pca

March 6, 2024

[26]:
```
import numpy as np
import imageio as iio
import DatasetSplitter
from sklearn.neighbors import KNeighborsClassifier
from sklearn.metrics import accuracy_score
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA as pca_test
```

1 Splitting images to training data and testing data

[2]:
```
trainingData, trainingLabels, testingData, testingLabels = DatasetSplitter.
 ↪splitData()
```

2 Computing covariance matrix

[3]:
```
def covariance(D):
    global mean_vector
    mean_vector = np.mean(D, axis=0)
    Z = D - mean_vector
    cov = (1/len(D)) * (Z.T @ Z)
    return cov
```

[4]:
```
# try
# def covariance(D):
#     global mean_vector
#     mean_vector = np.mean(D, axis=0)
#     global Z
#     Z = D - mean_vector
#     cov = (1/len(D))*(Z @ Z.T)
#     return cov
```

3 Testing computational correctness of covariance matrix

[5]:
```
# Test cov matrix
trainingData = np.array(trainingData)
myCov = covariance(trainingData)
numCov = np.cov(trainingData.T, bias=True)
# print(numCov)
```

4 Applying PCA algorithm to obtain P -> projection matrix

[6]:
```
def PCA(D, alpha):
    cov = covariance(D)
    eigenvalues, eigenvectors = np.linalg.eigh(cov)

    sorted_idx = eigenvalues.argsort()[::-1]  # Sort in descending order
    eigenvalues = eigenvalues[sorted_idx]
    eigenvectors = eigenvectors[:, sorted_idx]

    trace = np.trace(cov)
    c = 0 # Accumulator for sum of eigenvaleus
    lastIndex = 0
    for index in range(len(eigenvalues)):
        c += eigenvalues[index]
        if c / trace >= alpha :
            lastIndex = index
            break
    P = eigenvectors[:lastIndex + 1, :]# Projection Matrix
    return P
```

[7]:
```
# # try
# def PCA(D, alpha):
#     cov = covariance(D)
#     eigenvalues, eigenvectors = np.linalg.eigh(cov)

#     sorted_idx = eigenvalues.argsort()[::-1]  # Sort in descending order
#     eigenvalues = eigenvalues[sorted_idx]
#     eigenvectors = eigenvectors[:, sorted_idx]

#     eigenvectors = Z.T @ eigenvectors

#     eigenvectors = eigenvectors.T/ np.sqrt((eigenvectors.T ** 2).sum(axis=1,␣
↪keepdims=True))

#     trace = sum(eigenvalues)
#     c = 0 # Accumulator for sum of eigenvaleus
#     lastIndex = 0
```

```
#     for index in range(len(eigenvalues)):
#         c += eigenvalues[index]
#         if c / trace >= alpha :
#             lastIndex = index
#             break
#     P = eigenvectors[:lastIndex + 1, :]# Projection Matrix
#     return P
```

[8]:
```
def projectData(trainData, testData, alpha):
    P = PCA(trainData, alpha)
    # Project training data (correctly subtract mean before projection)
    projectedTrainingData = (trainData - mean_vector) @ P.T

    # Project testing data (correctly subtract mean before projection)
    projectedTestingData = (testData - mean_vector) @ P.T

    return projectedTrainingData, projectedTestingData
```

5 Using KNN classifier, tie breaking at distance strategy

[9]:
```
def classifyKNN(projectedTrainData, trainLabel, projectedTestData, testLabel,␣
 ↪k):
    knn = KNeighborsClassifier(n_neighbors=k, weights='distance')
    knn.fit(projectedTrainData, trainLabel)
    # Calc accuracy of clssification
    predictedClasses = knn.predict(projectedTestData)
    accuracy = accuracy_score(testLabel, predictedClasses)
    print(f"Accuracy {100 * (accuracy)}")
    return accuracy
```

