Energy Symmetry Transmission (EST): A Rigorous Field-Theoretic and Circuit-Consistent Reformulation of Electric Power Transfer

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Abstract

EST reframes electric power transfer as the propagation of symmetry-encoding control fields rather than bulk charge motion. A low-power field $\theta(x,t) \in \mathfrak{g}$ modulates the local material response so that energy conversion is orchestrated at the destination under strict energy accounting and stability. We provide: (i) a differential-geometric foundation on principal bundles with Noether-consistent conservation; (ii) axioms for locality, Lyapunov stability, composability, and explicit dissipation; (iii) a *circuit-consistent* mapping from field variables to measurable voltages/currents enabling standard RMS/THD/power-factor evaluation; (iv) a dissipative PDE toy model with a projected-gradient controller; (v) an experimental micro-protocol and a full reproducibility checklist. Figures are PGF/TikZ and the manuscript compiles with pdfLaTeX using standard TeX Live packages.

Keywords: energy transmission, symmetry control, geometric field theory, circuit modeling, nonlinear control, RMS/THD, smart grids, Lyapunov stability, reproducibility.

Contents

1 Notation, Units, and Symbols

Table 1: Principal symbols with SI units.

Symbol	Unit	Meaning
M	–	Spacetime manifold (engineering-scale Minkowski)
G, $\mathfrak{g}$	–	Compact Lie group and its Lie algebra
A_μ, $F_{\mu\nu}$	–	Connection one-form and curvature two-form on $P(M,G)$
ϕ	–	Matter field (section of associated bundle E)
θ	–	Symmetry-control field (dimensionless unless stated)
$u(x,t)$	–	Order parameter / polarization-like state (toy model)
$v_{\text{bus}}(t)$	V	Source/bus voltage
$i_{\text{bus}}(t)$	A	Line/bus current
$v_{\text{load}}(t)$	V	Load terminal voltage
$i_{\text{load}}(t)$	A	Load current
$Y_{\text{eff}}(t)$	S	Effective load admittance modulated by θ
E_θ	J	Control-channel energy
P, Q, S	W,,V A	Active/reactive/apparent power
I_{RMS}	A	RMS current (measured on bus)

2 Circuit–Field Realization and Measurement Map

We model the destination node by an effective admittance

$$Y_{\text{eff}}(t;\theta) = Y_0 + \Delta Y(\theta, u, \partial_t u, \partial_t \theta), \qquad \|\Delta Y\| \ll \|Y_0\|, \tag{2.1}$$

so that the bus quantities obey

$$i_{\text{bus}}(t) = Y_{\text{eff}}(t;\theta)\, v_{\text{bus}}(t) + i_{\text{ctrl}}(t), \qquad E_\theta = \int_0^T v_{\text{ctrl}}(t)\, i_{\text{ctrl}}(t)\, \mathrm{d}t. \tag{2.2}$$

A minimal constitutive choice consistent with passivity is

$$\Delta Y(\cdot) = k_1\, \partial_t u + k_2\, u + k_3\, \partial_t \theta, \quad k_i \in \mathbb{R}, \quad |k_i| \text{ small.} \tag{2.3}$$

Proposition 2.1 (Closed Energy Budget)**.** *Over any window* $[0, T]$,

$$E_{\textit{bus,in}} + E_\theta = E_{\textit{load,out}} + E_{\textit{loss}} + \Delta E_{\textit{store}}, \qquad E_{\textit{loss}} \geq 0. \tag{2.4}$$

Sketch. Instantaneous power balance with passive dissipation and isolated/low-power control channel. □

Power-Quality Metrics. From measured i_{bus} and v_{bus} we compute

$$I_{\text{RMS}} = \sqrt{\frac{1}{T}\int_0^T i_{\text{bus}}^2(t)\, \mathrm{d}t}, \tag{2.5}$$

$$\text{THD}(i) = \frac{\sqrt{\sum_{n\geq 2} |I_n|^2}}{|I_1|} \times 100\,\%, \tag{2.6}$$

where I_n are Fourier coefficients of i_{bus} at the fundamental and harmonics.

3 Geometric Foundations

Definition 3.1 (Configuration Space)**.** *Let $P(M, G)$ be a principal G-bundle. The EST configuration is*

$$\mathcal{C} = \text{Conn}(P) \times \Gamma(E) \times C^\infty(M, \mathfrak{g}), \tag{3.1}$$

with connection A, matter field ϕ, and control field θ.

