

Lecture NO(15)

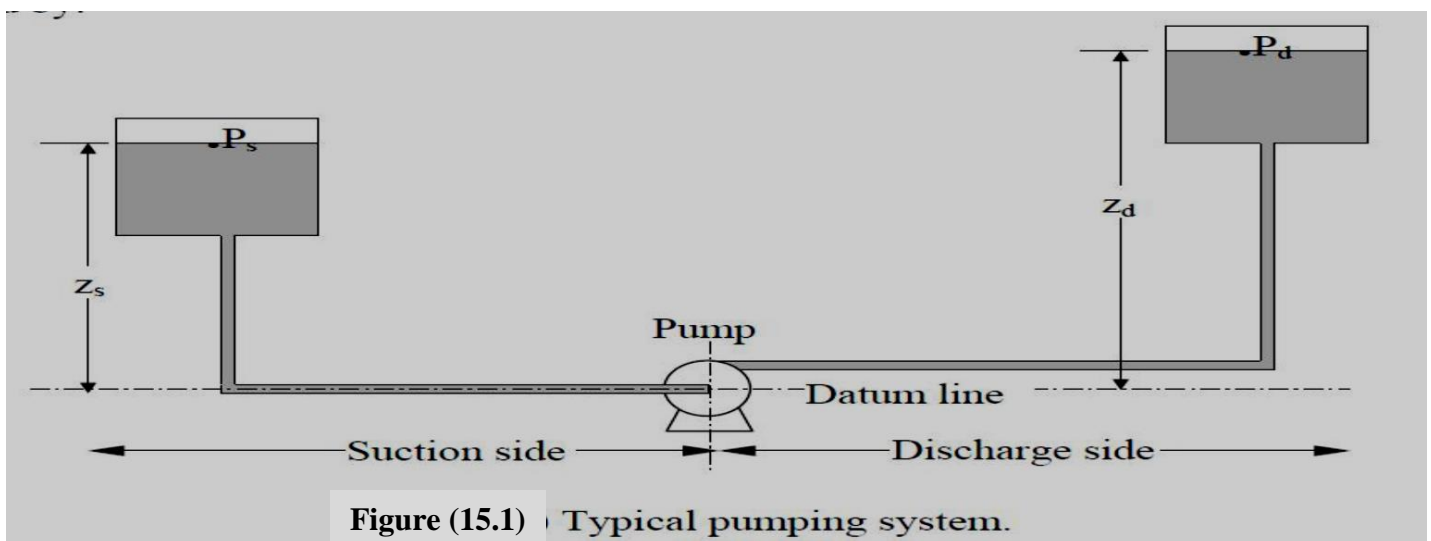
Pumping of Liquids

15.1 Introduction

Pumps are devices for supplying energy or head to a flowing liquid in order to overcome head losses due to friction and also if necessary, to raise liquid to a higher level. For the pumping of liquids or gases from one vessel to another or through long pipes, some form of mechanical pump is usually employed. **The energy required by the pump** will depend on the height through which the fluid is raised, the pressure required at delivery point, the length and diameter of the pipe, the rate of flow, together with the physical properties of the fluid, particularly its *viscosity and density*. The pumping of liquids such as sulphuric acid or petroleum products from bulk store to process buildings, or the pumping of fluids round reaction units and through heat exchangers, are typical illustrations of the use of pumps in the process industries. On the one hand, it may be necessary to inject reactants or catalyst into a reactor at a low, but accurately controlled rate, and on the other to pump cooling water to a power station or refinery at a very high rate. The fluid may be a gas or liquid of low viscosity, or it may be a highly viscous liquid, possibly with non-Newtonian characteristics. It may be clean, or it may contain suspended particles and be very corrosive. All these factors influence the choice of pump. Because of the wide variety of requirements, many different types are in use including centrifugal, piston, gear, screw, and peristaltic pumps, though in the chemical and petroleum industries the centrifugal type is by far the most important.

15.2 The Total Head (Δh)

The head imparted to a flowing liquid by a pump is known as the total head (Δh). If a pump is placed between points 1 and 2 in a pipeline, the head for steady flow are related by: -



$$\Delta h = \frac{\eta W_s}{g} = \left(\frac{p_2}{\rho g} + \frac{u_2^2}{2\alpha_2 g} + z_2 \right) - \left(\frac{p_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 \right) - h_f$$

$$\Rightarrow \Delta h = \frac{\Delta p}{\rho g} + \frac{\Delta u^2}{2\alpha g} + \Delta z + h_f \dots \dots \dots (15.1)$$

15.3 System Heads

The important heads to consider in a pumping system are: -

- 1- Suction head
- 2- Discharge head
- 3- Total head
- 4- Net positive suction head (NPSH)

The following definitions are given in reference to typical pumping system shown in preceding Figure, where the datum line is the centerline of the pump

$$1 - \text{Suction head } (h_s) \quad h_s = z_s + \frac{p_s}{\rho g} + (h_F)_s$$

$$2 - \text{Discharge head } (h_d) \quad h_d = z_d + \frac{p_d}{\rho g} + (h_F)_d$$