6 Main Flow

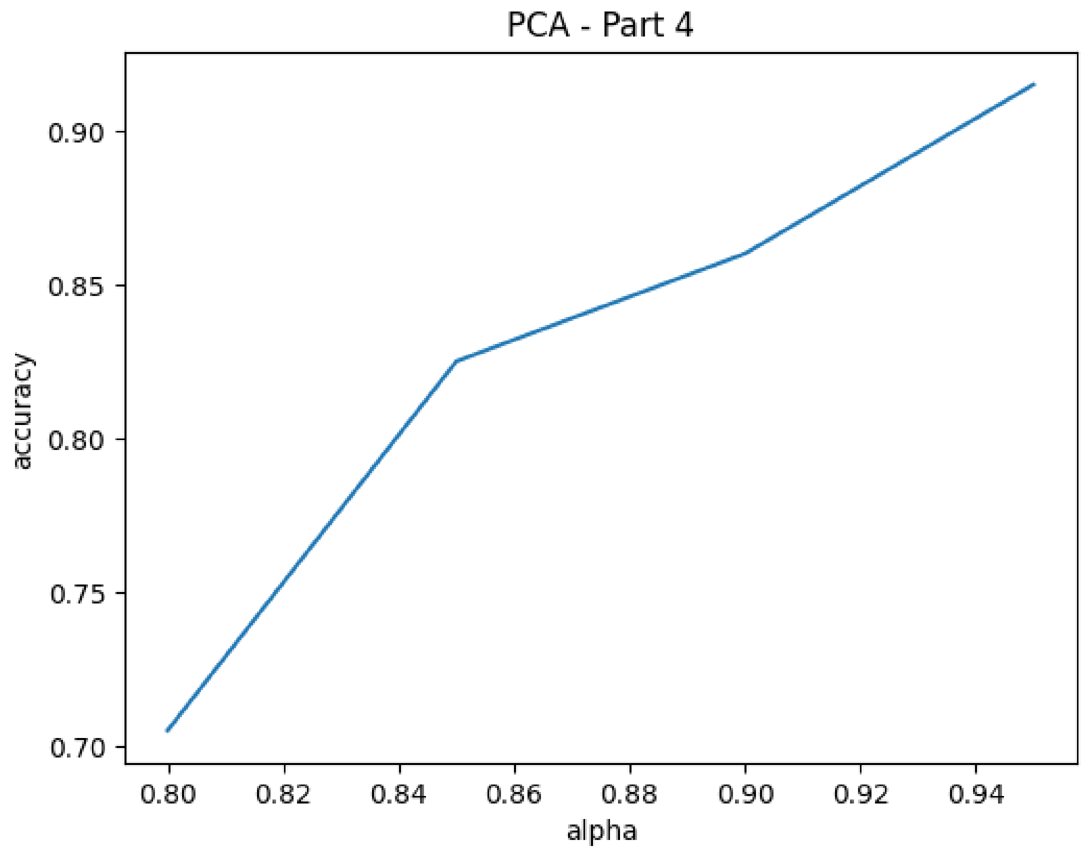
[10]:
```
alpha_list = np.array([0.8, 0.85, 0.9, 0.95])
accuracy_results = np.array([])
k = 1
for alpha in alpha_list:
    # Train the model
    projectedTrainingData, projectedTestingData = projectData(trainingData,␣
 ↪testingData, alpha)
    # Classify and test
    accuracy_results = np.append(accuracy_results, classifyKNN(
        projectedTrainingData, trainingLabels,
        projectedTestingData, testingLabels,
        k
    ))
```

```
Accuracy 70.5
Accuracy 82.5
Accuracy 86.0
Accuracy 91.5
```

[11]:
```
# Deep copy of results as it'll be used later
accuracyOfEqualSplit = np.array(accuracy_results)
```

7 Plot the results

[12]:
```
# plotting the points
plt.plot(alpha_list, accuracy_results)
plt.xlabel('alpha')
plt.ylabel('accuracy')
plt.title('PCA - Part 4')
plt.show()
```



According to this plot, accuracy is proportional to alpha.

