Introduction to **Calculus**

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What is Calculus?

- Small pebbles
- Used for counting in Abacus
- Continuous small Change
- One of the most widely used concept in Machine Learning Optimization

Rate of Change

Rate of Change

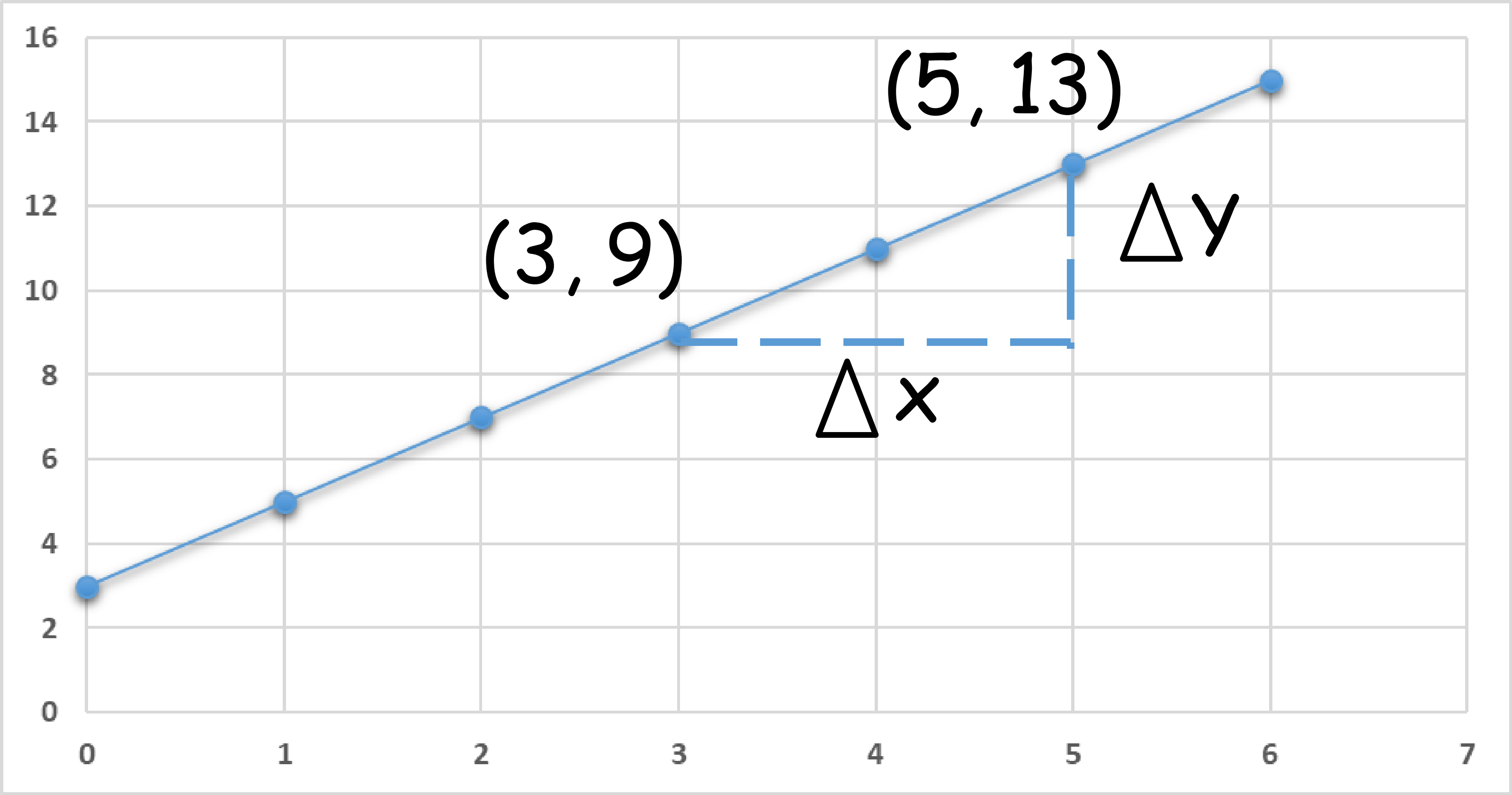
$y = 2x + 3$

Rate of Change

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{13 - 9}{5 - 3}$$

$$= 2$$

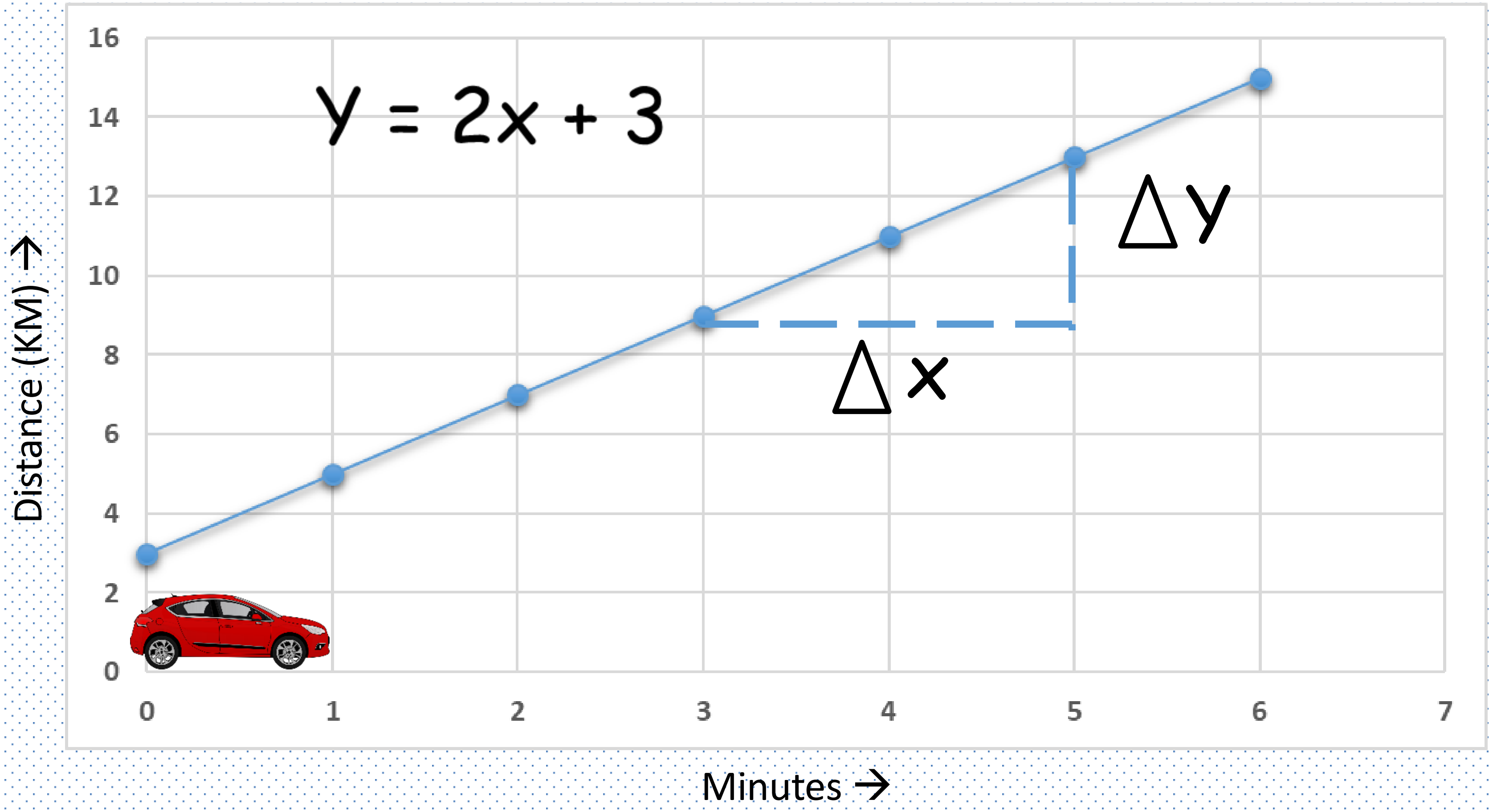


Rate of Change

Rate of Change

