

## **Lecture NO (5)**

### **Forces in Static Fluids (cont)**

#### **(5.1) Absolute and Relative Pressure**

The term pressure is sometimes associated with different terms such as *atmospheric*, *gauge*, *absolute*, and *vacuum*. The meanings of these terms have to be understood well before solving problems in hydraulic and fluid mechanics.

##### **(5.1.1) Atmospheric Pressure**

It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure is change as the density of air varies according to the altitudes. Greater the height lesser the density. Also it may vary because of the temperature and humidity of air. Hence for all purposes of calculations the pressure exerted by air at sea level is taken as standard and that is equal to: -

$$1 \text{ atm} = 1.01325 \text{ bar} = 101.325 \text{ kPa} = 10.328 \text{ m H}_2\text{O} = 760 \text{ torr (mm Hg)} = 14.7 \text{ psi}$$

##### **(5.1.2) Gauge Pressure or Positive Pressure**

It is the pressure recorded by an instrument. This is always above atmospheric. The zero mark of the dial will have been adjusted to atmospheric pressure. Gauge pressure is

$$p = \rho gh$$

##### **(5.1.3) Vacuum Pressure or Negative Pressure**

This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

##### **(5.1.4) Absolute Pressure**

Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure. Atmospheric pressure is usually considered as the datum line and all other pressures are recorded either above or below it. Absolute pressure is

$$p_{\text{Absolute}} = p_{\text{Atmospheric}} + \rho gh$$

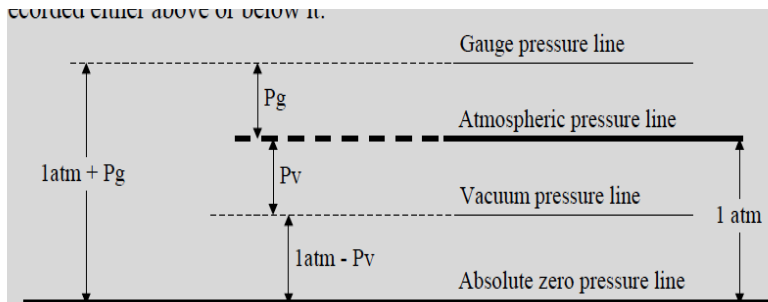
$$\text{Absolute Pressure} = \text{Atmospheric Pressure} + \text{Gauge Pressure}$$

$$\text{Absolute Pressure} = \text{Atmospheric Pressure} - \text{Vacuum Pressure}$$

For example if the vacuum pressure is 0.3 atm  $\Rightarrow$  absolute pressure =  $1.0 - 0.3 = 0.7$  atm

**Note: -**

**Barometric pressure** is the pressure that recorded from the barometer (apparatus used to measure atmospheric pressure).

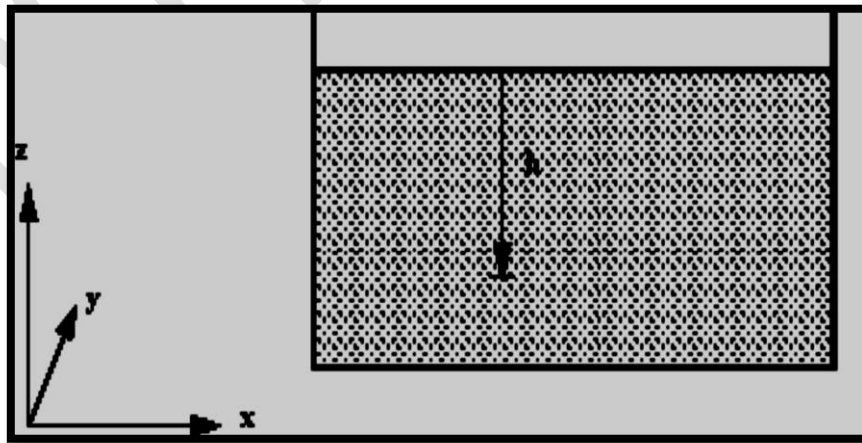


## (5.2) Pressure and Head

In a static fluid of constant density we have the relationship  $\frac{dp}{dz} = -\rho g$ , as shown in

$$p = -\rho g z + \text{constant}$$

In a liquid with a free surface the pressure at any depth  $z$  measured from the free surface so that  $z = -h$  (see the figure below)



