Lecture NO (12)

Fluid Friction in Pipes Cont.

12.1 Pipe roughness.

The question immediately arises as to how a roughness length scale ε can be assigned

to the surface of a particular wall material. The situation is idealized in Fig. 12.1, where ε is the diameter of the sand grains.

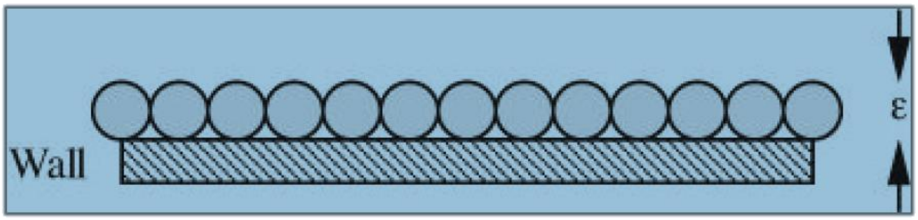

Fig. 12.1 Artificially roughened wall.

Nikuradse then deduced the friction factor for a large number of cases and was able to build up a friction-factor plot for artificially roughened pipes that was very similar to those shown in Fig. 11.7 for "real" surfaces. By comparing the two plots, it is then simple to assign an effective value of ε /D and, hence, ε for a "real" surface. Representative values are given in Table 12.1 for a variety of surfaces.

Table12.1 Effective Surface Roughness's

Surface	**ε (ft)**	**ε (mm)**
Concrete	0.001-0.01	0.3-3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel	0.00015	0.46
Drawn tubing	0.000005	0.0015

12.1.1 Friction-factor formulas. The following formula, known as the Colebrook and White equation, gives a good representation of the experimentally determined friction factor in the turbulent region,
and is the basis for that part of Fig.11.7:

$$\frac{1}{\sqrt{f_F}} = -4.0 log_{10}\left(\frac{\varepsilon}{D} + \frac{4.67}{Re\sqrt{f_F}}\right) + 2.28$$

$$= -1.737 ln log_{10}\left(0.269\frac{\varepsilon}{D} + \frac{1.257}{Re\sqrt{f_F}}\right) \dots\dots\dots\dots.(12.1)$$

in which the logarithm to base e is intended in the second version. Also, for the special case of a *hydraulically smooth surface*, the following relation, known as the Blasius equation, correlates experimental observations for turbulent flow at Reynolds numbers below 100,000:

$$f_F = 0.0790Re^{-\frac{1}{4}} \dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots\dots(12.2)$$

Hydraulically smooth means that the surface irregularities do not protrude beyond the laminar boundary layer immediately adjacent to the wall. For this reason, the roughness is unimportant—not the case if the irregularities are large enough to extend into the turbulent core, in which case they would enhance the degree of turbulence and therefore influence the friction factor. Olujić notes that Shacham has pointed out that, starting with a first estimate of f_F = 0.0075, the procedure converges with an average accuracy of less than one percent within just one iteration, for a very wide range of Reynolds numbers and roughness ratios.6 Therefore, it is reasonable to incorporate this starting estimate directly into Eqn. (12.1), which then gives the following explicit form for the friction factor:

$$f_F = \left\{-1.737 ln\left[0.269\frac{\varepsilon}{D} - \frac{2.185}{Re} ln\left(0.269\frac{\varepsilon}{D} + \frac{14.5}{Re}\right)\right]\right\}^{-2} \dots\dots\dots\dots\dots.(12.3)$$

In order to obtain the friction factor for rough pipes, we recommend:

1. For hand calculations, use Fig. 11.7
2. For computer programs and spreadsheet calculations, use Eqn. (12.3) for the turbulent region (Re > 4,000) and f_F = 16/Re for the laminar region (Re ≤ 2,000). Avoid designs in the uncertain transition region (2,000 < Re ≤ 4,000).

