TD1 TL1

Exercice 1 :

Rappel : V : alphabet : $\{a, b\}$ V^* : ens de mot sur cet alphabet $\{\varepsilon, a, aa, ab, b \ldots\}$

$w_1(w_2 \cdot w_3) = (w_1 \cdot w_2) \cdot w_3 = w_1 w_2 w_3$ Abus de notat° : aw : a suivi de w.

$|w|$ = nb de lettres ds w.

Pour définir par induction : • $\varepsilon \in L$

• $w_1, w_2 \in L \Rightarrow \begin{cases} aw, a \in L \\ bw, b \in L \\ w_1 w_2 \in L \end{cases}$

Q1 : $(\top \wedge \neg)$ → NON car ds les règles, il y a forcément une (avant le $\neg$

$(\top \wedge \top \wedge \top)$ → NON car il manque des parenthèses

Q2 : Notation :

(1) $\top \in E$ et $\bot \in E$

(2) $(\neg w) \in E$

(3) $(w_1 \wedge w_2) \in E$

(4) $(w_1 \vee w_2) \in E$

Arbre de preuve

$$\dfrac{(3)\dfrac{\top \in E \quad \top \in E}{(\top \wedge \top) \in E} \qquad (4)\dfrac{\bot \in E \quad \top \in E}{(\bot \vee \top) \in E}}{((\top \wedge \top) \vee (\bot \vee \top)) \in E}\,(4)$$

Q3 : $|w_1|_U = 1$, $|w_1|_B = 4$ $|w_1|_B = 5$

Q4. $\forall w \in E : |w|_N = 2|w|_B + |w|_U + 1$

• Base : $|\bot|_N = 1 = 2|\bot|_B + |\bot|_U + 1$ idem pour $\top$.

• Induction : 1) H : $|w|_N = 2|w|_B + |w|_U + 1$

$\underbrace{|(\neg w)|}_{1+|w|_N} \overset{?}{=} 2\underbrace{|(\neg w)|_B}_{|w|_B} + \underbrace{|(\neg w)|_U}_{|w|_U+1} + 1$

$= 2|w|_B + |w|_U + 1 + 1$

$= |w|_N + 1$

2) $|(w_1 \wedge w_2)|_N = 2|(w_1 \wedge w_2)|_B + |(w_1 \wedge w_2)|_U + 1$

HR : $|w_1|_N = 2|w_1|_B + |w_1|_U + 1$

$|w_2|_N = 2|w_2|_B + |w_2|_U + 1$

$|(w_1 \wedge w_2)|_N = |w_1|_N + |w_2|_N + 1$

$2|(w_1 \wedge w_2)|_B + |(w_1 \wedge w_2)|_U + 1 = 2|w_1|_N + 2|w_2|_N + 2 + |w_1|_U + |w_2|_U + 1$

$= |w_1|_N + |w_2|_N + 1$

Exercice 2 :

- $|\varepsilon|_A = 0$ ssi $\varepsilon \notin A$
  $|\varepsilon|_A = 1$ sinon
- $|aw|_A = 1 + |w|_A$ ssi $a \in A$
  $= |w|_A$ sinon

Exercice 3 :

Q1. • $\begin{cases} \varepsilon \in \mathcal{L}_1 \\ aa, ab, ba, bb \in \mathcal{L}_1 \end{cases}$

• $w_1, w_2 \in \mathcal{L}_1 \Rightarrow w_1 w_2 \in \mathcal{L}_1$

ou • $\varepsilon \in \mathcal{L}_1$
• $w \in \mathcal{L}_1 \Rightarrow aaw \in \mathcal{L}_1$ awa
$abw \in \mathcal{L}_1$ awb
$baw \in \mathcal{L}_1$ bwa
$bbw \in \mathcal{L}_1$ bwb

2. $a, \varepsilon \in \mathcal{L}_2$
$w \in \mathcal{L}_2 \Rightarrow abw \in \mathcal{L}_2$
$\Rightarrow bw \in \mathcal{L}_2$

3. $\varepsilon, a, b \in \mathcal{L}_3$
$w \in \mathcal{L}_3 \Rightarrow awa \in \mathcal{L}_3$
$\Rightarrow bwb \in \mathcal{L}_3$

Q2. $w \in \mathcal{L}_1 \Rightarrow |w|_{mod 2} = 0$ induction structurelle
$|w|_{mod 2} = 0 \Rightarrow w \in \mathcal{L}_1$ induct° sur la longueur.
$|w| = 0 \Rightarrow w = \varepsilon \in \mathcal{L}_1$
$|w|$ paire

Exercice 7 :

Rappel : $w \in L_1.L_2$ ssi $w = w_1.w_2$ $w_1 \in L_1, w_2 \in L_2$
$w \in L_1 \cup L_2$ ssi $w \in L_1 \vee w \in L_2$
$w \in L_1 \cap L_2$ ssi $w \in L_1 \wedge w \in L_2$
$w \in \overline{L}$ ssi $w \notin L$
$w \in L^*$ ssi $w = w_1 \ldots w_n$ où $(w_i)_{1 \leq i \leq n} \in L \vee w = \varepsilon$

