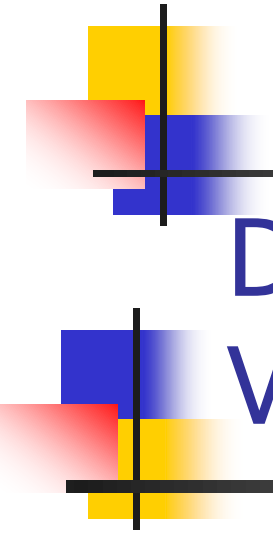
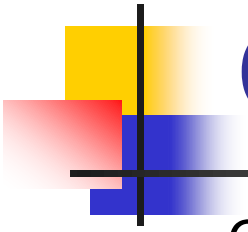


# DMET 702
# Visualization and Animation

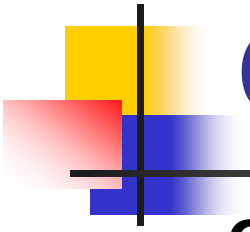
**Geometric Techniques**
**(Multi-dimensional Visualization)**

**Assoc. Prof. Dr. Rimon Elias**



# Contents

- Geometric techniques:
  - Scatterplot matrices
  - Projection pursuit technique
  - Prosection views
  - Hypersclices
  - Parallel coordinates



# Geometric Techniques

- **Geometric techniques** are used to visualize multi-dimensional data.

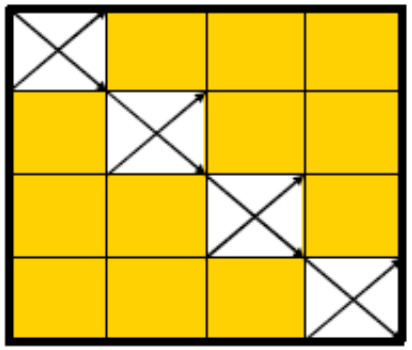
- Among the geometric techniques are:
  - Scatterplot matrices
  - Projection pursuit technique
  - Prosection views
  - Hypersclices
  - Parallel coordinates