$$\frac{\Delta \text{ Distance}}{\Delta \text{ Time}}$$

$=$ 2KM/minute



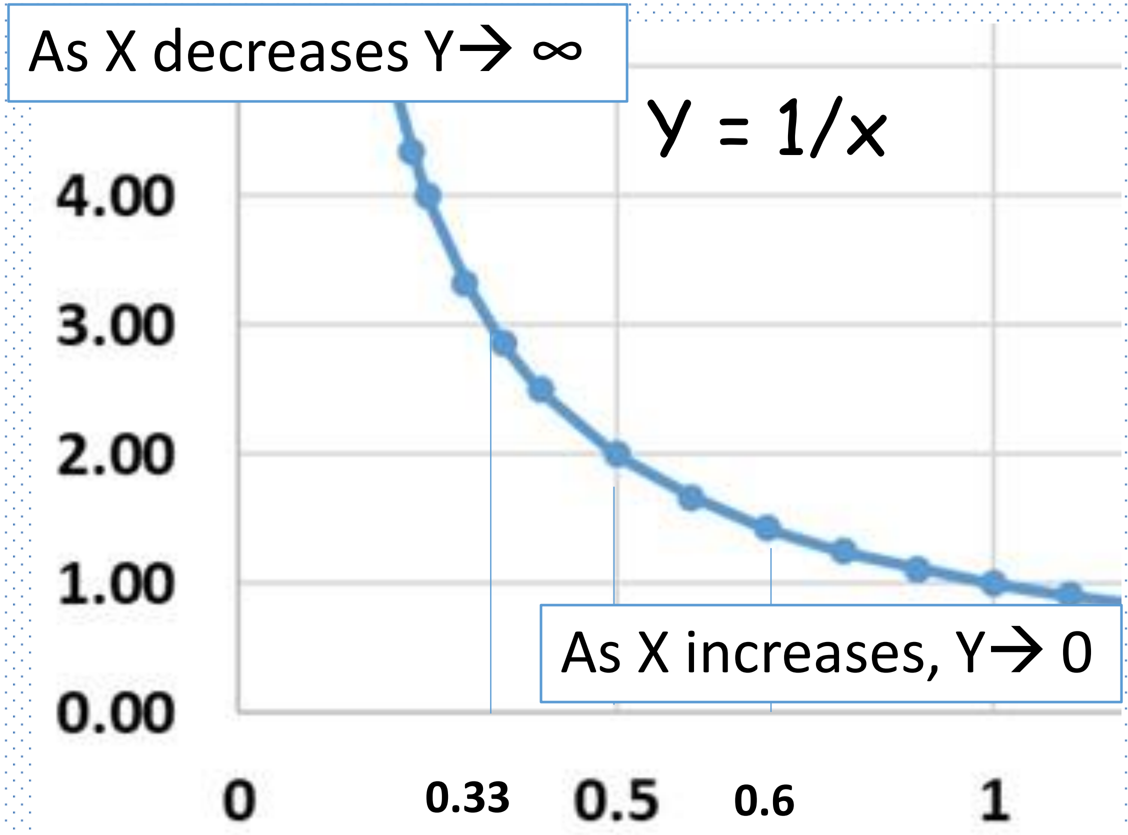
Limits

Limits

$y = 1/x$

$x \neq 0$

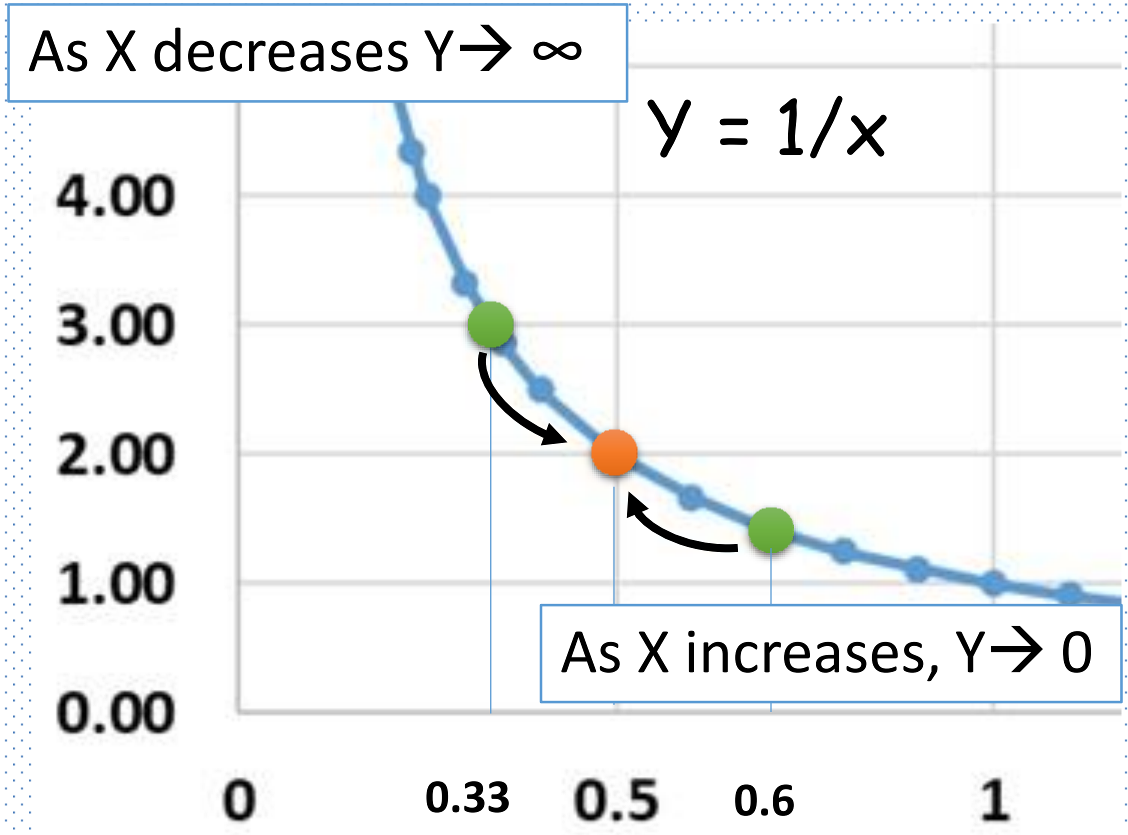
Limits



X	Y
1	1
10	0.1
100	0.01
1000	0.001
10,000	0.0001
1,000,000	0.000001

X	Y
1	1
0.5	2
0.01	100
0.001	1000
0.0001	10,000
0.000001	1,000,000

Limits



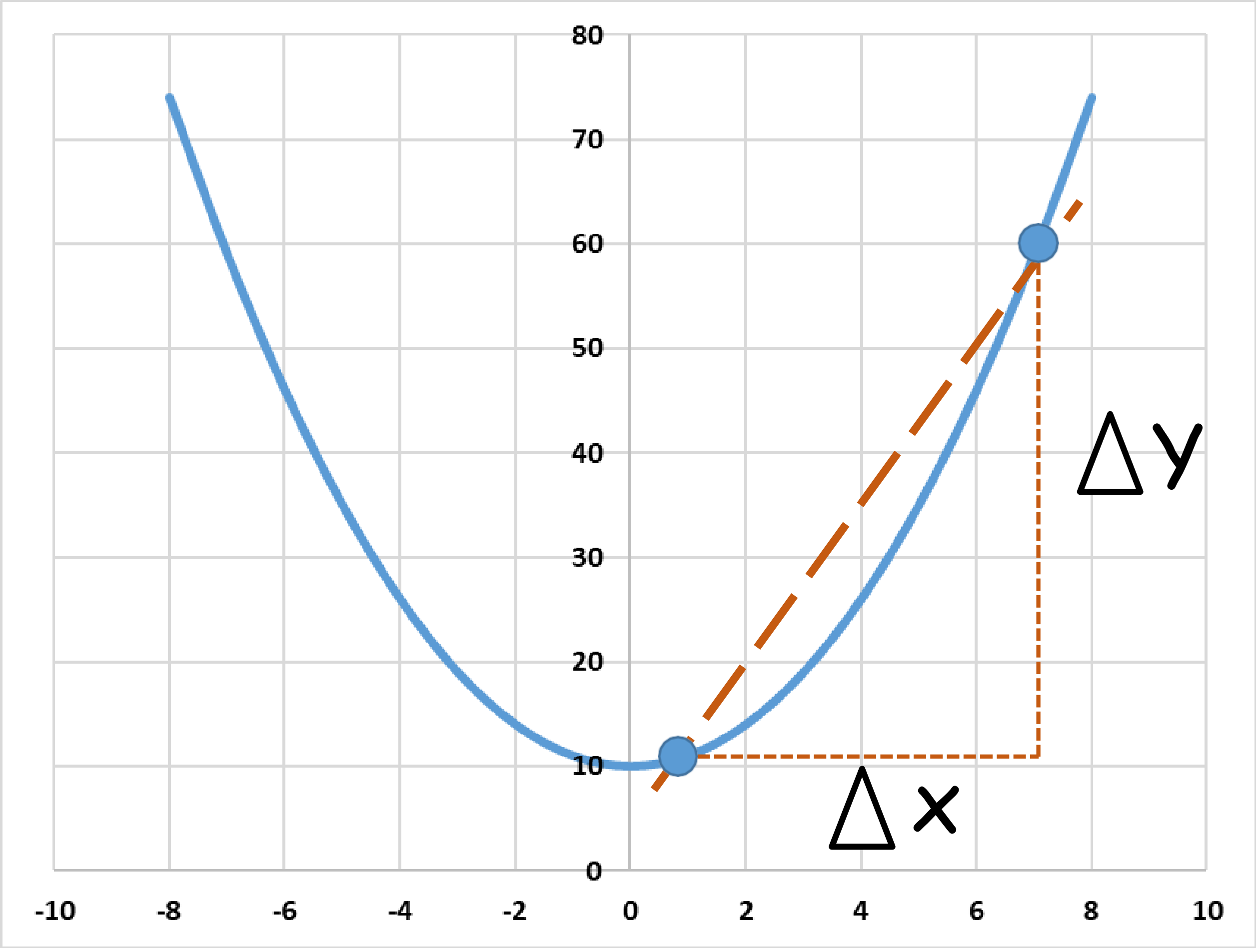
$$\lim_{x \to 0.5} \frac{1}{x} = 2$$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to 0} \frac{1}{x} = \infty$$