Q1. $L_1 = \{a\}^*\{b\}^*$

Q2. $\begin{cases} ab \in \mathcal{L}_2 \\ w \in \mathcal{L}_2 \Rightarrow awb \in \mathcal{L}_2 \end{cases}$

Q3. $\mathcal{L}_1 \setminus (\{\varepsilon\} \cup \mathcal{L}_2)$

Exercice 9 :

1. $L^* \overset{?}{=} (L^*)^*$ $L^* \subseteq (L^*)^*$ V
$w \in (L^*)^*$

2. $L^* \cup \Pi^* \overset{?}{=} (L \cup \Pi)^*$
- $L^* \cup \Pi^* \subseteq (L \cup \Pi)^*$ F
- $(L \cup \Pi)^* \not\subset L^* \cup \Pi^*$

$L = \{a\}$ $\Pi = \{b\}$ $ab \in (L \cup \Pi)^*$
$ab \notin L^* \cup \Pi^*$. aaaa...a ou bbb...b

3. $(L^* \cup \Pi^*)^* \overset{?}{=} (L \cup \Pi)^*$
* $(L^* \cup \Pi^*)^* \overset{?}{\subseteq} (L \cup \Pi)^*$
q2. $L^* \cup \Pi^* \subseteq (L \cup \Pi)^*$
$(L^* \cup \Pi^*)^* \subseteq ((L \cup \Pi)^*)^*$ q2
$\overset{q1}{=}$
$(L^* \cup \Pi^*)^* \subseteq (L \cup \Pi)^*$

$(L \cup \Pi)^* \subseteq (L^* \cup \Pi^*)^*$
$\left.\begin{array}{l} \cdot L \subseteq L^* \\ \Pi \subseteq \Pi^* \end{array}\right\}$ $L \cup \Pi \subseteq L^* \cup \Pi^*$
$(L \cup \Pi)^* \subseteq (L^* \cup \Pi^*)^*$ q1

TD1TL1

4. $L^+ \overset{?}{=} L^* - \{\varepsilon\}$ $\quad\underline{F}$

$L^* - \{\varepsilon\} \subseteq L^+$

Si $L = \{\varepsilon\}$, $L^+ = \{\varepsilon\}$ alors que $L^* - \{\varepsilon\} = \emptyset$

5. $(LM)^* \overset{?}{=} L^* M^*$ $\quad L = \{a\}, M = \{b\}$

$a^* \in L^* M^* \notin (LM)^*$ $\quad M^0 = \{\varepsilon\}$ $\quad\underline{F}$

$a+\varepsilon$ $\quad abab \in (LM)^* \notin (L^* M^*)$

6. $(L \cup M)^* \overset{?}{=} (L^* M^*)^*$

$\underline{\subseteq}$

$L \subseteq L^* M^*$

$M \subseteq L^* M^*$

$L \cup M \subseteq L^* M^*$

$(L \cup M)^* \subseteq (L^* M^*)^*$

$L^* \subseteq (L \cup M)^*$, $M^* \subseteq (L \cup M)^*$

$L\,\cancel{M} \subseteq (L \cup M)^* (L \cup M)^*$

$L^* M^* \subseteq ((L \cup M)^*)^*$

$(L^* M^*)^* \subseteq (((L \cup M)^*)^*)^*$

$(L^* M^*)^* \subseteq (L \cup M)^*$

$A^* A^* = A^*$

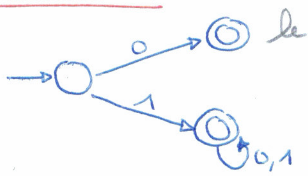
$(A^*)^*$

$\underline{V}$

TD2

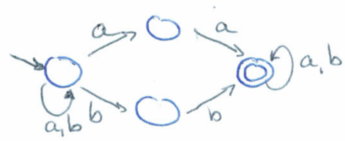
Exercice 1 :

1\.

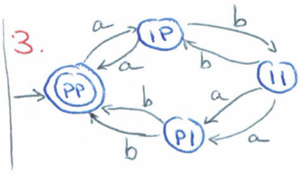


ne pas oublier le mot 0.

2\.

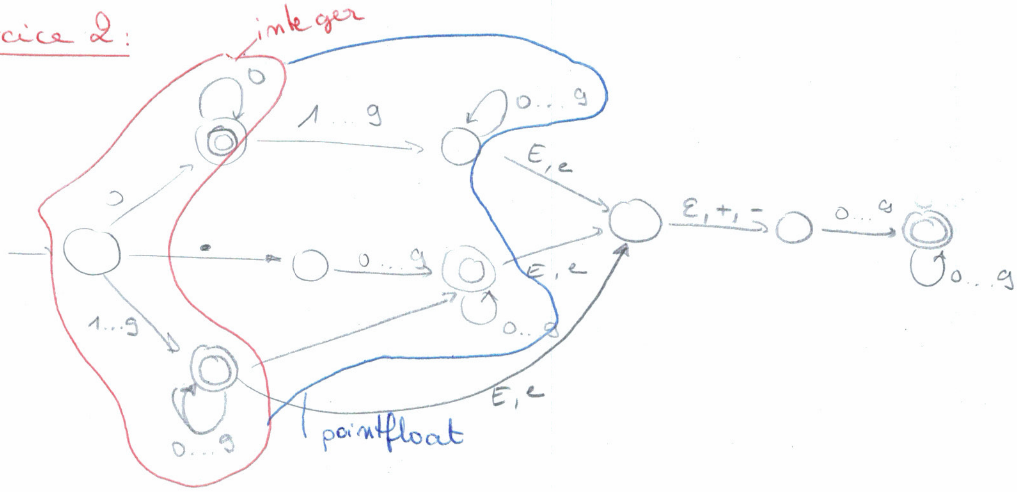


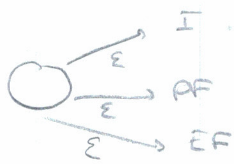
3\.