Fluid head measurement in a tank

This gives the pressure

$$p = \rho gh + \text{constant}$$

At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure,  $p_{\text{atmospheric}}$ . So

$$p = \rho gh + p_{\text{atmospheric}}$$

As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, it is convenient (and often done) to take atmospheric pressure as the datum. So we quote pressure as above or below atmospheric. Pressure quoted in this way is known as gauge pressure i.e.

As  $g$  is (approximately) constant, the gauge pressure can be given by stating the vertical height of any fluid of density  $\rho$  which is equal to this pressure.

$$p = \rho gh$$

This vertical height is known as **head** of fluid

**Note:** If pressure is quoted in *head*, the density of the fluid *must* also be given.

**Example:**

We can quote a pressure of  $500 \text{ kN m}^{-2}$  in terms of the height of a column of water of density,  $\rho = 1000 \text{ kg m}^{-3}$ . Using  $p = \rho gh$ ,

$$h = \frac{p}{g\rho} = \frac{500 \times 10^3}{9.81 \times 1000} = 50.95 \text{ m of water}$$

And in terms of Mercury with density,  $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$ .

$$h = \frac{p}{g\rho} = \frac{500 \times 10^3}{9.81 \times 13.6 \times 10^3} = 3.75 \text{ m of Mercury}$$

### (5.3) Pressure Measurement by Manometer

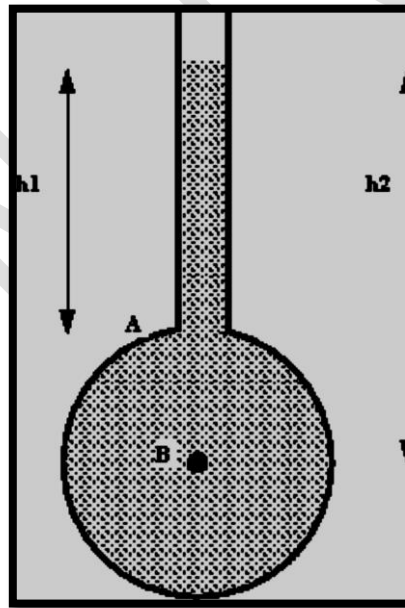
The relationship between pressure and head is used to measure pressure with a manometer (also known as a liquid gauge).

#### Objective:

To demonstrate the analysis and use of various types of manometers for pressure measurement.

#### (5.3.1) The Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured. An example can be seen in the figure below. This simple device is known as a **Piezometer tube**. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is gauge pressure.