Commercial steel pipe is manufactured in standard sizes, a selection of which is shown in Table 12.2. Note in Table 12.2 that the nominal size is roughly the same as the inside diameter, and that the wall thickness depends on the schedule number n, defined as:

$$n = 1{,}000\frac{p_{max}}{S_a} \dots\dots\dots\dots\dots\dots\dots\dots.(12.4)$$

Table 12.2 Representative Pipe Sizes

Size (in.)	Diameter (in.)	Number	Thickness (in.)	Diameter (in.)
1/2	0.840	40	0.109	0.622
		80	0.147	0.546
3/4	1.050	40	0.113	0.824
		80	0.154	0.742
1	1.315	40	0.133	1.049
		80	0.179	0.957
2	2.375	40	0.154	2.067
		80	0.218	1.939
3	3.500	40	0.216	3.068
		80	0.300	2.900
		160	0.437	2.626
4	4.500	40	0.237	4.026
		80	0.337	3.826
		160	0.531	3.438
6	6.625	40	0.280	6.065
		80	0.432	5.761
		160	0.718	5.189
8	8.625	40	0.322	7.981
		80	0.500	7.625
		160	0.906	6.813

Table 12.2 Representative Pipe Sizes (Cont.)

Size (in)	Diameter (in)	Number	Thickness (in)	Diameter (in)
10	10.75	40	0.365	10.020
		80	0.593	9.564
		160	1.125	8.500
12	12.75	40	0.406	11.938
		80	0.687	11.376
		160	1.312	10.126
16	16.00	40	0.500	15.000
		80	0.843	14.314
		160	1.562	12.876
24	24.00	40	0.687	22.626
		80	1.218	21.564
		160	2.312	19.376

in which p_{max} is the maximum allowable pressure in the pipe and S_a is the allowable tensile stress in the pipe wall. An interpretation of Eqn. (12.4) is readily apparent by studying Fig. 3.12, in which the hoop stress S in a pipe of diameter D subject to a pressure p is found by imagining the pipe to be split into two halves. The pressure force per unit length tending to blow away the right (or left) half is ***pD***. Also, if ***t*** is the wall thickness, there is a total restoring force 2St within the pipe wall. Equating the two forces when p and S have reached their respective limits p_{max} and S_a:

$$p_{max}D = 2S_a t \dots\dots\dots\dots\dots\dots\dots\dots (12.5)$$

$$\frac{t}{D} = \frac{p_{max}}{2S_a} \dots\dots\dots\dots\dots\dots\dots\dots (12.6)$$

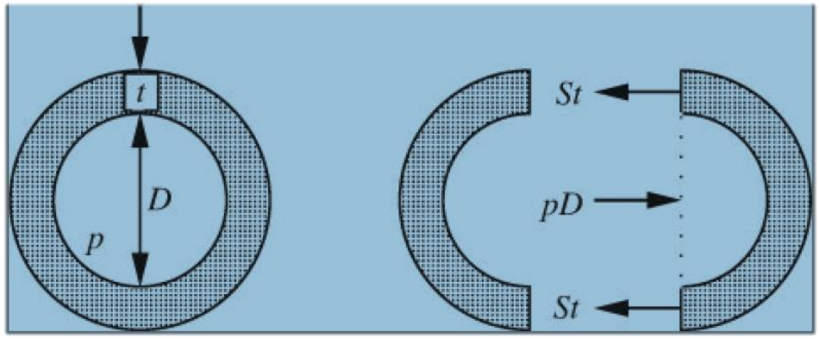

Fig. 12.2 Hoop stress in a pipe wall.

Thus, the schedule number of (12.4) is 2,000 times the ratio of the necessary wall thickness to the pipe diameter. (The above treatment is most accurate for *thin*-walled pipes.)

Solution of "simple" piping problems. Consider the transport of a fluid in a pipe from point 1 to point 2, as shown in Fig. 12.3, in which the following are assumed to be known—as will usually be the case in practice:

1. The elevation increase, $\Delta z = z_2 - z_1$, which may be positive, zero, or negative.

2. The length L of the pipe.

3. The material of construction of the pipe, and hence the pipe wall roughness ε.

4. The density ρ and viscosity μ of the fluid.

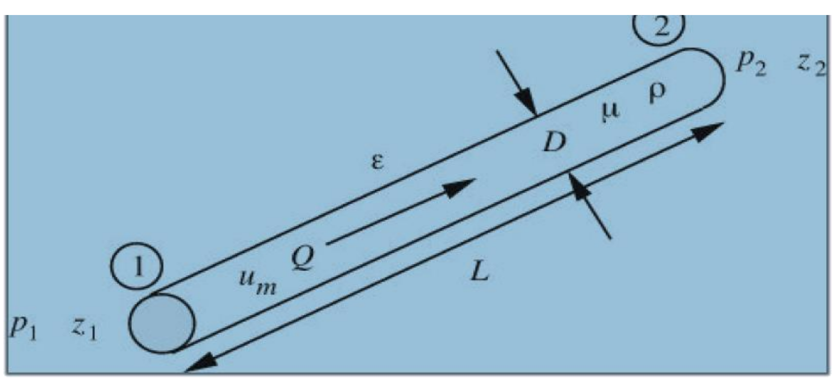

Fig. 12.3 Variables involved in pipe-flow problems.

The following additional variables are of prime importance:

1. The volumetric flow rate, Q.
2. The internal pipe diameter, D.
3. The pressure drop, $-\Delta p = p_1 - p_2$.

The specification of any *two* of Q, D, and $-\Delta p$ (there are three cases) then enables the *remaining one* of them to be determined. The following algorithms are equally applicable to solutions by hand calculator or by a computer application such as a spreadsheet.

The calculations are not equally straightforward in each of the three cases just identified, and we distinguish between the following possibilities and approaches, the second and third of which require *iterative* types of solution:

1. Known flow rate and diameter. The combination of specified Q and D represents the easiest situation, in which the following direct (noniterative) steps are needed to determine the required pressure drop—which would then typically enable the size of an accompanying pump to be found:

(a) Compute the mean velocity $u_m = 4Q/\pi D^2$, the Reynolds number $\text{Re} = \rho u_m D/\mu$, and the roughness ratio ε/D.

(b) Based on the values of Re and ε/D, determine the friction factor f_F from either the friction factor chart or from the equations that represent it.

(a) Compute the pressure drop from Eqn. (11.28):

$$-\Delta p = p_1 - p_2 = 2 f_F \rho u_m^2 \frac{L}{D} + \rho g \Delta z \; \ldots\ldots (11.28)$$

2. Known diameter and pressure drop. Given D and $-\Delta$p, the problem is now to find the flow rate Q. There is an immediate difficulty because—in the absence of the flow rate—the Reynolds number cannot be determined directly, so the friction factor is also unknown, even though the roughness ratio *is* known. The following steps are typically needed:

(a) Compute the roughness ratio ε/D.

(b) *Assume* or make a reasonable guess as to the first estimate for the Reynolds number Re. Since the majority of pipe-flow problems are in turbulent flow, a value such as Re = 10,000 or 100,000 should be considered. If the flow is of a viscous polymer, then the flow is probably laminar, in which case a value such as Re = 1,000 could be appropriate.

(c) Based on the values of ε/D and Re, compute the friction factor fF from either the friction factor

chart or from the equations that represent it.

(d) Compute the mean velocity um from:

$$-\Delta p = p_1 - p_2 = 2f_F\rho u_m^2\frac{L}{D} + \rho g\Delta z \dots\dots (11.28)$$

which can be rearranged to give um explicitly:

$$u_m = \sqrt{\frac{D}{L2f_F\rho} + [(p_1 - p_2) - \rho g\Delta z]} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (12.7)$$

(e) Compute the Reynolds number from Re = $\rho u_m D/\mu$. If this is acceptably close to the value used in Step (c), then the problem is essentially solved, in which case proceed with Step (f). If not, return to Step (c).

(f) Compute the flow rate from $Q = u_m\pi D^2/4$.

3. Known flow rate and pressure drop. Given Q and $-\Delta p$, the problem is to find the diameter D. The immediate difficulty is that—in the absence of the diameter—neither the Reynolds number *nor* the roughness ratio is known, so the friction factor is also unknown. The following steps are typically needed (other approaches are possible, but the formula given in Step (e) for the diameter is likely to lead to the quickest convergence):

(a) Estimate or guess the pipe diameter D. Hence, compute the corresponding mean velocity, $u_m = 4Q/\pi D^2$.