Exercice 2 :

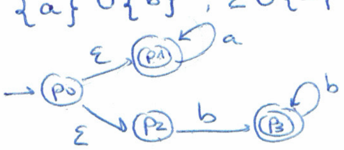
1\.



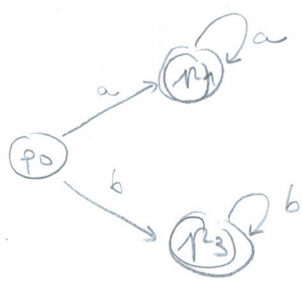


Rappel : ε-transition

$\{a\}^* \cup \{b\}^*$, $\varepsilon \cup \{a\}^* \cup \{b\}^*$

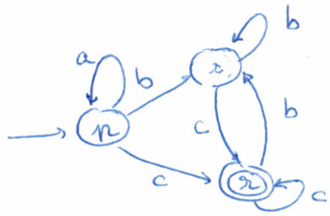


	$\varepsilon^* a$	$\varepsilon^* b$
p_0	p_1	p_3
p_1	p_1	/
p_2	/	p_3
p_3	/	p_3



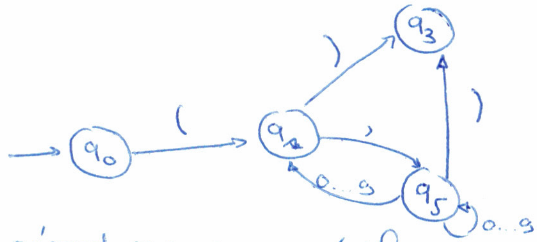
Exercice 5 :

	ε^*a	ε^*b	ε^*c
p	p	s	r
s	–	s	r
r	–	s	r



Exercice 6 :

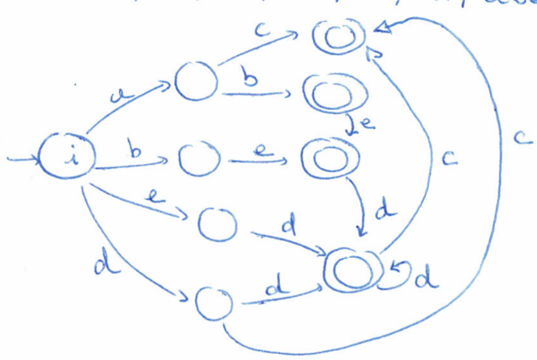
	$\varepsilon^*($	$\varepsilon^*)$	$\varepsilon^*0...9$	$\varepsilon^*,$
q_0	q_1	–	–	–
q_1	–	q_3	q_5	–
q_3	–	–	–	–
q_5	–	q_3	q_5	q_1



- Ne répond pas aux spécifications car cet automate reconnaît (123)

Exercice 7 :

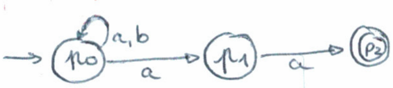
1. H_{R_0} : ab, be, dd, ac, dc, ed, abe, bed, ddc, edd, edc, ddd

2. 

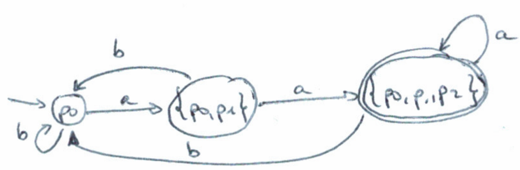
3. $I = \{i\}$. $F = \{q_a, a \in V\}$ $Q = I \cup F \cup \{p_a, a \in V\}$

$\delta = \{\{(i, a, p_a), (p_a, b, q_b)\}$ ssi $(a,b) \in R\} \cup \{(q_x, y, q_y)\ (x,y) \in R\}$

mot $\in H_R$ => induct° sur la longueur du mot



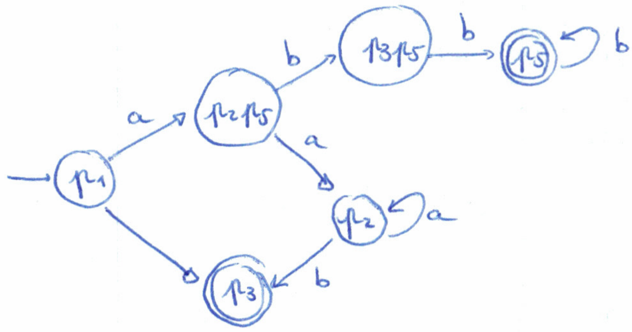
	a	b
p_0	$\{p_0, p_1\}$	p_0
$\{p_0, p_1\}$	$\{p_0, p_1, p_2\}$	p_0
F $\{p_0, p_1, p_2\}$	$\{p_0, p_1, p_2\}$	p_0