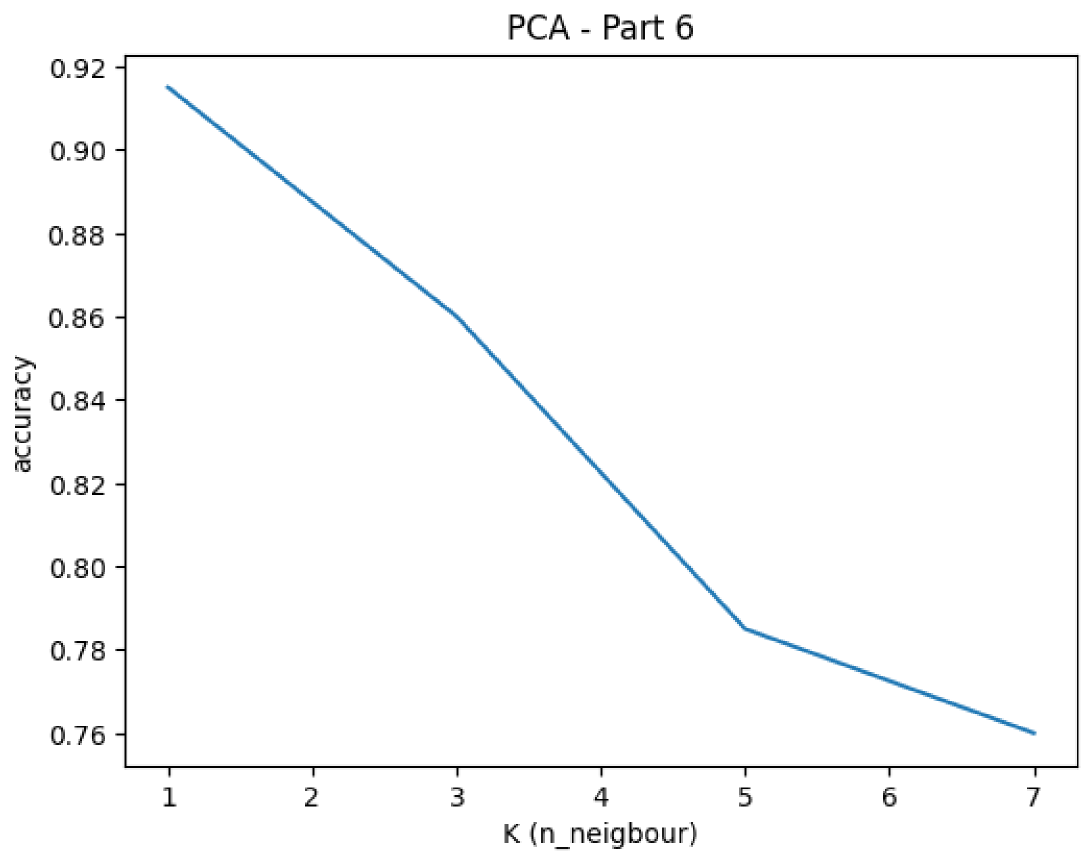
8 Hyper-parameter tuning

[13]:
```
k_list = np.array([1, 3, 5, 7])
accuracy_results = np.array([])
alpha = 0.95
for k in k_list:
    # Train the model
    projectedTrainingData, projectedTestingData = projectData(trainingData,␣
 ↪testingData, alpha)
    # Classify and test
    accuracy_results = np.append(accuracy_results, classifyKNN(
        projectedTrainingData, trainingLabels,
        projectedTestingData, testingLabels,
        k
    ))
```

```
Accuracy 91.5
Accuracy 86.0
```

```
Accuracy 78.5
Accuracy 76.0
```

[14]:
```python
# plotting the points
plt.plot(k_list, accuracy_results)
plt.xlabel('K (n_neigbour)')
plt.ylabel('accuracy')
plt.title('PCA - Part 6')
plt.show()
```