(b) Compute the Reynolds number, Re = $\rho u_m D/\mu$.

(c) Compute the roughness ratio ε/D.

(d) Based on the available values of ε/D and Re, compute the friction factor fF from either the
friction factor chart or from the equations that represent it.

(e) Compute the diameter from:

$$D = \left[\frac{32f_F\rho Q^2 L}{\pi^2(p_1 - p_2 - \rho g\Delta z)}\right]^{\frac{1}{5}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (12.8)$$

(Note that this equation results by eliminating the mean velocity between Eqn. (11.28) and $Q = u_m\pi D^2/4$.)

(f) Compute the mean velocity, um = 4Q/πD2.

(g) Compute the Reynolds number, Re = $\rho u_m D/\mu$. If this is acceptably close to the value used in Step (d), then the problem is essentially solved. If not, return to Step (c).

12.2 Flow in Noncircular Ducts

The cross section of a pipe is most frequently circular, but other shapes may be encountered. For example, the rectangular cross section of many domestic hot air heating ducts should be apparent to most people living in the United States. The situation for a *horizontal* duct is illustrated in **Fig. 12.4** ; the cross-sectional shape is quite arbitrary—it doesn't have to be rectangular as shown—as long as it is uniform at all locations. There, A is the cross-sectional area and P is the *wetted perimeter*— defined as the length of wall that is actually in contact with the fluid. For the flow of a gas, P will always be the length of the complete periphery of the duct; for liquids, however, it will be somewhat less than the periphery if the liquid has a free surface and incompletely fills the total cross section.

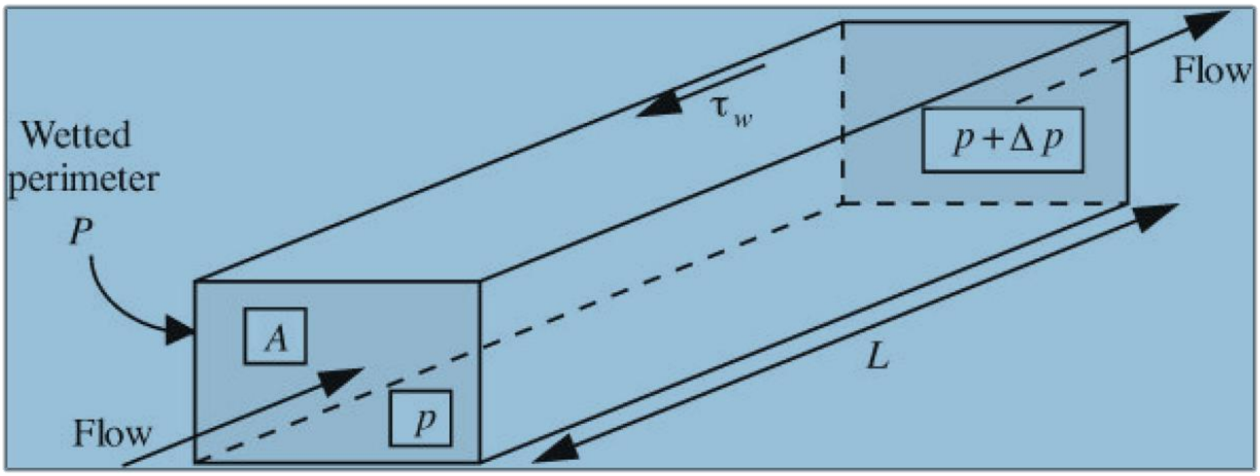

Fig. 12.4 Flow in a duct of noncircular cross section.

