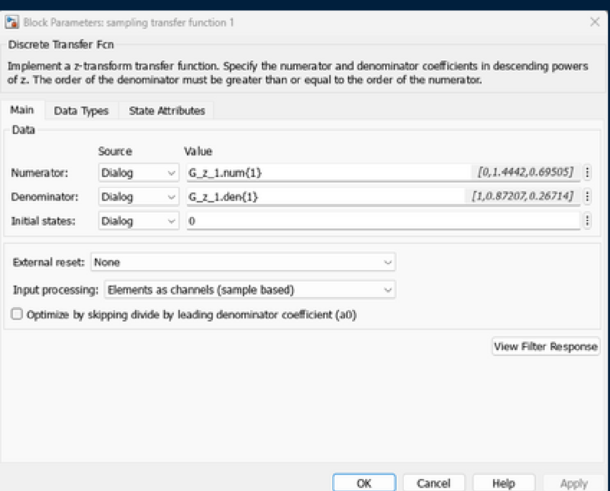
SIMULINK



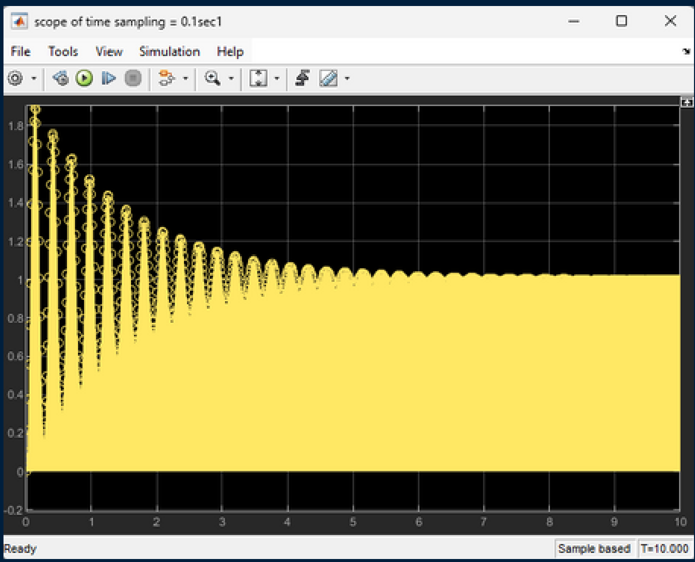
CONFIGURATIONS



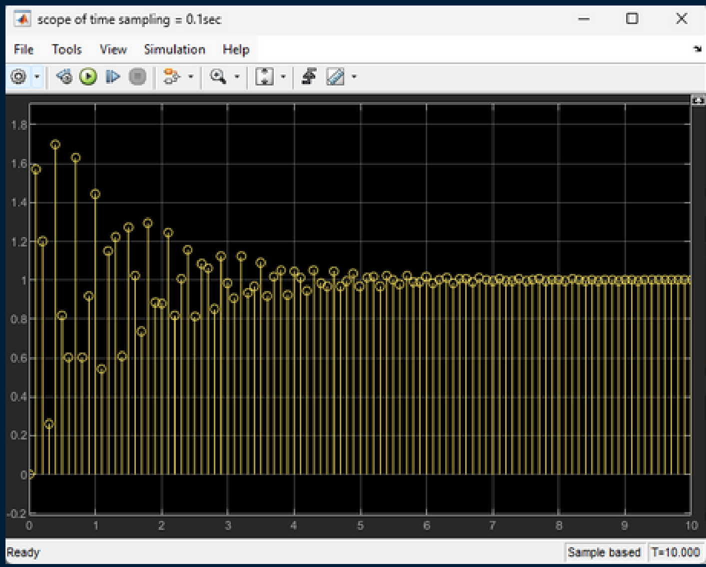




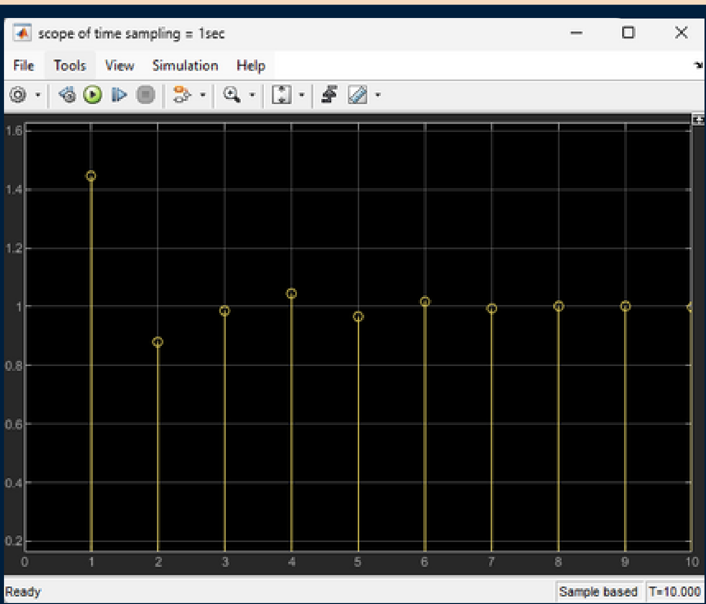
DISCRETE (STEM)