A simple piezometer tube manometer

Pressure at A = pressure due to column of liquid above A

$$p_A = \rho g h_1$$

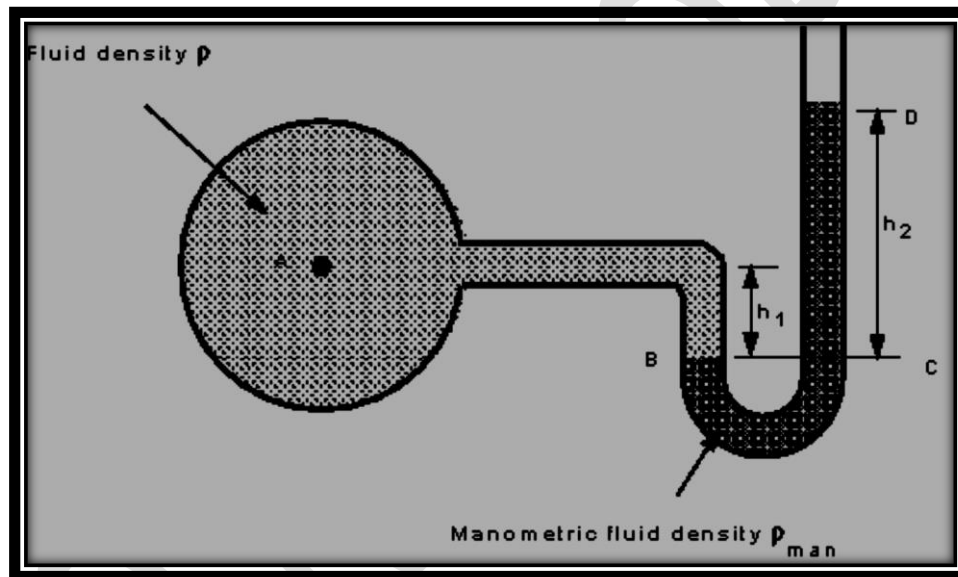
Pressure at B = pressure due to column of liquid above B

$$p_B = \rho g h_2$$

This method can only be used for liquids (i.e. not for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable.

### (5.3.2)The “U”-Tube Manometer

Using a “U”-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The “U” is connected as in the figure below and filled with a fluid called the *manometric fluid*. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



A “U”-Tube manometer

Pressure in a continuous static fluid is the same at any horizontal level so,

Pressure at B = Pressure at C

$$p_B = p_C$$

For the left hand arm

Pressure at B = Pressure at A + Pressure due to height  $h_1$  of fluid being measured

$$p_B = p_A + \rho g h_1$$

For the right hand arm

Pressure at C = Pressure at D + Pressure due to height  $h$  of manometric fluid

$$p_c = p_{atmospheric} + \rho_{man}gh_2$$

As we are measuring gauge pressure we can subtract  $p_{Atmospheric}$  giving

$$p_B = p_C$$

$$p_A + \rho gh_1 = p_{atmospheric} + \rho_{man}gh_2$$

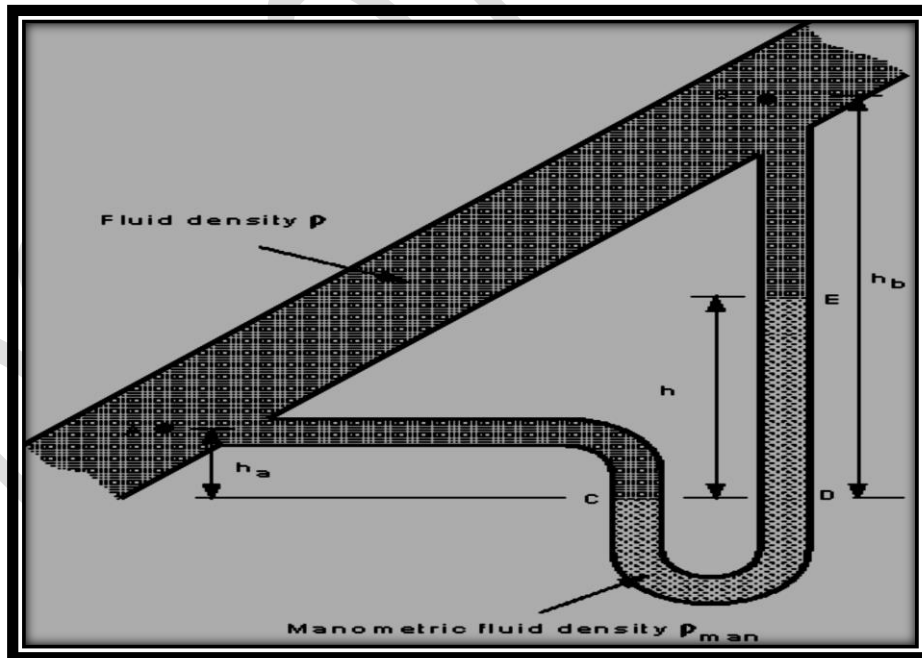
$$p_A = \rho_{man}gh_2 - \rho gh_1$$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid i.e.  $\rho_{man} \gg \rho$ . In this case the term  $\rho gh_1$  can be neglected, and the gauge pressure given by

$$p_A = \rho_{man}gh_2$$

### (5.3.2.1) Measurement of Pressure Difference Using a “U” Tube Manometer.

If the “U”-tube manometer is connected to a pressurized vessel at two points the pressure difference between these two points can be measured.