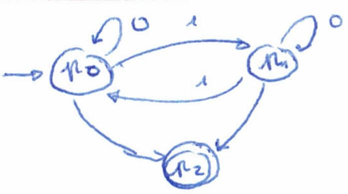
TD3 - T1

Exercice 1:

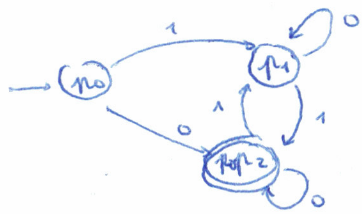
	$\varepsilon^* a$	$\varepsilon^* b$
$\{p_1\}$	$\{p_2, p_5\}$	$\{p_3\}$
$\{p_2, p_5\}$	$\{p_2\}$	$\{p_3, p_5\}$
$\{p_3\}$	–	–
$\{p_2\}$	$\{p_2\}$	$\{p_3\}$
$\{p_3, p_5\}$	~~$\emptyset$~~ –	$\{p_5\}$
$\{p_5\}$	–	$\{p_5\}$

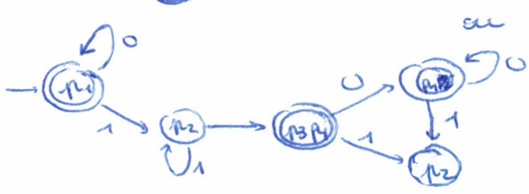


Exercice 2:

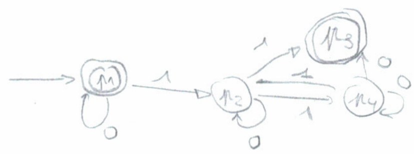


	0	1
p_0	p_0p_2	p_1
p_0p_2	p_0p_2	p_1
p_1	p_1	p_0p_2





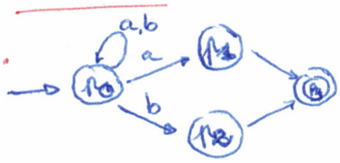
en partant de

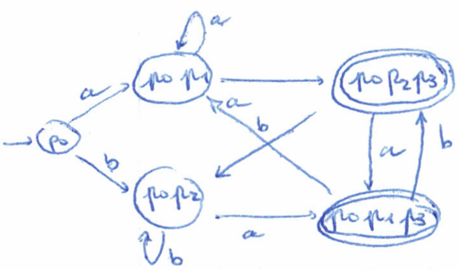


	0	1
p_1	$\{p_1\}$	$\{p_2\}$
p_2	$\{p_2\}$	$\{p_3, p_4\}$
$\{p_3, p_4\}$	$\{p_1 p_3\}$	$\{p_2\}$

Exercice 3:

1.





Rappel : Minimization

$p \equiv q \overset{def}{=} \forall w \in V^* : \delta(q,w) \in F \Leftrightarrow \delta(p,w) \in F$

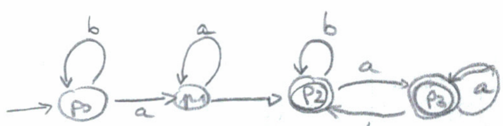
$p \equiv_0 q$: Q F → Soit p & q sont des états finaux, soit ils ne sont pas finaux

$p \equiv_{k+1} q$: $\forall a \in V^*$: $p \xrightarrow{a} p'$, $q \xrightarrow{a} q'$, $p' \equiv_k q'$

Ex : $\equiv_0$: $\{p_0, p_1\}, \{p_2, p_3\}$,

$\equiv_1$: $\{p_0\}, \{p_1\}, \{p_2, p_3\}$ $\quad p_2 \xrightarrow{b} p_2$, $p_0 \xrightarrow{b} p_0$

$\equiv_2$: $\{p_0\}, \{p_1\}, \{p_2, p_3\}$



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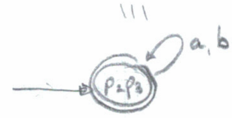
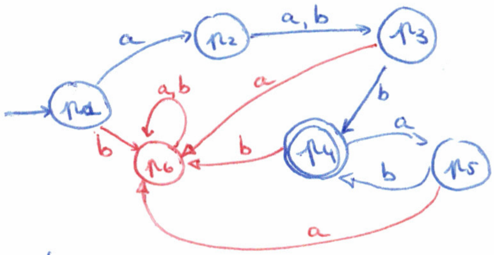


Exercice 6 :

• D'abord rendre cplt l'automate en rajoutant un état puit.