If τ_w is the wall shear stress and there is a pressure drop $-\Delta p$ over a length L, a momentum balance in the direction of flow yields:

$$p - (p + \Delta p)A - \tau_w PL = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (12.9)$$

The pressure drop is therefore:

$$-\Delta p = \frac{\tau_w PL}{A} = \underbrace{\frac{\tau_w}{\frac{1}{2}\rho u_m^2}}_{f_F} \frac{1}{2}\rho u_m^2 \frac{PL}{A} \ldots\ldots\ldots\ldots\ldots. (12.10)$$

$$= 2 f_F \rho u_m^2 \frac{L}{4A/_P} = 2 f_F \rho u_m^2 \frac{L}{D_e} \ldots\ldots\ldots\ldots\ldots\ldots (12.11)$$

Thus, the equation for the pressure drop is identical with that of Eqn. (11.28) for a circular pipe provided that D is replaced by the hydraulic mean diameter D_e, defined by:

$$D_e = \frac{4A}{P} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12.12)$$

The reader may wish to check that De = D for a circular duct. Following similar lines as those used previously, the frictional dissipation per unit mass can be deduced as:

$$f = 2f_F u_m^2 \frac{L}{D_e} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots (12.13)$$

and this expression can then be employed for *inclined* ducts of noncircular cross section.

Steady flow in open channels. A similar treatment follows for a liquid flowing steadily down a channel inclined at an angle θ to the horizontal, such as a river or irrigation ditch, shown in **Fig. 12.5.** Again, as long as the cross section is uniform along the channel, it can be quite arbitrary in shape, not necessarily rectangular. The driving force is now gravity, there being no variation of pressure because the free surface is uniformly exposed to the atmosphere.

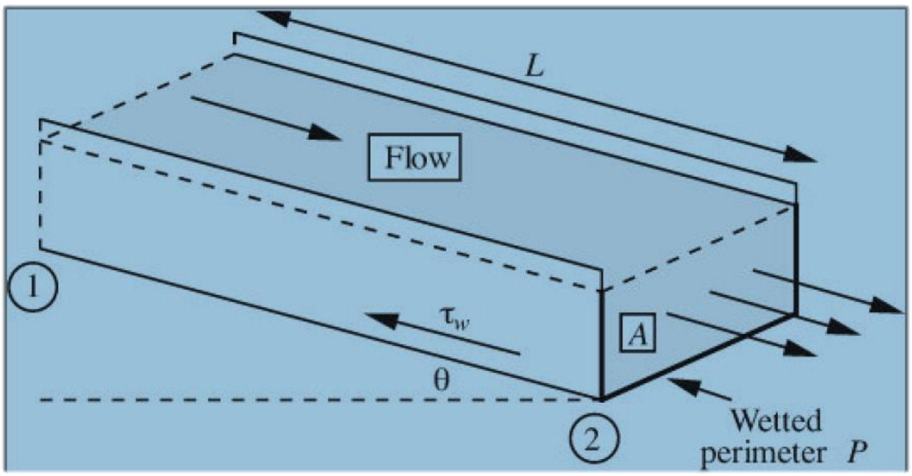

Fig. 12.5 Flow in an open channel.

If the wetted perimeter is again P and the cross-sectional area occupied by the liquid is A, a steady-state momentum balance in the direction of flow gives:

$$\rho A L g sin\theta - \tau_w P L = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots (12.13)$$

Noting that:

$$L g sin\theta = z_1 - z_2 = \Delta z \dots\dots\dots\dots\dots\dots\dots\dots\dots (12.14)$$

Division of Eqn (12.13) by -Pa gives

$$g\Delta z = \frac{\tau_w P L}{\rho A} = 0; \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (12.15)$$

in which the second term can be rearranged as:

$$2\frac{\tau_w}{\frac{1}{2}\rho u_m^2} u_m^2 \frac{L}{4A/P} = 2f_F u_m^2 \frac{L}{4A/P} \dots\dots\dots\dots (12.16)$$

Comparison with the overall energy balance:

$$g\Delta z + f = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (12.17)$$

$$f = 2f_F u_m^2 \frac{L}{D_e} \; where \; D_e = \frac{4A}{P} \dots\dots (12.18)$$

which has exactly the same form as Eqns. (12.12) and (12.13).