Pressure difference measurement by the “U”-Tube manometer

If the manometer is arranged as in the figure above, then

Pressure at C = Pressure at D

$$p_C = p_D$$

$$p_C = p_A + \rho g h_a$$

$$p_D = p_B + \rho g(h_b - h) + \rho_{man} g h$$

$$p_A + \rho g h_a = p_B + \rho g(h_b - h) + \rho_{man} g h$$

$$p_A - p_B = \rho g(h_b - h) + \rho_{man} g h - \rho g h_a$$

$$p_A - p_B = \rho g(h_b - h) + \rho_{man} g h - \rho g h_a$$

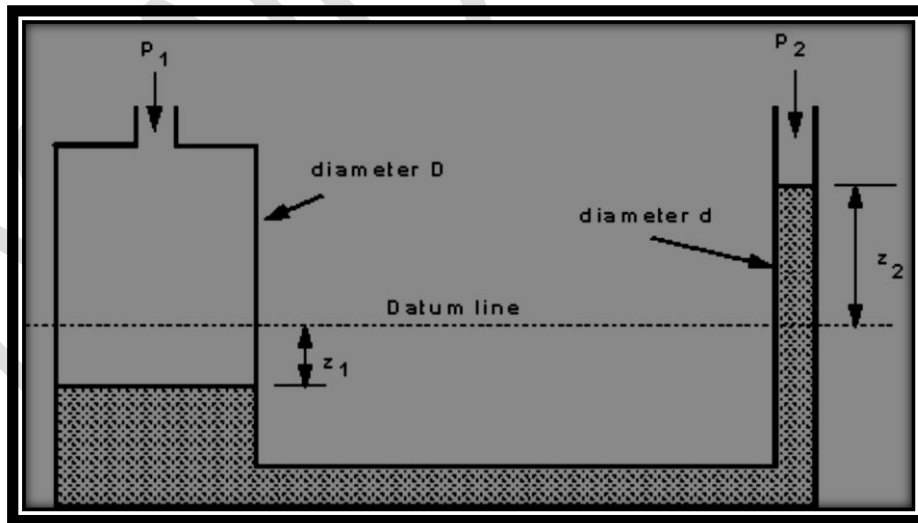
$$p_A - p_B = \rho g(h_b - h_a) + h g(\rho_{man} - \rho)$$

Again, if the fluid whose pressure difference is being measured is a gas and  $\rho \ll \rho_{man}$ , then the terms involving  $\rho$  can be neglected, so

$$p_A - p_B = \rho_{man} g h$$

#### (5.3.2.2) Advances to the “U” tube manometer.

The “U”-tube manometer has the disadvantage that the change in height of the liquid in both sides must be read. This can be avoided by making the diameter of one side very large compared to the other. In this case the side with the large area moves very little when the small area side moves considerably more



Assume the manometer is arranged as above to measure the pressure difference of a gas of (negligible density) and that pressure difference is  $p_1 - p_2$ . If the datum line indicates

the level of the manometric fluid when the pressure difference is zero and the height differences when pressure is applied is as shown, the volume of liquid transferred from the left side to the right =  $z_2 \times \left(\frac{\pi d^2}{4}\right)$