*



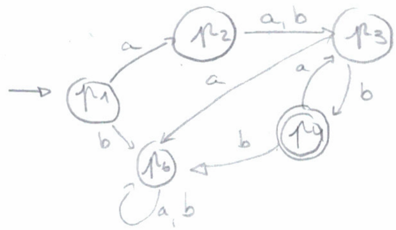
$\equiv_0$: $\{p_1, p_2, p_3, p_5, p_6\}$. $\{p_4\}$

$\equiv_1$: $\{p_3, p_5\}$, $\{p_4\}$, $\{p_1, p_2, p_6\}$

$\equiv_2$: $\{p_4\}$, $\{p_3, p_5\}$, $\{p_2\}$, $\{p_1, p_6\}$

$\equiv_3$: $\{p_4\}$, $\{p_3, p_5\}$, $\{p_2\}$, $\{p_1\}$, $\{p_6\}$

$\Longrightarrow$



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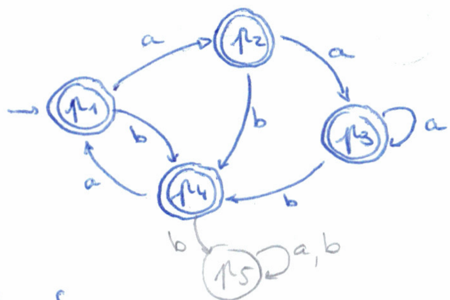
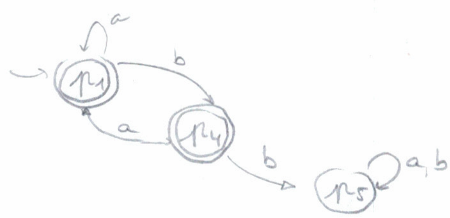


$\equiv_0$: $\{p_1, p_2, p_3, p_4\}$, $\{p_5\}$

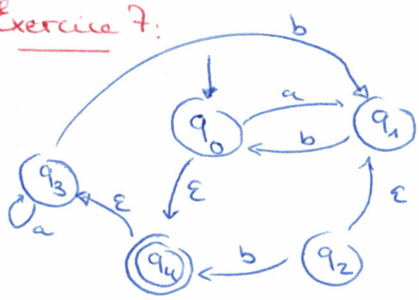
$\equiv_1$ $\{p_1, p_2, p_3\}$, $\{p_4\}$, $\{p_5\}$

$\equiv_2$: $\{p_1, p_2, p_3\}$, $\{p_4\}$, $\{p_5\}$

$\Longrightarrow$



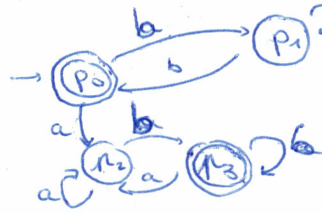
Exercice 7 :



	$\varepsilon^* a$	$\varepsilon^* b$
$p_0 = \{q_0\}$	$\{q_1, q_3\}$	$\{q_1\}$
$p_1 = \{q_1\}$	$\{q_1\}$	$\{q_0\}$
$p_2 = \{q_1, q_3\}$	$\{q_1, q_3\}$	$\{q_0, q_1\}$
$p_3 = \{q_0, q_1\}$	$\{q_1, q_3\}$	$\{q_0, q_1\}$

→ q_0 final car il va en q_4 final via ε.

$\Longrightarrow$



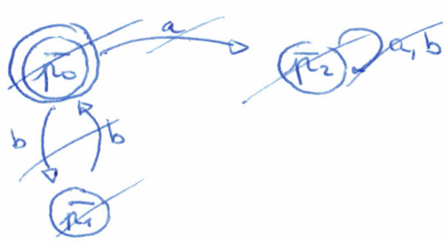
déterminisé, sans ε-trans.

$\equiv_0$: $\{p_0, p_3\}$ $\{p_1, p_2\}$

$\equiv_1$: $\{p_0\}$, $\{p_1\}$, $\{p_2\}$ $\{p_3\}$

$\equiv_2$: $\{p_0\}$, $\{p_1\}$, $\{p_2, p_3\}$

$\Longrightarrow$



TD4 - TL

Expressions régulières :

$E = \emptyset \mid \varepsilon \mid a \mid E_1.E_2 \mid E_1 + E_2 \mid E^*$

$[\![\emptyset]\!] = \emptyset$

$[\![\varepsilon]\!] = \{\varepsilon\}$

$[\![a]\!] = \{a\}$

$[\![E_1.E_2]\!] = \{w_1 w_2\},\ w_1 \in [\![E_1]\!]\ w_2 \in [\![E_2]\!]$

$[\![E_1 + E_2]\!] = [\![E_1]\!] \cup [\![E_2]\!]$

$[\![E^*]\!] = \{\varepsilon\} \cup [\![E]\!] \cup [\![E]\!][\![E]\!] \cup \ldots$

Exercice 1 :

1. $E.E^* + \varepsilon = E^*$ 2. $\varepsilon.E = E$ 3. $\emptyset^* = \varepsilon$ 4. $\varepsilon^* = \varepsilon$ 5. $\emptyset.E = \emptyset$ 6. $\emptyset + E = E$

Exercice 2 :

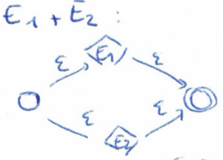
1. Les nombres composés de 0 & de 1 avec ~~un nombre multi~~ 3k nombre de 1. ($k \geq 0$)
2. Les mots sur $\{0,1\}$ ayant au max 2 zéros consécutifs.
3. les mots sur $\{0,1\}$ ———— 1 zéro.

