Robotics Modelling
Modeling and control of underwater vehicle: Sparus

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Abstract

This document allows to model "Sparus", the last autonomous underwater vehicle of IQUA Robotics (Girona). This report is designed for students, so they can swiftly understand theory and codes that have been written. All the work done here can be adapted to an other shape if future work needs it.

Contents

1 Introduction

1.1 Main caracteristic of the Sparus

The Sparus AUV is represented by the figure Fig.1. It allows to define the main axes and the position of the gravity center.

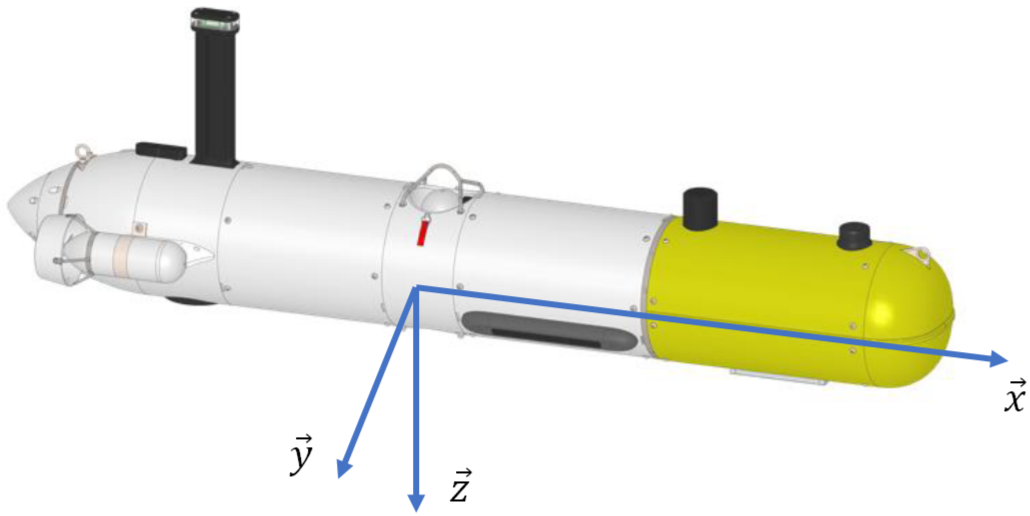

Figure 1: The Sparus in 3D

Therefore we consider the origin of the base "body" in the gravity center of the Sparus. It is placed at the x-position of the central thruster and at middle of the cylinder.

$$\overrightarrow{r}_g = {}_b\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{1}$$

Moreover, the coordinates of the buoyancy center are:

$$\overrightarrow{r}_b = {}_b\begin{bmatrix}0\\0\\-0.02\,m\end{bmatrix} \tag{2}$$

1.2 Notations

For your study, the following scalar notations has to be used :

ρ_w	water mass density	$1000\ kg.m^{-3}$
R	Radius	$0.115\ m$
L	Length	$1.6\ m$
m	Mass	$52\ Kg$
m_b	Buoyancy mass	$52,1\ Kg$
g	Earth gravity	$9.81\ m.s^{-2}$
P	Weight	$P = m.g$

And the following vector and matrix notations has to be defined :

G^b	Gravity vector
B^b	Buoyancy vector
U^b	Thrust mapped vector
K^b	Friction matrix
E^b	Mapping matrix
$M_G^b = M_B^b + M_A^b$	Generalized mass matrix
M_B^b	Body mass matrix
M_A^b	Added mass matrix
$C_G^b = C_B^b + C_A^b$	Generalised Coriolis matrix
v	Velocity vector in Body frame
η	Position vector in Earth frame
J_θ	Generalised rotation matrix

1.3 Sensor and Thruster positions

Moreover, the values of the sensors and thrusters positions (represented on the figure Fig.2) are :

	Coordinates in basse b	Values in m
Motor 1	$\overrightarrow{r}^b_{t1} = (d_{1x}, d_{1y}, d_{1z})^T$	$(0, 0, 0.08)$
Motor 2	$\overrightarrow{r}^b_{t2} = (d_{2x}, d_{2y}, d_{2z})^T$	$(-0.59, 0.17, 0)$
Motor 3	$\overrightarrow{r}^b_{t3} = (d_{3x}, d_{3y}, d_{3z})^T$	$(-0.59, -0.17, 0)$
DVL	$\overrightarrow{r}^b_{t3} = (d_{dx}, d_{dy}, d_{dz})^T$	$(-0.4145, 0, 0.11)$
IMU and Depth	$\overrightarrow{r}^b_{t3} = (d_{ix}, d_{iy}, d_{iz})^T$	$(0.364, -0.021, -0.085)$
USBL	$\overrightarrow{r}^b_{t3} = (d_{ux}, d_{uy}, d_{uz})^T$	$(0.44, 0, -0.14)$

1.4 Global real mass matrix

All terms are in SI units.

$$M_{RB}^{CO} = \begin{bmatrix} 52 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 52 & 0 & 0.1 & 0 & -1.3 \\ 0 & 0 & 52 & 0 & 1.3 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0 & 0 \\ -0.1 & 0 & 1.3 & 0 & 9.4 & 0 \\ 0 & -1.3 & 0 & 0 & 0 & 9.5 \end{bmatrix} \quad (3)$$

2 Dynamic Modelling

The objective of this section is to help you to developp the dynamic model of the Sparus.