# Scatterplot Matrices

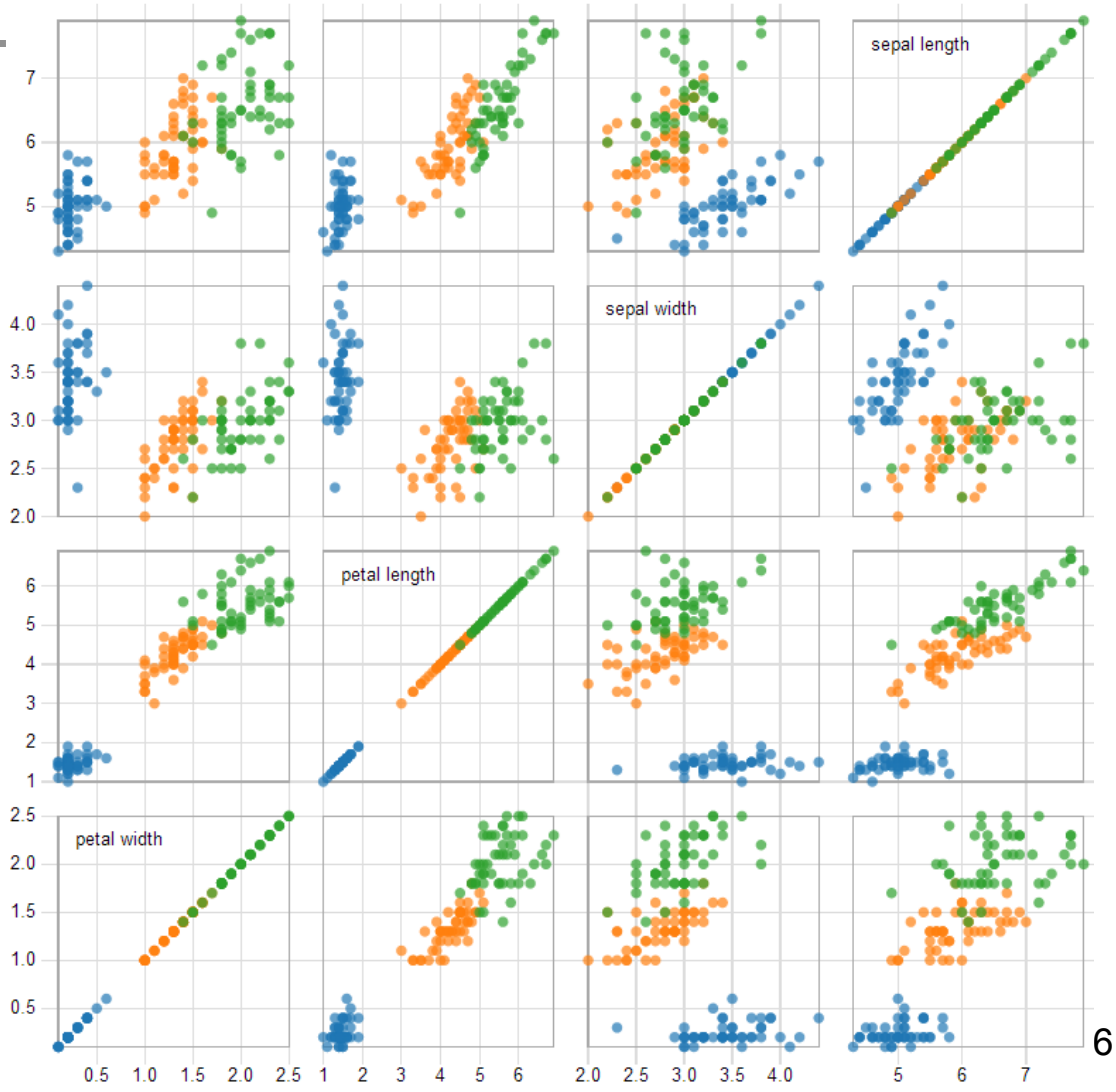
- Papers:
  - Andrews D. F., "Plots of High-Dimensional Data," in: Biometrics, vol. 29, pp. 125-136, 1972.
  - Cleveland W. S., "Visualizing Data," AT&T Bell Laboratories, Murray Hill, NJ, Hobart Press, Summit NJ, 1993.

# Scatterplot Matrices

- The idea:
  - Build a matrix of scatterplots where each attribute is plotted against each other attribute.
  - For n attributes, there should be nxn plots.
  - Diagonal maps an attribute to itself.
  - Each pair is plotted twice; on each side of the diagonal.
  - For *n*D data, the number of unique scatterplots is: $\frac{n(n-1)}{2}$
- Using scatterplots, it is easy to compare one attribute to other attributes.



# Scatterplot Matrices: An Example



# Projection Pursuit Technique

- Papers:
  - Friedman, J.H., and Tukey, J.W., "A projection pursuit algorithm for exploratory data analysis," IEEE Trans. on Computers, Vol. C-23, No. 9, pp. 881-889, 1974.
  - S. Crawford and T. Fall, "Projection pursuit techniques for visualizing high-dimensional data sets," in G. M. Nielson, B. Shriver and L. J. Rosenblum (eds.) Visualization in Scientific Computing IEEE Computer Society Press, pp. 94-108, 1990.

# Projection Pursuit Technique

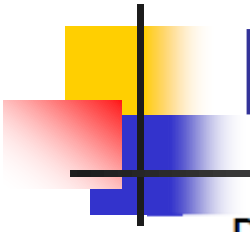
- Such techniques are for finding meaningful projections of multidimensional data so as to project the data to 1D, 2D or 3D space for easy pattern visualization.
- The simplest form is to display data over two dimensions at a time, which is the scatterplot form we have just studied.
- A given projection can be characterized by a numerical index that indicates the amount of structure that is present.
- Such an index can be used as the basis to find interesting projections.