Exercice 3 :

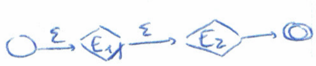
1. $(0+1)^* (00 + 11)(0+1)^*$ ✓
2. $1^* (01)^* 1^*$ $(1+01)^*$
trop d'étoiles.
3. $(0+1)^* 0 (0+1)^*$ ✓ ou $1^* 0 (0+1)^*$
↳ auto. nom. det ↳ auto. det.
4. $\cancel{(0.1)^* (1.0)^*}$ $\cancel{[(01)^*(\varepsilon+0)] + [(10)^*(\varepsilon+1)]}$ $(0+\varepsilon)(10)^*(1+\varepsilon)$ ou $(1+\varepsilon)(01)^*(0+\varepsilon)$
5. $(00 + 10 + 01 + 11)^*$ $((0+1)(0+1))^*$

Rappel :

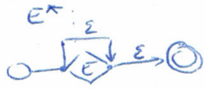
$E_1 + E_2$:

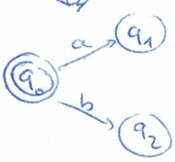


$E_1.E_2$:



E^* :





$\begin{cases} x_0 = a x_1 + b x_2 + \varepsilon \\ \cancel{x = (a+b)^*(ab)^*} \end{cases}$ $\cancel{x = ax + w}$ $\cancel{= a^* w}$

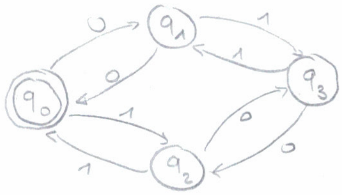
Exercice 6 : Rappel $X = AX + B$ sol° $A^* B$

$\begin{cases} x_0 = 0 x_0 + 1 x_1 + \varepsilon \\ x_1 = 1 x_1 + 0 x_2 \\ x_2 = 0 x_0 + 1 x_1 \end{cases} \iff \begin{cases} x_0 = \\ x_1 = 1 x_1 + 00 x_0 + 01 x_1 \\ x_2 = \end{cases} \iff \begin{cases} x_0 = \\ x_1 = (1+01) x_1 + 00 x_0 \\ x_2 = \end{cases} \iff \begin{cases} x_0 = \\ x_1 = (1+01)^* 00 x_0 \\ x_2 = \end{cases}$

$\iff \begin{cases} x_0 = 0 x_0 + 1(1+01)^* 00 x_0 + \varepsilon \\ x_1 = \\ x_2 = \end{cases} \iff \begin{cases} x_0 = (0+1(1+01)^* 00)^* \\ x_1 = \\ x_2 = \end{cases}$

Exercice 7 :

Rappel de l'automate.



$$\begin{cases} x_0 = 0x_1 + 1x_2 + \varepsilon \\ x_1 = 0x_0 + 1x_3 \\ x_2 = 1x_0 + 0x_3 \\ x_3 = 1x_1 + 0x_2 \end{cases}$$

1. Eliminat° de x_1.

$$\begin{cases} x_0 = 00x_0 + 01x_3 + 1x_2 + \varepsilon \\ x_1 = \\ x_2 = \\ x_3 = 10x_0 + 11x_3 + 0x_2 \end{cases}$$

2. el de x_2

$$\Leftrightarrow \begin{cases} x_0 = 00x_0 + 01x_3 + 11x_0 + 10x_3 + \varepsilon \\ x_1 = \\ x_2 = \\ x_3 = 10x_0 + 11x_3 + 01x_0 + 00x_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_0 = (00 \mid 11)x_0 + (01 \mid 10)x_3 + \varepsilon \\ x_1 = \\ x_2 = \\ x_3 = (10 \mid 01)x_0 + (11 \mid 00)x_3 \end{cases}$$

3. el de x_3

$$\Leftrightarrow \begin{cases} x_0 = (01 \mid 10)(11 \mid 00)^*(10 \mid 01)x_0 + (00 \mid 11)x_0 + \varepsilon \\ x_1 = \\ x_2 = \\ x_3 = (11 \mid 00)^*(10 \mid 01)x_0 \end{cases}$$

$$\Leftrightarrow \quad x_0 = \left(\left((01+10)(11+00)^*(10+01)\right) + (00+11)\right)^*$$

Rappel :

① L_1, L_2 reguliers $\Rightarrow$ L_1L_2, L_1^*, $L_1 \cup L_2$, $L_1 \cap L_2$, $\overline{L_1}$, $L_1 \setminus L_2$ régulier

Montrer que L régulier : - Trouver $R_1 \dots R_n$ régulier $L = R_1 \, op_1 \, R_2 \, op_2 \, R_3 \dots$

L non régulier : - Prendre $\underset{\text{non régulier}}{N}$ $R_1 \dots R_n$ reguliers $N = L \, op_1 \, R_1 \, op_2 \, R_2 \dots$

② Substitution regulières : $S(a) = L$ ← régulier

$S(aa) \rightarrow L^2$

$S(a) = L$

$h : V \rightarrow W^*$

$h(a) = w \in W^*$

$h(w_1w_2) = h(w_1)h(w_2)$

L régulier $\Rightarrow h(L)$ régulier

Ⓡ R regulier tq $h(R) = L$

ⓃⓇ N non regulier tq $h(L) = N$

Exercice 8 :

1. $L_1 = \{w \in \{a,b\}^*, |w|_a = |w|_b\}$

L_1 est non régulier car : En le supposant régulier comme $\{a^nb^n, n \geq 0\} = \underset{\text{reg}}{L_1} \cap \underset{\text{reg}}{(a^*b^*)}$

alors $\{a^nb^n\}$ serait regulier.