3- Total head(Δh)

Total head(Δh) , which is required to impart to the flowing liquid is the difference between the discharge and suction head

Thus

$$\Delta h = h_d - h_s \dots \dots \dots (15.2)$$

$$\Delta h = (z_d - z_s) + \left(\frac{p_d - p_s}{\rho g} \right) + [(h_F)_d - (h_F)_s] \dots \dots \dots (15.3)$$

Where

$$(h_F)_d = 4f_d + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_d + \frac{u_d^2}{2g}$$

$$(h_F)_s = 4f_s + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_s + \frac{u_s^2}{2g}$$

The suction head (h_s) decreases and the discharge head (h_d) increases with increasing liquid flow rate because of the increasing value of the friction head loss

terms $(h_f)_s$ and $(h_f)_d$. Thus the total; head (Δh) which the pump is required to impart to the flowing liquid increases with increasing the liquid pumping rate.

Note:

If the liquid level on the suction side is below the centerline of the pump, z_s is negative.

4- Net positive suction head (NPSH)

Available net positive suction head

$$\text{NPSH} = z_s + \left(\frac{p_s - p_v}{\rho g} \right) - (h_f)_s \dots \dots \dots (15.4)$$

This equation gives the head available to get the liquid through the suction piping. P_v is the vapor pressure of the liquid being pumped at the particular temperature in question.

The available net positive suction head (NPSH) can also be written as:

$$\text{NPSH} = h_s - \frac{p_v}{\rho g} \dots \dots \dots (15.5)$$

The available net positive suction head (NPSH) in a system should always be positive i.e. the suction head always be capable of overcoming the vapor pressure (P_v) since the frictional head loss $(h_f)_s$ increases with increasing pumping rate. At the boiling temperature of the liquid P_s and P_v are equal and the available NPSH becomes $[z_s - (h_f)_s]$. In this case no suction lift is possible since z_s must be positive. If the term $(P_s - P_v)$ is sufficiently large, liquid can lift from below the centerline of the pump. In this case z_s is negative. From energy consideration it is immaterial whether the suction pressure is below atmospheric pressure or well above it, as long as the fluid remains liquid. However, if the suction pressure is only slightly greater than the vapor pressure, some liquid may flash to vapor inside the pump, a process called “Cavitation”, which greatly reduces the pump capacity and severe erosion. If the suction pressure is actually less than the vapor pressure, there will be vaporization in the suction line, and no liquid can be drawn into the pump. To avoid cavitation, the pressure at the pump inlet must exceed the vapor pressure by a certain value, called the “ net positive suction head (NPSH)”. The required values of NPSH is about 2-3 m H_2O for small pump; but it increases with pump capacity and values up to 15 m H_2O are recommended for very large pump.

5.4 Power Requirement

The power requirement to the pump drive from an external source is denoted by (P). It is calculated from W_s by:

$$\dot{m}W_s = \frac{Q\Delta p}{\eta} = \frac{Q\Delta h\rho g}{\eta} = \frac{\dot{m}\Delta h g}{\eta} \dots\dots\dots (15.6)$$

The mechanical efficiency (η) decreases as the liquid viscosity and hence the frictional losses increase. The mechanical efficiency is also decreased by power losses in gear, bearing, seals, etc. These losses are not proportional to pump size. Relatively large pumps tend to have the best efficiency whilst small pumps usually have low efficiencies. Furthermore, high-speed pumps tend to be more efficient than low-speed pumps. In general, high efficiency pumps have high NPSH requirements.

15.5 Types of Pumps

Pumps can be classified into: -

1- Centrifugal pumps. 2- Positive displacement pumps.

1- Centrifugal pumps

This type depends on giving the liquid a high kinetic energy, which is then converted as efficiently as possible into pressure energy. It is used for liquid with very wide-ranging properties and suspensions with high solid content including, for example, cement slurries, and may be constructed from a very wide range of corrosion resistant materials. Process industries commonly use centrifugal pumps. The whole pump casing may be constructed from plastics such as polypropylene or it may be fitted with a corrosion resistant lining. Because it operates at high speed, it may be directly coupled to an electric motor and it will give a high flow rate for its size. They are available in sizes about 0.004 to 380 m³/min [1-100,000 gal/min] and for discharge pressures from a few m H₂O head to 5,000 kPa. In this type of pump (Figure 15.2), the fluid is fed to the center of a rotating impeller and is thrown outward by centrifugal action. As a result of the high speed of rotation the liquid acquires a high kinetic energy and the pressure difference between the suction and delivery sides arises from the interconversion of kinetic and pressure energy.