# Projection Pursuit Technique

- There is a clear separation of these data into two clusters.
- Note that the direction of maximum variance is vertical. This is called the principal component.

- If you project on the principal component, separation will not be clear.
- Optimal separation would be produced when horizontal direction is used. This is the projection pursuit direction.

Source: `http://www.cis.hut.fi/aapo/papers/NCS99web/node8.html`

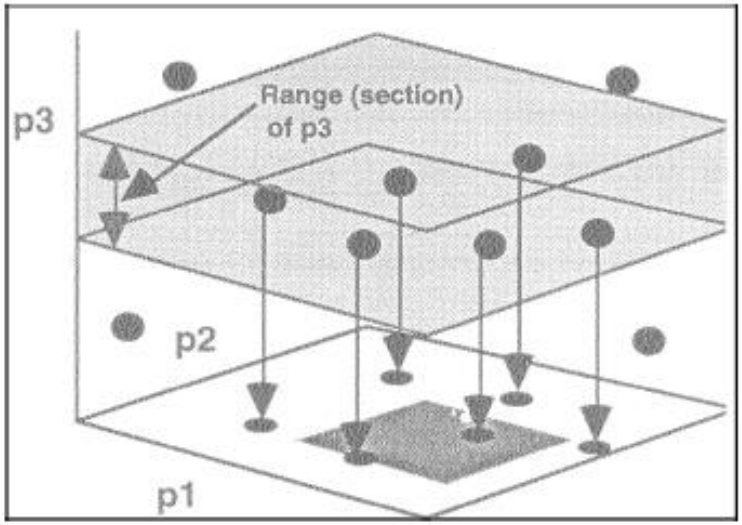


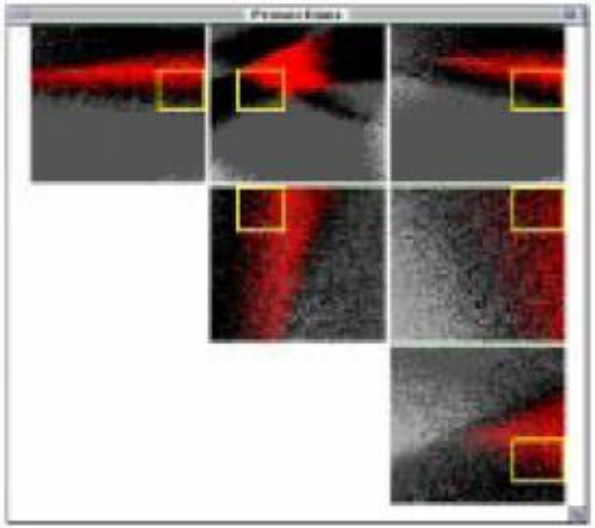
# Prosection Views

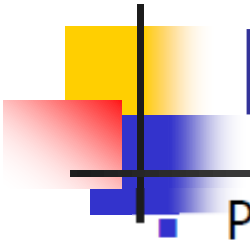
- Papers:
  - Furnas G. W., Buja A., "Prosections Views: Dimensional Inference through Sections and Projections," Journal of Computational and Graphical Statistics, vol. 3, no. 4, pp. 323-353, 1994.
  - Su H., Dawkes H., Tweedie L., Spence R., "An Interactive Visualization Tool for Tolerance Design," Technical Report, Imperial College, London, 1995.
- **Prosection Views** is an example of both geometric as well as interactive techniques.

# Prosection Views

- **Prosection** stands for **pro**jection of a **section** of space.
- The idea:
  - Orthogonal projections are built.
  - The user selects a range.
  - The selected range gets colored differently.