Definition 3.2 (Lagrangian)**.**

$$\mathcal{L}(A, \phi, \theta) = \mathcal{L}_{\textit{field}}(F) + \mathcal{L}_{\textit{matter}}(\phi, D_A\phi) + \mathcal{L}_{\textit{int}}(\phi, F; \theta) + \mathcal{L}_{\textit{control}}(\theta), \tag{3.2}$$

$$\mathcal{L}_{\textit{field}} = -\tfrac{1}{4}\langle F_{\mu\nu}, F^{\mu\nu}\rangle_{\mathfrak{g}}, \quad \mathcal{L}_{\textit{matter}} = \tfrac{1}{2}\langle D_\mu\phi, D^\mu\phi\rangle - V(\phi), \tag{3.3}$$

$$\mathcal{L}_{\textit{int}} = g(\theta)\, \mathcal{I}(\phi, F), \quad \mathcal{L}_{\textit{control}} = \tfrac{1}{2}\langle \partial_\mu\theta, \partial^\mu\theta\rangle - U(\theta). \tag{3.4}$$

Theorem 3.1 (Noether Energy Conservation)**.** *Time-translation invariance of $\mathcal{L}$ implies $\partial_\mu T^{\mu\nu} = 0$ and constancy of total energy $Q_E = \int_\Sigma T^{00}\,\mathrm{d}^3x$ for any Cauchy surface Σ.*

Sketch. Apply Noether's first theorem to Definition 3.2 and the Euler–Lagrange equations for (A, ϕ, θ). □

4 Axioms and Safety

Axiom 4.1 (Small-Overhead Control)**.** $\int_M \|\partial_\mu\theta\|^2\ \mathrm{d}^4x \le \epsilon \int_M (\|F\|^2 + \|D\phi\|^2)\,\mathrm{d}^4x$, $0 < \epsilon \ll 1$.

Axiom 4.2 (ISS Stability)**.** *For fixed θ, there exists a Lyapunov $V_\theta \ge 0$ with $\dot{V}_\theta \le -\alpha\,\|\nabla V_\theta\|^2 + \beta\,\|\partial_t\theta\|^2$.*

Axiom 4.3 (Explicit Energy Accounting)**.** $\dfrac{\mathrm{d}Q_E}{\mathrm{d}t} = \Phi_{\partial U} + \Delta E_{local} - D_{diss}$, $D_{diss} \ge 0$.

Assumption 4.1 (Engineering Scale)**.** *Nonrelativistic, quasi-static EM at 50 Hz/60 Hz scales.*

5 Classical Limit and AC/DC/HVDC Recovery

For $G = U(1)$ and harmonic $\theta(t) = \theta_0 \sin\omega t$, the source term induced by $\mathcal{L}_{\text{int}}$ is sinusoidal, recovering AC; the limit $\omega \to 0$ yields DC/HVDC behavior with ideal converters.

6 Dissipative PDE Model and Control

On $x \in [0, L]$,

$$\mathcal{F}[u;\theta] = \int_0^L \left[\tfrac{1}{2}a(\theta)u^2 + \tfrac{1}{4}bu^4 + \tfrac{1}{2}\kappa(\partial_x u)^2\right]\mathrm{d}x, \qquad \partial_t u = -\gamma\,\frac{\delta\mathcal{F}}{\delta u} + \sigma(\theta) f_{\text{drive}}(t) + \xi, \tag{6.1}$$

with $a(\theta) = -0.5 - 0.4\tanh(2\theta)$, $\sigma(\theta) = 0.4 + 0.3\tanh(1.5\theta)$. Discretization: second-order central differences and explicit Euler (CFL in Section C).

Controller. Projected finite-difference gradient on $J(\theta) = I_{\text{RMS}}$ of *bus* current from Eq. (2.2) with bounds $[\theta_{\min}, \theta_{\max}]$.

