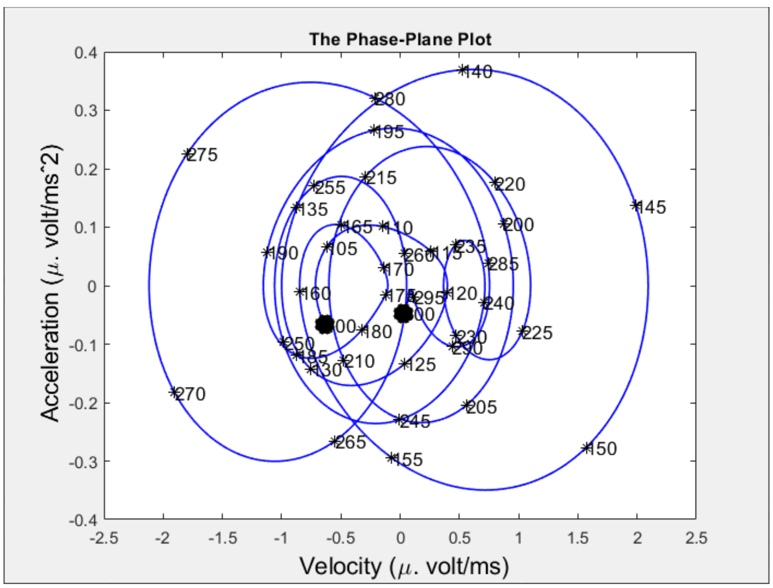
Getting Started with EEGlab Plugin

FDA

(eegplugin_fda version 0.1)



EEGLAB

>> FDA

(September 2021)