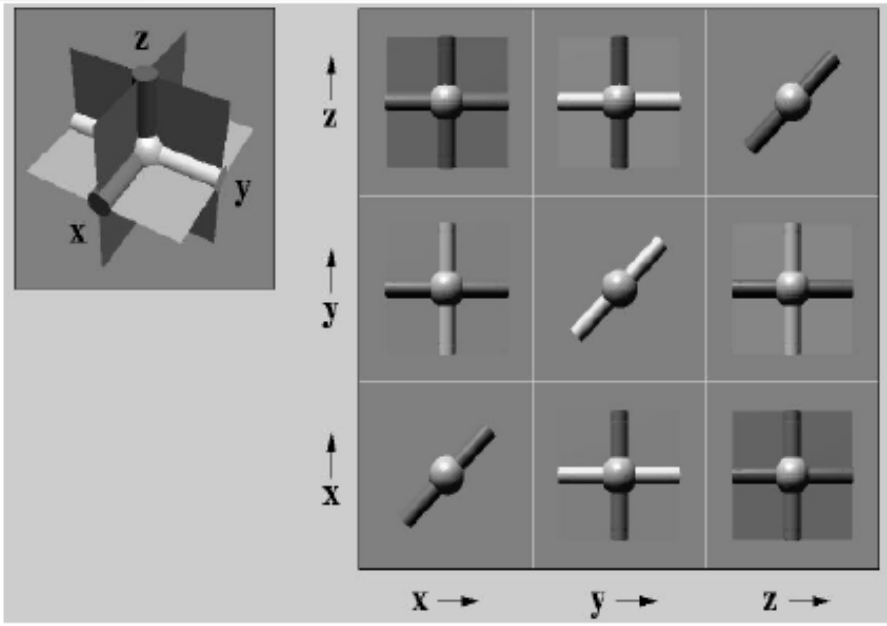
# HyperSclice

- Paper:
  - van Wijk J. J., van Liere R., "HyperSlice," Proc. Visualization'93, San Jose, CA, pp. 119-125, 1993.

- **HyperSlice** is an example of both geometric as well as interactive techniques.

# HyperSclice

- Idea:
  - The user selects a single nD point of interest. This is indicated by the sphere in the example shown.
  - An area of interest surrounding the point can be determined.
  - A 2D slice can be displayed for each pair of variables.
  - The matrix of slice panels is called HyperSlice.



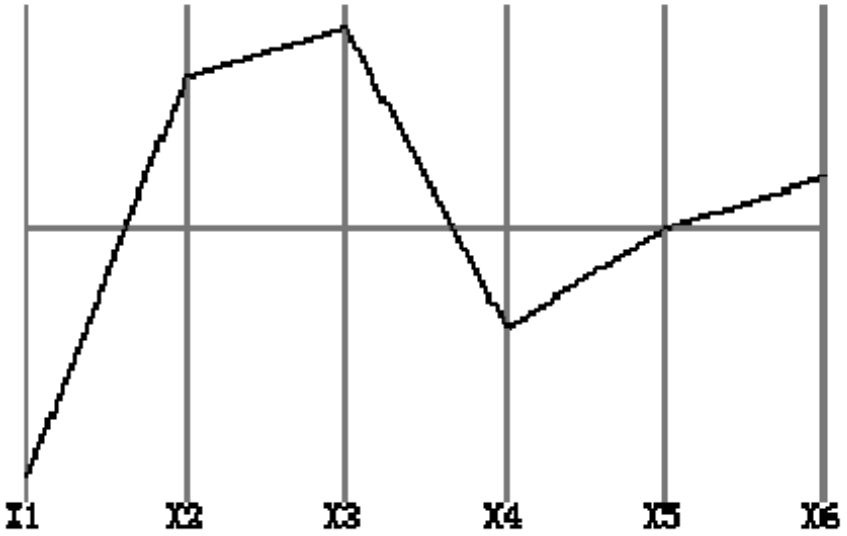

The concept of HyperSlice for n=3.

# Parallel Coordinates

- Papers:
  - A. Inselberg, "**n**-dimensional graphics, part I--lines and hyperplanes," Technical Report G320-2711, IBM Los Angeles Scientific Center, IBM Scientific Center, 9045 Lincoln Boulevard, Los Angeles (CA), 900435, 1981.
  - Inselberg A., "The Plane with Parallel Coordinates, Special Issue on Computational Geometry," The Visual Computer, Vol. 1, 1985, pp. 69-97.

