

Energy Symmetry Transmission (EST): A Rigorous Field-Theoretic and Circuit-Consistent Reformulation of Electrical Energy Transfer

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Abstract

EST reframes electric power transfer as the propagation of symmetry-encoding control fields rather than bulk charge motion. A low-power field $\theta(x, t) \in \mathfrak{g}$ modulates the local material response so that energy conversion is orchestrated at the destination under strict energy accounting and stability. We provide: (i) a differential-geometric foundation on principal bundles with Noether-consistent conservation; (ii) axioms for locality, Lyapunov stability, composability, and explicit dissipation; (iii) a *circuit-consistent* mapping from field variables to measurable voltages/currents enabling standard RMS/THD/power-factor evaluation; (iv) a dissipative PDE toy model with a projected-gradient controller; (v) an experimental micro-protocol and a full reproducibility checklist. Figures are PGF/TikZ and the manuscript compiles with pdfLaTeX using standard TeX Live packages.

Keywords: energy transmission, symmetry control, geometric field theory, circuit modeling, nonlinear control, RMS/THD, smart grids, Lyapunov stability, reproducibility.

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1 Notation, Units, and Symbols

Table 1: Principal symbols with SI units.

Symbol	Unit	Meaning
M	–	Spacetime manifold (engineering-scale Minkowski)
G, \mathfrak{g}	–	Compact Lie group and its Lie algebra
$A_\mu, F_{\mu\nu}$	–	Connection one-form and curvature two-form on $P(M, G)$
ϕ	–	Matter field (section of associated bundle E)
θ	–	Symmetry-control field (dimensionless unless stated)
$u(x, t)$	–	Order parameter / polarization-like state (toy model)
$v_{\text{bus}}(t)$	V	Source/bus voltage
$i_{\text{bus}}(t)$	A	Line/bus current
$v_{\text{load}}(t)$	V	Load terminal voltage
$i_{\text{load}}(t)$	A	Load current
$Y_{\text{eff}}(t)$	S	Effective load admittance modulated by θ
E_θ	J	Control-channel energy
P, Q, S	W,,V A	Active/reactive/apparent power
I_{RMS}	A	RMS current (bus)

2 Circuit–Field Realization and Measurement Map

We model the destination node by an effective admittance

$$Y_{\text{eff}}(t; \theta) = Y_0 + \Delta Y(\theta, u, \partial_t u, \partial_t \theta), \quad \|\Delta Y\| \ll \|Y_0\|, \quad (2.1)$$

so that the bus quantities obey

$$i_{\text{bus}}(t) = Y_{\text{eff}}(t; \theta) v_{\text{bus}}(t) + i_{\text{ctrl}}(t), \quad E_\theta = \int_0^T v_{\text{ctrl}}(t) i_{\text{ctrl}}(t) dt. \quad (2.2)$$

A minimal constitutive choice consistent with passivity is

$$\Delta Y(\cdot) = k_1 \partial_t u + k_2 u + k_3 \partial_t \theta, \quad k_i \in \mathbb{R}, \quad |k_i| \text{ small}. \quad (2.3)$$

Proposition 2.1 (Closed Energy Budget). *Over any window $[0, T]$,*

$$E_{\text{bus}, \text{in}} + E_\theta = E_{\text{load}, \text{out}} + E_{\text{loss}} + \Delta E_{\text{store}}, \quad E_{\text{loss}} \geq 0. \quad (2.4)$$

Sketch. Instantaneous power balance with passive dissipation and isolated/low-power control channel. \square

Power–Quality Metrics. From measured i_{bus} and v_{bus} we compute

$$I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T i_{\text{bus}}^2(t) dt}, \quad (2.5)$$

$$\text{THD}(i) = \frac{\sqrt{\sum_{n \geq 2} |I_n|^2}}{|I_1|} \times 100\%, \quad (2.6)$$

where I_n are Fourier coefficients of i_{bus} .