Contents

Menus

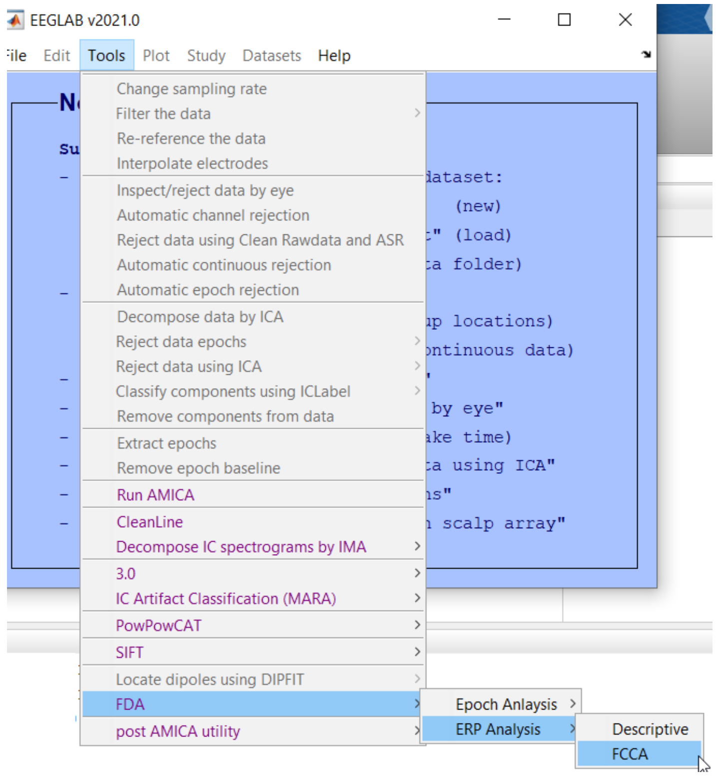


Figure A: FDA plugin in EEGLAB

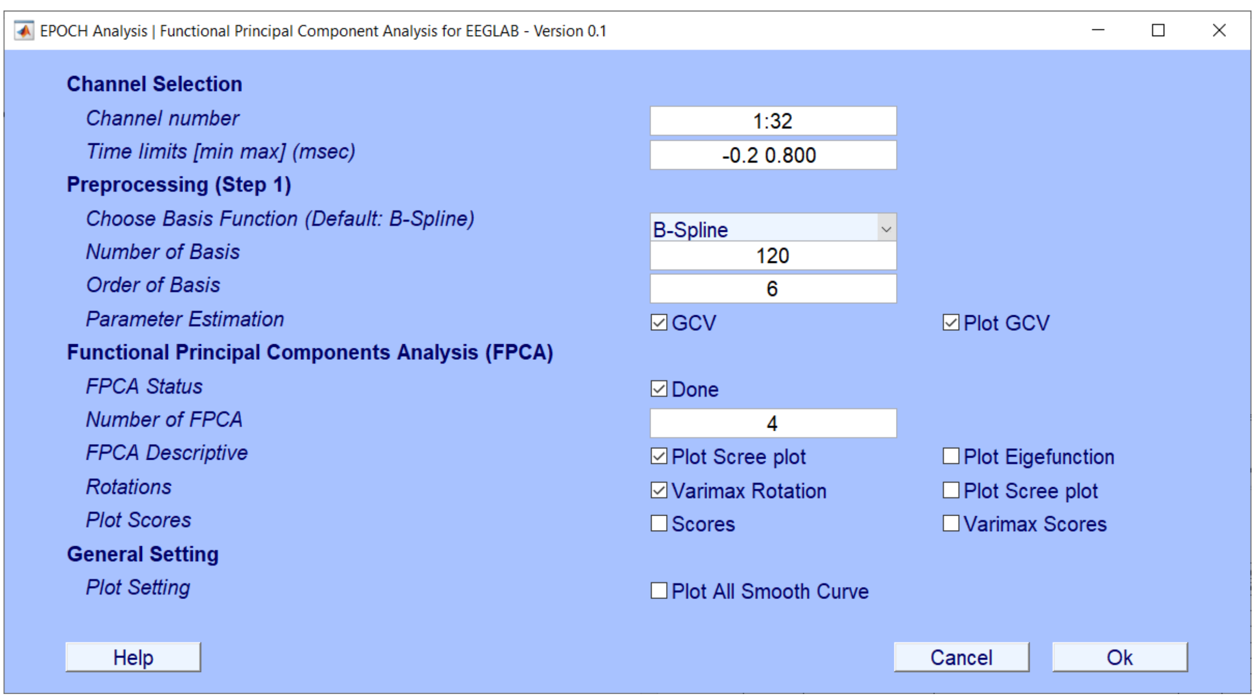


Figure B: Epoch Analysis | Functional Principal Component Analysis Menu

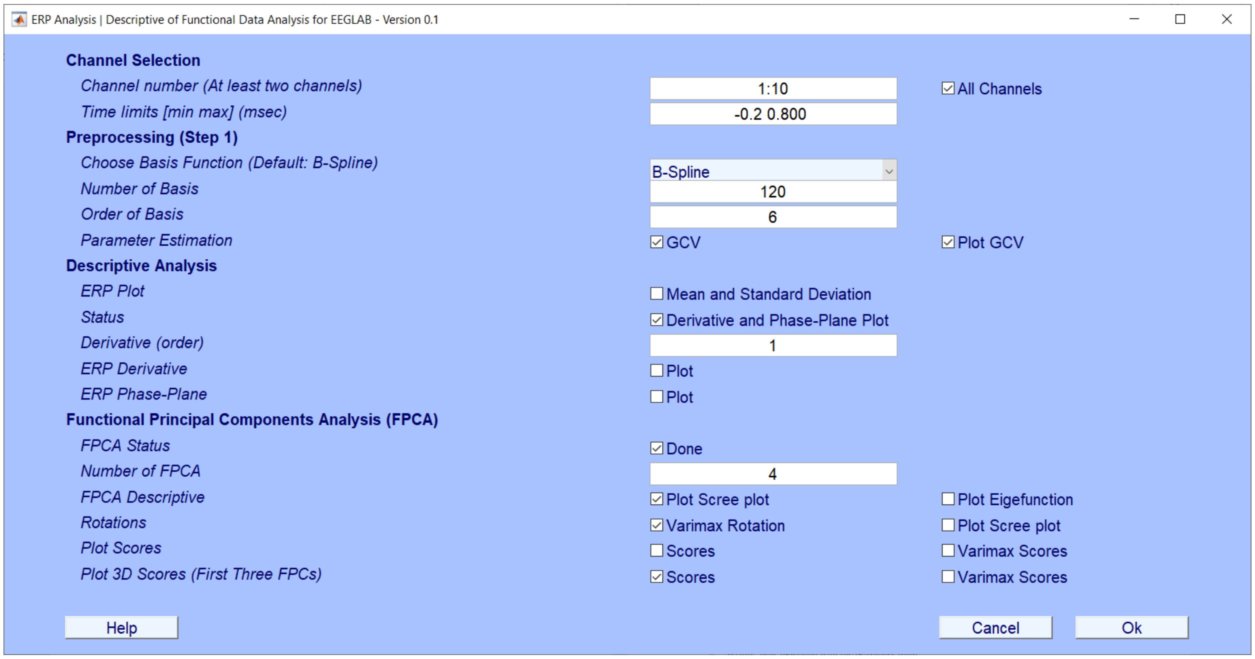


Figure C: ERP Analysis | Descriptive of Functional Data Analysis Menu

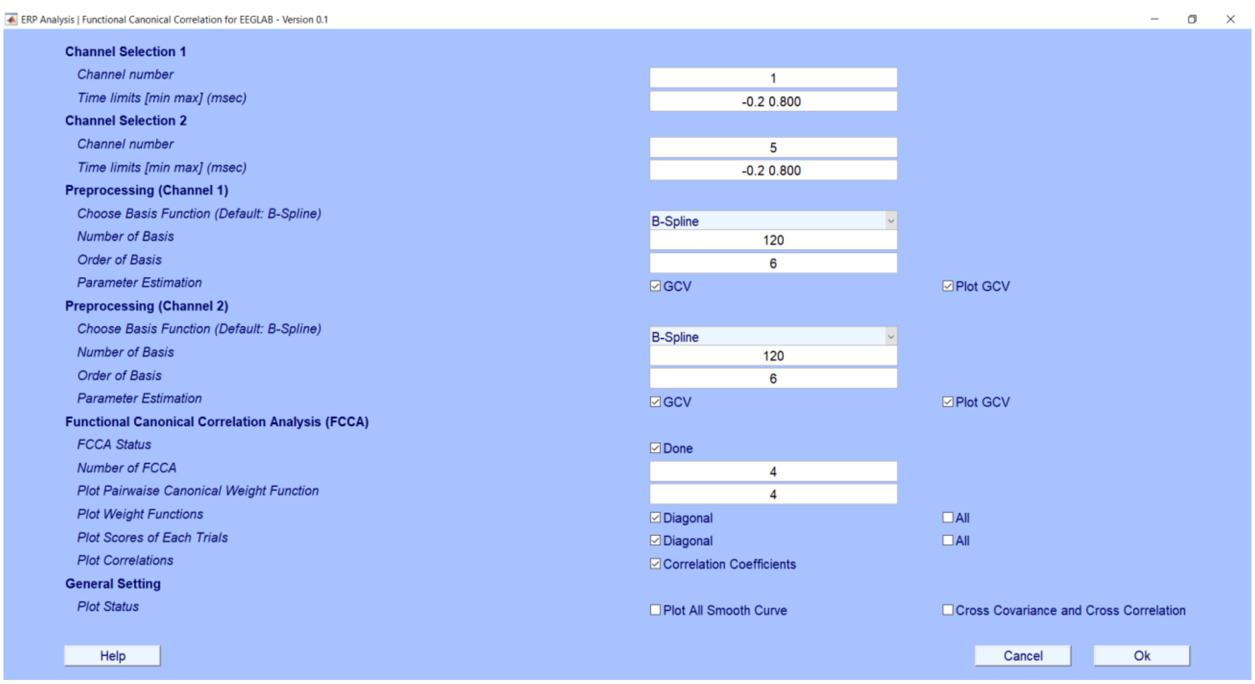


Figure D: ERP Analysis | Functional Canonical Correlation Analysis Menu

FDA Plugin

Version 0.1
(2021-09-20)

Title FDA_EEGLAB Plug-in

Description Doing Functional Data Analysis (FDA) in EEGLAB.

Version 0.1 (Updated six months with adding new functions and enhancing the previous menus.)

Procedures This plug-in has two procedures for developing menus:

1- Calling MATLAB functions that were previously published by other authors. In this case, the permission of using their codes in EEGLAB was obtained before implementing them in the EEGLAB. And their papers, books, and codes are cited.
2- Writing new functions.

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Plan: Statistical Methods

Items	Menu	Sub-menu	Versions		
			0.1	0.6	1.1
1.1	Epoch Analysis	FPCA	✓	×	×
1.2		Registration		×	×
2.1	ERP Analysis	Descriptive	✓	×	×
2.2		FCCA	✓	×	×
2.3		Multi-Dimensional FPCA		×	×
3	Regression	Functional Regression		×	×
4	Bayesian	Bayesian Functional Regression			×
5	Machine Learning	Functional Machine Learning			×

✓: Available in this version. ×: Available in new versions.

Plan: EEGLAB Features

Items	Menu	Description	Versions		
			0.1	0.6	1.1
1	Developing GUI	Developing GUI for task-specific EEG analysis	✓	×	×
2	Standalone Menu	Calling each menu separately.	✓	×	×
3	Exporting Data	Exporting the result of FDA analysis into the matrices		×	×
4	Mapping on the Scalp	Show the outputs on the scalp.		×	×
5	Data transfer	Data is made in one menu and calling in the next menus		×	×
6	Calling functions on STUDY	Apply FDA functions on STUDY			×

✓: Available in this version. ×: Available in new versions.

Short Description

1 – Epoch Analysis

In this menu, the epochs of ERP curves of each channel are considered. Usually, there are many epochs for each ERP for each channel in each trial. In this version, we only consider the time domain ERPs. The following figure shows an example of epochs of ERP curves.

<table>
<tr><td rowspan="2"></td><td>Channel Number</td><td colspan="4">1</td></tr>
<tr><td>Time window of ERP (ms)</td><td>-200 …</td><td>0 …</td><td>100…</td><td>800</td></tr>
<tr><td rowspan="4">Epochs</td><td>1</td><td colspan="4">Epochs 1 ERP 1 of Channel 1</td></tr>
<tr><td>2</td><td colspan="4">Epochs 2 ERP 1 of Channel 1</td></tr>
<tr><td>.
.
.</td><td colspan="4">Epochs … ERP 1 of Channel 1</td></tr>
<tr><td>n</td><td colspan="4">Epochs n ERP 1 of Channel 1</td></tr>
</table>

Example Dataset for Channel 1.

1.1 – FPCA

Aim doing function principal component analysis (FPCA) of ERP curve of selected channel(s) among their epochs.

Pre requires The data is cleaned and their artifact is removed and the epoch is specified.

Pre-Analysis The ERP curves are smoothed with two methods (B-Spline or Fourier transform) with a specific number of basis functions (recommended between 30-40 or overfitted with more than 100) and specified degree (default is 6). To avoid overfitting, the penalty term is estimated with generalized cross-validation (optional).

Analysis The number of FPCs (Harmonics) is specified. The scree plot shows the fraction of variance for each FPC. The eigenfunctions are plotted. The varimax transformation can be applied to them for interpretations. The FPCA scores are plotted against each FPCs. Each point in this plot is the epoch's number.