SAMPLING =0.01S

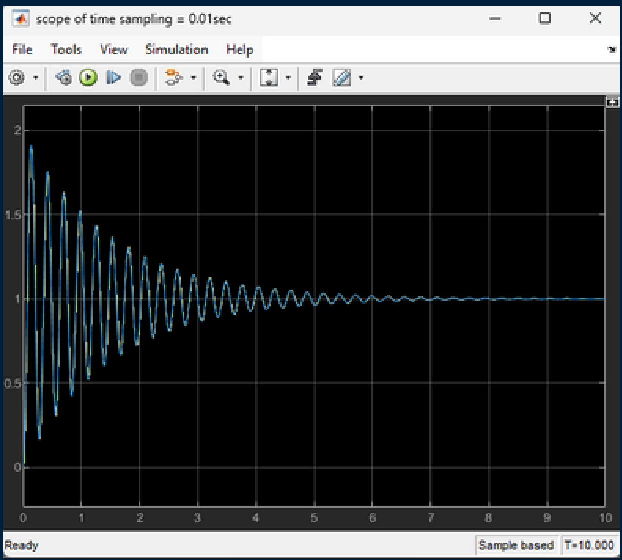


SAMPLING =0.1S

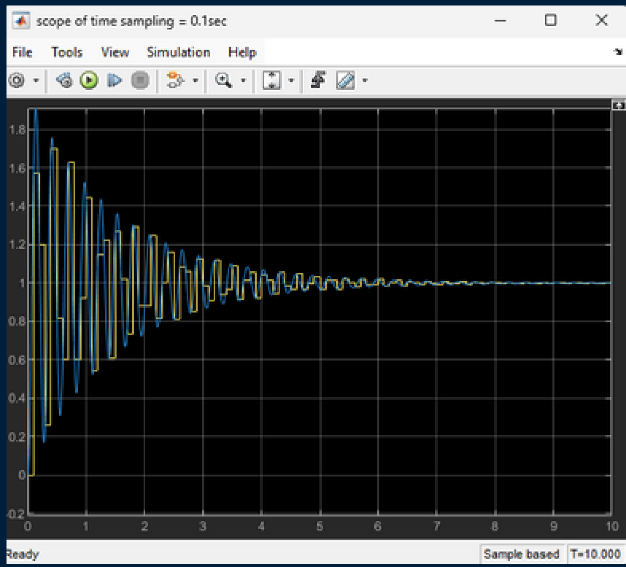


SAMPLING =1S

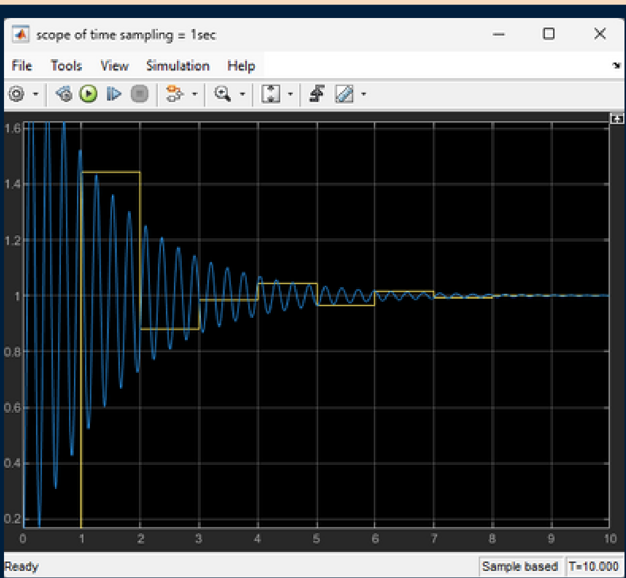
GRAPHS



SAMPLING =0.01S

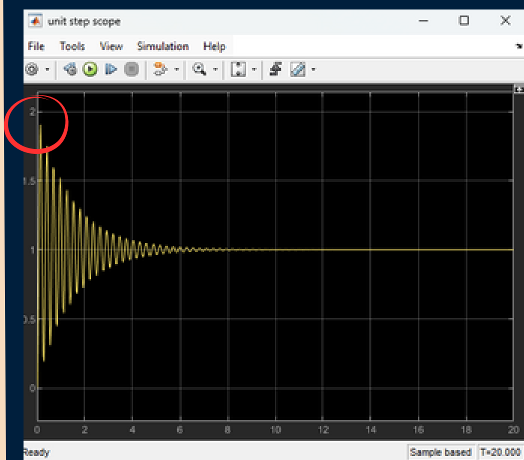


SAMPLING =0.1S



SAMPLING =1S

COMPARE WITH PART E-2



UNIT STEP SCOPE

COMMENT

- THE DISCRETE UNIT STEP RESPONSES WERE OBTAINED USING SIMULINK FOR SAMPLING PERIODS OF 0.01 S, 0.1 S, AND 1 S AND COMPARED WITH THE CONTINUOUS-TIME STEP RESPONSE IN PART E-2.

- FOR TS = 0.01 S, THE DISCRETE RESPONSE CLOSELY MATCHES THE CONTINUOUS RESPONSE, INDICATING ACCURATE REPRESENTATION OF THE SYSTEM DYNAMICS.

- FOR TS = 0.1 S, SLIGHT DEVIATIONS APPEAR DUE TO THE LOWER SAMPLING RATE, BUT THE OVERALL BEHAVIOR REMAINS SIMILAR.

- FOR TS = 1 S, THE DISCRETE RESPONSE SIGNIFICANTLY DEVIATES FROM THE CONTINUOUS RESPONSE, SHOWING LOSS OF DYNAMIC INFORMATION.

- CONCLUSION: SMALLER SAMPLING PERIODS RESULT IN BETTER APPROXIMATION OF THE CONTINUOUS-TIME SYSTEM.

THE END