# Parallel Coordinates

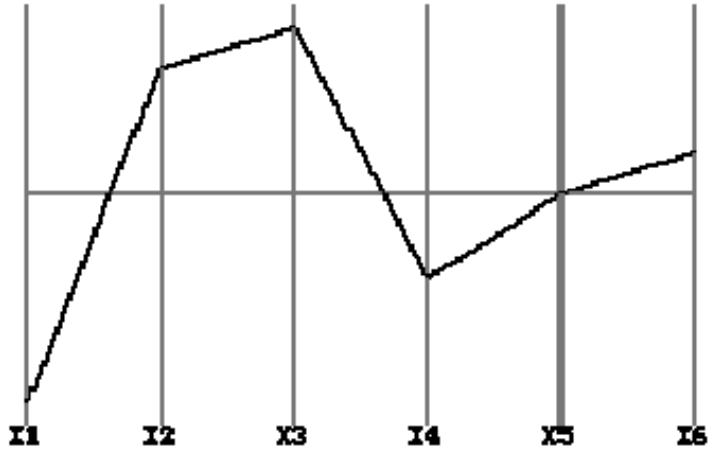
- In **parallel coordinates**, a multidimensional point [$x_1$, $x_2$, $x_3$, ...] is drawn by plotting $x_1$ on axis $X_1$, $x_2$ on axis $X_2$ and so on.



# Parallel Coordinates

- In contrary to **Cartesian** coordinates where all axes are **perpendicular** to each other, axes in **parallel** coordinates are **parallel** to each other. Also, the distances between each two axes are equal.

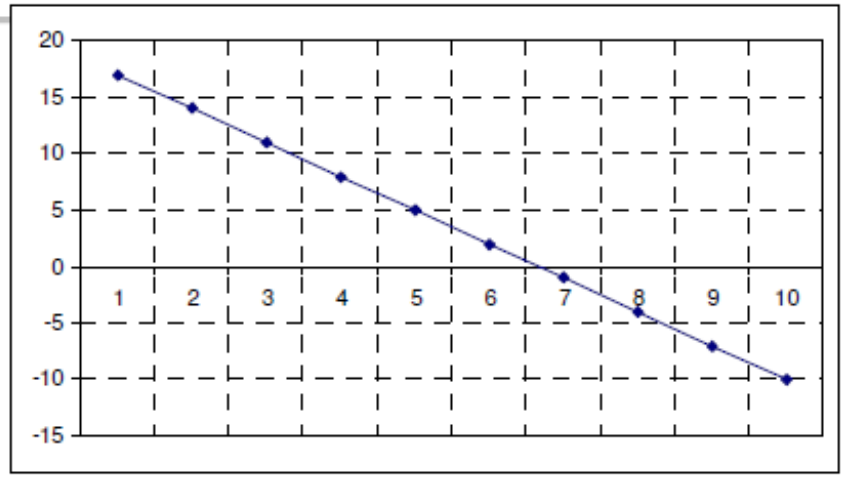
- Example: Shown is a 6D point [-5,3,4,-2,0,1] represented in Parallel coordinates.

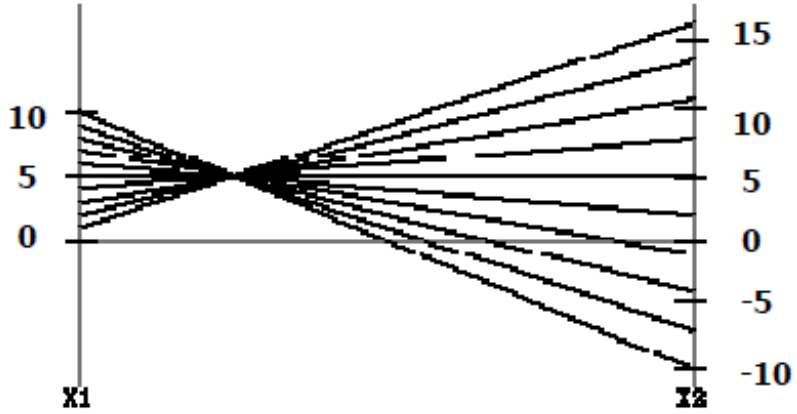



Six equally-spaced parallel coordinates

# Parallel Coordinates: An Example

- Draw the 2D line represented by the equation $x_2$=-3$x_1$+20 in parallel coordinates.
- In Cartesian coordinates, it appears as ➔
- This is achieved by considering a series of points on the line.
- In Parallel coordinates, it appears as ➔