Usage Studying the variability between epochs with FPCA. The main benefit of FPCA against PCA is that it considers the time dependency between points by considering the FDA mythology. It reduces the number of extracted FPCs and the eigenfunctions are estimated. The FPCA scores are used for clustering the epochs and finding the outliers.

2 – ERP Analysis

In this menu, the ERP curves of different channels (electrodes) are smoothed over epochs. Then, two methods including functional principal component analysis and functional canonical correlation among electrodes are done within this menu. The following table is an example of this type of dataset for channel numbers of 1 to m.

	Channel Number	1			
	Time window of ERP (ms)	-200 …	0 …	100…	800
Epochs	1	Epochs 1 ERP 1 of Channel 1			
	2	Epochs 2 ERP 1 of Channel 1			
	. . .	Epochs … ERP 1 of Channel 1			
	n	Epochs n ERP 1 of Channel 1			

.
.
.

	Channel Number	m			
	Time window of ERP (ms)	-200 …	0 …	100…	800
Epochs	1	Epochs 1 ERP 1 of Channel 2			
	2	Epochs 2 ERP 1 of Channel 2			
	. . .	Epochs … ERP 1 of Channel 2			
	n	Epochs n ERP 1 of Channel 2			

Example Dataset for Channel 1 to m

2.1 – FPCA

Aim doing functional principal component analysis (FPCA) of smoothed ERP curve over a selected channel(s).

Pre requires The data is cleaned and their artifact is removed and the epoch is specified.

Pre-Analysis The ERP curves are smoothed with two methods (B-Spline or Fourier transform) with a specific number of basis functions (recommended between 30-40 or overfitted with more than 100) and specified degree (default is 6). To avoid overfitting, the penalty term is estimated with generalized cross-validation (optional). The functional mean of all smoothed ERP cures over epochs for each selected channel is estimated. The FPCA is done over these functional means of selected channels.

Analysis The functional mean and standard deviation of smoothed ERP curves are estimated and compared with the point-wise mean and standard deviation of raw ERP. The derivative of smoothed functional means is calculated and plotted for first, second, third, and degree-1 order as specified by the user.

The Phase-Plane curve of smoothed functional means is calculated. The x-axis is the first derivative (velocity) and the y-axis is the second derivative (acceleration). The number of FPCs (Harmonics) of smoothed functional means is specified. The scree plot shows the fraction of variance for each FPC. The eigenfunctions are plotted. The varimax transformation can be applied to them for interpretations. The

FPCA scores are plotted against each FPCs. Each point in this plot is the epoch's number. The 3D plot of FPCA scores of the first three scores is plotted.

Usage Studying the effect of smoothing on the mean and standard deviation of curves. It shows that if the parameters are correctly specified, the functional mean and raw point-wise mean are the same as each other, but functional standard deviations are lower than the point-wise standard deviation. The derivatives of the smoothed functional mean show the velocity (order 1), acceleration (order 2), and … (degree-1). The Phase-Plane plot is used for studying the velocity and acceleration on mean smoothed curves. In this plot, values before the trials start (-negative value until 0) are colored in red too and the remaining points (for example 0 to 800 (ms)) are colored in black. There is some number on the plot to show the time of ERPs. It is useful to detect the peaks and changes in the mean smoothed function especially in studying N1, P1, and … (P300, etc.).

Studying the variability between channels with FPCA. The main benefit of FPCA against PCA is that it considers the time dependency between points by considering the FDA mythology. It reduces the number of extracted FPCs and the eigenfunctions are estimated. The FPCA scores are used for clustering the channels and finding the groups of channels. In many application, the most variability between channels are estimated with the first three FPCs (FVE > 85%), therefore the 3D plot of first three eigenfunctions is very useful to study the most of the variations between channels.

Data structure the following table is an example of smoothed functional mean among electrodes. The FPCA is done on this table (the functional part is the ERP time).

<table>
<tr><td></td><td>Time window of ERP (ms)</td><td>-200 …</td><td>0 …</td><td>100…</td><td>800</td></tr>
<tr><td rowspan="4">Channels</td><td>1</td><td colspan="4">Mean of functional smoothed ERP of Channel 1</td></tr>
<tr><td>2</td><td colspan="4">Mean of functional smoothed ERP of Channel 2</td></tr>
<tr><td>.
.
.</td><td colspan="4"></td></tr>
<tr><td>m</td><td colspan="4">Mean of functional smoothed ERP of Channel m</td></tr>
</table>

Example Dataset for Channel 1 to m

2.2 – FCCA

Aim doing functional canonical correlation analysis (FCCA) of ERP curve of two selected channel(s).

Pre requires The data is cleaned and their artifact is removed and the epoch is specified.

Pre-Analysis The ERP curves are smoothed with two methods (B-Spline or Fourier transform) with a specific number of basis functions (recommended between 30-40 or overfitted with more than 100) and specified degree (default is 6). To avoid overfitting, the penalty term is estimated with generalized cross-validation (optional).

Analysis The number of FCC is specified. The FCC weight functions of two selected channels are estimated and plotted against each other in two ways: 1) Diagonal and 2) All. In the diagonal, only the same number of FCC with functions are plotted against each other for example the first FCC weight function of channel 1 against the first FCC weight function of channel 2, etc. In the All option, all FCC weight functions are plotted against each other. For example, the first FCC weight function of channel 1 against the first FCC weight function of channel 2, the second FCC weight function of channel 2, etc.…. The canonical correlation coefficients are calculated and plotted. The point-wise covariance surface and image are also plotted between ERPs of two selected channels.

Usage In many applications, we like to study the correlation between the ERP of two channels. The FCC weight functions show the pattern of two channels over time and the correlation coefficients are estimated. If the FCC weight functions have the same pattern over time, it means that they behave the same. But if the FCC weight functions have a different or vise-versa pattern over time, it means that they behave differently. We study the first FCC weight functions, then the second and …, respectively. The cross-covariance surface and image on smoothed ERPs are plotted and they showed the relations between two ERP curves of selected channels on each time point.

Tutorial

In this tutorial, we do the same examples with FDA_EEGLAB Plug-in to show the functionality of this GUI.

Tutorial Dataset

An example dataset is downloaded from the study of internal attention in the brain with two oddball tasks for visual and auditory tasks [1].

The dataset is for the first subject and first trial of the auditory task. It has two stimuli: 1) standard stimuli (CODE 124 - 390 Hz pure tone), 2) target stimuli (CODE 150 - laser gun). The dataset is cleaned, but we do the preprocessing steps as re-references, remove DC offset, high-pass filter (1 and 50 Hz), remove line noise with Cleanline, Clean continuous date using ASR, ICA with runica algorithm, extract epochs, and automatic epoch rejection with find improbable data and using ICLabel plugin to remove the artifacts. One channel is removed (34-1=33) and some ICA were excluded from the analysis and the reconstrued data are saved. [1:5]

The final clean datasets with epochs are available as:

Row	**Stimuli**	**Name**	**# Epoches**	**Time Window**	**Size (MB)**
1	Standard	a1_1_5_125_ICA_Cleaned.set	95	-200 800 ms	15.3
2	Target	a1_2_5_150_ICA_Cleaned.set	23	-200 800 ms	4.1

Name of datasets for tutorial

Installing FDA_EEGLAB

In FDA_EEGLAB version 0.1, we call some MATLAB source codes for FDA analysis with permissions [6]. The main functions are listed below (the equivalent R codes are available in the fda package in cran. www.r-project.org with the same authors):

Row	MATLAB function	Short Description	Chapters in [6]
1	`create_bspline_basis`	Creating B-Spline basis	3
2	`create_fourier_basis`	Creating Fourier basis	3
3	`fd`	Budling functional data	4
4	`smooth_basis`	Smoothing curves	5
5	`mean`	Functional mean	6
6	`var_fd`	Variance-Covariance	6
7	`cor_fd`	Cross-Correlation	6
8	`std_fd`	Functional Standard Deviation	6
9	`eval_fd`	Evaluation of functional object	6
10	`deriv_fd`	Derivative of functional object	6
11	`Getcoef`	Get coefficients of functional object	6
12	`pca_fd`	Functional principal component analysis	7
13	`varmax_pca_fd`	Varimax Rotation of FPCA	7
14	`cca_fd`	Functional Canonical Correlation Aanlysis	7

MATLAB functions, their description, and chapters.