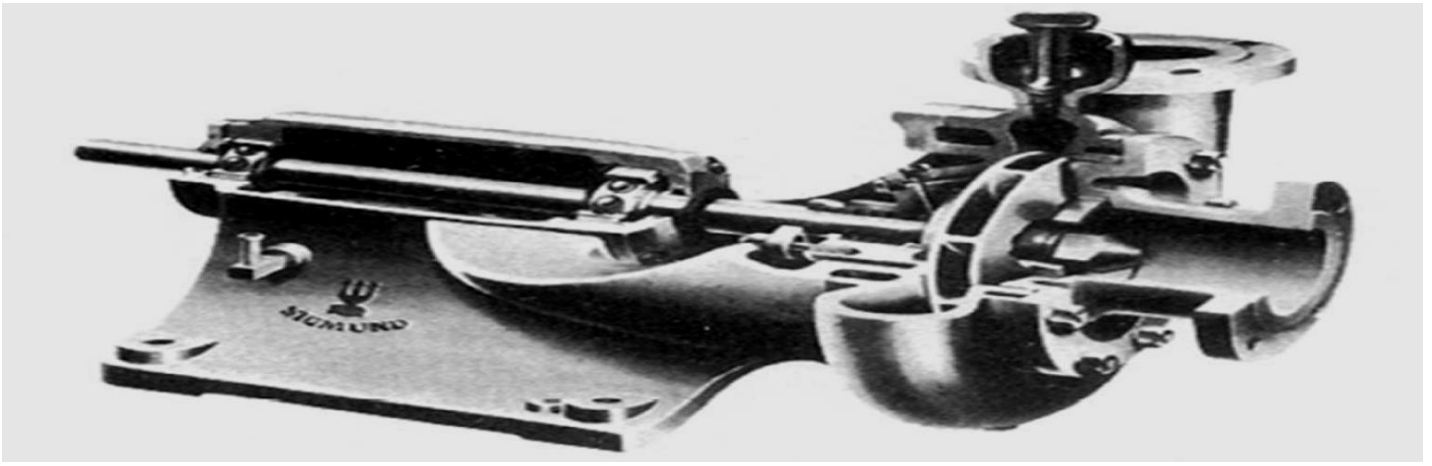


Figure (15.2) section of centrifugal pump

The impeller (Figure 15.3) consists of a series of curved vanes so shaped that the flow within the pump is as smooth as possible. The greater the number of vanes on the impeller, the greater is the control over the direction of motion of the liquid and hence the smaller are the losses due to turbulence and circulation between the vanes. In the open impeller, the vanes are fixed to a central hub, whereas in the closed type the vanes are held between two supporting plates and leakage across the impeller is reduced. The liquid enters the casing of the pump, normally in an axial direction, and is picked up by the vanes of the impeller. In the simple type of centrifugal pump, the liquid discharges into a volute, a chamber of gradually increasing cross-section with a tangential outlet. A volute type of pump is shown in Figure 15.4.

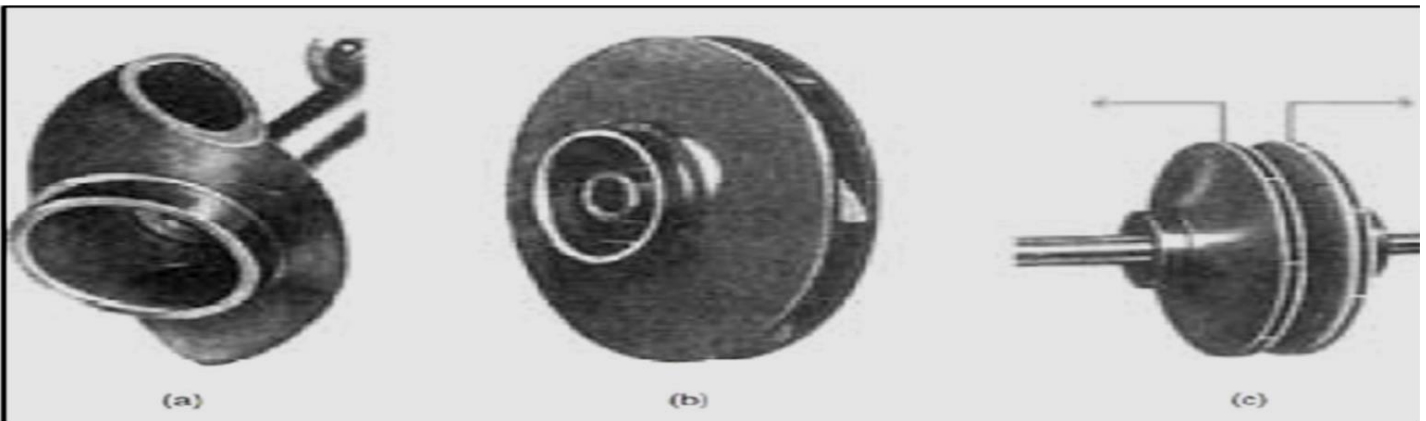


Figure (15.3) Types of impeller (a) for pumping suspensions (b) standard close impeller (c) double impeller