3 Geometric Foundations

Definition 3.1 (Configuration Space). *Let $P(M, G)$ be a principal G -bundle. The EST configuration is*

$$\mathcal{C} = \text{Conn}(P) \times \Gamma(E) \times C^\infty(M, \mathfrak{g}), \quad (3.1)$$

with connection A , matter field ϕ , and control field θ .

Definition 3.2 (Lagrangian).

$$\mathcal{L}(A, \phi, \theta) = \mathcal{L}_{\text{field}}(F) + \mathcal{L}_{\text{matter}}(\phi, D_A \phi) + \mathcal{L}_{\text{int}}(\phi, F; \theta) + \mathcal{L}_{\text{control}}(\theta), \quad (3.2)$$

$$\mathcal{L}_{\text{field}} = -\frac{1}{4} \langle F_{\mu\nu}, F^{\mu\nu} \rangle_{\mathfrak{g}}, \quad \mathcal{L}_{\text{matter}} = \frac{1}{2} \langle D_\mu \phi, D^\mu \phi \rangle - V(\phi), \quad (3.3)$$

$$\mathcal{L}_{\text{int}} = g(\theta) \mathcal{I}(\phi, F), \quad \mathcal{L}_{\text{control}} = \frac{1}{2} \langle \partial_\mu \theta, \partial^\mu \theta \rangle - U(\theta). \quad (3.4)$$

Theorem 3.1 (Noether Energy Conservation). *Time-translation invariance of \mathcal{L} implies $\partial_\mu T^{\mu\nu} = 0$ and constancy of total energy $Q_E = \int_\Sigma T^{00} d^3x$ for any Cauchy surface Σ .*

Sketch. Apply Noether's first theorem to Definition 3.2 and the Euler–Lagrange equations for (A, ϕ, θ) . \square

4 Axioms, Passivity, and Safety

Axiom 4.1 (Small-Overhead Control). $\int_M \|\partial_\mu \theta\|^2 \, d^4x \leq \epsilon \int_M (\|F\|^2 + \|D\phi\|^2) \, d^4x$, $0 < \epsilon \ll 1$.

Axiom 4.2 (ISS Stability). *For fixed θ , there exists a Lyapunov $V_\theta \geq 0$ with $\dot{V}_\theta \leq -\alpha \|\nabla V_\theta\|^2 + \beta \|\partial_t \theta\|^2$.*

Axiom 4.3 (Explicit Energy Accounting). $\frac{dQ_E}{dt} = \Phi_{\partial U} + \Delta E_{local} - D_{diss}$, $D_{diss} \geq 0$.

Proposition 4.1 (Passivity of Y_{eff} with Small Couplings). *If $|k_i| \leq \bar{k} \ll 1$ in Eq. (2.3) and the medium is dissipative, then the map $v_{bus} \mapsto i_{bus}$ is passive and finite-gain; hence the interconnection with a passive grid preserves stability margins.*

Sketch. Dissipativity inequality on storage $S = \frac{1}{2} C v_{bus}^2$ with small perturbation in admittance; use incremental passivity and small-gain. \square

Assumption 4.1 (Engineering Scale). *Nonrelativistic, quasi-static EM at 50 Hz/60 Hz scales.*

5 Classical Limit and AC/DC/HVDC Recovery

For $G = U(1)$ and harmonic $\theta(t) = \theta_0 \sin \omega t$, the source term induced by \mathcal{L}_{int} is sinusoidal, recovering AC; the limit $\omega \rightarrow 0$ yields DC/HVDC behavior with ideal converters. A full derivation is provided in Section A.

