Exo ②

$U_k(w) = E(u_k(w)) = E(w) \pi_k Var(w) \qquad \pi_k \in \mathbb{R}$

$Var(w) \nearrow \Rightarrow E(w) \nearrow$ riscophobe ✓

$$\frac{\frac{\partial E(w)}{\partial w}}{\frac{\partial Var(\sigma)}{\partial w}} = \pi_k \gamma \frac{\partial U}{\partial w} = \frac{\partial E}{\partial U} \quad \pi_k \quad \frac{\partial r}{\partial w}$$

$$\frac{\partial \epsilon}{\partial U}$$

2) $x_k = (\omega_i)_{i \in [1..N]} \qquad \sum \omega_i = 1$

x_k^* sol opt $\qquad x_k^*$ tq

$$\boxed{W_k(1) = w_k(0)(1 + R)}$$

$\arg\min Var(R) \qquad \arg\max \mathbb{E}(R)$

$\mathbb{E}(w_f) = m \qquad Var(w_f) = 0$

$\mathbb{1}_N x_k^* = 1 \qquad \mathbb{1}_N x_k^* = 1$

3 - $\mathcal{L}(Var(R), \mathbb{E}(w_f)) = Var(R) - \lambda_1(\mathbb{E}(w_f) - m) - \lambda_2(\sum x_i - 1)$

sol Td : 1

1) $W_f = W_0(1+R)$

$U_k(W_f) = W_0 + W_0 \mathbb{E}(R) - \Pi_k W_0^2 Var(R)$

$U_k(W_f) = V(\mathbb{E}(R), Var(R))$

$V(m, v) = W_0 + W_0 m - \Pi_k W_0^2 v$

$\frac{\partial V}{\partial v} = -\Pi_k W_0^2 < 0 \Rightarrow \Pi_k > 0$

2) l'investisseur maximise l'espérance d'utilité de sa richesse finale

$W_k(1) = W_k(0)(1 + R_{P,k}(x))$

x = composition du portefeuille

$W_0 = W_k(0)$

$$x_k^* = \underset{\substack{x \in \mathbb{R}^N \\ {}^t\mathbb{1}x = 1}}{\operatorname{Arg\,Max}} \left(W_0 + W_0 \mathbb{E}(R_{P,k}(x)) - \Pi_k W_0^2 Var(R_{P,k}(x))\right)$$

${}^t\mathbb{1}x = 1$ composition du portefeuille !!

Lagrangien $L(x, \lambda) = \mathbb{E}(R_{P,k}(x)) - \Pi_k W_k(0) Var(R_{P,k}(x))$

$- \lambda({}^t\mathbb{1}x - 1)$

$\mathbb{E}(R_{P,k}(x)) = {}^t\varepsilon x \qquad Var(R_{P,k}(x)) = {}^tx \Lambda x$

$\underset{x}{\overrightarrow{grad}}\, L(x^*, \lambda^*) = 0_{\mathbb{R}} = {}^t\varepsilon - \Pi_k W_k(0) \times 2 \Lambda x^* - \lambda^* {}^t\mathbb{1}$ (1)

$\underset{\lambda}{grad}\, L(x^*, \lambda^*) = 0 = {}^t\mathbb{1}x - 1$ (2)

(1) $\Rightarrow x_k^* = \frac{1}{2\Pi_k W_k(0)}\left[\Lambda^{-1}{}^t\varepsilon - \lambda^* \Lambda^{-1} {}^t\mathbb{1}\right]$

reporte dans (2)

$\frac{1}{2\Pi_k W_k(0)}\left[b - \lambda^* a\right] = 1 \Rightarrow b - \lambda^* a = 2\Pi_k W_k(0)$

$$\lambda^* = \frac{b - 2\pi_k w_k(0)}{a}$$

donc $x_k^* = \frac{1}{2\pi_k w_k(0)} \left[b\mu - \frac{b - 2\pi_k w_k(0)}{a} \times a \right]$

$$= \frac{b}{2\pi_k w_k(0)} \mu + \left(1 - \frac{b}{2\pi_k w_k(0)}\right) v$$

$x = \alpha z + (1-\alpha) y$ v : la composition du porte feuille minimale

4- le porte feuill de l'investisseur k est moyenne - variance efficient car combinaison convexe de deux porte feuill moyenne variance efficient

5- $\mu_k^* = \mu^*(\pi_k)$ et $\sigma_k^{*2} = \sigma^{*2}(\pi_k)$

$$= {}^t\mathcal{E}\, x_k^* = \alpha_k\, {}^t\mathcal{E}\, \mu + (1-\alpha_k)\, {}^t\mathcal{E}\, v$$

$$= \alpha_k \frac{c}{b} + (1-\alpha_k) \frac{b}{a}$$

$$= \alpha_k \left(\frac{c}{b} - \frac{b}{a} \right) + \frac{b}{a} = \frac{\cancel{b}}{2\pi_k w_k(0)} \left(\frac{ac - b^2}{a\cancel{b}} \right) + \frac{b}{a}$$

$$\sigma_k^{*2} = {}^t x_k^* \Lambda x_k^* = \alpha_k^2\, {}^t\mu \Lambda \mu + 2\alpha_k(1-\alpha_k)\, {}^t\mu \Lambda v + (1-\alpha_k)^2\, {}^t v \Lambda v$$

$$= \alpha_k^2 \frac{c}{b^2} + 2\alpha_k(1-\alpha_k) \frac{b}{ab} + (1-\alpha_k)^2 \frac{1}{a}$$

$$= \alpha_k^2 \left[\frac{c}{b^2} - \frac{2}{a} + \frac{1}{a} \right] + \frac{1}{a}$$

$$= \frac{b^2}{4\pi_k^2 w_k^2(0)} \frac{ac - b^2}{ab^2} + \frac{1}{a}$$

$$\mu_k^* = m_{min} + \frac{ac - b^2}{a} \times \frac{1}{2\pi_k w_k(0)}$$

$$\sigma_k^{*2} = \sigma_{min}^2 + \frac{ac-b^2}{a} \frac{1}{(2\pi_k w_k(0))^2}$$

$$\sigma_k^{*2} = \sigma_{min}^2 + \frac{ac-b^2}{a}\left[\frac{\mu_k^* - m_{min}}{\frac{ac-b^2}{a}}\right]^2$$

$$\sigma_k^{*2} = \sigma_{min} + \frac{a}{ac-b^2}\left(\mu_k^* - m_{min}\right)^2$$

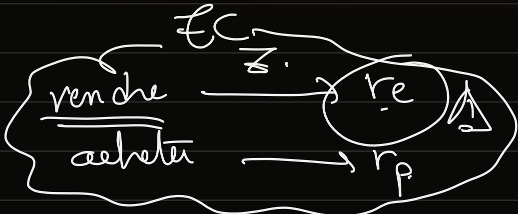
$$\frac{d(\sigma_k^*)^2}{d\mu_k^*} = \frac{2a}{ac-b^2}\left(\mu_k^* - m_{min}\right)$$

$$= \frac{2a}{ac-b^2}\,\frac{ac-b^2}{a}\,\frac{1}{2\pi_k w_k(0)} = \frac{1}{2\pi_k w_k(0)}$$

$$\frac{d\mu_k^*}{d(\sigma_k^2)} = \pi_k w_0(k)$$

exo (1) emprunter avec r_e $\quad r_e > r_p$
prêter avec r_p
rentabilité espérée

$r_e < m_0^\sigma$



r_p $\quad B_p$: zero coupon acheté

vendre ZC B_e à taux r_e

1) un investisseur ne