2. $L_2 = \{a^ib^jc^k, \; i+j = k \geq 0\}$

L_2 non régulier car en posant :

$h : \begin{cases} a \rightarrow x \\ b \rightarrow x \\ c \rightarrow y \end{cases}$: $h(L_2) = \{x^ny^n, n \geq 0\}$

ou

$\{b^nc^n, n \geq 0\} = L_2 \cap (b^*c^*)$

$\{a^nc^n, n \geq 0\} = L_2 \cap (a^*c^*)$

3. $L_3 = \{(ab)^{2n}(cd)^{2n}, n \geq 0\}$

$h : \begin{cases} a \rightarrow x \\ b \rightarrow \varepsilon \\ c \rightarrow y \\ d \rightarrow \varepsilon \end{cases}$ $h(L_3) = \{x^{2n}y^{2n}, n \geq 0\} = L_4$

$L_4 \cup \{x\} L_4 \{y\}$

TD4-TL

Exercice 9 :

Rappel : Lemme de l'étoile

L régulier : $\exists N \geq 1, \forall w : |w| \geq N : w = xyz, |y| \geq 1, |xy| \leq N$
$\forall i \geq 0 : xy^iz \in L$

Utilisés pour montrer qu'un langage est non régulier en trouvant un w qui app. au lang.

1 . $L_1 = \{wb^n, n \in \mathbb{N}, w \in \{a,b\}^n\}$

Prenons $a^N b^N = xyz$ $|xy| \leq N$ et $xy^iz \in L$

$a^{|x|} a^{|y|}$ $xy^2z = a^{N+|y|} b^{|N|} \notin L$ contradiction.

Si $|y|$ pair faux car $|w| = 2n$

Si $|y|$ impair faux car il doit y avoir au moins moitié de b.

2 . $L_2 = \{w \in \{ab\}^*, w$ est un palyndrome$\}$

Prenons $a^N b a^N$ $\begin{cases} |xy| \leq N \\ |y| \geq 1 \end{cases}$ $xy^0z = a^{N-|y|} b \ a^N \notin L$

Partie 2 - TD1

Grammaire : $G = \langle T, N, S, R \rangle$ non terminaux, terminaux, axiome, règles $w \in L(G)$ ssi $S \xrightarrow{*} w$

Hiérarchie de Chomsky		
	G	0
	SC	1
	HC	2
	Regulières	3

$\{u \to v\}$ $(T \cup N)^* N (T \cup N)^*$ $(T \cup N)^*$

3 . $S \to aS$
$\to b$ ou $S \to Sa$
$S \to b$

2 . $S \to aSb \mid \ldots$

1 . $u \to v$ $|u| \leq |v|$

0 . $u \to v$

$a^* b a^*$

$S \to aS \mid bT$
$T \to aT \mid \varepsilon$

aaba

$S \to aS \to aaS \to aabT \to aabaT \to aaba$

Exercice 1 :

Q1. $\{a^n b^p \mid n \geq p \geq 0\}$: $S \to aSb \mid aS \mid \varepsilon$

Q2. $\{a^n b^p \mid n \neq p\}$:
$S \to P \mid \Pi$
$P \to aPb \mid aP \mid \varepsilon$ + de a que de b
$\Pi \to a\Pi b \mid \Pi b \mid \varepsilon$ + de b que de a

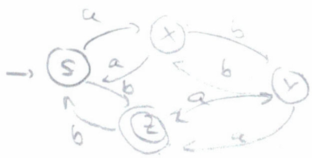
ou
$S \to aSb \mid X \mid Y$
$X \to aX \mid a$ que des a
$Y \to Yb \mid b$ que des b.

Q3. $\{a^n b^p \mid 2p \geq n \geq p\}$ $S \to aSb \mid aaSb \mid \varepsilon$

Q4. $\{a^n b^p c^q \mid n+p = q\}$ $S \to aSc \mid X$
$X \to bXc \mid \varepsilon$
$S \xrightarrow{*} a^n X c^n \xrightarrow{*} a^n b^p X c^p c^n$

Question 2 :

$S \to aX \mid bZ$
$X \to aS \mid bY$
$Y \to aZ \mid bX$
$Z \to aY \mid bS \mid \varepsilon$



Exercice 2 :

aaa bbb ccc

Question 1 :

2. $S \xrightarrow{(2)} aSBc \xrightarrow{(1)} aabcBc \xrightarrow{(3)} aabBcc \xrightarrow{(4)} aabbcc$