Therefore, we need to do the following steps:

1- Download the Zip folder name fdaM from
(Link: https://www.psych.mcgill.ca/misc/fda/downloads/FDAfuns/Matlab/fdaM.zip)
2- Unzip it on your PC
3- Open the MATLAB
4- Run "`addtopath('...\fdaM');`" or in the MATLAB , Home tab → Environment → set path → add with subfolder → fdaM .

We also provide the fdaM in the plug-in folder and you can set the path manually for further analysis in MATLAB.

Some other references for studying Functional Data Analysis:

- [Book] James O. Ramsay and Bernard W. Silverman, "Functional Data Analysis: Methods and Case Studies", 2005, Springer
- [Book] James O. Ramsay and Bernard W. Silverman, "Applied Functional Data Analysis: Methods and Case Studies", 2002, Springer
- [Website] https://www.psych.mcgill.ca/misc/fda/index.html
- [Book] Ferraty, Frédéric, Vieu, Philippe, "Nonparametric Functional Data Analysis, Theory and Practice", 2006, springer
- [Book] Matthew Reimherr and Piotr Kokoszka, "Introduction to Functional Data Analysis", 2017, Chapman and Hall/CRC
- [Article] Wang, Jane-Ling, Jeng-Min Chiou, and Hans-Georg Müller. "Functional data analysis." Annual Review of Statistics and Its Application 3 (2016): 257-295.
- [Article] Sørensen, Helle, Jeff Goldsmith, and Laura M. Sangalli. "An introduction with medical applications to functional data analysis." Statistics in Medicine 32.30 (2013): 5222-5240.

Getting Familiar with FDA_EEGLAB Menu

After installing/moving FDA_EEGLAB in the plugin folder. Run the EEGLAB in MATLAB:

```
>> eeglab()
```

The FDA plug-in is in the Tools and has the following menus (figure 1)

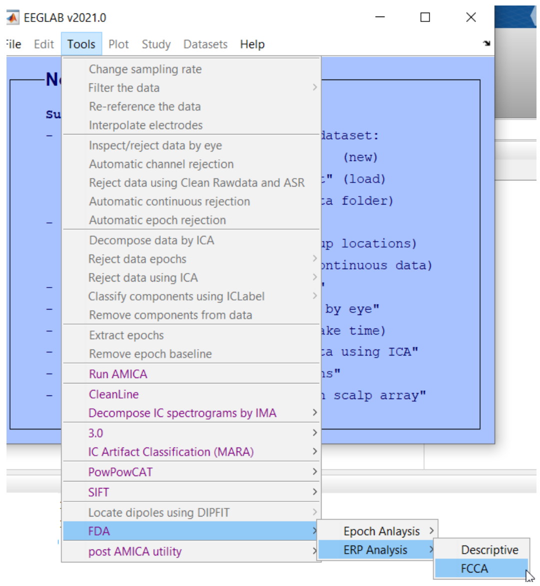


Figure 1- The FDA_EEGLAB Menu and submenu

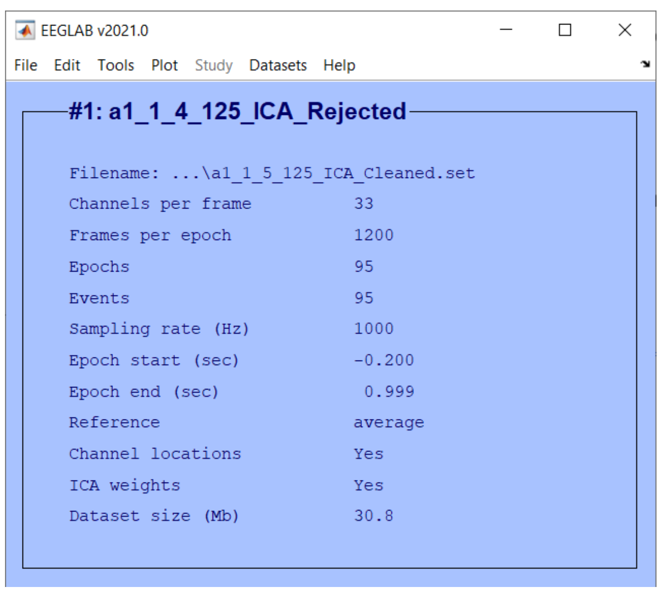


Figure 2- Loading .set files.

Epoch Analysis Examples

Example 1: Epoch Analysis – Smoothing all ERP of epochs.

In this analysis, we want to smooth all ERP curves of channel 1. In this regard, we do it with three scenarios.

First scenario

1- Load the 'a1_1_5_125_ICA_Cleaned.set'
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_125_ICA_Cleaned.set .
2- It has 33 channels, 95 Epochs, and a time window from -200 to 1000 ms. (Figure 2) It shows that the channel location is available, ICA weights exist, the reference method is average and the sampling rate is 1000 Hz.
3- Click Tools → FDA → Epoch Analysis → FPCA.
4- In this GUI, we want to smooth **channel 1** ERP curves for each epoch. The **time windows** are set to **-100 to 600** ms, the **basis function type** is **B-Spline**, the **number of bases** is **20**, **order of basis** is **2**, uncheck all other options except **Plot All Smooth Curve**. (figure 3) and then **OK**.

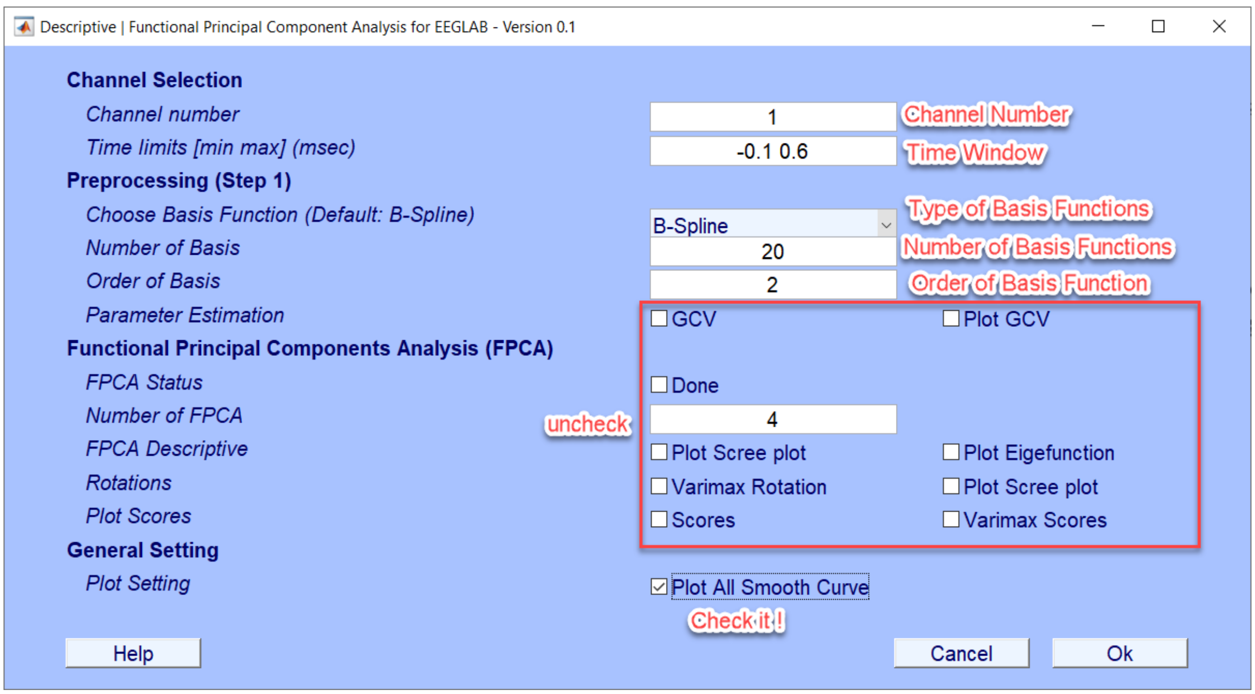


Figure 3 – Smoothing ERP curves (Scenario 1)