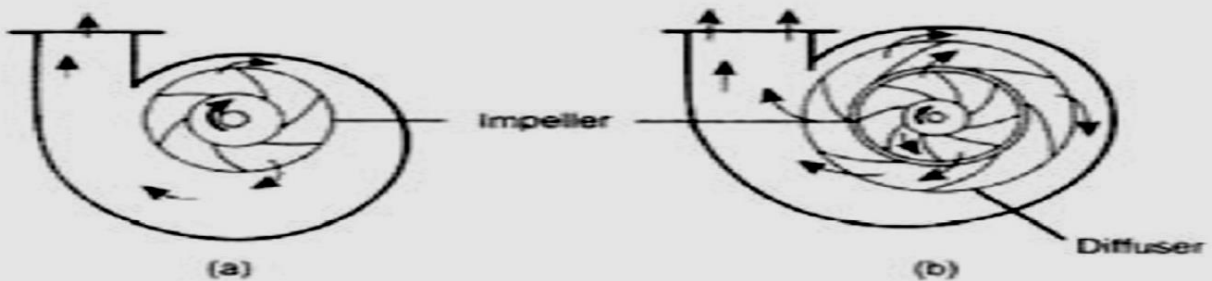


Figure (15.4) Radial flow pumps (a) with volute (b) with diffuser vanes

In the turbine pump (Figure 15.4(b)) the liquid flows from the moving vanes of the impeller through a series of fixed vanes forming a diffusion ring. This gives a more gradual change in direction to the fluid and more efficient conversion of kinetic energy into pressure energy than is obtained with the volute type.

2- Positive Displacement Pumps

In this type, the volume of liquid delivered is directly related to the displacement of the piston and therefore, increases directly with speed and is not appreciably influenced by the pressure. It is used for *high pressure and constant rates this type can be classified into: -*

2.1-Reciprocating Pumps, such as

a- The Piston Pump

This pump may be single-acting, with the liquid admitted only to the portion of the cylinder in front of the piston or double-acting, in which case the feed is admitted to both sides of the piston. The majority of pumps are of the single acting type typically giving a low flow rate of say 0.02 m³/s at a high pressure of up to 100 Mpa.

b- The Plunger (or Ram) Pump

This pump is the same in principle as the piston type but differs in that the gland is at one end of the cylinder making its replacement easier than with the standard piston type. The piston or ram pump may be used for injections of small quantities of inhibitors to polymerization units or of corrosion inhibitors to high-pressure systems, and also for boiler feed water applications.

c- The Diaphragm Pump

The diaphragm pump has been developed for handling corrosive liquids and those containing suspensions of abrasive solids. It is in two sections separated by a diaphragm of rubber, leather, or plastics material. In one section a plunger or piston operates in a cylinder in which a non-corrosive fluid is displaced. The particularly simple and inexpensive pump results, capable of operating up to 0.2 Mpa.

d- The Metering (or Dosing) Pump

Metering pumps are driven by constant speed electric motors. They are used where *a constant and accurately controlled rate of delivery of a liquid is required, and they will maintain this constant rate irrespective of changes in the pressure against which they operate. The pumps are usually of the plunger type for low throughput and high-pressure applications; for large volumes and lower pressures a diaphragm is used. In either case, the rate of delivery is controlled by* adjusting the stroke of the piston element, and this can be done whilst the pump is in operation. A single-motor driver may operate several individual pumps and, in this way, give control of the actual flows and of the flow ratio of several streams at the same time. The output may be controlled from zero to maximum flow rate, either manually on the pump or remotely. These pumps may be used for the dosing of works effluents and water supplies, and the feeding of reactants, catalysts, or inhibitors to reactors at controlled rates, and although a simple method for controlling flow rate is provided, high precision standards of construction are required.

2.2-Rotary Pumps, such as

a- The Gear Pump

Gear and lobe pumps operate on the principle of using mechanical means to transfer small elements or "packages" of fluid from the low pressure (inlet) side to the high pressure (delivery) side. There is a wide range of designs available for achieving this end. The general characteristics of the pumps are similar to those of reciprocating piston pumps, but the delivery is more even because the fluid stream is broken down into so much smaller elements. The pumps are capable of delivering to a high pressure, and the pumping rate is approximately proportional

to the speed of the pump and is not greatly influenced by the pressure against which it is delivering. Again, it is necessary to provide a pressure relief system to ensure that the safe operating pressure is not exceeded.

b- The Cam Pump

A rotating cam is mounted eccentrically in a cylindrical casing and a very small clearance is maintained between the outer edge of the cam and the casing. As the cam rotates it expels liquid from the space ahead of it and sucks in liquid behind it. The delivery and suction sides of the pump are separated by a sliding valve, which rides on the cam. The characteristics again are similar to those of the gear pump.

c- The Vane Pump

The rotor of the vane pump is mounted off center in a cylindrical casing. It carries rectangular vanes in a series of slots arranged at intervals round the curved surface of the rotor. The vanes are thrown outwards by centrifugal action and the fluid is carried in the spaces bounded by adjacent vanes, the rotor, and the casing. Most of the wear is on the vanes and these can readily be replaced.

d- The Flexible Vane Pump

The pumps described above will not handle liquids containing solid particles in suspension, and the flexible vane pumps has been developed to overcome this disadvantage. In this case, the rotor is an integral elastomer moulding of a hub with flexible vanes which rotates in a cylindrical casing containing a crescent-shaped block, as in the case of the internal gear pump.

