MAT 2384 Summary Sheet

Order 1 ODE's

Standard Form: $y' = f(x,y)$
Differential Form: $M(x,y)dx + N(x,y)dy = 0$

Seperable First Order ODE's

Definition. A first order ODE is called seperable if it can be written in the form

$$F(x)dx = G(y)dy$$

Steps to Solving Seperable ODE's

1. Write $y' = \frac{dy}{dx}$
2. Seperate the ODE to write it in the form $F(x)dx = G(y)dy$
3. Integrate both sides
4. If an initial condition is given, solve for the integration constant C.

First Order ODE's with Homogeneous Coefficients

Definition. A function $F(x,y)$ is called homogeneous of degree k if

$$F(\lambda x, \lambda y) = \lambda^k \cdot F(x,y)$$

Definition. A first order ODE given in differential form is called homogeneous if both $M(x,y)$ and $N(x,y)$ are homogeneous of the same degree.

Theorem. A first order ODE of homogeneous coefficients can be made seperable by changing the function by substituting $u = \frac{y}{x} \implies y = xu$ or $u\frac{x}{y} \implies y = \frac{x}{u}$.

Exact First Order ODEs

Definition. Given a function $F(x,y)$, the differential of F denoted by dF is defined by

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$$

Remark. $dF = 0 \iff F(x,y) = C$.

Definition. A first order ODE is called exact if there exists $F(x,y)$ such that

$$\frac{\partial F}{\partial x} = M(x,y) \text{ and } \frac{\partial F}{\partial x} = N(x,y)$$

Theorem. A first order ODE is exact if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Iterative Methods to Solving $f(x) = 0$

Theorem. (Intermediate Value Theorem). Let $f : [a,b] \to \mathbb{R}$ be a continuous function. Let $y \in \mathbb{R}$ between $f(a)$ and $f(b)$. Then there exists $z \in [a,b]$ such that $f(z) = y$.

Fixed Point Iteration

Definition. The value $x = r$ is a fixed point for $g(x)$ if $g(r) = r$.

Definition. The iteration sequence is given as

$$x_{n+1} = g(x_n)$$

with x_0 being given.

Theorem. Assume that the function $g(x)$ has a fixed-point s on an interal I, if

- $g(x)$ is continuous on I,
- $g'(x)$ is continuous on I, and
- $|g'(x)| < 1$ for all $x \in I$.

Then then the iteration sequence converges.

Steps to Solve Using Fixed-Point Iteration

Then the steps for solving are as follows,

1. Start with $f(x) = 0$
2. Rewrite $f(x) = 0$ under the form $x = g(x)$
3. Verify the iteration sequence $x_0, x_1 = g(x_0), \ldots, x_n = g(x_{n-1})$ converges using the above theorem
4. Compute the terms of the sequence and stop when the required accuracy is reached (i.e when 2 consecutive terms have the same k decimal places where k is the desired accuracy).