Differential Calculus

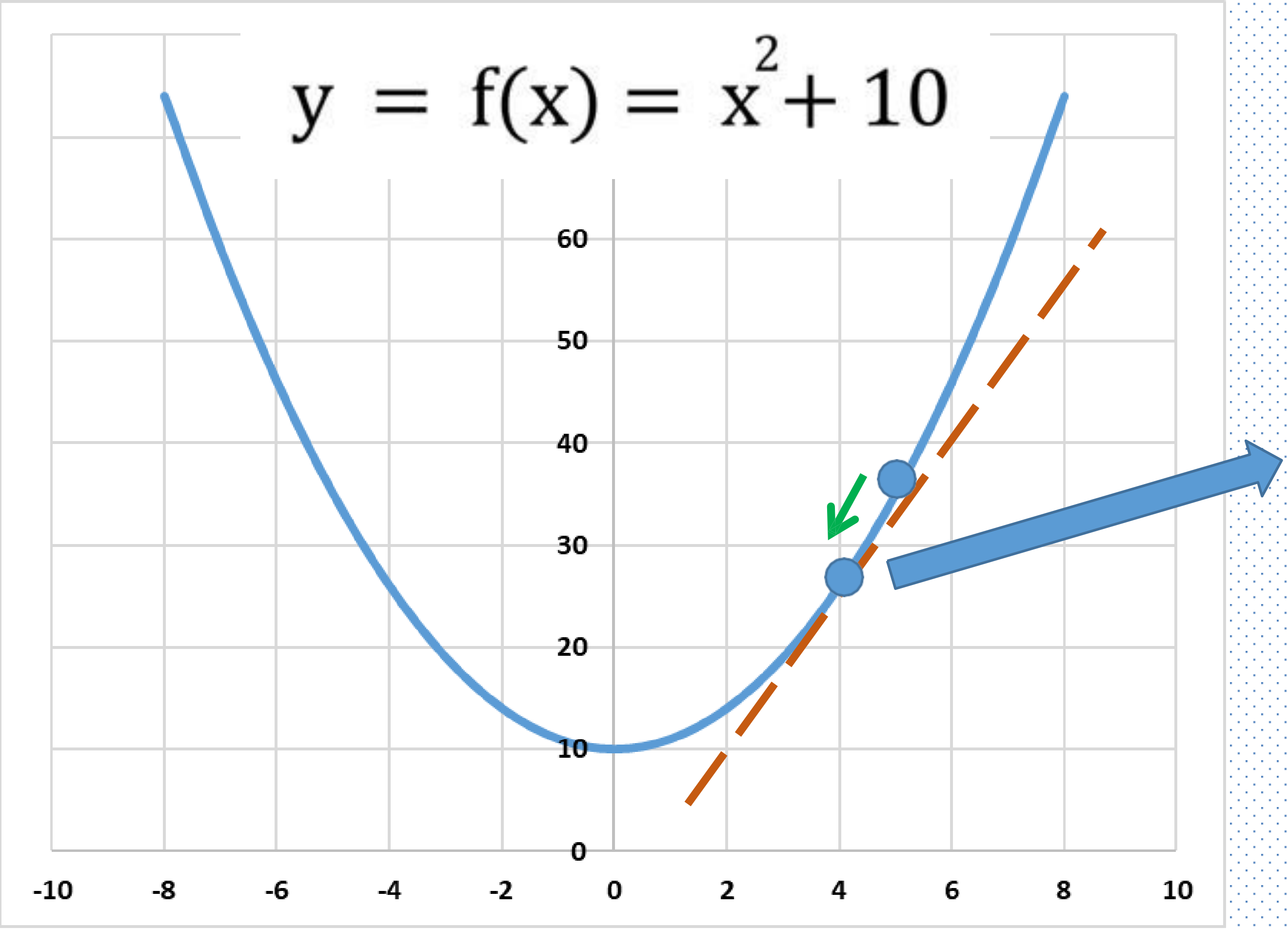
Slope between two points



Average Slope

$$= \frac{\Delta y}{\Delta x}$$

Derivative



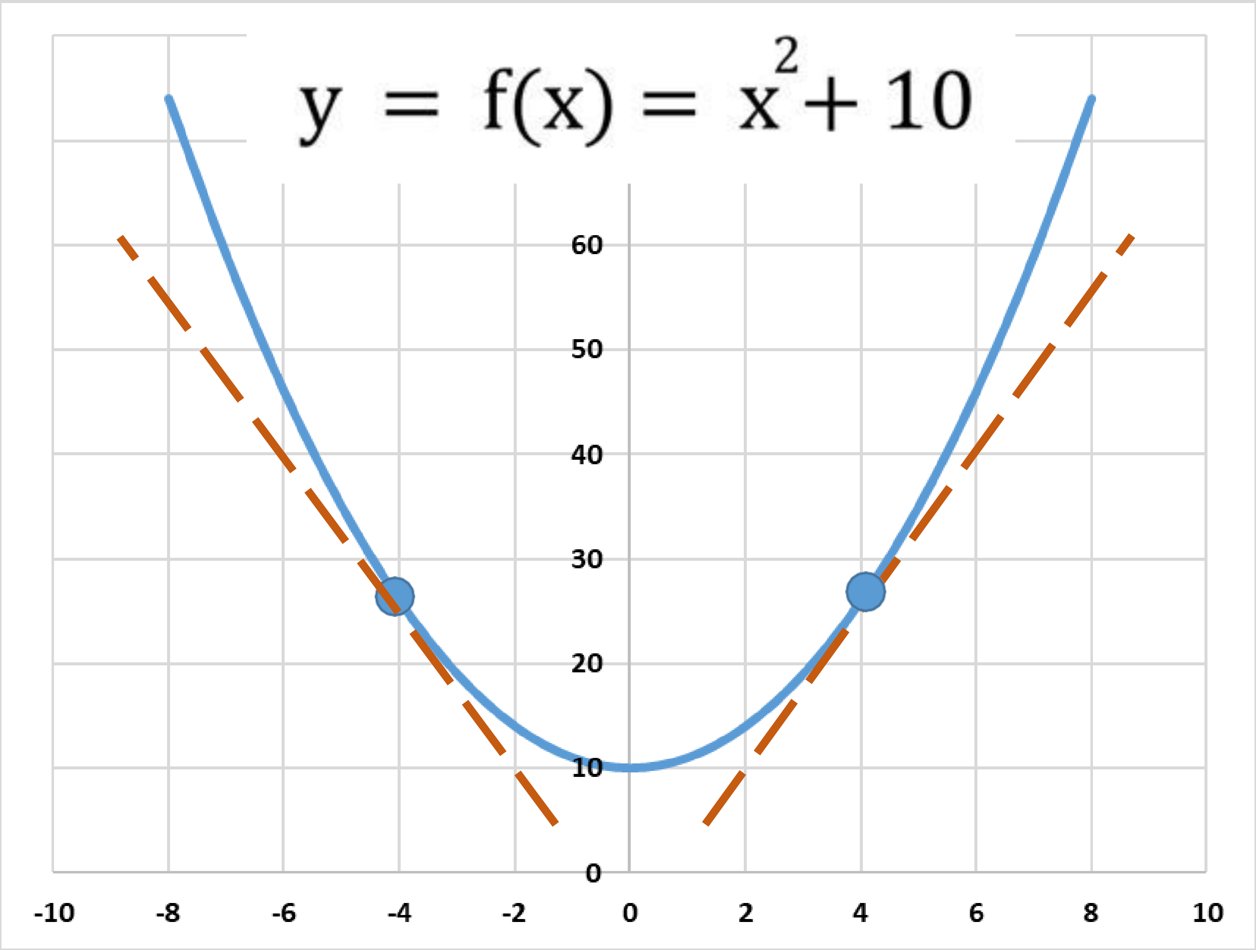
$$\text{Slope} = 2x + \Delta x$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

0

$$\frac{dy}{dx} = 2x$$

Derivative



$$\frac{dy}{dx} = 2x$$

1. $x = 4$; slope = 8
2. $x = -4$; slope = -8

Differentiability and Rules

Derivative rules

- Derivative of a vertical line
- Derivative of a horizontal line
- Differentiability for various functions
- Power rule of derivative

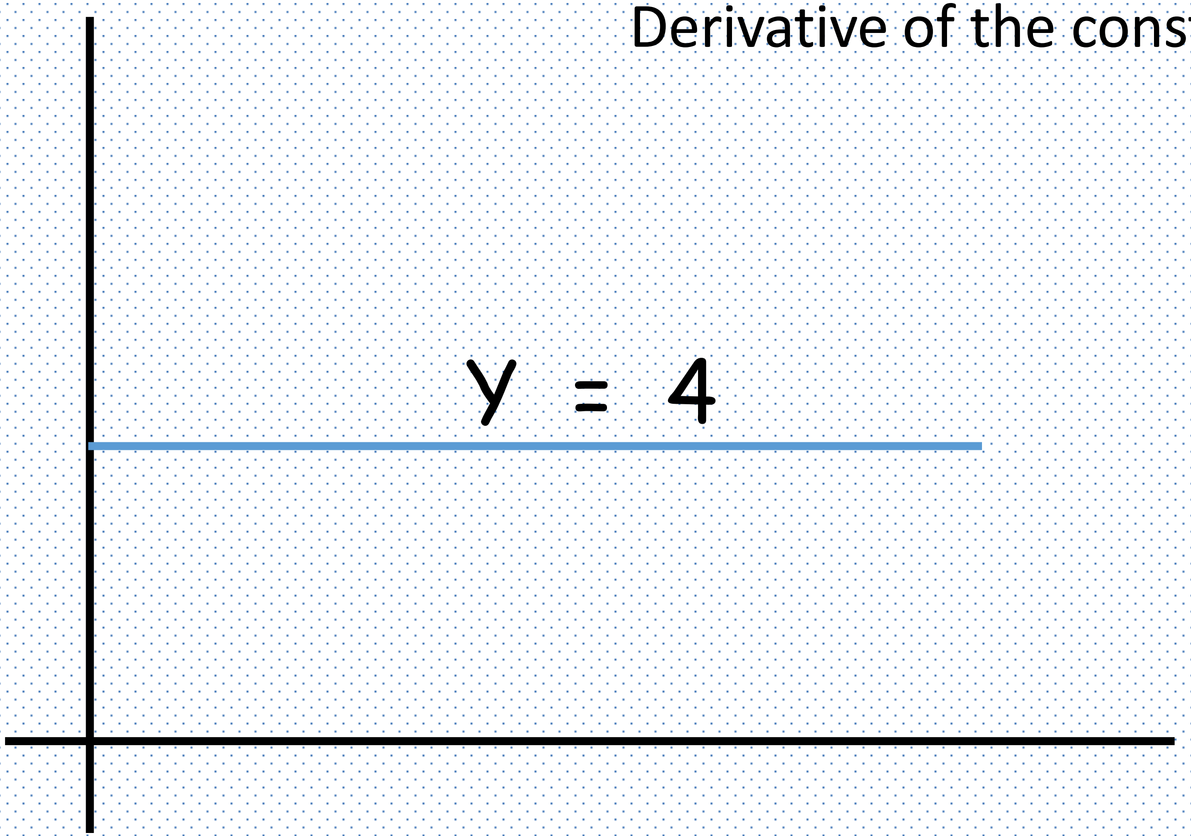
Derivative Rules

$$\Delta x = 0$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \infty \text{ or Undefined}$$

Derivative Rules – Constant

Derivative of the constant is ZERO.