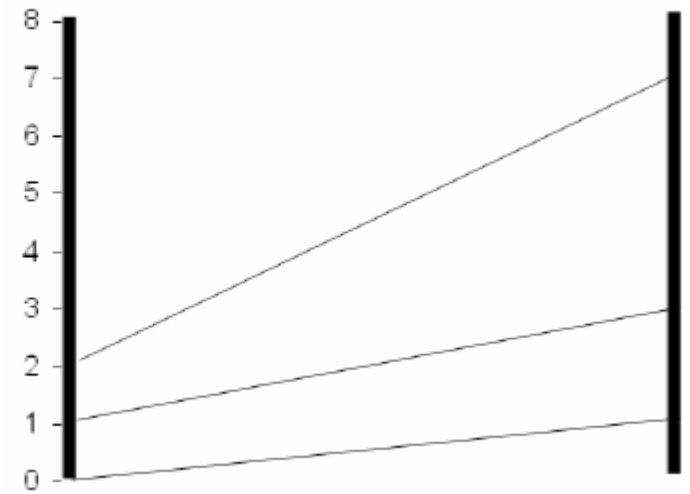
# Parallel Coordinates: An Example

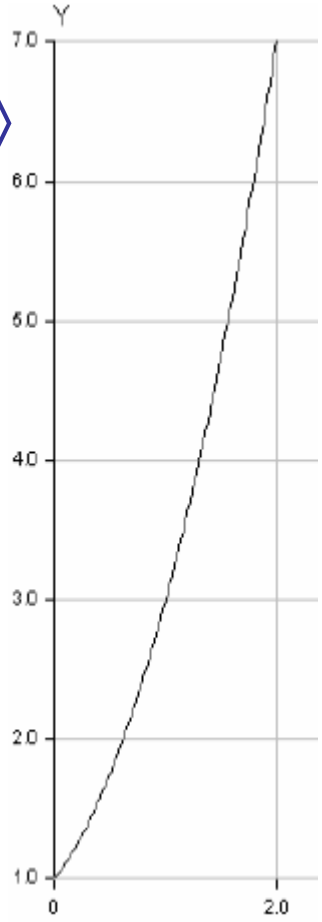
- Example: The curve shown in drawn in Cartesian coordinates. Re-draw it in parallel coordinates. The curve equation is:

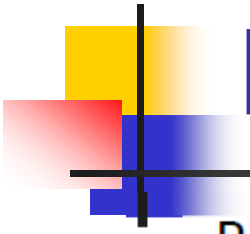
$$y = x^2 + x + 1$$

Answer:

| x | y |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 7 |



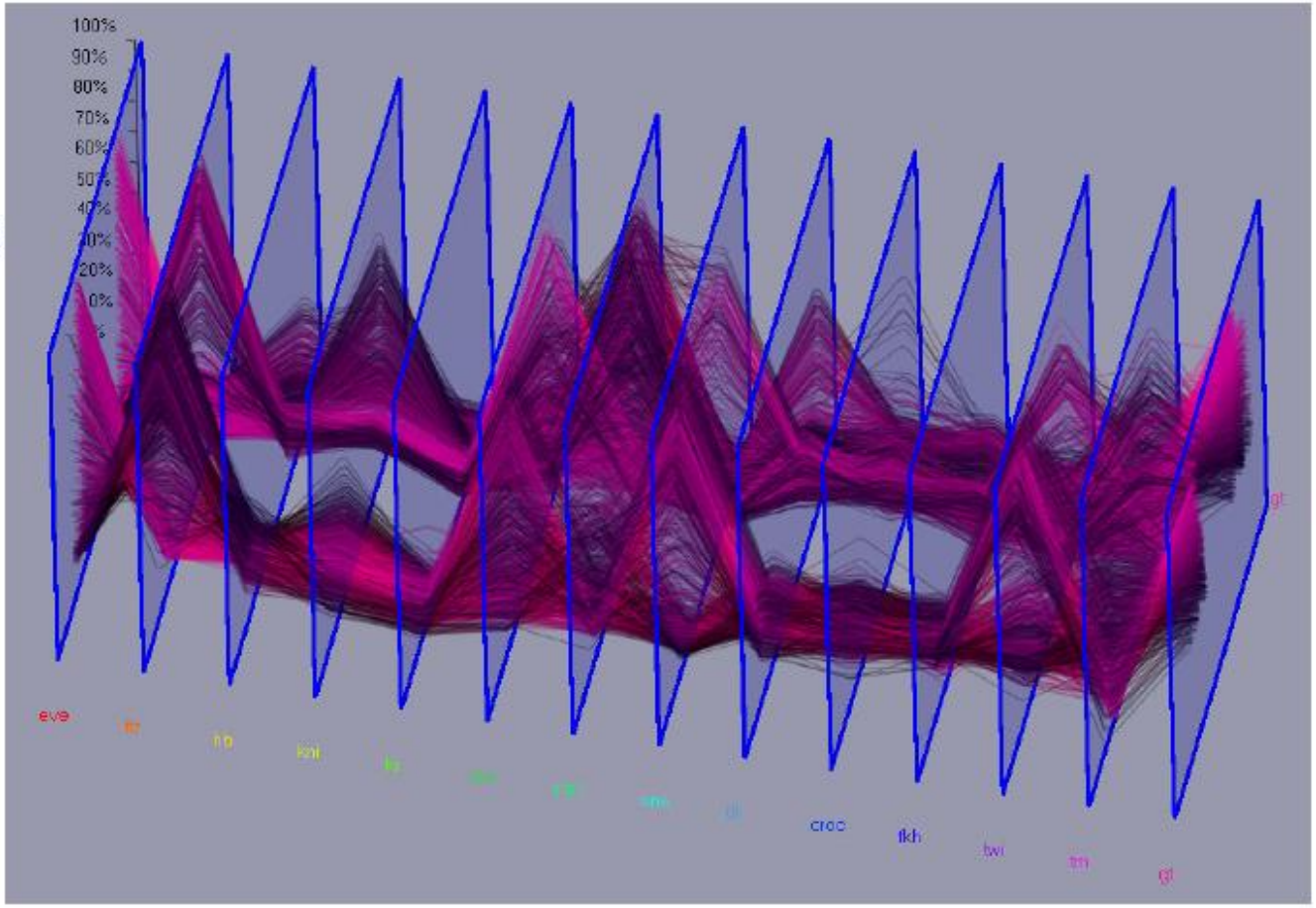


# Parallel Coordinates

- Rotations in Cartesian coordinates turn to translation in parallel coordinates.
- Parallel coordinate plot may be redrawn to show changes of multidimensional data over time so as to produce an animated plot.

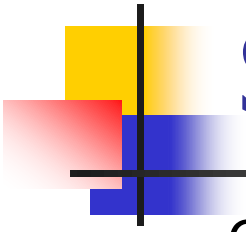
# 3D Parallel Coordinates

- In **3D parallel coordinates**, parallel planes are used instead of axes.



# Other Techniques

- Landscapes:
  - Wright W., "Information Animation Applications in the Capital Markets," Proc. Int. Symposium on Information Visualization, Atlanta, GA, 1995, pp. 19-25.



# Summary

- Geometric techniques are:
  - Scatterplot matrices
  - Projection pursuit technique
  - Prosection views
  - Hypersclices
  - Parallel coordinates