Lecture NO (7)
Fluid Dynamics

Objectives

- Introduce concepts necessary to analyses fluids in motion
- Identify differences between Steady/unsteady uniform/non-uniform compressible/incompressible flow
- Demonstrate streamlines and stream tubes
- Introduce the Continuity principle through conservation of mass and control volumes
- Derive the Bernoulli (energy) equation
- Demonstrate practical uses of the Bernoulli and continuity equation in the analysis of flow
- Introduce the momentum equation for a fluid
- Demonstrate how the momentum equation and principle of conservation of momentum is used to predict forces induced by flowing fluids

This section discusses the analysis of fluid in motion - fluid dynamics. The motion of fluids can be predicted in the same way as the motion of solids are predicted using the fundamental laws of physics together with the physical properties of the fluid. It is not difficult to envisage a very complex fluid flow. Spray behind a car; waves on beaches; hurricanes and tornadoes or any other atmospheric phenomenon are all example of highly complex fluid flows which can be analyzed with varying degrees of success (in some cases hardly at all!). There are many common situations which are easily analyzed.

(7.1)Uniform Flow, Steady Flow

It is possible - and useful - to classify the type of flow which is being examined into small number of groups.

If we look at a fluid flowing under normal circumstances - a river for example - the conditions at one point will vary from those at another point (e.g. different velocity) we have non-uniform flow. If the conditions at one point vary as time passes then we have unsteady flow.

Under some circumstances the flow will not be as changeable as this. He following terms describes the states which are used to classify fluid flow:

- ***Uniform flow:*** If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be *uniform*.
- ***Non-uniform:*** If at a given instant, the velocity is not the same at every point the flow is *non-uniform*. (In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform – as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.)
- ***Steady:*** A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but **DO NOT** change with time.

• ***unsteady:*** If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practice there are always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.

Combining the above we can classify any flow in to one of four types:

1. ***Steady uniform flow.*** Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

Steady non-uniform flow. Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

3. ***Unsteady uniform flow.*** At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. ***Unsteady non-uniform flow.*** Every condition of the flow may change from point to point and with time at every point. For example waves in a channel. If you imaging the flow in each of the above classes you may imagine that one class is more complex than another. And this is the case - steady uniform flow is by far the most simple of the four. You will then be pleased to hear that this course is restricted to only this class of flow. We will not be encountering any non-uniform or unsteady effects in any of the examples (except for one or two quasi-time dependent problems which can be treated at steady).

(7.2)Compressible or Incompressible

All fluids are compressible - even water - their density will change as pressure changes. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density. As you will appreciate, liquids are quite difficult to compress - so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account
- even for liquids. Gasses, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

(7.3) Three-dimensional flow

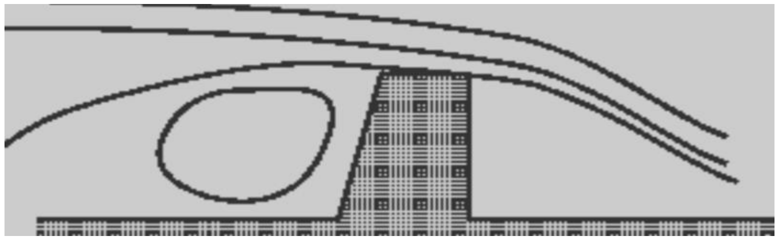
Although in general all fluids **flow three-dimensionally,** with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much simpler.

Flow is ***one dimensional*** if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section. The flow may be unsteady, in this case the parameter vary in time but still not across the cross-section. An example of one-dimensional flow is the flow in a pipe. Note that since flow must be zero at the pipe wall - yet non-zero in the center – there is a difference of parameters across the cross-section. Should this be treated as two-dimensional flow?

Possibly - but it is only necessary if very high accuracy is required. A correction factor is then usually applied.

Flow is ***two-dimensional*** if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir foe which typical streamlines can be seen in the figure below.

Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.