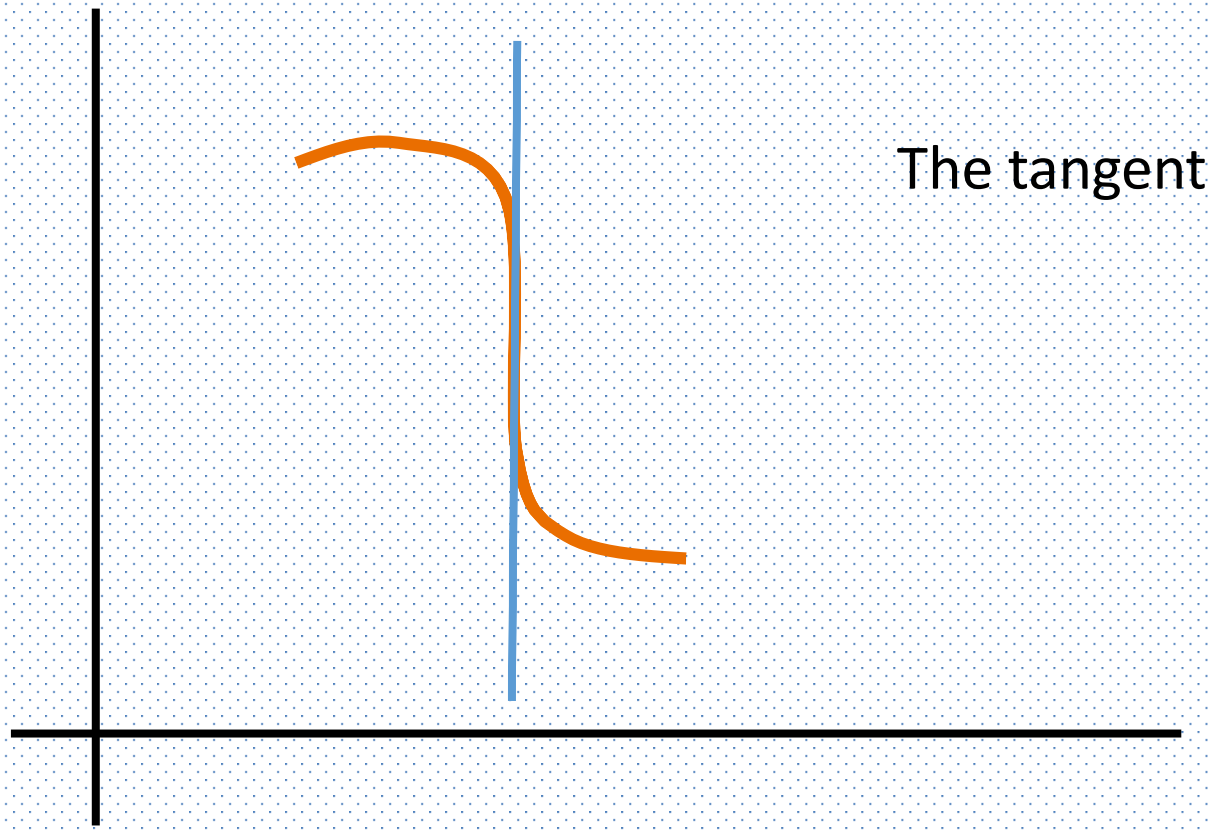


$$\Delta y = 0$$

$$\frac{dy}{dx} = 0$$

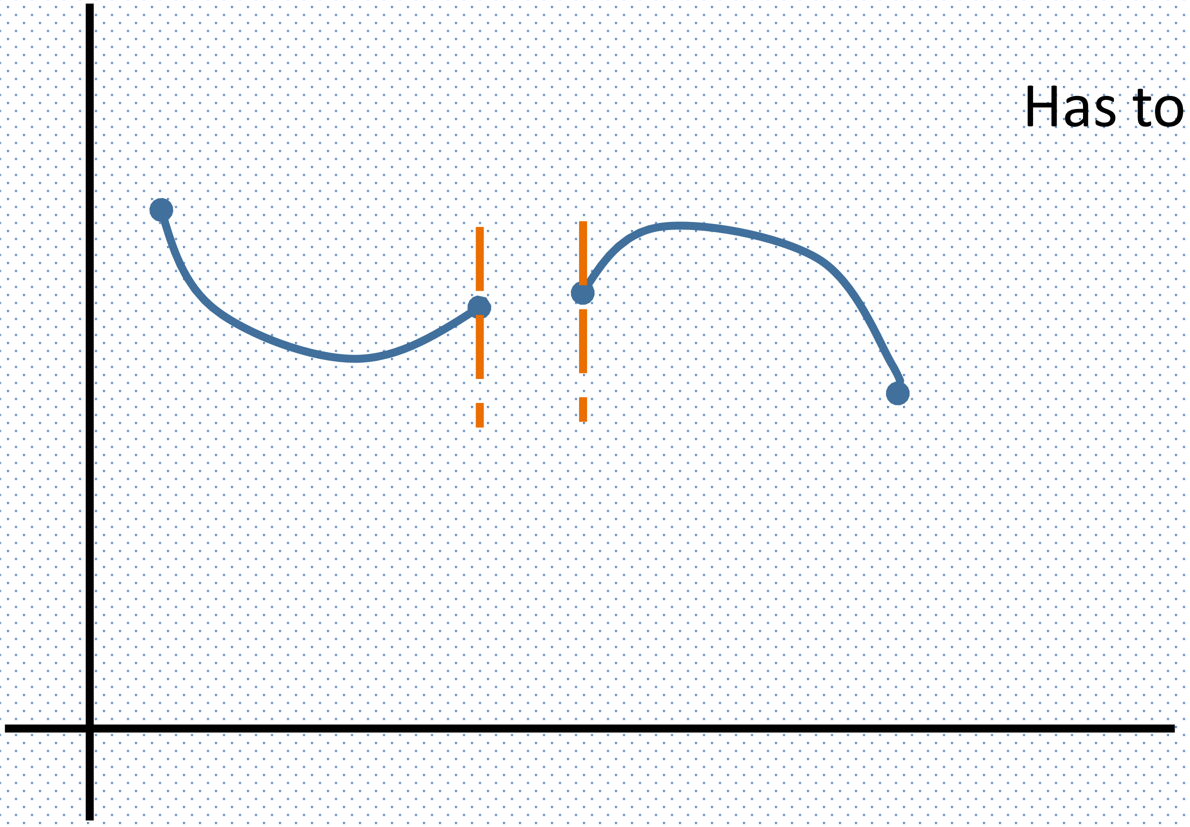
$$\frac{d\,(4)}{dx} = 0$$

Differentiability



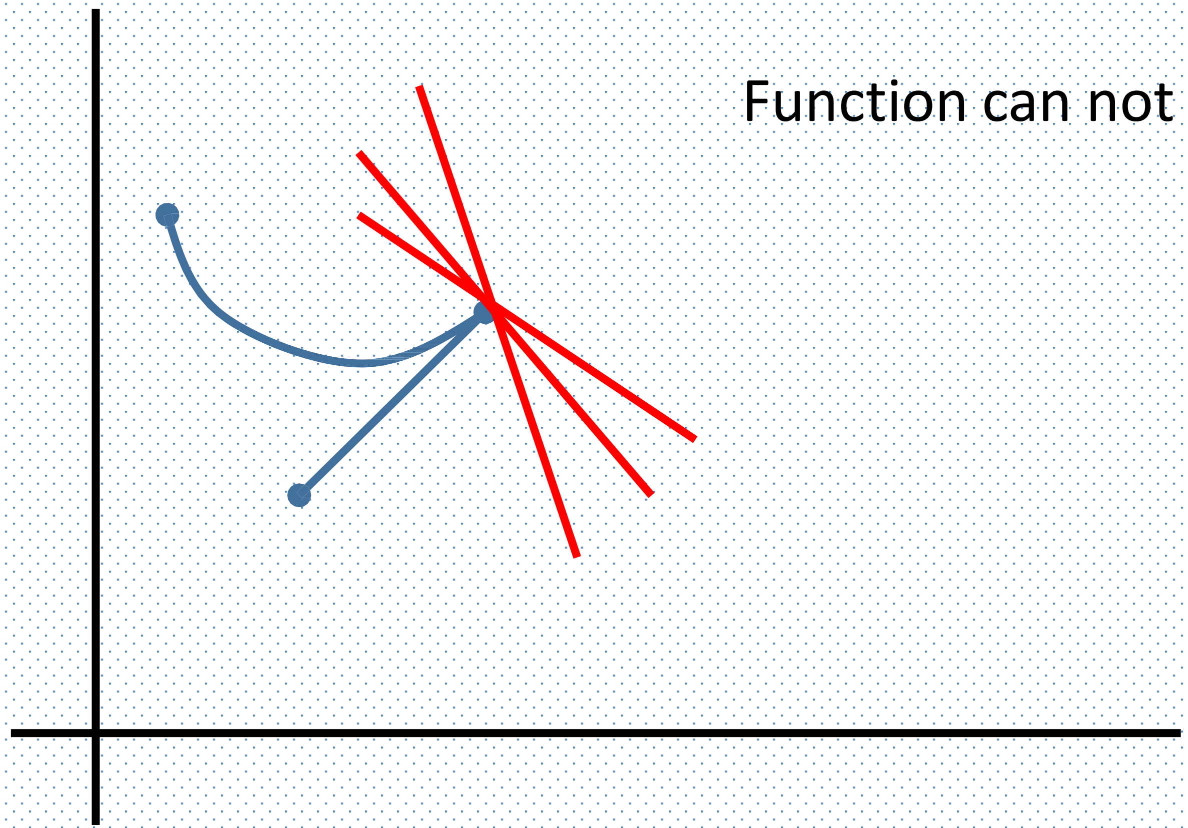
The tangent line can not be vertical.