The following plot is produced:

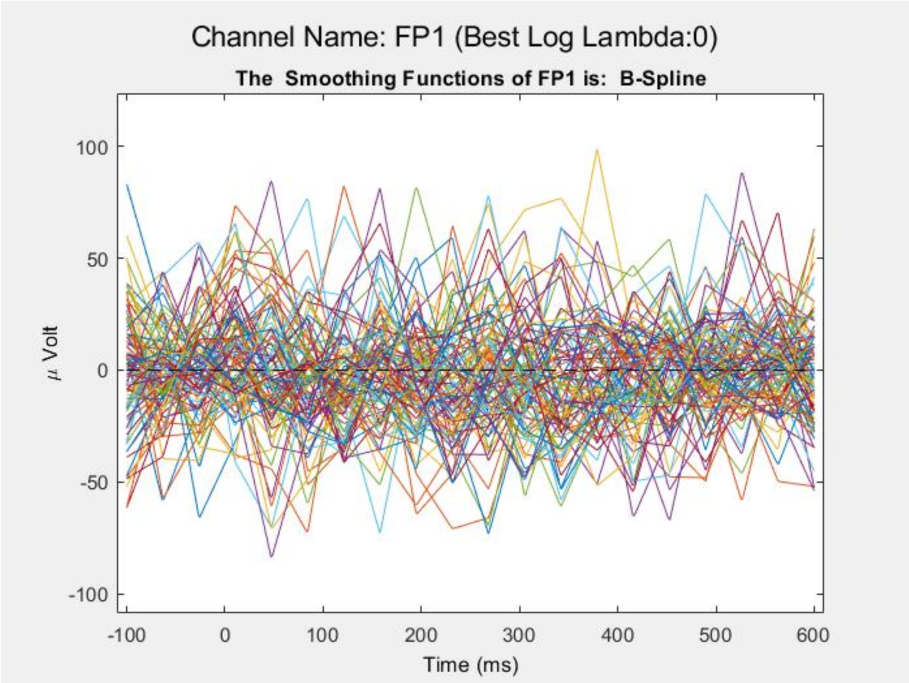


Figure 4 – The smoothed ERP curves with B-Spline with order 2 output of scenario 1.

In figure 4, the x-axis is the time window from -100 to 600 ms and the y-axis is the value of smoothed curves. Each color is related to each epoch. We conclude from this plot that the order of B-Spline is not appropriate, because it cannot smooth the curves very well, therefore we increase the **order of basis function** to **6**. (**Figure 5**)

Figure 5 – Changing the order of B-Spline to 6 (default).

The result is in figure 6. The ERP curves are smoother than in figure 4.

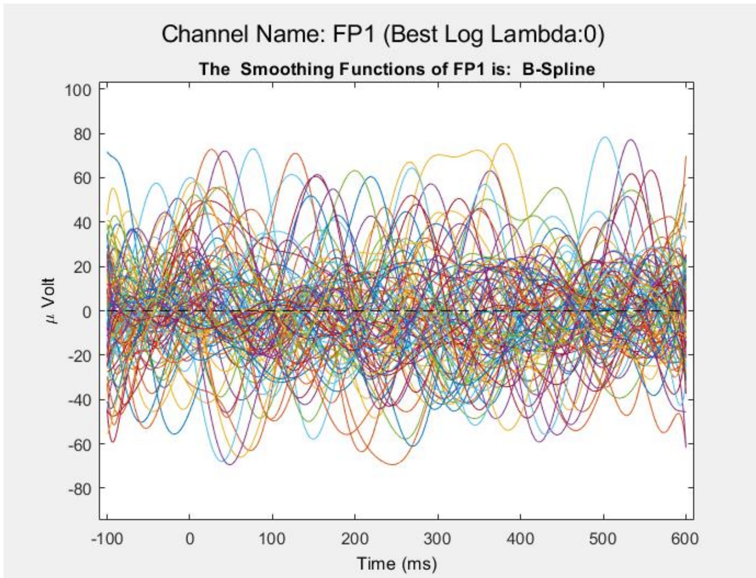


Figure 6 – The smoothed ERP curves with B-Spline with order 6 output of scenario 1.

But there is one question that remains. How good the fit of the curves are? We can tackle the overfitted problem by adding the penalty term and finding its value with generalized cross-validation (GCV). In this regard, we check the **GCV** and **plot GCV**. (figure 7)

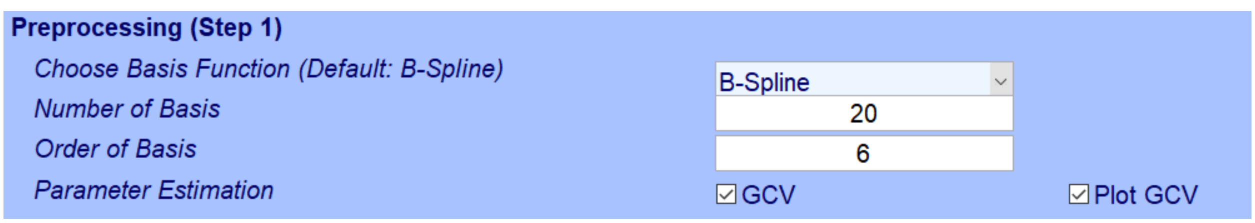


Figure 7 – Check the GCV and plot GCV (default).

It produces two plots:

1- Figure 8. It shows the GCV values for different lambda terms and the minimum GCV is shown with a dotted line.
2- Figure 9. The smoothed curves with the selected lambda.

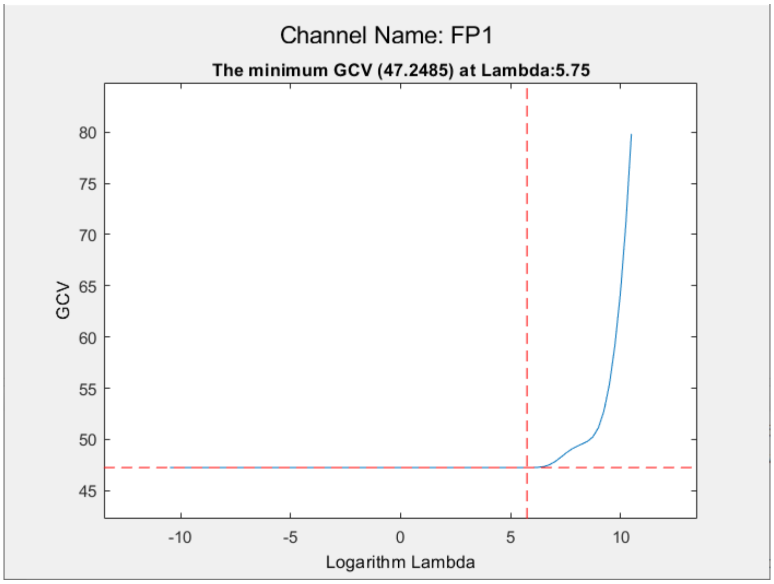


Figure 8 – The GCV plot, the minimum GCV is 47.2485 that reaches in logarithm lambda equals 5.75. (Top of the plot log lambda)

Figure 9 is very smooth that is not very useful for modeling the ERP curves. We can add several basis functions and increase them to the large values for example **120** and do it again with **GCV**. The result is shown in figure 10 (GCV) and 11 (smooth curves).