6 Dissipative PDE Model and Control

On $x \in [0, L]$,

$$\mathcal{F}[u; \theta] = \int_0^L \left[\frac{1}{2} a(\theta) u^2 + \frac{1}{4} b u^4 + \frac{1}{2} \kappa (\partial_x u)^2 \right] dx, \quad (6.1)$$

$$\partial_t u = -\gamma \frac{\delta \mathcal{F}}{\delta u} + \sigma(\theta) f_{\text{drive}}(t) + \xi, \quad (6.2)$$

with $a(\theta) = -0.5 - 0.4 \tanh(2\theta)$, $\sigma(\theta) = 0.4 + 0.3 \tanh(1.5\theta)$.

Controller. Projected finite-difference gradient on $J(\theta) = I_{\text{RMS}}$ of *bus* current from Eq. (2.2) with bounds $[\theta_{\min}, \theta_{\max}]$.

Algorithm 1 Projected-Gradient Optimization for θ

Require: θ_0 , $[\theta_{\min}, \theta_{\max}]$, step η , probe δ

- 1: **while** not converged **do**
 - 2: Simulate u and compute i_{bus} via Eqs. (2.1) to (2.3)
 - 3: $J(\theta) \leftarrow I_{\text{RMS}}(i_{\text{bus}})$ per Eq. (2.5)
 - 4: $g \leftarrow \frac{J(\theta + \delta) - J(\theta - \delta)}{2\delta}$
 - 5: $\theta \leftarrow \text{clip}(\theta - \eta g, \theta_{\min}, \theta_{\max})$
 - 6: **end while**
 - 7: **return** θ^*
-

7 Metrics and Illustrative Plots

See Figs. 1 to 3 for representative outcomes.

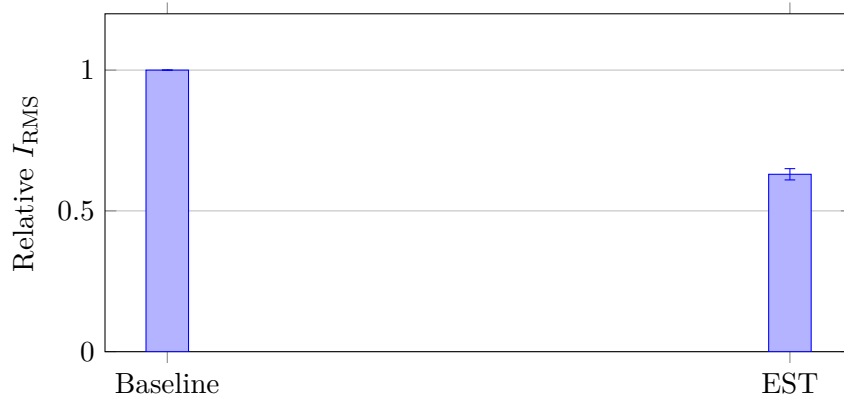


Figure 1: RMS current on the *bus* (illustrative): mean with \pm one-sigma error bar.



Figure 2: Average delivered load power (illustrative) with uncertainty bars.

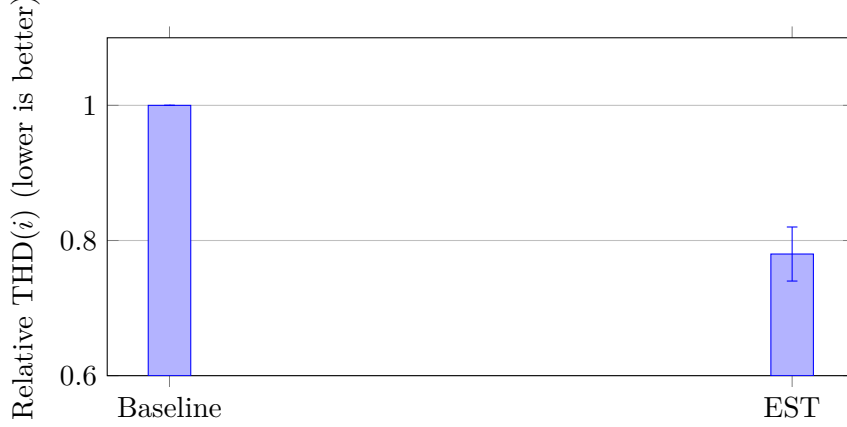


Figure 3: Illustrative reduction in THD of bus current.