Differentiability



Has to be a continuous function.

Differentiability



Function can not take sudden change in direction.

Power Rule of Derivative

$$y = f(x) = ax^{n} \Rightarrow \frac{dy}{dx} = a*n\,x^{n-1}$$

Power Rule of Derivative

$$y = f(x) = x^2 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 2x$$

$$y = f(x) = x^3 + 10 \quad \Rightarrow \quad \frac{dy}{dx} = 3x^2$$

(Original Index −1)

Remove constant

(Original Index * Original Coefficient)

Power Rule of Derivative

$$y = f(x) = x^3 + 10 \Rightarrow \frac{dy}{dx} = 3x^2$$

(Original Index −1)

Remove constant

(Original Index * Original Coefficient)

$$y = f(x) = 2x^3 + 4x^2 - 7x + 9 \Rightarrow \frac{dy}{dx} = 6x^2 + 8x - 7$$

Direction, Maxima and Minima

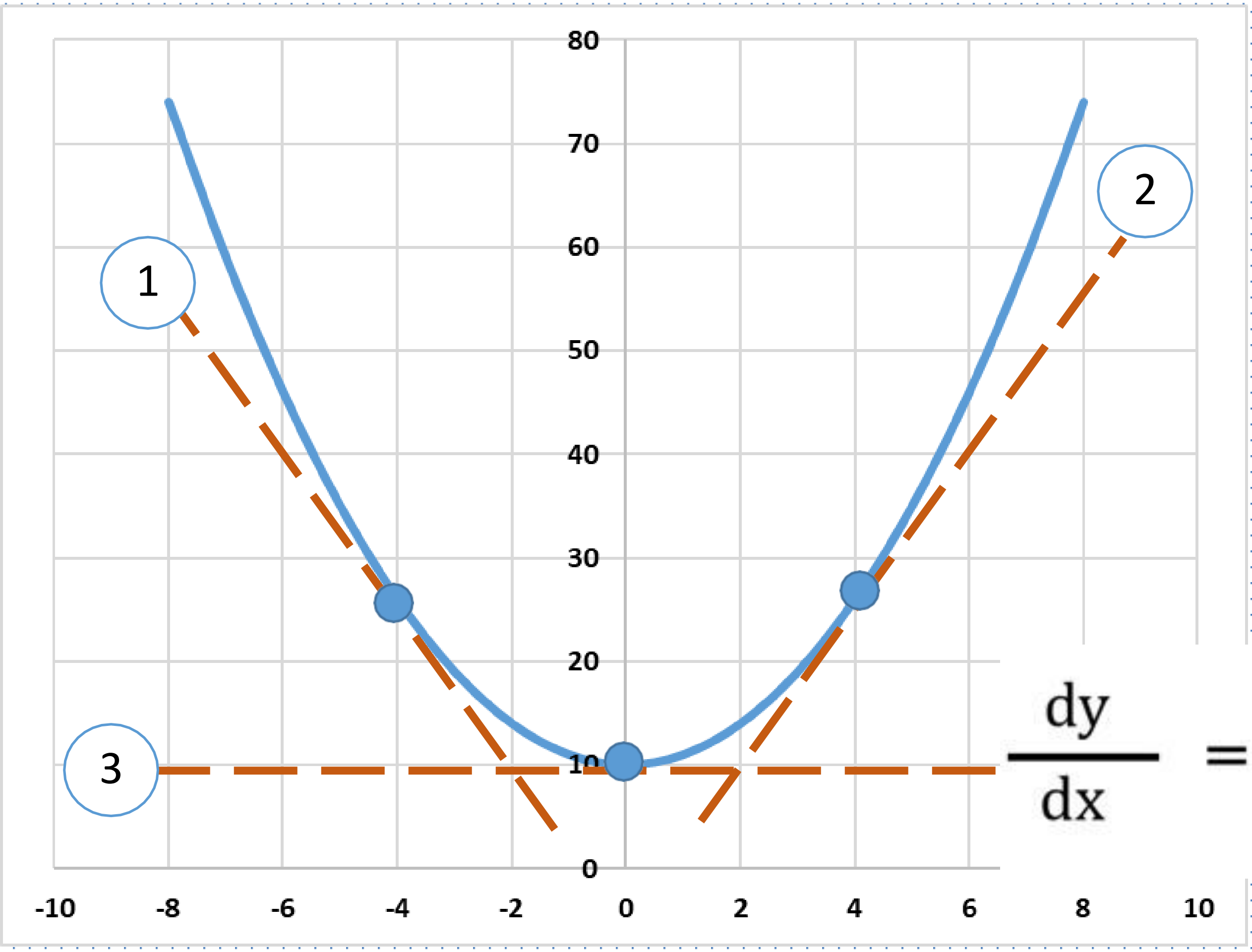
Derivative for directions

$$y = f(x) = x^2 + 10$$

$$\frac{dy}{dx} = 2x$$

1 $$\frac{dy}{dx} = -8$$

2 $$\frac{dy}{dx} = +8$$



$$\frac{dy}{dx} = 0$$

Second Order Derivative

$$\frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d^2y}{dx^2}$$

Second Order Derivative

$$y = f(x) = x^2 + 10 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow \frac{d^2y}{dx^2} = 2$$

$$y = f(x) = -x^2 + 10 \Rightarrow \frac{dy}{dx} = -2x \Rightarrow \frac{d^2y}{dx^2} = -2$$

Rules for Maxima and Minima

Second Derivative < 0 $\Rightarrow$ Local Maxima

Second Derivative > 0 $\Rightarrow$ Local Minima

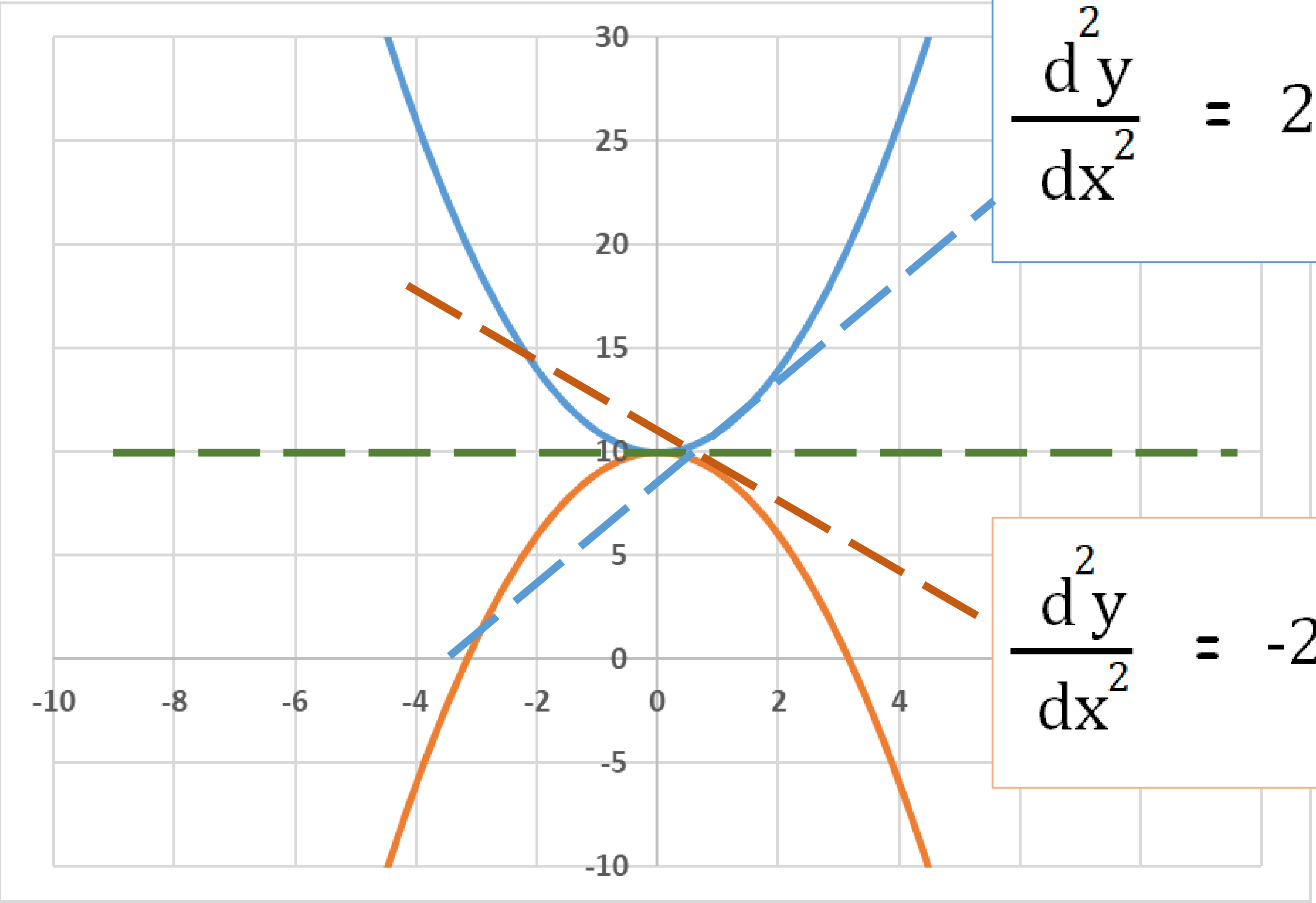
Maxima or Minima?