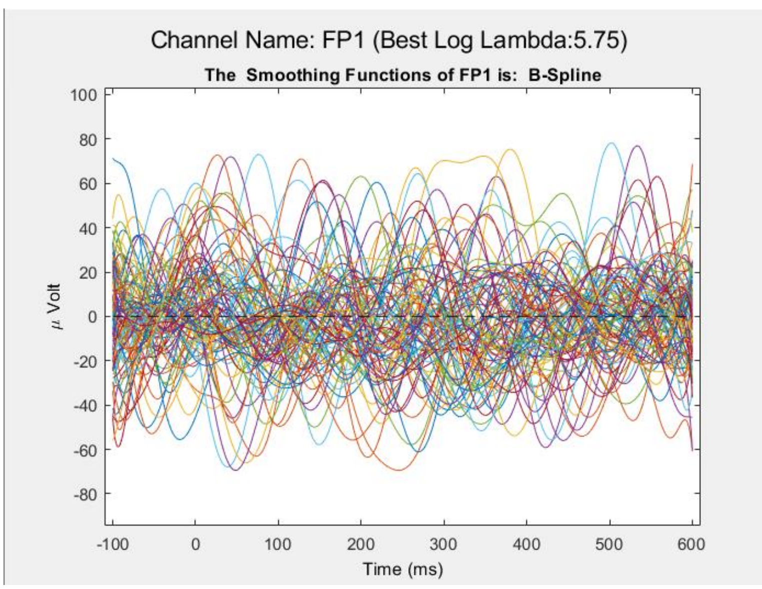


Figure 9 – The smoothed curves with the minimum GCV.

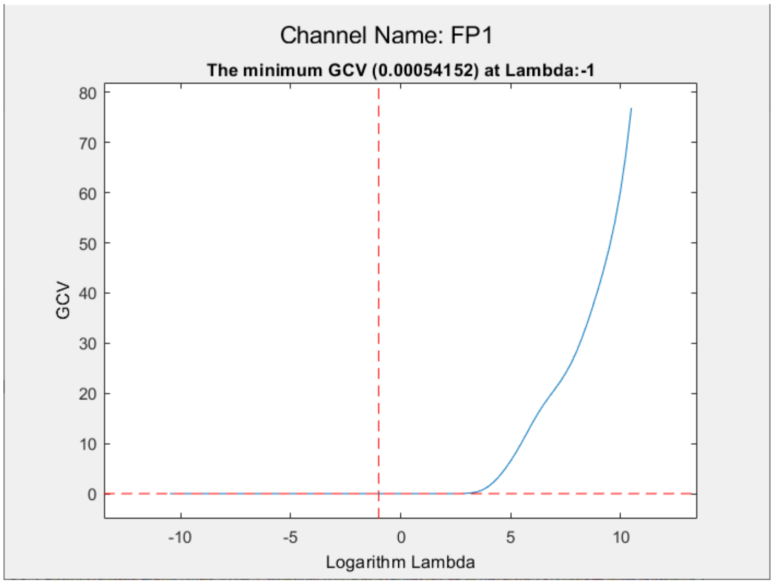


Figure 10 – The GCV curve with 120 B-spline basis function.

The figure 11 shows the smoothed curves that can model the complex behaviors of each ERP.

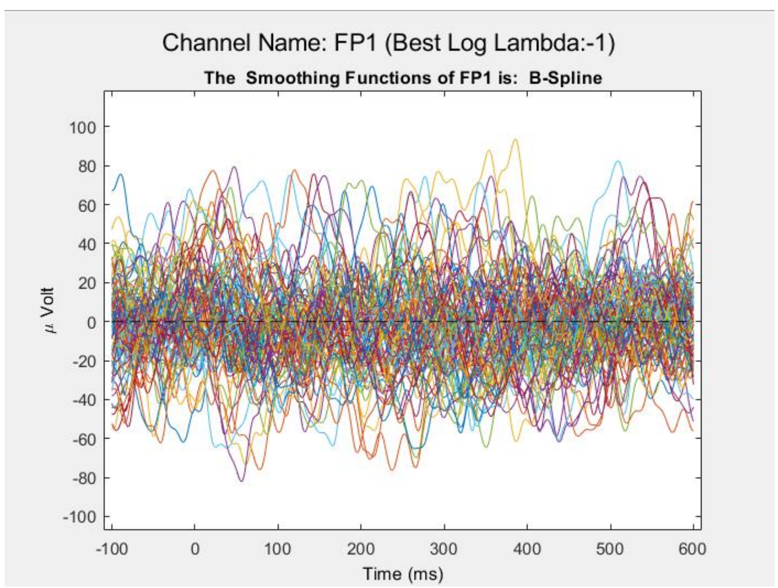


Figure 11 – The smoothed curves with the minimum GCV (number of basis function = 120, order = 6).

Second scenario

In this scenario, we want to smooth all channel ERPs with 120 number of basis functions, and Fourier basis functions, the time window -200 to 800 ms. (figure 12)

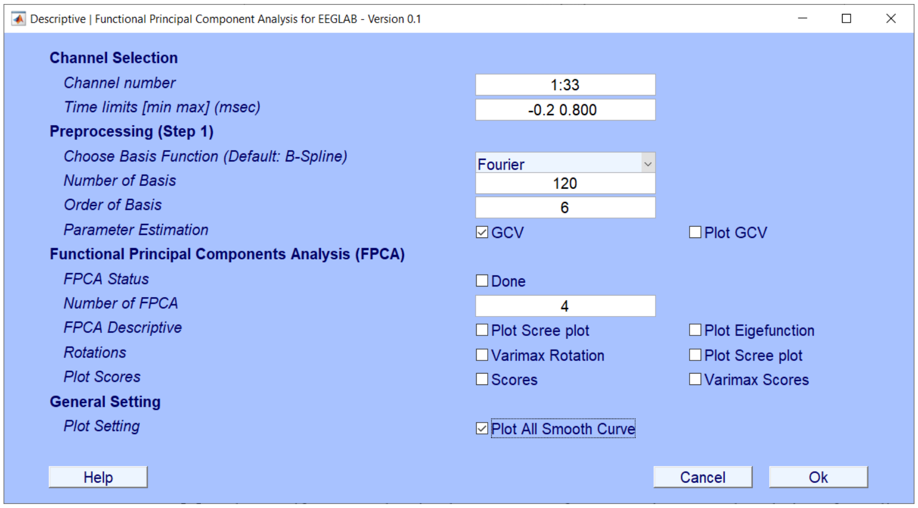


Figure 12 – All channels (1:33), time window (-0.200 800 s), Basis function (Fourier), Parameter Estimation (GCV)

The result produces 33 plots and takes some minutes. The MATLAB command shows the result as in the output windows.:

```
Channel Number: 10, 2-Log Lambda: -10.25, Mean GCV of Epochs: 0.3626, Mean SSE of
Epochs: 281.1217

Channel Number: 10, 3-Log Lambda: -10, Mean GCV of Epochs: 0.3626, Mean SSE of Epochs:
281.1217
```

Recommendations:

1 - B-Spline is used for non-periodic curves but Fourier is recommended for periodic curves.

2 – When working with a large number of channels, uncheck the plot options to avoid producing a lot of plots on the console.

Third scenario

In this scenario, we want to smooth some specific channels number 1, 10, 22, and 31. In this regard, we write it like figure 13.

Figure 13– some channels (1, 10, 22, 31), time window (-0.200 800 s).