e- The Flow Inducer or Peristaltic Pump

This is a special form of pump in which a length of silicone rubber or other elastic tubing, typically of 3 to 25 mm diameter, is compressed in stages by means of a rotor as shown in Figure . The tubing is fitted to a curved track mounted concentrically with a rotor carrying three rollers. As the rollers rotate, they flatten the tube against the track at the points of contact. These "flats" move the fluid by positive displacement, and the flow can be precisely controlled by the speed of the motor. These pumps have been particularly useful for biological fluids where all forms of contact must be avoided. They are being increasingly used and are suitable for pumping emulsions, creams, and similar fluids in laboratories and small plants where the freedom from glands, avoidance of aeration, and corrosion resistance are valuable, if not essential. Recent

developments[^] have produced thick-wall, reinforced moulded tubes which give a pumping performance of up to $0.02 \text{ m}^3/\text{s}$ at 1 MN/m^2 . The control is such that these pumps may conveniently be used as metering pumps for dosage processes.

f- The Mono pump

Another example of a positive acting rotary pump is the single screw-extruder pump typified by the Mono pump, in which a specially shaped helical metal rotor revolves eccentrically within a double-helix, resilient rubber stator of twice the pitch length of the metal rotor. A continuous forming cavity is created as the rotor turns — the cavity progressing towards the discharge, advancing in front of a continuously forming seal line and thus carrying the pumped material with it. The Mono pump gives a uniform flow and is quiet in operation. It will pump against high pressures; the higher the required pressure, the longer are the stator and the rotor and the greater the number of turns. The pump can handle corrosive and gritty liquids and is extensively used for feeding slurries to filter presses. It must never be run dry. The Mono Merlin Wide Throat pump is used for highly viscous liquids.

g- The Screw pumps

A most important class of pump for dealing with highly viscous material is represented by the screw extruder used in the polymer industry. The screw pump is of more general application and will be considered first. The fluid is sheared in the channel between the screw and the wall of the barrel. The mechanism that generates the pressure can be visualized in terms of a model consisting of an open channel covered by a moving plane surface. If a detailed analysis of the flow in a screw pump is to be carried out, then it is also necessary to consider the small but finite leakage flow that can occur between the flight and the wall. With the large pressure generation in a polymer extruder, commonly 100 bar (107 N/m^2), the flow through this gap, which is typically about 2 per cent of the barrel internal diameter, can be significant. The pressure drops over a single pitch length may be of the order of 10 bar (106 N/m^2), and this will force fluid through the gap. Once in this region the viscous fluid is subject to a high rate of shear (the rotation speed of the screw is often about 2 Hz), and an appreciable part of the total viscous heat generation occurs in this region of an extruder.

15.6 The advantages and disadvantages of the centrifugal pump

The main advantages are:

- (1) It is simple in construction and can, therefore, be made in a wide range of materials.
- (2) There is a complete absence of valves.
- (3) It operates at high speed (up to 100 Hz) and, therefore, can be coupled directly to an electric motor. In general, the higher the speed the smaller the pump and motor for a given duty.
- (4) It gives a steady delivery.
- (5) Maintenance costs are lower than for any other type of pump.
- (6) No damage is done to the pump if the delivery line becomes blocked, provided it is not ran in this condition for a prolonged period.
- (7) It is much smaller than other pumps of equal capacity. It can, therefore, be made into a sealed unit with the driving motor, and immersed in the suction tank.
- (8) Liquids containing high proportions of suspended solids are readily handled. The main

disadvantages are:

- (1) The single-stage pump will not develop a high pressure. Multistage pumps will develop greater heads but they are very much more expensive and cannot readily be made in corrosion-resistant material because of their greater complexity. It is generally better to use very high speeds in order to reduce the number of stages required.
- (2) It operates at a high efficiency over only a limited range of conditions: this applies especially to turbine pumps.
- (3) It is not usually self-priming.
- (4) If a non-return valve is not incorporated in the delivery or suction line, the liquid will run back into the suction tank as soon as the pump stops.
- (5) Very viscous liquids cannot be handled efficiently.

15.7 Priming the Pump

The theoretical head developed by a centrifugal pump depends on *the impeller speed, the radius of the impeller, and the velocity of the fluid leaving the impeller. If these factors are constant, the developed head is the same for fluids of all densities and is the same for liquids and gases. A centrifugal pump trying to operate on air, then can neither draw liquid upward from an initially empty suction line nor force liquid a full discharge line. Air can be displaced by priming the pump.*

For example, if a pump develops a head of 100 ft and is full of water, the increase in pressure is $[100 \text{ ft} (62.3 \text{ lb/ft}^3) (\text{ft}^2 / 144 \text{ in}^2)] = 43 \text{ psi} (2.9 \text{ atm})$. If full of air the pressure increase is about 0.05 psi (0.0035 atm).