In our case the best K is K = 1

9 Bonus

10 Using randomized PCA for fast computation

The time complexity of randomized PCA is O(n * p^2) + O(p^3) where p is the number of PCs. Meanwhile, The time complexity of traditional PCA is O(n * d^2) + O(d^3)

[15]:
```
k = 1
numPca = pca_test(n_components=114, svd_solver='randomized')
transformedTrainingData = numPca.fit_transform(trainingData)
transformedTestingData = numPca.transform(testingData)
accuracy = classifyKNN(
        transformedTrainingData, trainingLabels,
        transformedTestingData, testingLabels,
        k
    )
print(f"Randomized PCA at K = 1")
```

```
Accuracy 93.0
Randomized PCA at K = 1
```

The accuracy is better than mine but In general it's less than the traditional PCA.

The reason why our PCA is less better tha pyhon's one is the propagation error (truncation error) at calculating the covariance matrix

11 Using different split of data (70% for training - 30 % for testing)

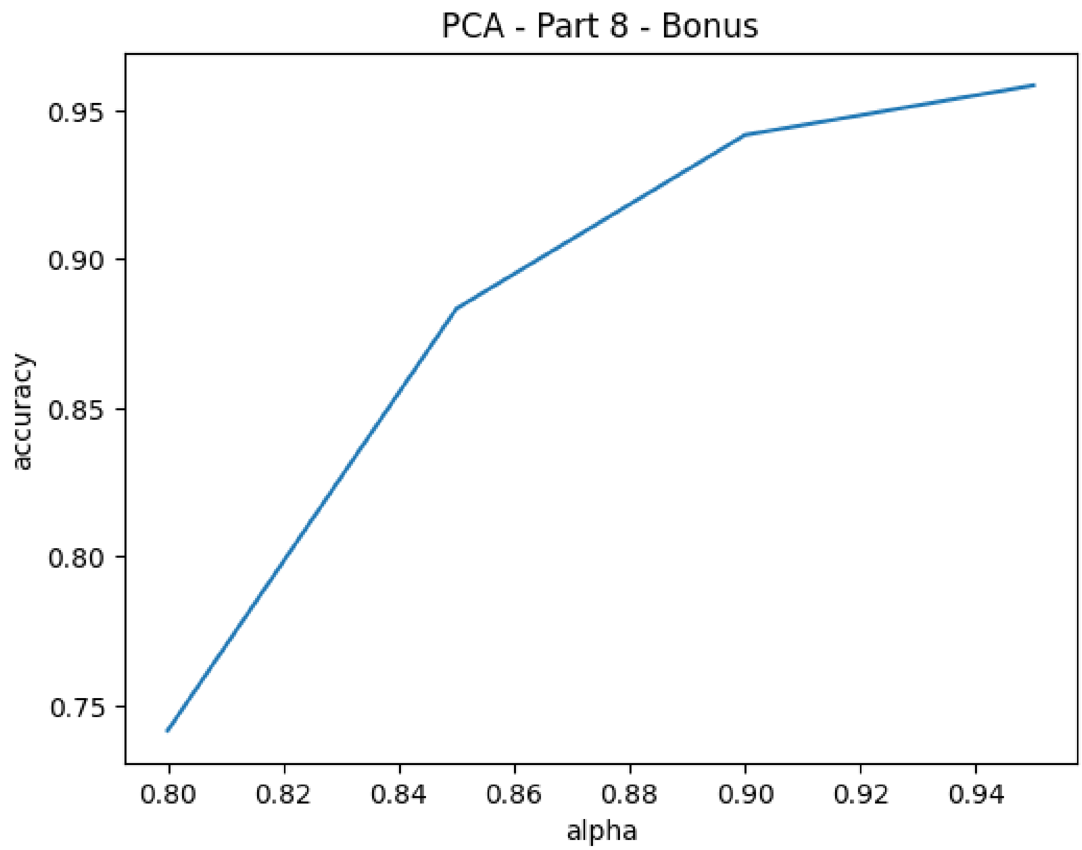
[16]:
```
trainingData, trainingLabels, testingData, testingLabels = DatasetSplitter.
 ↪differentSplitData()
```