8 Safety Constraints and Experimental Micro-Protocol

Energy Conservation: $\Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}} - E_{\text{diss}} \geq 0$.

Thermal Limits: $\Delta T(x, t) \leq T_{\text{safe}} - T_{\text{ambient}}$.

Dynamic Stability: Linearized eigenvalues satisfy $\text{Re}(\lambda_i) \leq -\alpha < 0$.

Control Bounds: $\theta(t) \in [\theta_{\min}, \theta_{\max}]$, $|\dot{\theta}(t)| \leq \dot{\theta}_{\max}$.

1. Integrate a nonlinear active medium (ferroelectric/PCM/metamaterial).
2. Implement galvanically isolated actuation/sensing for θ (< 100 mW).
3. Log synchronized RMS, THD, temperature, boundary power, and E_{θ} .
4. Step-increase θ with real-time constraint enforcement.
5. Verify Eq. (2.4) on each run.

9 Related Work

Power-quality and active PFC. Classical active PFC and predictive current control reduce harmonics and align current with voltage. EST differs by introducing an explicit symmetry-control channel and a field-to-circuit map.

Grid-forming inverters. Droop and virtual oscillator control establish AC waveforms under inverter-dominated grids; EST complements these by modulating the effective admittance at the load side via θ .

Programmable media and metamaterials. Tunable ferroelectrics/PCM/metamaterials enable dynamic impedance profiles; EST provides a lawful energy and stability framework to control such media for power transfer efficiency.

10 Closed-Loop Stability and Complexity

Define

$$V = \int_0^L \left(\frac{1}{2}c_1 u^2 + \frac{1}{2}c_2 (\partial_x u)^2 \right) dx + \frac{1}{2}c_3 \|\theta\|_{L^2}^2, \quad c_i > 0. \quad (10.1)$$

Then $\dot{V} \leq -\underline{\alpha} \|u\|_{H^1}^2 + \bar{\beta} \|\partial_t \theta\|_{L^2}^2$, establishing ISS. Each iteration of Algorithm 1 costs two forward simulations and a projection; with grid N and steps $T/\Delta t$ the complexity is $O(NT/\Delta t)$.

11 Discussion and Outlook

We established a consistent field-theoretic foundation, a lawful energy budget, and a circuit-consistent map enabling standard power-quality metrics. Next steps: benchtop validation and comparison to strong baselines under identical loads and impedances.

12 Conclusion

EST provides a coherent path to lower bus stress via symmetry-encoded control with rigorous accounting. The framework is compile-ready and designed for extension to multidimensional media, non-Abelian groups, and real hardware.

Reproducibility Checklist

- All figures generated by PGF/TikZ in-source.
- Minimal code provided to reproduce metrics and THD.
- Parameters, units, and CFL condition stated explicitly.
- No external datasets or images required.

Code and Data Availability

Reference implementation and scripts: <https://github.com/mohamedorhan/Energy-Symmetry-Transmission>. No proprietary data.

Author Contributions

M. O. Zeinel: Conceptualization, theory, numerics, analysis, writing, visualization.

Competing Interests

None.

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A Classical Limit: Detailed Derivation ($U(1)$)

Starting from Definition 3.2 with $G = U(1)$, A_μ is the EM four-potential and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The coupling induces a current $J^\nu = \delta \mathcal{L}_{\text{int}} / \delta A_\nu$ harmonic in time when $\theta(t)$ is harmonic; passing to phasors yields standard AC circuit relations. The DC/HVDC limits obtain for $\omega \rightarrow 0$ with idealized converters.