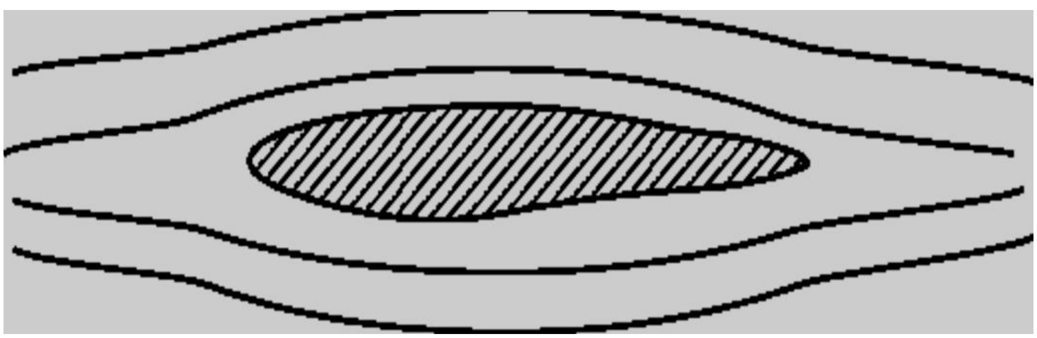


Two-dimensional flow over a weir.

In this course we will only be considering steady, incompressible one and two-dimensional flow.

(7.4)Streamlines and stream tubes

In analyzing fluid flow it is useful to visualize the flow pattern. This can be done by drawing lines joining points of equal velocity - velocity contours. These lines are known as streamlines. Here is a simple example of the streamlines around a cross-section of an aircraft wing shaped body:



Streamlines around a wing shaped body

When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.

Close to a solid boundary streamlines are parallel to that boundary

At all points the direction of the streamline is the direction of the fluid velocity: this is how they are defined. Close to the wall the velocity is parallel to the wall so the streamline is also parallel to the wall.

It is also important to recognize that the position of streamlines can change with time - this is the case in unsteady flow. In steady flow, the position of streamlines does not change.

Some things to know about streamlines

- Because the fluid is moving in the same direction as the streamlines, fluid cannot cross a streamline.
- Streamlines cannot cross each other. If they were to cross this would indicate two different velocities at the same point. This is not physically possible.
- The above point implies that any particles of fluid starting on one streamline will stay on that same streamline throughout the fluid.

A

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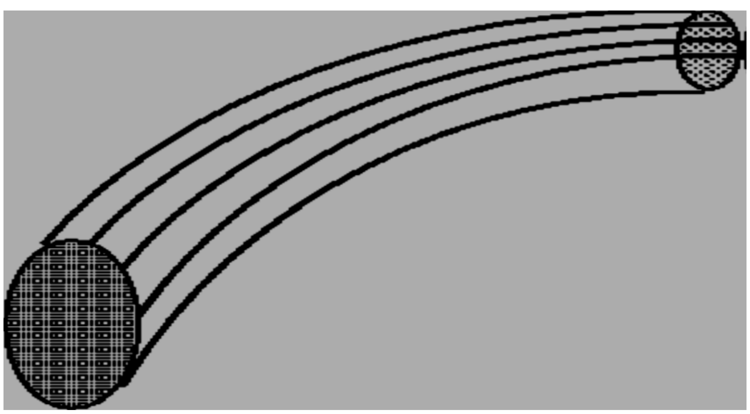
streamlines along which the fluid flows. This tubular surface is known as a ***stream tube***.



A Stream tube

And in a two-dimensional flow we have a stream tube which is flat (in the plane of the paper):

A two dimensional version of the stream tube

The “walls” of a stream tube are made of streamlines. As we have seen above, fluid cannot flow across a streamline, so fluid cannot cross a stream tube wall. The stream tube can often be viewed as a solid walled pipe. A stream tube is not a pipe - it differs in unsteady flow as the walls will move with time. And it
differs because the “wall” is moving with the fluid.