[17]:
```
alpha_list = np.array([0.8, 0.85, 0.9, 0.95])
accuracy_results = np.array([])
k = 1
for alpha in alpha_list:
    # Train the model
    projectedTrainingData, projectedTestingData = projectData(trainingData,␣
 ↪testingData, alpha)
    # Classify and test
    accuracy_results = np.append(accuracy_results, classifyKNN(
        projectedTrainingData, trainingLabels,
        projectedTestingData, testingLabels,
        k
    ))
```

```
Accuracy 74.16666666666667
Accuracy 88.33333333333333
Accuracy 94.16666666666667
Accuracy 95.83333333333334
```

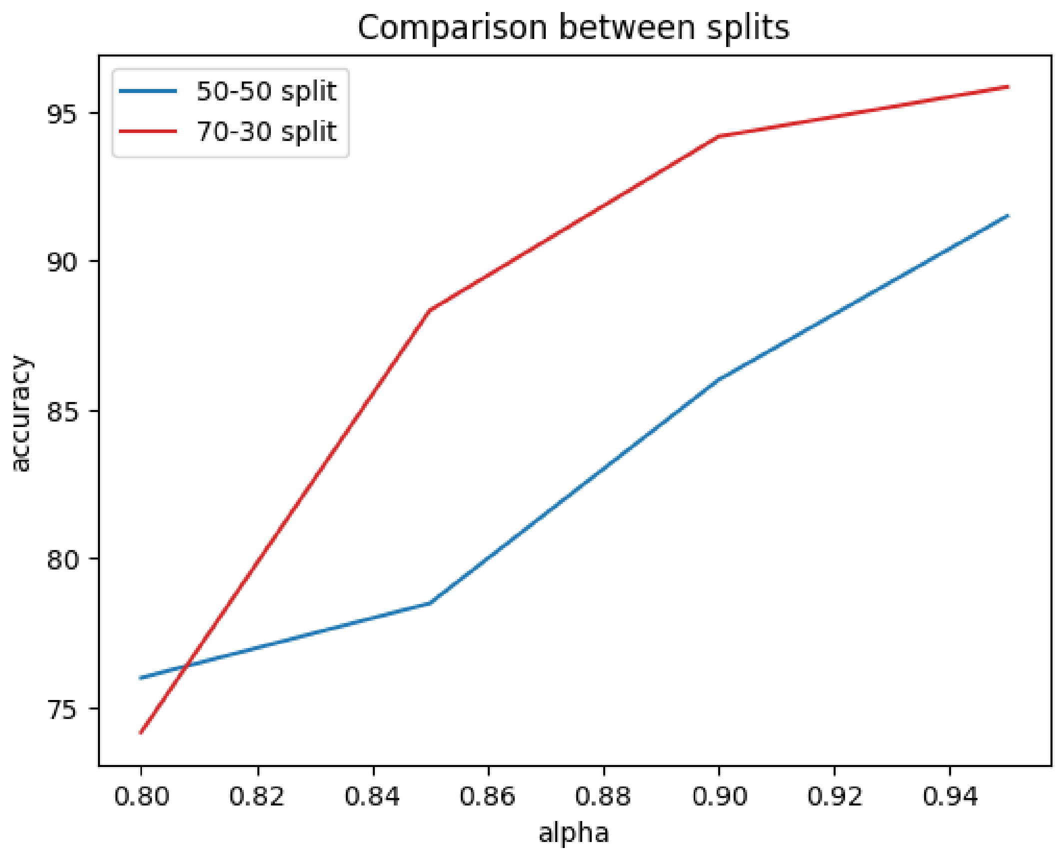
12 Plot the results

[18]:
```
# plotting the points
plt.plot(alpha_list, accuracy_results)
plt.xlabel('alpha')
plt.ylabel('accuracy')
plt.title('PCA - Part 8 - Bonus')
plt.show()
```