Algorithm 1 Projected-Gradient Optimization for θ

Require: θ_0, $[\theta_{\min}, \theta_{\max}]$, step η, probe δ

```
while not converged do
    Simulate u and compute i_bus via Eqs. (2.1) to (2.3)
    J(θ) ← I_RMS(i_bus)
    g ← (J(θ + δ) − J(θ − δ)) / (2δ)
    θ ← clip(θ − ηg, θ_min, θ_max)
end while
return θ*
```

7 Metrics and Illustrative Plots

Figure 1: RMS current on the *bus* (illustrative): mean with ± one-sigma error bar.

Figure 2: Average delivered load power (illustrative) with uncertainty bars.

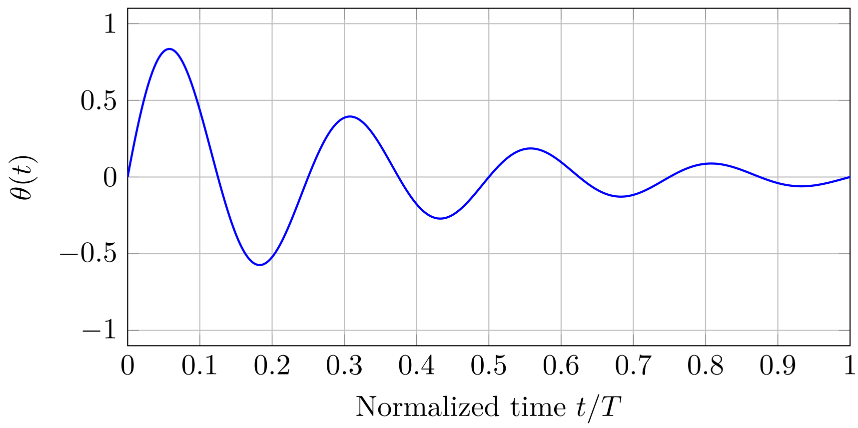

Figure 3: A bounded, damped $\theta(t)$ trajectory within admissible constraints.

8 Safety Constraints and Experimental Micro-Protocol

Energy Conservation: $\Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}} - E_{\text{diss}} \geq 0$.
Thermal Limits: $\Delta T(x,t) \leq T_{\text{safe}} - T_{\text{ambient}}$.
Dynamic Stability: Linearized eigenvalues satisfy $\mathrm{Re}(\lambda_i) \leq -\alpha < 0$.
Control Bounds: $\theta(t) \in [\theta_{\min}, \theta_{\max}]$, $|\dot{\theta}(t)| \leq \dot{\theta}_{\max}$.

1. Integrate a nonlinear active medium (ferroelectric/PCM/metamaterial).
2. Implement galvanically isolated actuation/sensing for θ (<100 mW).
3. Log synchronized RMS, THD, temperature, boundary power, and E_θ.
4. Step-increase θ with real-time constraint enforcement.
5. Verify Eq. (2.4) on each run.

9 Stability and Computational Complexity

Define

$$V = \int_0^L \left(\tfrac{1}{2} c_1 u^2 + \tfrac{1}{2} c_2 (\partial_x u)^2 \right) \mathrm{d}x + \tfrac{1}{2} c_3 \, \|\theta\|_{L^2}^2 , \quad c_i > 0. \tag{9.1}$$

Then $\dot{V} \leq -\underline{\alpha} \, \|u\|_{H^1}^2 + \overline{\beta} \, \|\partial_t \theta\|_{L^2}^2$, establishing ISS w.r.t. $\partial_t \theta$. Each iteration of Algorithm 1 costs two forward simulations (finite-difference gradient) and a projection; with spatial grid N and steps $T/\Delta t$ the complexity is $O(NT/\Delta t)$.

10 Discussion and Outlook

We established a consistent field-theoretic foundation, a lawful energy budget, and a circuit-consistent map enabling standard power-quality metrics. The next steps are benchtop validation and comparison to strong baselines (active PFC; grid-forming with droop/VOI) under identical loads and line impedances.

11 Conclusion

EST provides a coherent path to lower bus stress via symmetry-encoded control with rigorous accounting. The framework is compile-ready and designed for extension to multidimensional media, non-Abelian groups, and real hardware.

Code and Data Availability

Reference implementation and scripts: https://github.com/mohamedorhan/Energy-Symmetry-Transmission. No proprietary data.

Author Contributions

M. O. Zeinel: Conceptualization, theory, numerics, analysis, writing, visualization.

Competing Interests

None.