(7.5) Flow rate

(7.5.1)Mass flow rate

If we want to measure the rate at which water is flowing along a pipe. A very simple way of doing this is to catch all the water coming out of the pipe in a bucket over a fixed time period. Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is known as the *mass flow rate.*

For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$Mass\ flow\ rate = \frac{Mass\ of\ fliud\ in\ bucket}{Time\ taken\ to\ collect\ the\ fluid}$$

$$= \frac{8.0 - 2.0}{7} = 0.857\,\frac{\text{kg}}{\text{s}}\,(\text{kg.}\,\text{s}^{-1})$$

Performing a similar calculation, if we know the mass flow is 1.7kg/s, how long will it take to fill a container with 8kg of fluid?

$$Time\ = \frac{Mass}{Mass\ flow\ rate} = \frac{8}{1.7} = 4.7\ s$$

(7.5.2) Volume flow rate - Discharge.

More commonly we need to know the volume flow rate - this is more commonly known as *discharge*. (It is also commonly, but inaccurately, simply called flow rate). The symbol normally used for discharge is Q.

The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid gives us the mass flow rate. Consequently, if the density of the fluid in the above example is 850 kgm^3 then: discharge

$$Discharge\ Q = \frac{Volume\ of\ fluid}{Time}$$
$$= \frac{Mass\ \ of\ fluid}{Density\ \times Time}$$

$$= \frac{Mass\ \ of\ flow\, rate}{Density}$$

$$\frac{0.857}{850} = 0.001008\frac{m^3}{s}\ (m^3s^{-1}) = 1.008 \times 10^{-3}\ \frac{m^3}{s} = 1.008\frac{l}{s}$$

An important aside about units should be made here:

As has already been stressed, we must always use a consistent set of units when applying values to equations. It would make sense therefore to always quote the values in this consistent set. This set of units will be the SI units. Unfortunately, and this is the case above, these actual practical values are very small or very large (0.001008m^3/s is very small). These numbers are difficult to imagine physically. In these cases it is useful to use *derived units,* and in the case above the useful derived unit is the liter. (1 liter = $10^{-3}m^3$). So the solution becomes $1.008 l / s$. It is far easier to imagine 1 liter than 1.0 ´ $10^{-3}m^3$. Units must always be checked, and converted if necessary to a consistent set before using in an equation.

(7.5.2.1)Discharge and mean velocity.

If we know the size of a pipe, and we know the discharge, we can deduce the mean velocity

Discharge in a pipe

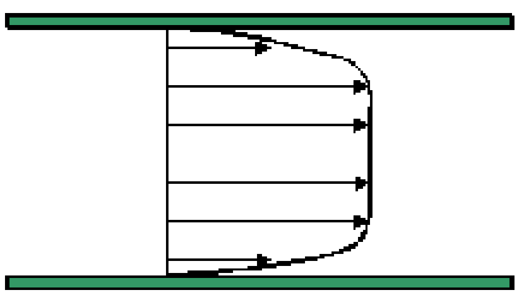
If the area of cross section of the pipe at point X is A, and the mean velocity here is u_m . During a time t, a cylinder of fluid will pass point X with a volume A×u_m×t. The volume per unit time (the discharge) will thus be

$$Q = \frac{Volume}{Time} = \frac{Au_m t}{t} = Au_m$$

So if the cross-section area, A, is $1.2 \times 10^{-3} m^2$ and the discharge, Q is 2.4 *l* / s , then the mean velocity, u_m , of the fluid is

$$u_m = \frac{Q}{A} = \frac{2.4 \times 10^{-3}}{1.2 \times 10^{-3}} = 2.0\ m/s$$

Note how carefully we have called this the mean velocity. This is because the velocity in the pipe is not constant across the cross section. Crossing the centerline of the pipe, the velocity is zero at the walls increasing to a maximum at the center then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution. A typical one is shown in the figure below.



A typical velocity profile across a pipe

This idea, that mean velocity multiplied by the area gives the discharge, applies to all situations - not just pipe flow

(7.6)Continuity

Matter cannot be created or destroyed - (it is simply changed in to a different form of matter). This principle is known as the conservation of mass and we use it in the analysis of flowing fluids.

The principle is applied to fixed volumes, known as control volumes (or surfaces).