[19]:
```
alpha_list = np.array([0.8, 0.85, 0.9, 0.95])
accuracyOfEqualSplit = np.array([76.0, 78.5, 86.0, 91.5])
accuracyOfDifferentSplit = np.array([74.16666666666667, 88.33333333333333, 94.
↪16666666666667, 95.83333333333334])

fig, ax = plt.subplots()

# Plot both lines on the same axes
ax.plot(alpha_list, accuracyOfEqualSplit, color='tab:blue', label='50-50 split')
ax.plot(alpha_list, accuracyOfDifferentSplit, color='tab:red', label='70-30␣
↪split')
```

```python
# Set labels and title
ax.set_xlabel('alpha')
ax.set_ylabel('accuracy')
plt.title('Comparison between splits')

# Add a legend
ax.legend()

plt.show()
```



13 Dual PCA

Theoretically, basic PCA and Dual PCA have similar accuracy. In practice, if the number of samples (n) is much smaller than the number of dimensions (d), Dual PCA is more computationally efficient.

However, if n is larger than or equal to d, basic PCA is more efficient and avoids potential issues with finding inverse of singular values diagonal matrix.

Time complexity of basic PCA : O(n * d^2) + O(d^3) -> Z.T @ Z, eig(cov)

Time complexity of dual PCA : O(d * n^2) + O(n^3) -> Z @ Z.T, svd(cov)

other terms are order of r (reduced dimensions) so it's negligible.

Calculate the scatter-matrix to make use of singular value decomposition (SVD)

[21]:
```
def covariance(D):
    global mean_vector
    mean_vector = np.mean(D, axis=0)
    Z = D - mean_vector
    cov = (Z @ Z.T)
    return cov
```

14 Dual PCA implementation

[22]:
```
def DualPCA(D, testingData, alpha): # D is n x d
    cov = covariance(D)
    global eigenvectors, eigenvalues, eigenvectorsTranspose                # U␣
 ↪-> n x rank
    eigenvectors, eigenvalues, eigenvectorsTranspose = np.linalg.svd(cov) #␣
 ↪With resc pect to Z @ Z.T
    eigenvalues = np.sqrt(eigenvalues) # Sigma

    sorted_idx = eigenvalues.argsort()[::-1]  # Sort in descending order
    eigenvalues = eigenvalues[sorted_idx]
    eigenvectors = eigenvectors[:, sorted_idx]

    # Project training Data Z . V = U . Sigma
    trace = sum(eigenvalues)

    c = 0 # Accumulator for sum of eigenvaleus
    lastIndex = 0
    for index in range(len(eigenvalues)):
        c += eigenvalues[index]
        if c / trace >= alpha :
            lastIndex = index
            break

    # r -> reduced
    Ur = eigenvectors[:, :lastIndex + 1] # U` (reduced)
    sigmar = np.diag(eigenvalues[:lastIndex + 1])
    projectedTrainingData = Ur @ sigmar

    # Project testing
    Z = D - mean_vector
    testingData = testingData - mean_vector

    epsilon = 1e-10  # Small constant
```

```
    # Check if an element is zero consider it eps o.w leave it as it is.
    inverse_matrix = np.diag(np.where(eigenvalues[:lastIndex + 1] == 0, 1 /␣
↪epsilon,
                            np.reciprocal(eigenvalues[:lastIndex + 1])))

    projectionMatrix  =  Z.T @ Ur @ inverse_matrix
    projectedTestingData = testingData @ projectionMatrix

    return projectedTrainingData, projectedTestingData
```