5.8 Operating Characteristics

The operating characteristics of a pump are conveniently shown by plotting the head (h), power (P), efficiency (η), and sometimes required NPSH against the flow (or capacity) (Q) as shown in Figure (5). These are known as characteristic curves of the pump. It is important to note that the efficiency reaches a maximum and then falls, whilst the head at first falls slowly with Q but eventually falls off rapidly. The optimum conditions for operation are shown as the duty point, i.e. the point where the head curve cuts the ordinate through the point of

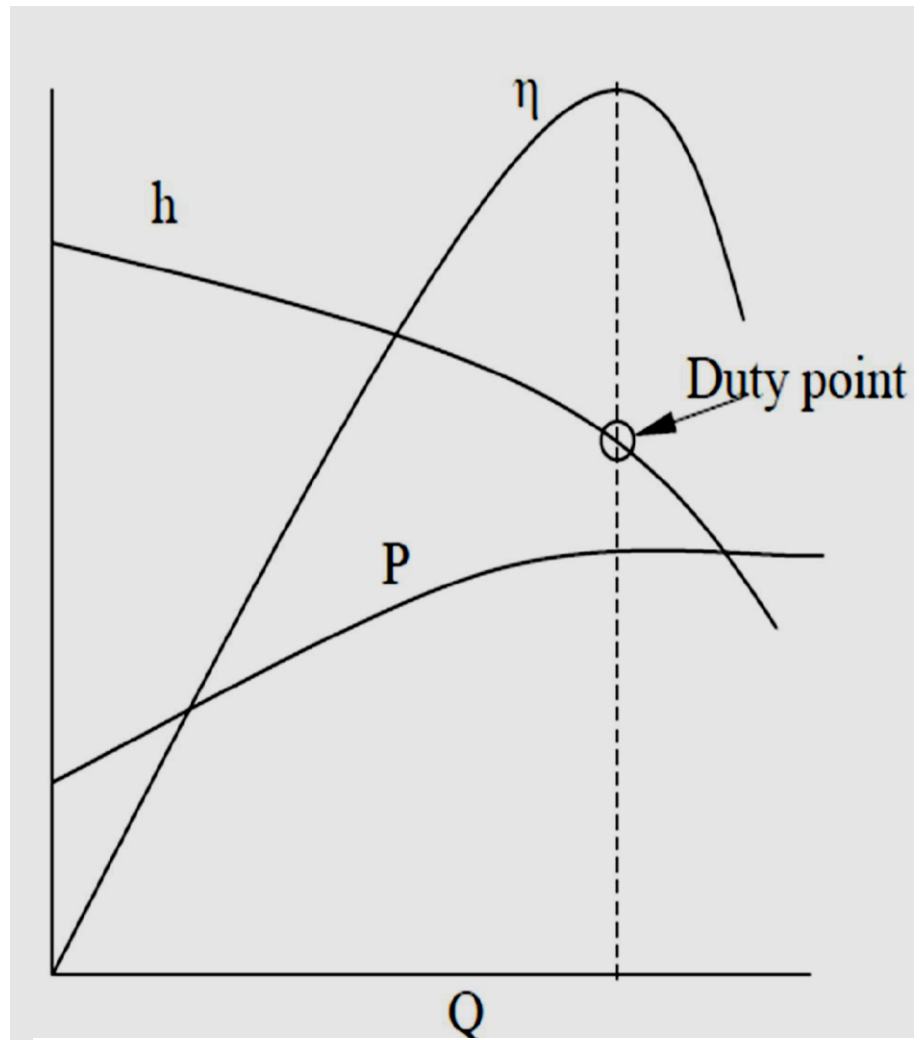


Figure (15.5) Radial flow pumps characteristics Δh

Characteristic curves have a variety of shapes depending on *the geometry of the impeller and pump casing*. Pump manufactures normally supply the curves only for operation with water.

In a particular system, a centrifugal pump can only operate at one point on the Δh against Q curve and that is the point where the Δh against Q curve of the pump intersects with the Δh against Q curve of the system as shown in Figure. The system total head at a particular liquid flow rate

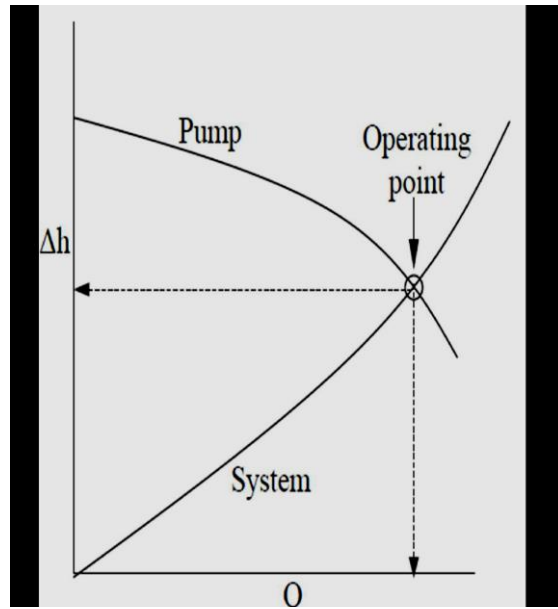


Figure (15.6) system and pump total head against capacity

$$\Delta h = (z_d - z_s) + \left(\frac{p_d - p_s}{\rho g} \right) + [(h_F)_d - (h_F)_s] \dots \dots \dots (15.3)$$