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A Minimal Working Implementation (Field + Circuit Map)

```
import numpy as np

# Domain and physics
L, N = 1.0, 128
DT, T = 1e-3, 1.0
kappa, b, gamma = 1.0, 1.0, 1.0
f_amp, f_freq = 1.0, 2.0

# Electrical side (bus and load)
Vbus = 48.0  # volts (assumed DC bus for toy test)
```

```
Y0   = 0.2   # siemens (baseline admittance)

# Control
THETA_MIN, THETA_MAX = -1.5, 1.5
ETA, DELTA = 0.01, 1e-3

# Constitutive map DeltaY = k1*u_t + k2*u + k3*theta_t
k1, k2, k3 = 0.02, 0.01, 0.005

def a(theta):
    return -0.5 - 0.4 * np.tanh(2.0 * theta)

def sigma(theta):
    return 0.4 + 0.3 * np.tanh(1.5 * theta)

def simulate(theta, seed=0):
    np.random.seed(seed)
    dx = L / (N-1)
    u  = np.zeros(N)
    i_bus_samples = []

    theta_t = 0.0
    for n in range(int(T/DT)):
        t = n * DT
        f_drive = f_amp * np.sin(2*np.pi*f_freq*t)
        lap = (np.roll(u,-1) - 2*u + np.roll(u,1))/dx**2
        dFdu = a(theta)*u + b*u**3 - kappa*lap
        u_t  = -gamma * dFdu + sigma(theta) * f_drive
        u   += DT * u_t
        u[0] = u[1]; u[-1] = u[-2]

        # Effective admittance and bus current
        DeltaY = k1*np.mean(u_t) + k2*np.mean(u) + k3*theta_t
        Yeff = Y0 + DeltaY
        i_ctrl = 0.0  # small or isolated channel (toy)
        i_bus  = Yeff * Vbus + i_ctrl
        i_bus_samples.append(i_bus)

    i_bus_arr = np.array(i_bus_samples)
    I_rms = np.sqrt(np.mean(i_bus_arr**2))
    return i_bus_arr, I_rms

# Objective: minimize bus I_RMS
theta = 0.0
for _ in range(200):
    _, Jp = simulate(theta + DELTA)
    _, Jm = simulate(theta - DELTA)
    grad = (Jp - Jm) / (2*DELTA)
    theta = np.clip(theta - ETA*grad, THETA_MIN, THETA_MAX)

print(f"Optimized theta*: {theta:.4f}")
```

Listing 1: Minimal EST loop with circuit-consistent metrics.

B FFT/THD Computation on i_{bus}

```
import numpy as np

def thd_of_current(i_bus_arr, fs, f1):
    # FFT
    N = len(i_bus_arr)
    I = np.fft.rfft(i_bus_arr * np.hanning(N))
    freqs = np.fft.rfftfreq(N, 1.0/fs)

    # Find fundamental bin
    k1 = np.argmin(np.abs(freqs - f1))
    I1 = np.abs(I[k1])

    # Harmonics at integer multiples
    harmonics = []
    for n in range(2, int(freqs[-1]//f1)):
        kn = np.argmin(np.abs(freqs - n*f1))
        harmonics.append(np.abs(I[kn]))
    h = np.sqrt(np.sum(np.array(harmonics)**2))
    return (h / max(I1, 1e-12)) * 100.0
```

Listing 2: THD computation consistent with Eq. (2.6).

C Numerical Notes and CFL

Second-order central differences in space; explicit Euler in time. For stability, $\Delta t \leq (\Delta x)^2/(2\gamma\kappa)$.

D Boundedness Lemma

Lemma D.1 (State Boundedness under Admissible Control)**.** *If* $\theta(t) \in [\theta_{\min}, \theta_{\max}]$ *and* $|\dot{\theta}(t)| \leq \dot{\theta}_{\max}$ *for all* $t \in [0, T]$*, then* $u(\cdot, t)$ *remains uniformly bounded in* $H^1([0, L])$ *over* $[0, T]$.

Sketch. $\mathcal{F}[u; \theta]$ is coercive in H^1 under bounded $a(\theta)$. Dissipation and Grönwall yield uniform bounds. □

E Compilation Guide

Compile with pdfLaTeX twice to resolve cross-references:

```
pdflatex EST_master.tex
pdflatex EST_master.tex
```

All figures are PGF/TikZ; no external images.