Example 2: Epoch Analysis – Doing FPCA on ERP of Epochs

In this analysis, we first smooth all ERP curves of channel 14. And then we do functional principal component analysis (FPCA) on smoothed ERP curves of each epoch. Following the steps as below:

1- Load the 'a1_1_5_125_ICA_Cleaned.set'
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_125_ICA_Cleaned.set .
2- It has 33 channels, 95 Epochs, and a time window from -200 to 1000 ms. (Figure 2) It shows that the channel location is available, ICA weights exist, the reference method is average and the sampling rate is 1000 Hz.
3- Click Tools → FDA → Epoch Analysis → FPCA.
4- In this GUI, the **Channel Selection** part, we want to smooth **channel 14** ERP curves for each epoch. The **time window** is set to **-200 to 800** ms.
5- In **Preprocessing (Step 1)** part, the **basis function type** is **B-Spline**, the **number of basis** is **120**, **order of basis** is **6**, check **GCV** and check **Plot GCV**.
6- In **Functional Principal Component Analysis (FPCA)** part, check **FPCA Status**, **the number of FPCA** is **20**, and check **Plot Scree Plot** and uncheck others.
7- In the **General Setting** part, uncheck the **Plot All Smooth Curve.**

The menu is like Figure 14.

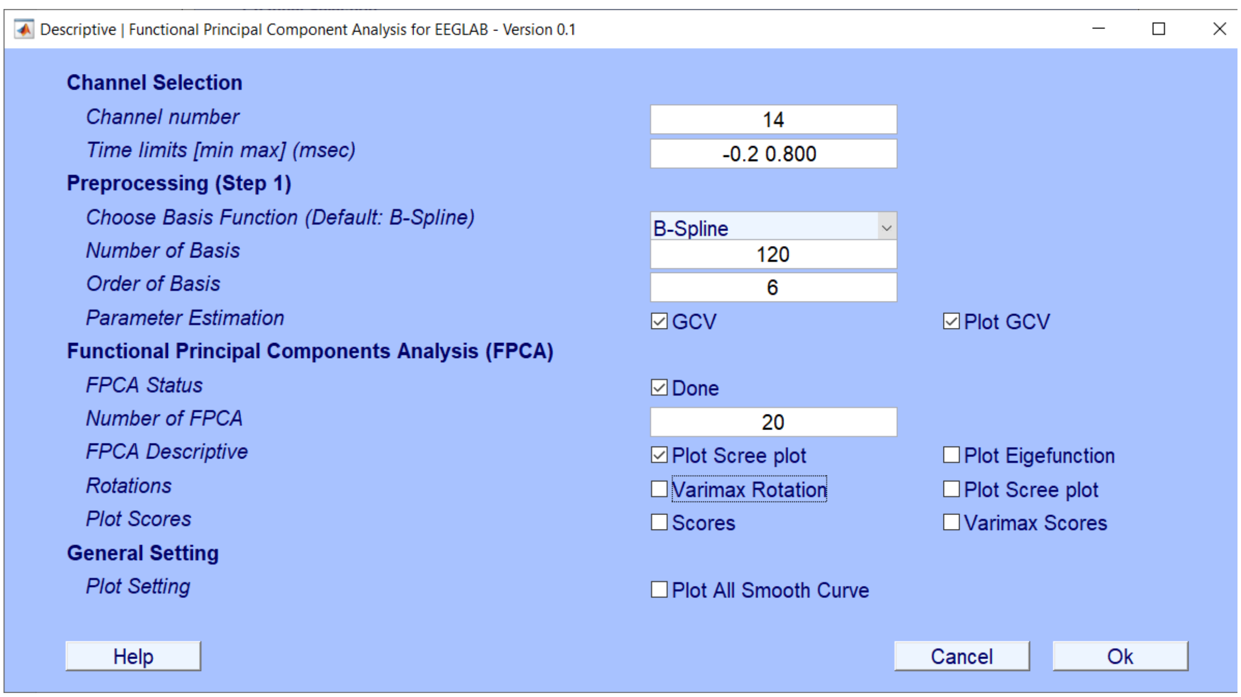


Figure 14– FPCA Setting for channel 14.

EEGLAB
>> FDA

The scree plot (figure 14) and output table in the console are presented:

```
tableReport =
  20×4 table
    Channel     Number_FPCAs      FVE      Cumulative_FVE

    _______     ____________     ______    ______________
    14            1              13.085    13.085
    14            2              7.5363    20.622
    14            3              7.3306    27.952
    14            4              6.0753    34.027
    14            5              5.7506    39.778
    14            6              3.7737    43.552
    14            7              3.5745    47.126
    14            8              3.3622    50.488
    14            9              3.0085    53.497
    14           10              2.7881    56.285
    14           11               2.642    58.927
    14           12              2.5156    61.442
    14           13               2.143    63.585
    14           14              1.9818    65.567
    14           15              1.8446    67.412
    14           16              1.7483     69.16
    14           17               1.634    70.794
    14           18              1.5082    72.302
    14           19              1.4508    73.753
    14           20              1.4073    75.161
```

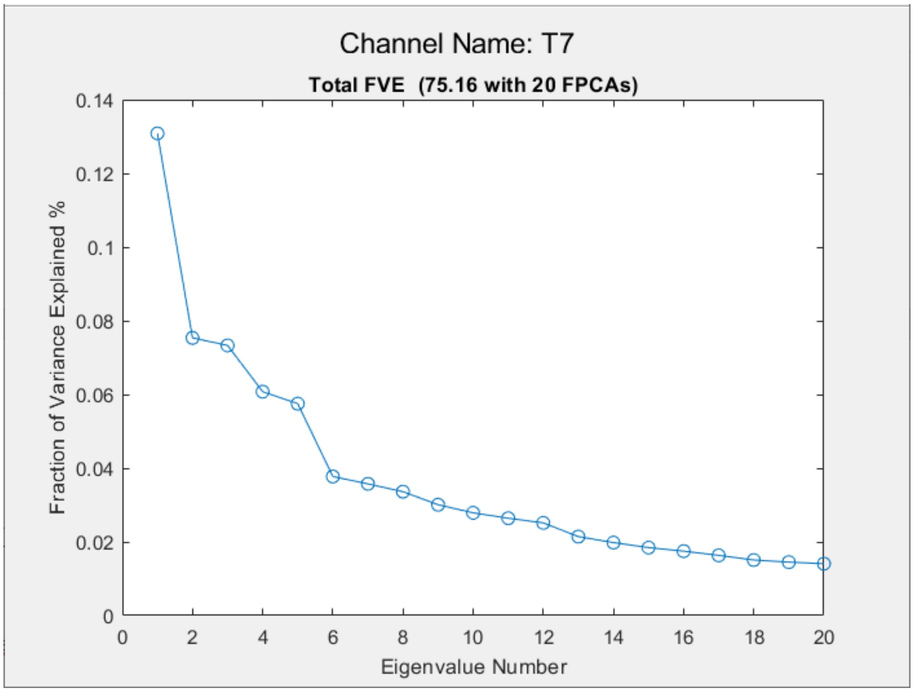


Figure 14– The Scree plot for channel 14 (T7)

They showed that the first FPCA has 13.085 FVE that is the greatest value. With the five first FPCs, about 39.778 variations are explained. We run the menu again and check the eigenfunctions as figure 15:

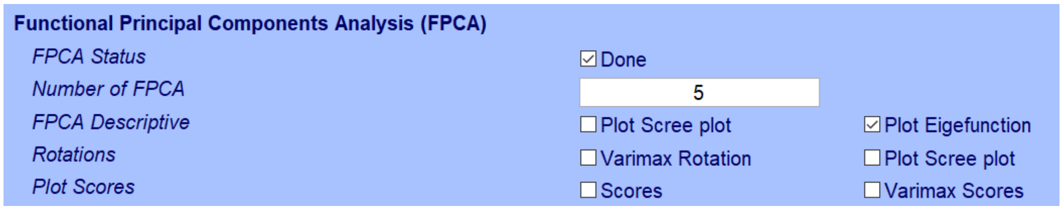


Figure 14– Plot the eigenfunction for the first 5 FPCs.

