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- Determine whether the system $g(x,y)=xyf(x,y)$ is (1) linear and/or (2) shift-invariant.

- Find the transfer function of PSF given as $h(x)=e^{-x^2/30}$. Then determine the MTF of the system and sketch it roughly. On the sketch indicate the region which is responsible for the resolution of the system, explain it.

Basic Fourier transform pairs

Signal	Fourier Transform
1	$\delta(u, v)$
$\delta(x, y)$	1
$\delta(x - x_0, y - y_0)$	$e^{-j2\pi(ux_0 + vy_0)}$
$\delta_s(x, y; \Delta x, \Delta y)$	$\text{comb}(u\Delta x, v\Delta y)$
$e^{j2\pi(u_0x + v_0y)}$	$\delta(u - u_0, v - v_0)$
$\sin[2\pi(u_0x + v_0y)]$	$\frac{1}{2j} [\delta(u - u_0, v - v_0) - \delta(u + u_0, v + v_0)]$
$\cos[2\pi(u_0x + v_0y)]$	$\frac{1}{2} [\delta(u - u_0, v - v_0) + \delta(u + u_0, v + v_0)]$
$\text{rect}(x, y)$	$\text{sinc}(u, v)$
$\text{sinc}(x, y)$	$\text{rect}(u, v)$
$\text{comb}(x, y)$	$\text{comb}(u, v)$
$e^{-\pi(x^2 + y^2)}$	$e^{-\pi(u^2 + v^2)}$

Properties of the Fourier transform

Property	Signal	Fourier Transform
	$f(x, y)$	$F(u, v)$
	$g(x, y)$	$G(u, v)$
Linearity	$a_1f(x, y) + a_2g(x, y)$	$a_1F(u, v) + a_2G(u, v)$
Translation	$f(x - x_0, y - y_0)$	$F(u, v)e^{-j2\pi(ux_0 + vy_0)}$
Conjugation	$f^*(x, y)$	$F^*(-u, -v)$
Conjugate symmetry	$f(x, y)$ is real-valued	$F(u, v) = F^*(-u, -v)$
		$F_R(u, v) = F_R(-u, -v)$
		$F_I(u, v) = -F_I(-u, -v)$
		$ F(u, v) = F(-u, -v) $
		$\angle F(u, v) = -\angle F(-u, -v)$
Signal reversing	$f(-x, -y)$	$F(-u, -v)$
Scaling	$f(ax, by)$	$\frac{1}{ ab } F\left(\frac{u}{a}, \frac{v}{b}\right)$
Rotation	$f(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$	$F(u \cos \theta - v \sin \theta, u \sin \theta + v \cos \theta)$
Circular symmetry	$f(x, y)$ is circularly symmetric	$F(u, v)$ is circularly symmetric
		$ F(u, v) = F(u, v)$
		$\angle F(u, v) = 0$
Convolution	$f(x, y) * g(x, y)$	$F(u, v)G(u, v)$
Product	$f(x, y)g(x, y)$	$F(u, v) * G(u, v)$
Separable product	$f(x)g(y)$	$F(u)G(v)$
Parseval's theorem	$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) ^2 dx dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) ^2 du dv$	

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux + vy)} dx dy$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux + vy)} du dv$$

$$g(x, y) = h(x, y) * f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi, \eta) h(x - \xi, y - \eta) d\xi d\eta$$

$$\int_{-\infty}^{\infty} e^{-a^2 \tau^2} d\tau = \frac{\sqrt{\pi}}{a}, \quad \text{for } a \neq 0$$