$$y = f(x) = x^2 + 10$$

Minima at y = 10

$$y = f(x) = -x^2 + 10$$

Maxima at y = 10



$$\frac{d^2y}{dx^2} = 2$$

$$\frac{d^2y}{dx^2} = -2$$

Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1$$

Step 1 – Get the first Derivative

Step 2 – Get the Second Derivative

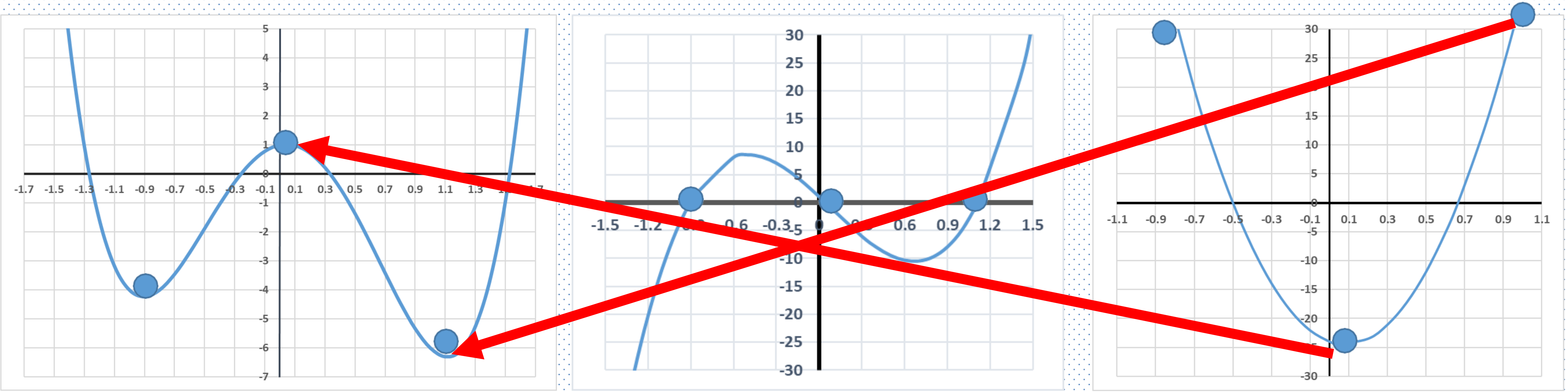
Step 3 – Identify points where slope is zero

Step 4 – Get the second derivative when slope is zero

Step 5 – Apply the rules for maxima and minima

Derivative for Maxima and Minima

$$y = 6x^4 - 2x^3 - 12x^2 + x + 1 \qquad \frac{dy}{dx} = 24x^3 - 6x^2 - 24x + 1 \qquad \frac{d^2y}{dx^2} = 72x^2 - 12x - 24$$



-0.9054	0.04131	1.1141	-0.9054	0.04131	1.1141	-0.9054	0.04131	1.1141

Partial Derivative

Partial Derivative

$$f(x, y) = x^2 + y^2$$

$$x \Rightarrow \frac{d(f(x,y))}{dx} = \frac{d(x^2 + y^2))}{dx} = \frac{d(x^2 + c))}{dx} = 2x$$

$$y \Rightarrow \frac{d(f(x,y))}{dy} = \frac{d(x^2 + y^2))}{dy} = \frac{d(c + y^2))}{dy} = 2y$$

Partial Derivative

$$f(x, y) = x^2 + y^2$$

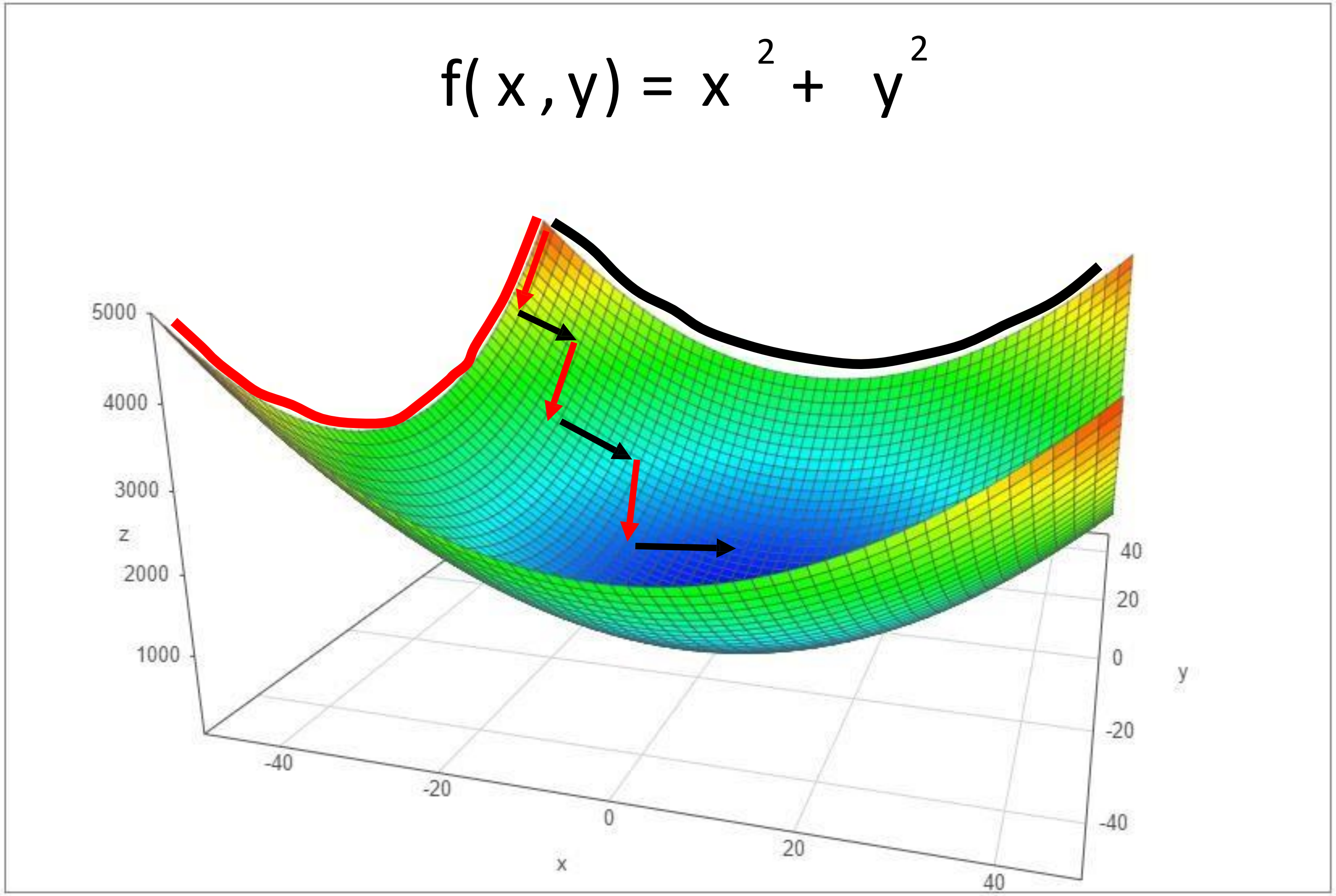
x

$$\frac{\partial (f(x,y))}{\partial x} = 2x$$

y

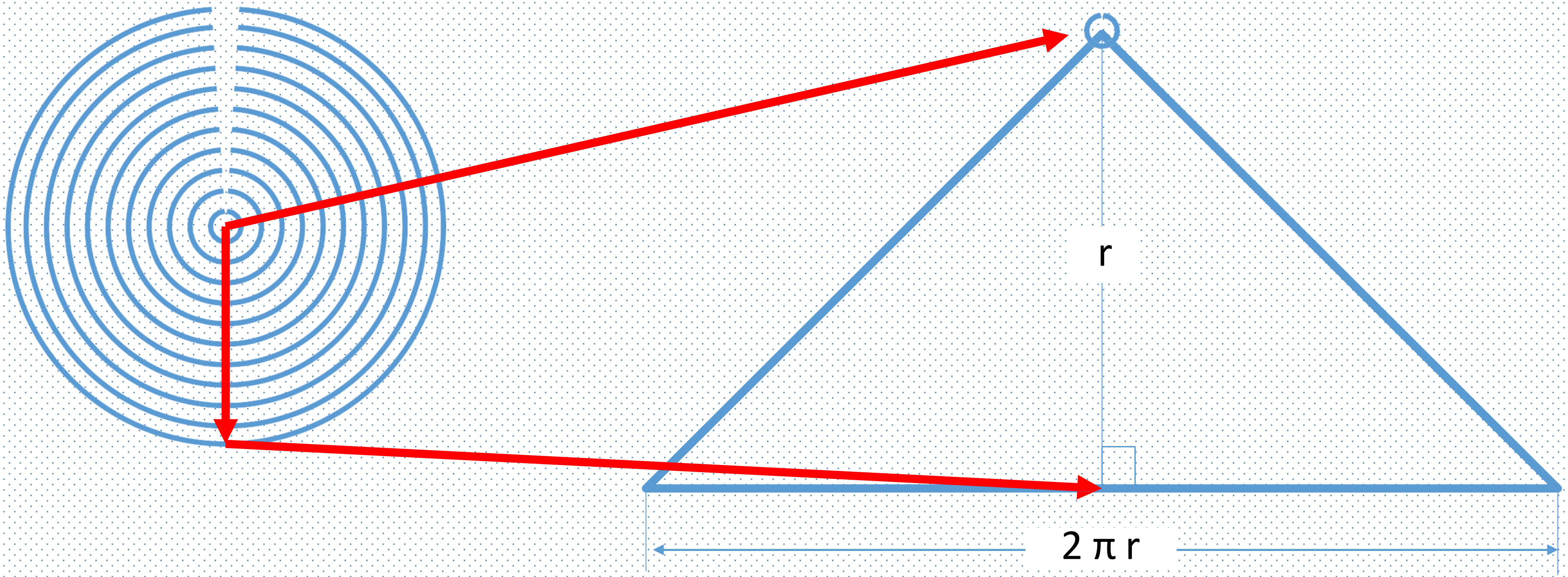
$$\frac{\partial (f(x,y))}{\partial y} = 2y$$