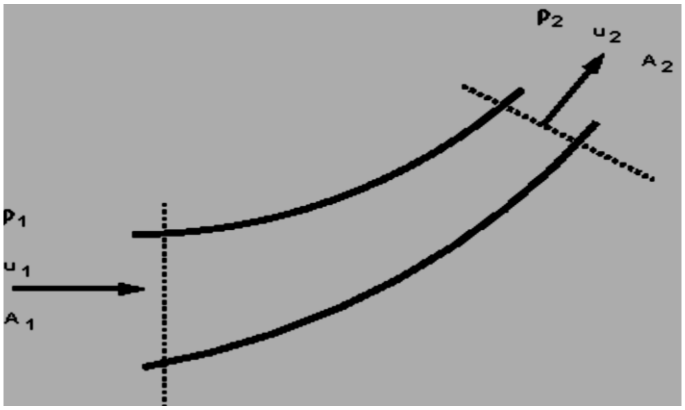
For any control volume the principle of *conservation of mass* says

Mass entering per unit time = Mass leaving per unit time + Increase of mass in the control volume per unit time

For steady flow there is no increase in the mass within the control volume, so

Mass entering per unit time = Mass leaving per unit time

This can be applied to a stream tube such as that shown below. No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this stream tube section.



A streamtube

We can then write

Mass entering per unit time at end 1 = Mass leaving per unit time at end 2

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2$$

Or for steady flow

$$\boldsymbol{\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = m = constant}$$

This is the equation of continuity

The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall - in this case we can use the mean velocity and write

$$\rho_1 A_1 u_{m1} = \rho_2 A_2 u_{m2} = m$$

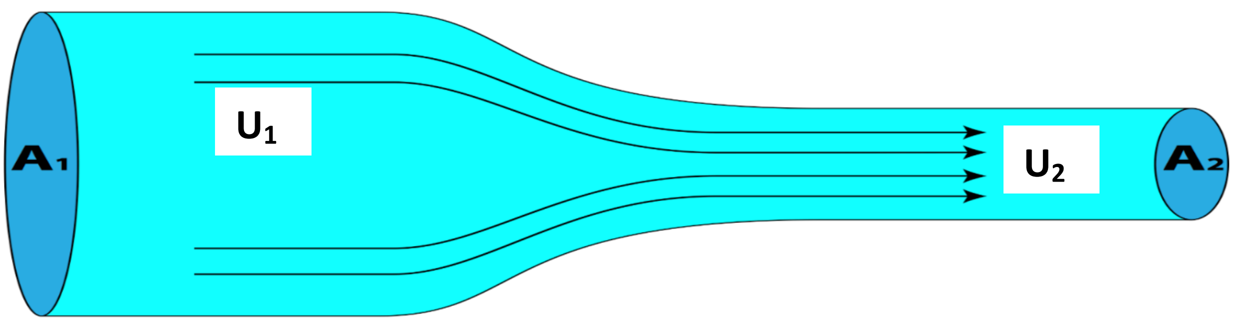
When the fluid can be considered incompressible, i.e. the density does not change, $\rho_1 = \rho_2 = \rho$ so (dropping the m subscript)

$$\boldsymbol{A_1 u_1 = A_2 u_2 = Q}$$

This is the form of the continuity equation most often used.

(7.6.1)Some example applications

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity principle, the *mass flow rate* must be the same at each section - the mass going into the pipe is equal to the mass going out of the pipe. So we can write:

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 = m$$

(With the sub-scripts 1 and 2 indicating the values at the two sections)

As we are considering a liquid, usually water, which is not very compressible, the density changes very little so we can say $\rho_1 = \rho_2 = \rho$. This also says that the volume flow rate is constant or that

Discharge at section 1 = Discharge at section 2

$$Q_1 = Q_2$$

$$A_1 u_1 = A_2 u_2$$

For example if the area A m $_1$ = 10 × 10$^{-3\ 2}$ and A m $_2$ = 3 × 10$^{-3\ 2}$ and the upstream mean velocity, u_1 = 2.1 m/s, then the downstream mean velocity can be calculated by