Where

$$(h_F)_d = 4f_d + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_d + \frac{u_d^2}{2g}$$

$$(h_F)_s = 4f_s + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_s + \frac{u_s^2}{2g}$$

For the same pipe and diameter for suction and discharge line

$$\Delta h = \Delta z + \frac{\Delta p}{\rho g} + 4f \left[\left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_d + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_s \right] \frac{u^2}{2g} \dots \dots \dots (15.7)$$

But

$$u = \frac{Q}{\pi/4d^2}$$

$$\Delta h = \Delta z + \frac{\Delta p}{\rho g} + \frac{4f}{2g} \left[\left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_d + \left(\frac{L}{d} + \sum \frac{L_e}{d} \right)_s \right] \left(\frac{Q}{\pi/4d^2} \right)^2 \dots \dots \dots (15.8)$$

5.9 Centrifugal Pump Relations

The power (PE) required in an ideal centrifugal pump can be expected to be a function of the liquid density (ρ), the impeller diameter (D), and the rotational speed of the impeller (N). If the relationship is assumed to be given by the equation,

$$P_E = c\rho^a N^b D^c \dots\dots\dots (1)$$

then it can be shown by dimensional analysis that

$$P_E = c_1 \rho N^3 D^5 \dots\dots\dots (2)$$

where, c_1 is a constant which depends on the geometry of the system. The power (P_E) is also proportional to the product of the volumetric flow rate (Q) and the total head (Δh) developed by the pump.

$$P_E = c_2 Q \Delta h \dots\dots\dots (3)$$

where, c_2 is a constant. The volumetric flow rate (Q) and the total head (Δh) developed by the pump are: -

$$Q = c_3 N D^3 \dots\dots\dots (4)$$

$$\Delta h = c_4 N^2 D^2 \dots\dots\dots (5)$$

where, c_3 and c_4 are constants.

Equation (5) could be written in the following form,

$$\Delta h^{3/2} = c_4^{3/2} N^3 D^3 \dots\dots\dots (6)$$

Combine equations (4) and (6) [eq. (4) divided by eq. (6)] to give;

$$\frac{Q}{\Delta h^{3/2}} = \frac{c_3}{c_4^{3/2}} \frac{1}{N} \Rightarrow \frac{QN^2}{\Delta h^{3/2}} = const \dots\dots (7)$$

Or

$$\frac{N\sqrt{Q}}{\Delta h^{3/4}} = const = N_s \dots\dots\dots (8)$$

When the rotational speed of the impeller N is (rpm), the volumetric flow rate Q in (USgalpm) and the total head Δh developed by the pump is in (ft), the constant N_s in equation (8) is known as *the specific speed of the pump. The specific speed is used as an index of pump types and always evaluated at the best efficiency point (bep) of the pump. Specific speed vary in the range (400 – 10,000) depends on the impeller type, and has the dimensions of $(L/T^2)^{3/4}$. [British gal=1.2USgal, ft³=7.48USgal, m³=264US gal]*

15.9.1 Homologous Centrifugal Pumps

Two different size pumps are said to be geometrically similar when the ratios of corresponding dimensions in one pump are equal to those of the other pump. Geometrically similar pumps are

said to be homologous. A set of equations known as the *affinity laws govern the performance of homologous centrifugal pumps at various impeller speeds.*

For the tow homologous pumps, equations (4), and (5) are given

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2} \right) \left(\frac{D_1}{D_2} \right)^3 \text{-----(9)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2} \right)^2 \left(\frac{D_1}{D_2} \right)^2 \text{-----(10)}$$

Similarly for the tow homologous pumps equation (2) can be written in the form;

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2} \right)^3 \left(\frac{D_1}{D_2} \right)^5 \text{-----(11)}$$

And by analogy with equation (10),

$$\frac{NPSH_1}{NPSH_2} = \left(\frac{N_1}{N_2} \right)^2 \left(\frac{D_1}{D_2} \right)^2 \text{-----(12)}$$

Equations (9), (10), (11), and (12) are the affinity law for homologous centrifugal pumps.

For a particular pump where the impeller of diameter D_1 , is replaced by an impeller with a slightly different diameter D_2 the following equations hold

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2} \right) \left(\frac{D_1}{D_2} \right) \text{-----(13)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2} \right)^2 \left(\frac{D_1}{D_2} \right)^2 \text{-----(14)}$$

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2} \right)^3 \left(\frac{D_1}{D_2} \right)^3 \text{-----(15)}$$

The characteristic performance curves are available for a centrifugal pump operating at a given rotation speed, equations (13), (14), and (15) enable the characteristic performance curves to be plotted for other operating speeds and for other slightly impeller diameters.