Multiple variables in a function



Integration

Calculating the area of a circle



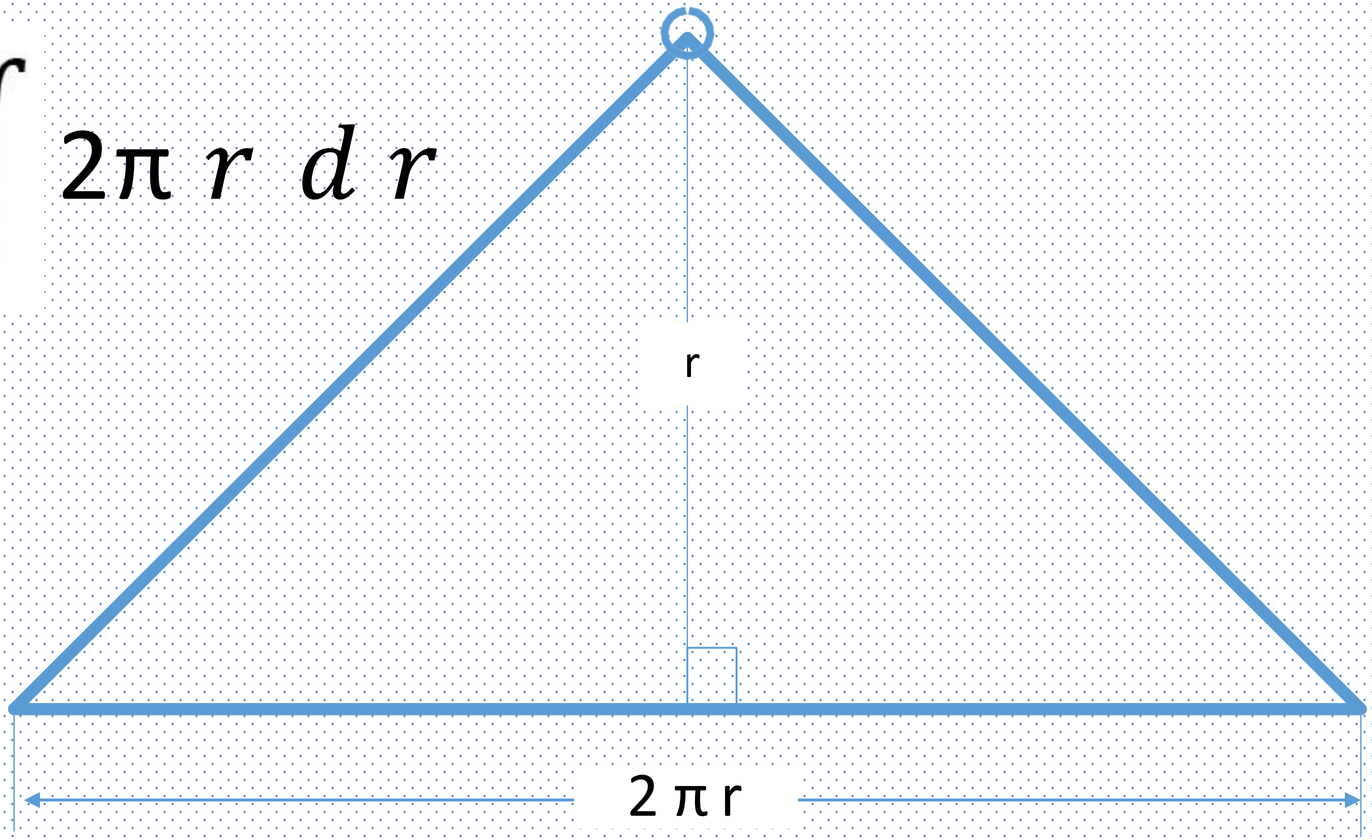
Calculating the area of a circle

$$\text{Area} = \frac{\text{Height} \; * \; \text{Base}}{2}$$

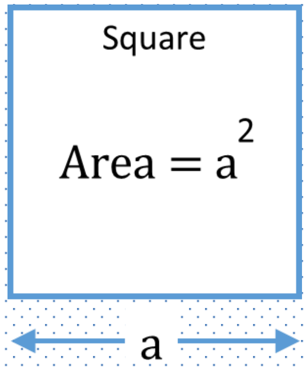
$$= \frac{2\pi r \; * \; r}{2}$$

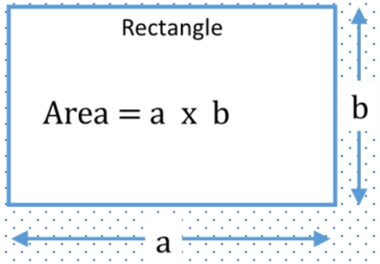
$$= \pi r^2$$

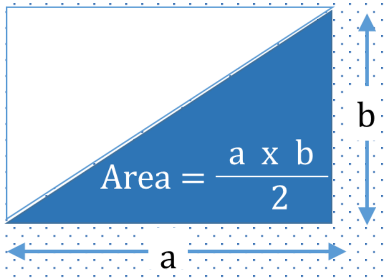
$$\int 2\pi r \; dr$$

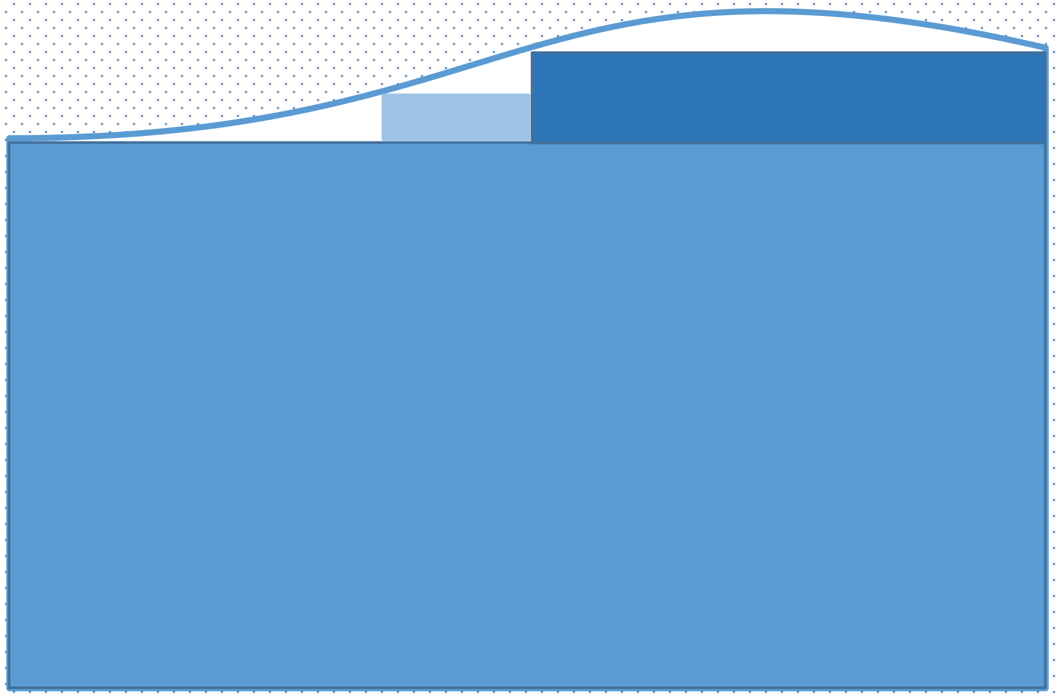


Understanding the problem

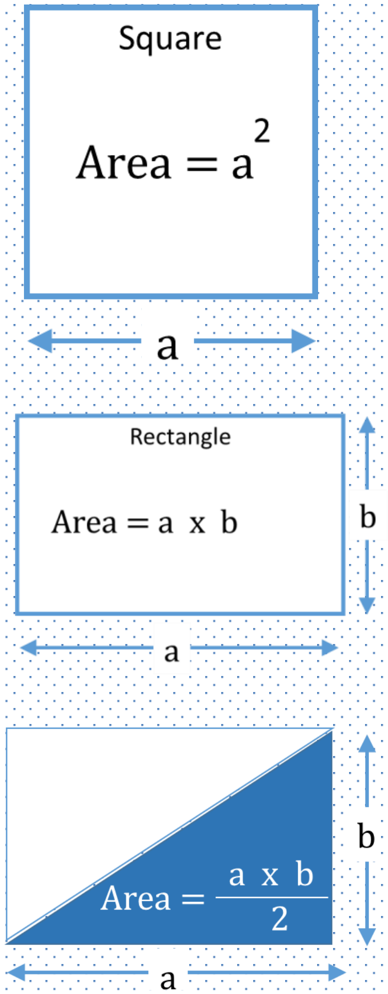