3. $S \xrightarrow{(2)} aSBc \xrightarrow{(2)} aaSBcBc \xrightarrow{(2)} aaabcBcBc \xrightarrow{(3)} aaabBcBcc \xrightarrow{(3)} aaabBB\ldots$
$\xrightarrow{(4)} aaabbBccc$
$\xrightarrow{(4)} aaabbbccc$

$S \xrightarrow{(2)\times n-1} a^{n-1} S \overbrace{BcBcBc\ldots}^{(n-1)\times} \xrightarrow{(1)} a^n bc\overbrace{BcBc\ldots}^{(n-1)\times}$

$\xrightarrow{\frac{n(n-1)}{2}\times(3)} a^n b\overbrace{B\ldots B}^{n-1}c^n$

$\xrightarrow{(n-1)\times(4)} a^n b^n c^n$

Rappel : $G = \langle T, N, S, R \rangle$

$w \in L(G) \iff S \overset{*}{\underset{R}{\Rightarrow}} w$
$\in T^*$

$S \to aX \mid \ldots$	3	regulier
$S \to aSb \mid \ldots$	2	HC
$bX \to bb \ldots$	1	SC
$u \to v$	0	

Exercice 3 :

Q1. Def 2 : $S \to aX \mid \varepsilon$
$X \to bS$

Def 1 :

Q2. $G_1 \in$ Def 1, $G_2 \in$ Def 2

* Def$_2$ ⊆ Def$_1$ (juste un cas particulier de def$_1$ où les mots et des lettres

Est-ce que Def$_1$ ⊆ Def$_2$?
Si $G_1 \in$ Def$_1$ est-ce il existe $G_2 \in$ Def$_2$ tq $L(G_1) = L(G_2)$

$G_1 = \langle T, N, S, R \rangle$
$G_2 = \langle T, N \cup A_i, S, \cup [R_i] \rangle$

$[A \to wB]_1 = A \to aB \quad w = a \quad w = aw'$
$= \{A \to aA_1\} \cup [A_1 \to w'B]$

$w \in L(G_1) \Rightarrow w \in L(G_2) \quad w = w_1 \ldots w_n$
$X_i \to w_{i+1} X_{i+1}$

$S \Rightarrow_1 w_1 X_1 \Rightarrow_1 w_1 w_2 X_2 \ldots \Rightarrow w_1 \ldots w_n X_n$

$[X_1 \to w_{i+1} X_{i+1}] = X_1 \Rightarrow$

TD 4 - TL

Exercice 4 :

Q1. $Y \to 0Y01 \mid 1Y01 \mid 0Y11 \mid 1Y11 \mid 0$ ou $Y \to NYN \mid 0$
$N \to 0 \mid 1$

Q2. $w \in YZ$

$a_1 \ldots 0_{n+1} \ldots a_{2n+1} b_1 \ldots 1_{m+1} \ldots b_{2m+1}$

$|w| = 2n + 2m + 2 = 2(n+m+1)$

$|w_1| = |w_2| = n + m + 1$

Le 0 apparaît en $n+1^e$ place de w_1. Rq le 1 apparaît en $n+1^e$ place de w_2.

Le 1 est $2n+1 + m+1^e$ place de w
$= 2n + m + 2$ donc en $2n + m + 2 - (n + m + 1)$ place de w_2 / $n + m + 1 - m$
$= n + 1^e$ place de w_2 ✓

Q3.

$a_1 \ldots a_n a_{n+1} \ldots a_{2n}$

$\exists i : \begin{cases} a_i = 0 \\ a_{n+i} = 1 \end{cases}$ (ou $\begin{cases} a_i = 1 \\ a_{n+i} = 0 \end{cases}$) (2i peut aussi $\hat{e} >$ à n aussi) mais par contre $2i < n+i$

$a_1 - a_{i-1} 0 a_{i+1} \ldots a_{2i} \ldots a_n a_{n+1} \ldots a_{n+i-1} 1 a_{n+i+1} \ldots a_{2n} \in YZ$

$\in Y$ | $w_1 \in Z$ | w_2

$\left.\begin{array}{l} n+i - 2i = n - i \\ 2n - (n+i) = n - i \end{array}\right\}$ ok $\hat{m}$ nbr de chaque côté du 1 → lg impaire + 1 au milieu

$|w_1| = n + i - 2i = n - i$ $\quad |w_2| = 2n - (n+i) = n - i$

$C \to YZ \mid ZY \mid Y \mid Z$

Exercice 5 :

$G_1 = \langle S_1, N_1, T_1, R_1 \rangle$ HC
$G_2 = \langle S_2, N_2, T_2, R_2 \rangle$ HC
$N_1 \cap N_2 = \emptyset$

1. Union : $G_1 \cup G_2$: $S \to S_1 \mid S_2$
$S_1 \to \ldots$ $\quad S_2 \to \ldots$ ok

Concaténation : $G_1 G_2$ $\quad S \to S_1 S_2$
$S_1 \to \ldots$ $\quad S_2 \to \ldots$ ok

Itération : G_1^* : $S \to S_1 S \mid \varepsilon$ ok

Vrai : fermé : on reste dans la classe des langages hors-contexte

2. Intersection : Faux
$a^* b^n c^n \cap a^n b^n c^* = a^n b^n c^n$

Complémentaire : Faux cf exo 4.