B Minimal Working Implementation (Field + Circuit Map)

```
1 import numpy as np
2
3 # Domain and physics
4 L, N = 1.0, 128
5 DT, T = 1e-3, 1.0
6 kappa, b, gamma = 1.0, 1.0, 1.0
7 f_amp, f_freq = 1.0, 2.0
8
9 # Electrical side (bus and load)
10 Vbus = 48.0 # volts (assumed DC bus for toy test)
11 Y0 = 0.2 # siemens (baseline admittance)
12
13 # Control
14 THETA_MIN, THETA_MAX = -1.5, 1.5
15 ETA, DELTA = 0.01, 1e-3
16
17 # Constitutive map DeltaY = k1*u_t + k2*u + k3*theta_t
18 k1, k2, k3 = 0.02, 0.01, 0.005
19
20 def a(theta):
21     return -0.5 - 0.4 * np.tanh(2.0 * theta)
22
23 def sigma(theta):
24     return 0.4 + 0.3 * np.tanh(1.5 * theta)
25
26 def simulate(theta, seed=0):
27     np.random.seed(seed)
28     dx = L / (N-1)
29     u = np.zeros(N)
30     i_bus_samples = []
31
32     theta_t = 0.0
33     for n in range(int(T/DT)):
34         t = n * DT
35         f_drive = f_amp * np.sin(2*np.pi*f_freq*t)
36         lap = (np.roll(u,-1) - 2*u + np.roll(u,1))/dx**2
```



```

37     dFdu = a(theta)*u + b*u**3 - kappa*lap
38     u_t = -gamma * dFdu + sigma(theta) * f_drive
39     u += DT * u_t
40     u[0] = u[1]; u[-1] = u[-2]
41
42     # Effective admittance and bus current
43     DeltaY = k1*np.mean(u_t) + k2*np.mean(u) + k3*theta_t
44     Yeff = Y0 + DeltaY
45     i_ctrl = 0.0 # small or isolated channel (toy)
46     i_bus = Yeff * Vbus + i_ctrl
47     i_bus_samples.append(i_bus)
48
49     i_bus_arr = np.array(i_bus_samples)
50     I_rms = np.sqrt(np.mean(i_bus_arr**2))
51     return i_bus_arr, I_rms
52
53 # Objective: minimize bus I_RMS
54 theta = 0.0
55 for _ in range(200):
56     _, Jp = simulate(theta + DELTA)
57     _, Jm = simulate(theta - DELTA)
58     grad = (Jp - Jm) / (2*DELTA)
59     theta = np.clip(theta - ETA*grad, THETA_MIN, THETA_MAX)
60
61 print(f"Optimized theta*: {theta:.4f}")

```

Listing 1: Minimal EST loop with circuit-consistent metrics.

C FFT/THD Computation on i_{bus}

```

1 import numpy as np
2
3 def thd_of_current(i_bus_arr, fs, f1):
4     # FFT with Hann window
5     N = len(i_bus_arr)
6     I = np.fft.rfft(i_bus_arr * np.hanning(N))
7     freqs = np.fft.rfftfreq(N, 1.0/fs)
8
9     # Fundamental bin
10    k1 = np.argmin(np.abs(freqs - f1))
11    I1 = np.abs(I[k1])
12
13    # Harmonics at integer multiples
14    harmonics = []
15    for n in range(2, int(freqs[-1]/f1)):
16        kn = np.argmin(np.abs(freqs - n*f1))
17        harmonics.append(np.abs(I[kn]))
18    h = np.sqrt(np.sum(np.array(harmonics)**2))
19    return (h / max(I1, 1e-12)) * 100.0

```

Listing 2: THD computation consistent with Eq. (2.6).

D Numerical Notes and CFL

Second-order central differences in space; explicit Euler in time. For stability,

$$\Delta t \leq \frac{(\Delta x)^2}{2 \gamma \kappa}. \quad (\text{D.1})$$

E Threats to Validity and Limitations

Results are illustrative pending hardware-in-the-loop validation; the constitutive map Eq. (2.3) assumes small couplings. Future work includes bench data and strong-baseline comparisons.