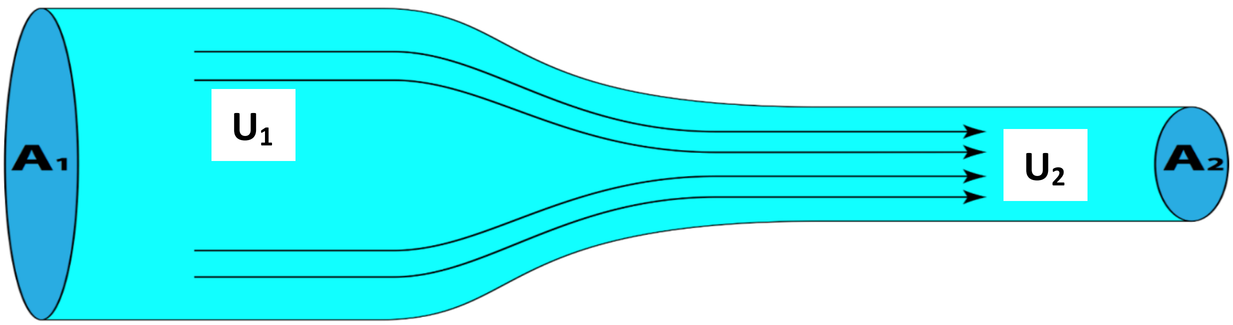
$$A_1 u_1 = A_2 u_2$$

$$u_2 = \frac{A_1 u_1}{A_2} = 7.0 m/s$$

Notice how the downstream velocity only changes from the upstream by the ratio of the two areas of the pipe. As the area of the circular pipe is a function of the diameter we can reduce the calculation further,

$$u_2 = \frac{A_1 u_1}{A_2} = \frac{A_1}{A_2} u_1 = \frac{\frac{\pi d_1^2}{4}}{\frac{\pi d_2^2}{4}} u_1 = \frac{d_1^2}{d_2^2} u_1 = \left(\frac{d_1}{d_2}\right)^2 u_1$$

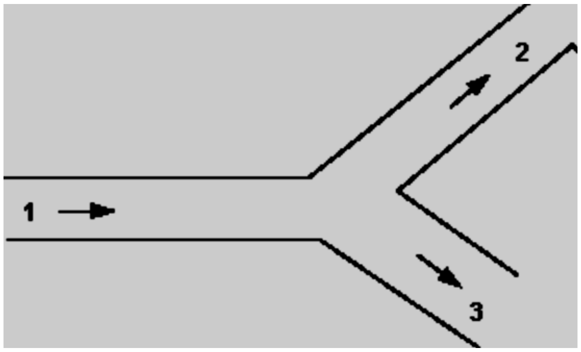
Now try this on a *diffuser*, a pipe which expands or diverges as in the figure below,



If the diameter at section 1 is $d_1 = 30mm$ and at section 2 $d_2 = 40\ mm$ and the mean velocity at section 2 is $u_2 = 3.0\ m/s$. The velocity entering the diffuser is given by,

$$u_1 = \left(\frac{40}{30}\right)^2 3.0 = 5.3 m/s$$

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



Total mass flow into the junction = Total mass flow out of the junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

When the flow is incompressible (e.g. if it is water) $\rho_1 = \rho_2 = \rho$

$$Q_1 = Q_2 + Q_3$$

$$u_1 A_1 = u_2 A_2 + u_3 A_3$$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm. What are the values of discharge and mean velocity in each pipe?

$$Q_1 = u_1 A_1 = \left(\frac{\pi d^2}{4}\right) u_1 = 0.00392 m^3/s$$

$$Q_2 = 0.3 Q_1 = 0.001178 m^3/s$$

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - 0.3 Q_1 = 0.7 Q_1 = 0.00275 m^3/s$$

$$Q_2 = u_2 A_2$$

$$u_2 = 0.936 m/s$$

$$Q_3 = u_3 A_3$$

$$u_3 = 0.972 m/s$$