12.3 Pressure drop across pipe fittings.

A variety of auxiliary hardware such as valves and elbows is associated with most piping installations. These *fittings* invariably cause the flow to deviate from its normal straight course and hence induce additional turbulence and frictional dissipation. Indeed, the resulting additional pressure drop is sometimes comparable to that in the pipeline itself. The basic procedure is to recognize that the fitting causes an additional pressure drop that would be produced by a certain length of pipe into which the fitting is introduced. Therefore, we substitute for the fitting an extra contribution to the length of the pipe, based on the *equivalent length* (*L/D*)*e* of the fitting. For example, referring to Table 12.3 three standard 90° elbows in a 6–in.-diameter line cause a pressure drop that is equivalent to an extra 45 ft of pipe.

Table 3.4 Equivalent Lengths of Pipe

Type of Fitting	$(L/D)_e$
Angle valve (open)	160
Close return bend	75
Gate valve (open)	6.5
Globe valve (open)	330
Square 90° elbow	70
Standard 90° elbow	30
Standard "T" (through side outlet)	70
45° elbow	15
Sudden contraction, 4:1	15
Sudden contraction, 2:1	11
Sudden contraction, 4:3	6.5
Sudden expansion, 1:4	30
Sudden expansion, 1:2	20
Sudden expansion, 3:4	6.5

Laminar and turbulent velocity profiles. The parabolic velocity profile already encountered in laminar flow in a pipe is again illustrated on the left of Fig. 12.6. On the right, we see for the first time the general shape of the velocity profile for *turbulent* flow.

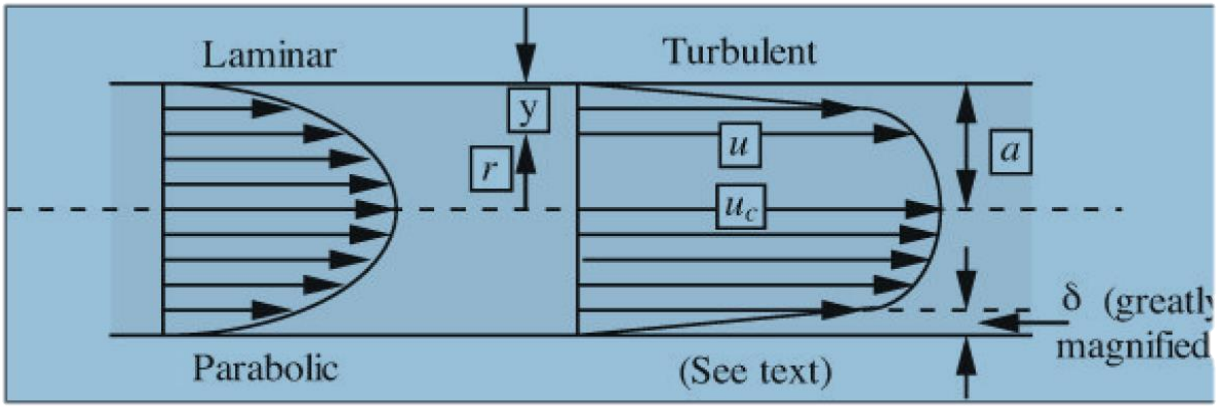

Fig. 12.6 Laminar and turbulent velocity profiles.

1. A very thin region, known as the *laminar sublayer*, in which turbulent effects are essentially absent, the shear stress is virtually constant, and there is an extremely steep

The following equation is derived for the thickness δ of the laminar sublayer relative to the pipe diameter *D* as a function of the Reynolds number in the pipe:

$$\frac{\delta}{D} = 62Re^{-7/8} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(12.19)$$

2. A turbulent core, which extends over nearly the whole cross section of the pipe. Here, the velocity profile is relatively flat because rapid turbulent radial momentum transfer tends to "iron out" any differences in velocity. A representative equation for the ratio of the velocity u at a distance y from the wall to the centerline velocity u_c is:

$$\frac{u}{u_c} = \left(\frac{y}{a}\right)^{\frac{1}{n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots.(12.20)$$

in which n in the exponent varies somewhat with the Reynolds number, as in Table 12.5. For n = 1/7, Eqn. (12.20).

Table 3.5 Exponent n for Equation

Re	n
4.0×10^3	6
2.3×10^4	6.6
1.1×10^4	7
1.1×10^6	8.8
2.0×10^6	10
3.2×10^6	10