2.1 Kinematics

First, we need to describe two frames:

- Earth inertial frame (E-frame)
- Body-fixed frame (B-frame)

Then we will define the relationship between those two frames using the Euler angle theory in order to be able to describe a velocity belonging to the B-frame in the E-frame. We don't speak about a relationship between positions in those two frames because position in body-fixed frame is physically irrelevant and has no meaning. So, the equation is:

$$\dot{\overrightarrow{\eta}} = J_\theta . \overrightarrow{\upsilon} \quad (4)$$

With,

$$\dot{\overrightarrow{\eta}} = \begin{pmatrix} \overrightarrow{v^e} \\ \overrightarrow{\omega^e} \end{pmatrix} = \begin{pmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{pmatrix}^T \quad (5)$$

$$\dot{\overrightarrow{\upsilon}} = \begin{pmatrix} \overrightarrow{v^b} \\ \overrightarrow{\omega^b} \end{pmatrix} = \begin{pmatrix} \dot{u} & \dot{v} & \dot{w} & \dot{p} & \dot{q} & \dot{r} \end{pmatrix}^T \quad (6)$$

Where $\dot{\overrightarrow{\eta}}$ the generalised velocity is vector in E-frame and it is composed of Nessie linear velocity vector $\overrightarrow{v^e}$ and Nessie angular velocity vector $\overrightarrow{\omega^e}$.

And $\overrightarrow{\upsilon}$ is the generalised velocity is vector in B-frame and it is composed of Nessie linear velocity vector $\overrightarrow{v^b}$ and Nessie angular velocity vector $\overrightarrow{\omega^b}$.

In addition, the generalized rotation and transfer matrix J_θ is composed of 4 sub-matrixes according to equation:

$$J_\theta = \begin{bmatrix} R_\theta & 0_{3*3} \\ 0_{3*3} & T_\theta \end{bmatrix} \tag{7}$$

Where the rotation R_θ and the transfer T_θ matrices are defined according in the lesson.

2.2 Dynamics

The dynamics is described in the B-frame :

$$M_G^b \dot{v} + C_G^b v = G^b - B^b - K^b + U^b \tag{8}$$

where :

- M_G^b is the general mass matrix of the body
- C_G^b is the general body Coriolis matrix.
- $G^b + B^b$: Gravity and Buoyancy
- $K^b = \sum K_i^b$: Sum of all body drag forces
- U^b: Thruster

2.2.1 Gravity and Buoyancy

The gravity G^b and buoyancy B^b vector can be defined as follow:

$$G^b = \begin{pmatrix} \overrightarrow{F}_g^b \\ \overrightarrow{r}_g \wedge \overrightarrow{F}_g^b \end{pmatrix} = \begin{pmatrix} R_\theta^{-1} \overrightarrow{F}_g^e \\ \overrightarrow{r}_g \wedge R_\theta^{-1} \overrightarrow{F}_g^e \end{pmatrix} \tag{9}$$

and

$$B^b = \begin{pmatrix} \overrightarrow{F}_b^b \\ \overrightarrow{r}_b \wedge \overrightarrow{F}_b^b \end{pmatrix} = \begin{pmatrix} R_\theta^{-1} \overrightarrow{F}_b^e \\ \overrightarrow{r}_b \wedge R_\theta^{-1} \overrightarrow{F}_b^e \end{pmatrix} \tag{10}$$

where $\overrightarrow{F}_g^e$ and $\overrightarrow{F}_b^e$ are respectively the force vectors of the gravity and buoyancy expressed in the earth frame.

2.2.2 Thruster

The Thruster forces and moments can be represented by the following equation:

$$U^b = E^b F_T^b = E_{6*3}^b \begin{bmatrix} F_{T1}^b \\ F_{T2}^b \\ F_{T3}^b \end{bmatrix} \tag{11}$$

Where F^b_{T1} is the force of thruster (or motor) 1 in Body-fixed frame. We have the same definition for the six forces.

E^b has to be determined thanks to the position of the thruster (see section 1.3). For exemple, the moment produced by the thruster 1 is $\overrightarrow{r}^b_{t1} \wedge \overrightarrow{F^b_{T1}}$.

After adjusting the model behaviour with experimental data, we have chosen the following first order transfer function :

$$G(s) = \frac{Kt}{\tau s + 1} \tag{12}$$

Consequently, the input is a percentage of the maximal force ($F_{max} = Kt$ at 100%).

The values of the gain and constant times are defined below:

	Forward	rearward		
Kt_1	55 N	28.5 N	τ_1	0.4 s
Kt_2	71.5 N	30 N	τ_2	0.8 s
Kt_3	71.5 N	30 N	τ_3	0.8 s

3 Work to do

You have to send me back your work on moodle. You have to realize in order of importance before the deadline (to be determined):

1. With the figure 2, compute all the dimension of the different bodies
2. Considering the given global real mass matrix explains all the terms: from which part the various terms in the matrix originate.
3. Compute each added mass matrix at the buoyancy center of the sparus. Excepted the main body, the CG and CB of the other bodies are at the same point.
4. Compare this matrix at CG and CO. Is it important to take into account the distance between the two points ?
5. Compare the values of the main solid with the others and conclude.
6. Compare the added and real mass matrix and conclude.
7. Estimate all drag matrices.
8. Complete the simulator with some simple experiments to validate it.
9. Find some simulations to highlight the impact of the different coefficients in the global mass matrix. (Impose linear accelerations)
10. Find some simulations to highlight the impact of the drag forces of the different bodies (Impose constant linear speed)