And the fall in level of the left side is

$$z_1 = \left( \frac{\text{volume move}}{\text{Area of left side}} \right)$$

$$z_1 = \frac{\frac{z_2 \pi d^2}{4}}{\frac{\pi D^2}{4}}$$

$$z_1 = \left(\frac{d}{D}\right)^2 z_2$$

We know from the theory of the “U” tube manometer that the height different in the two columns gives the pressure difference so

$$p_1 - p_2 = \rho g \left[ z_2 + z_2 \left(\frac{d}{D}\right)^2 \right]$$

$$p_1 - p_2 = \rho g z_2 \left[ 1 + \left(\frac{d}{D}\right)^2 \right]$$

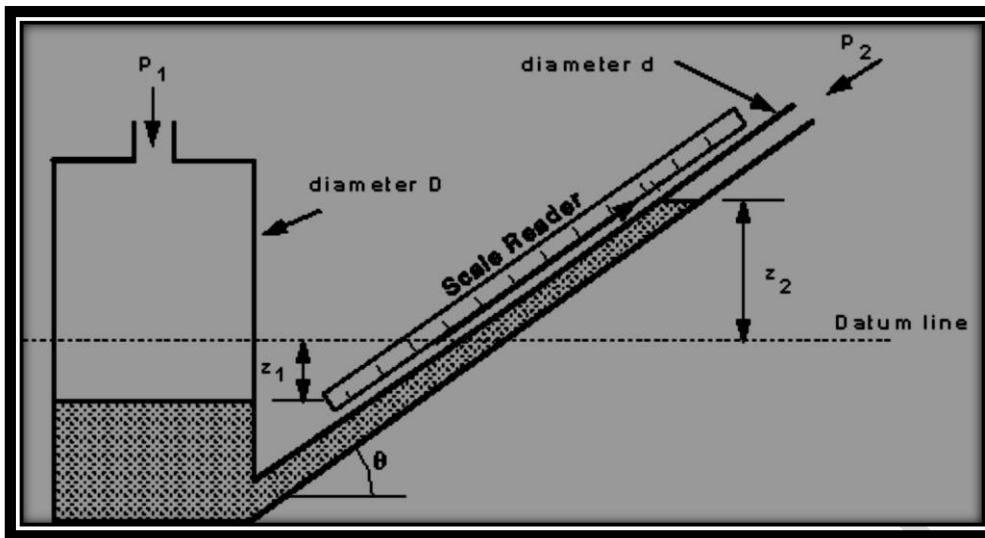
Clearly if D is very much larger than d then  $(d/D)^2$  is very small so

$$p_1 - p_2 = \rho g z_2$$

So only one reading need be taken to measure the pressure difference.

If the pressure to be measured is very small then tilting the arm provides a convenient way of obtaining a larger (more easily read) movement of the manometer. The above arrangement with a tilted arm is shown in the figure below.





Tilted manometer.

The pressure difference is still given by the height change of the manometric fluid but by placing the scale along the line of the tilted arm and taking this reading large movements will be observed. The pressure difference is then given by

$$p_1 - p_2 = \rho g z_2$$
$$\rho g x \sin \theta$$

The sensitivity to pressure change can be increased further by a greater inclination of the manometer arm, alternatively the density of the manometric fluid may be changed.

#### **(5.4)Choice of Manometer**

Care must be taken when attaching the manometer to vessel, no burrs must be present around this joint. Burrs would alter the flow causing local pressure variations to affect the measurement.

#### **Some disadvantages of manometers:**

- Slow response - only really useful for very slowly varying pressures - no use at all for fluctuating pressures;
- For the “U” tube manometer two measurements must be taken simultaneously to get the h value. This may be avoided by using a tube with a much larger cross-sectional area on one side of the manometer than the other;

- It is often difficult to measure small variations in pressure - a different manometric fluid may
- be required - alternatively a sloping manometer may be employed; It cannot be used for very large pressures unless several manometers are connected in series;
- For very accurate work the temperature and relationship between temperature and  $\rho$  must be known.

**(5.5)Some advantages of manometers:**

- They are very simple.
- No calibration is required - the pressure can be calculated from first principles