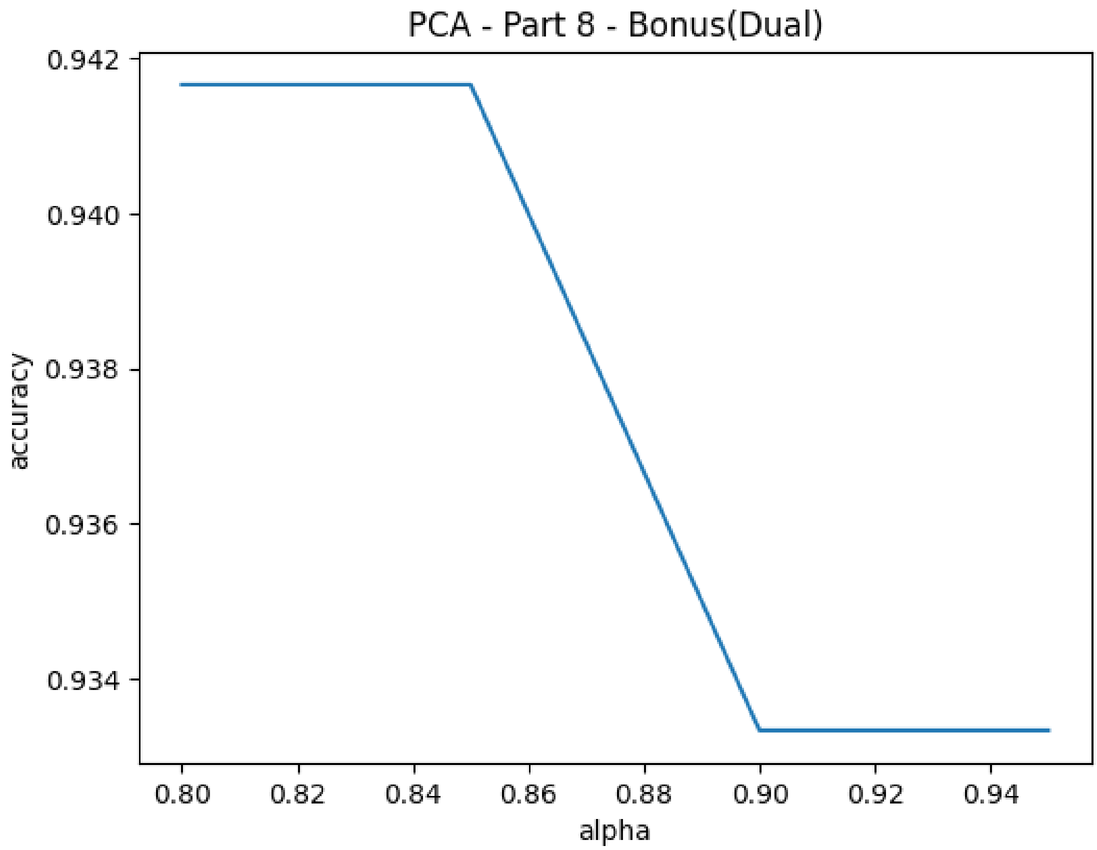
15 Testing dual PCA

[30]:
```
alpha_list = np.array([0.8, 0.85, 0.9, 0.95])
accuracy_results_dual = np.array([])
k = 1
for alpha in alpha_list:
    # Train the model
    projectedTrainingData, projectedTestingData = DualPCA(trainingData,␣
↪testingData, alpha)
    # Classify and test
    accuracy_results_dual = np.append(accuracy_results_dual, classifyKNN(
        projectedTrainingData, trainingLabels,
        projectedTestingData, testingLabels,
        k
    ))
```

```
Accuracy 94.16666666666667
Accuracy 94.16666666666667
Accuracy 93.33333333333333
Accuracy 93.33333333333333
```

16 Plotting

[32]:
```
# plotting the points
plt.plot(alpha_list, accuracy_results_dual)
plt.xlabel('alpha')
plt.ylabel('accuracy')
plt.title('PCA - Part 8 - Bonus(Dual)')
plt.show()
```


PCA - Part 8 - Bonus(Dual)
0.942
0.940
0.938
0.936
0.934
accuracy
0.80
0.82
0.84
0.86
0.88
0.90
0.92
0.94
alpha