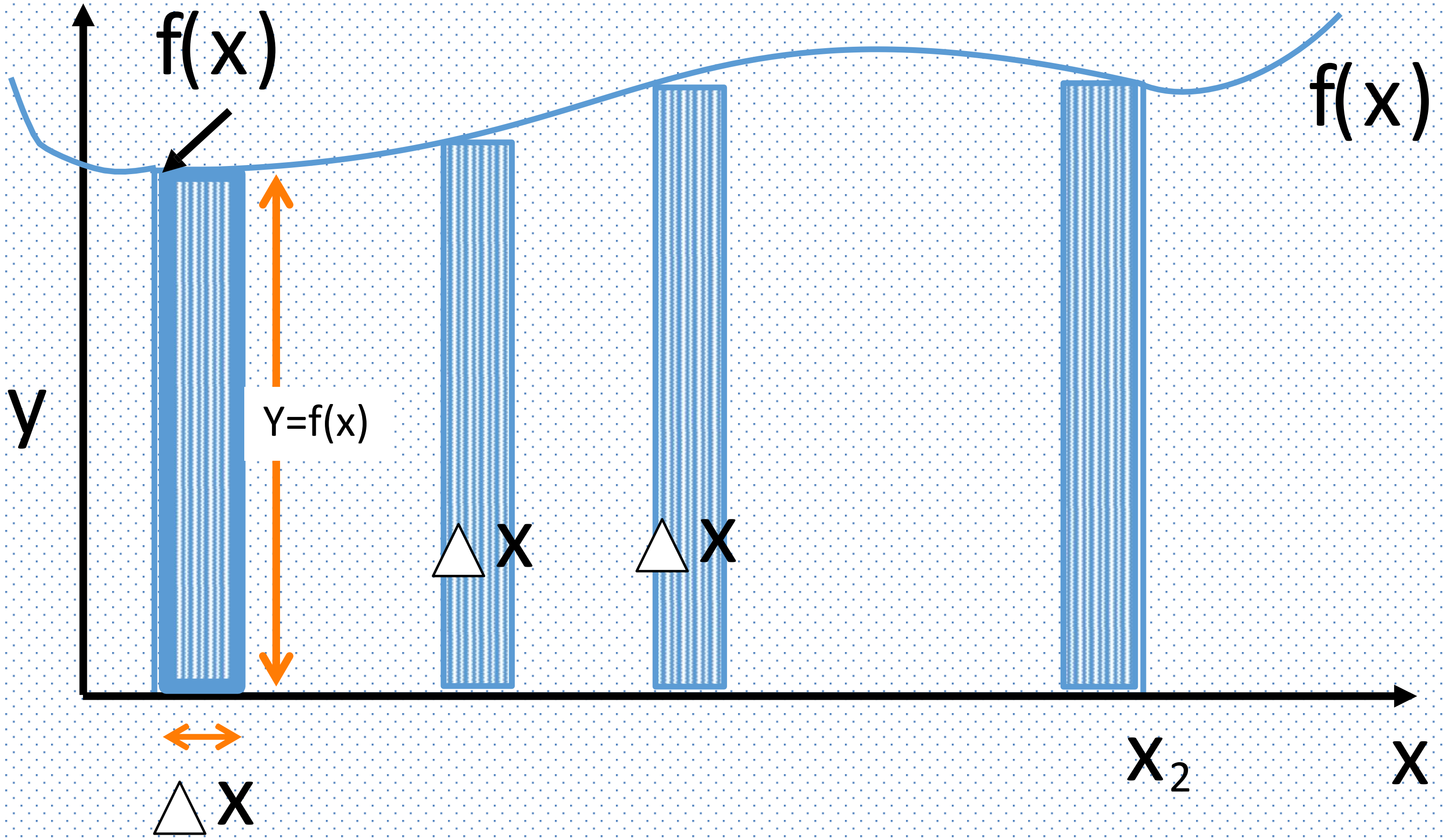






Understanding the problem



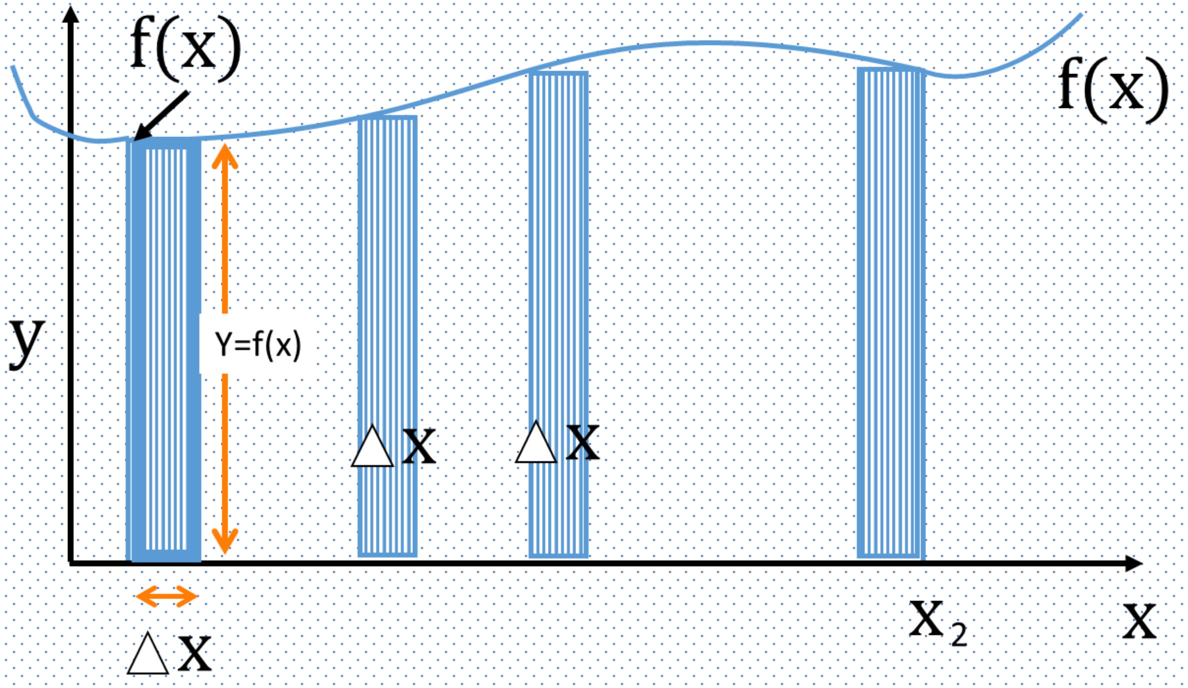


Understanding the problem

$$\text{Area} = f(x) * \triangle x$$

$$\text{Area} = \sum_{i=1}^{n} f(x_i) * \triangle x_i$$

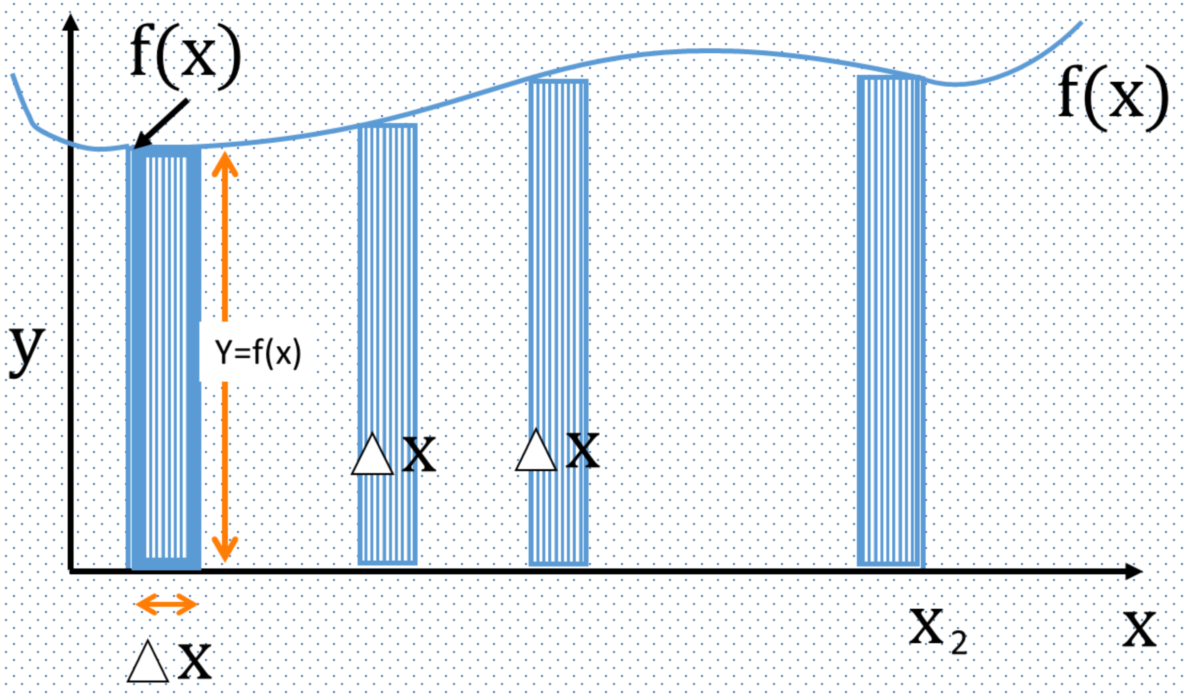
$$\boxed{\text{Area} = \lim_{\triangle x \to 0} \sum_{i=1}^{n} f(x_i) * \triangle x_i}$$



Integration

$$\text{Area} = \lim_{\triangle x \to 0} \sum_{i=1}^{n} f(x_i) * \triangle x_i$$

$$\boxed{\int_{x1}^{x2} f(x)\,dx}$$



Thank You!