5.10 Centrifugal Pumps in Series and in Parallel

5.10.1 Centrifugal Pumps in Parallel

Consider two centrifugal pumps in *parallel*. The total head for the pump combination (Δh_T) is the same as the total head for each pump,

$$\Delta h_T = \Delta h_1 + \Delta h_2$$

$$Q_T = Q_1 + Q_2$$

The operating characteristics curves for two pumps in parallel are: - Solution by trial and error

- 1- Draw Δh versus Q for the two pumps and the system.
- 2- Draw horizontal Δh_T line and determine Q_1 , Q_2 , and Q_S .
- 3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system).
- 4- If $Q_T \neq Q_S$ repeat steps 2, 3, and 4 until $Q_T = Q_S$.

Another procedure for solution

- 1- The same as above.
- 2- Draw several horizontal lines (4 to 6) for Δh_T and determine their Q_T .
- 3- Draw Δh_T versus Q_T .
- 4- The duty point is the intersection of Δh_T curve with Δh_S curve.

15.10.2 Centrifugal Pumps in Series

Consider two centrifugal pumps in *series*.

The total head for the pump combination

(Δh_T) is the sum of the total heads for the two pumps,

$$\Delta h_T = \Delta h_1 + \Delta h_2$$

$$Q_T = Q_1 + Q_2$$

The operating characteristics curves for two pumps in series are: -

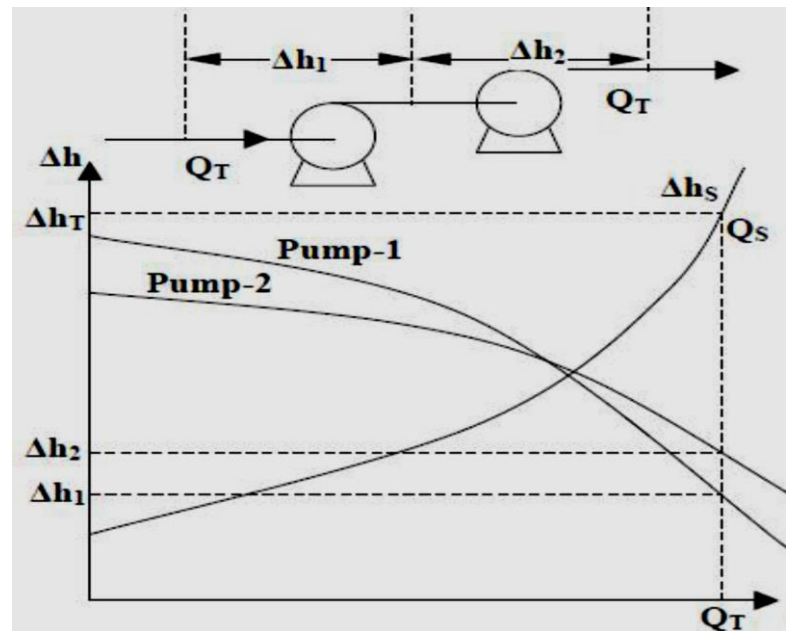
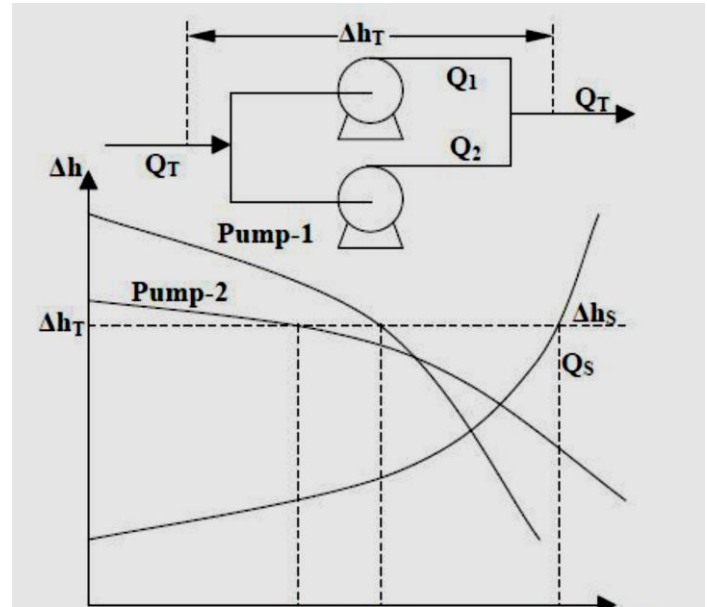
Solution by trial and error

- 1- Draw Δh versus Q for the two pumps and the system.
- 2- Draw vertical Q_T line and determine Δh_1 , Δh_2 , and Δh_S .
- 3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system)
- 4- If $\Delta h_T \neq \Delta h_S$ rep

repeat steps 2, 3, and 4 until $\Delta h_T = \Delta h_S$.

Another procedure for solution

- 1- The same as above.
- 2- Draw several Vertical lines (4 to 6) for Q_T and determine their Δh_T .
- 3- Draw Δh_T versus Q_T .



4- The duty point is the intersection of Δh_T curve with Δh_s curve.