Five plots of eigenfunctions are plotted like in figure 15. It shows the two peaks between 0 to 100 and 350 to 400 ms.

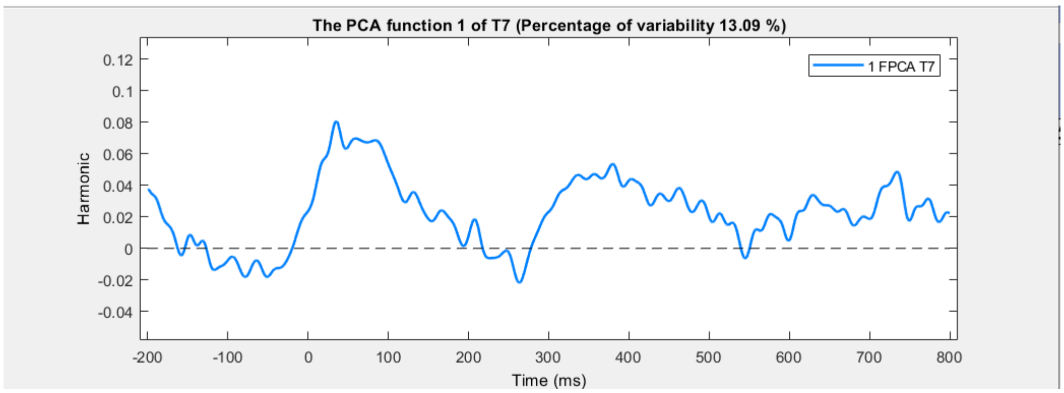


Figure 15– First eigenfunction that captures 13.09% of FVE

We can also rotate this eigenfunction with VARIMAX and plot them again(figure 16):

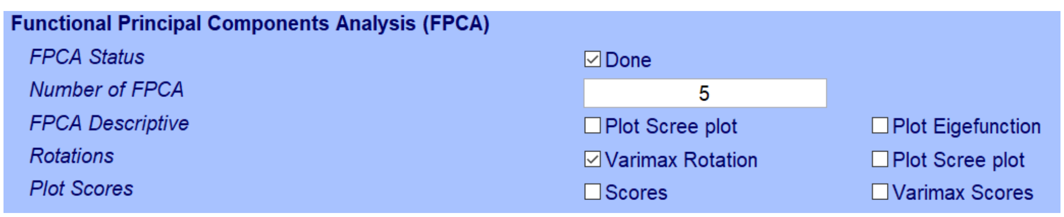


Figure 16– The VARIMAX rotation

It produces five eigenfunctions, the first one is in plot 17. Its interpretation is easy and it showed two big peaks 0 to 100 ms and 300 to 400 ms. But one point is that the FVE is not the same as before. In this eigenfunction, FVE is reduced from 13.09% to 11.77%. but total FVE is the same in all 5 eigenfunctions.

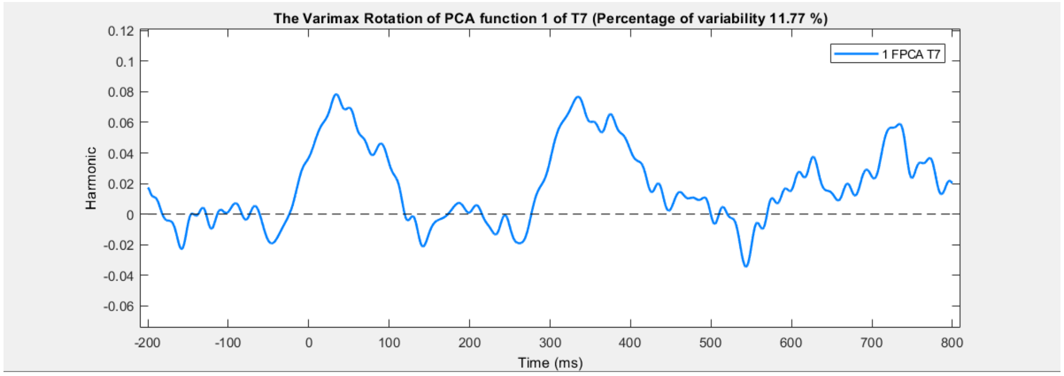


Figure 17– The VARIMAX rotation of eigenfunction 1

We can also plot the FPCs score against each other. In this case, we want to plot the first two FPC scores and their VARIMAX rotations. We change the menu like in figure 18.

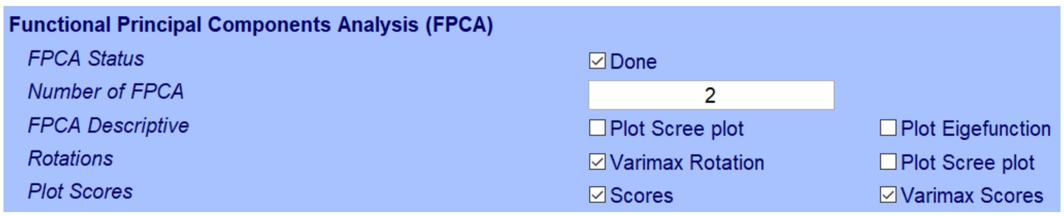


Figure 17– Plot scores of FPCAs of the first two FPCAs.

The FPCs scores of the first two eigenfunctions (figure 17) and the first two VARIMAX rotated eigenfunctions (figure 18):

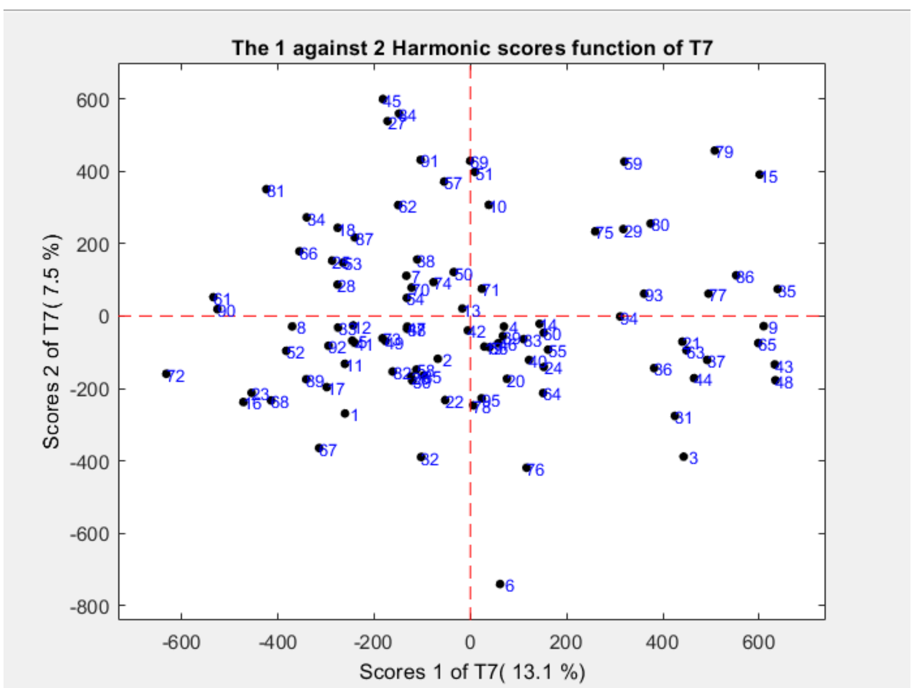


Figure 17– Plot scores of FPCAs of the first two FPCAs.

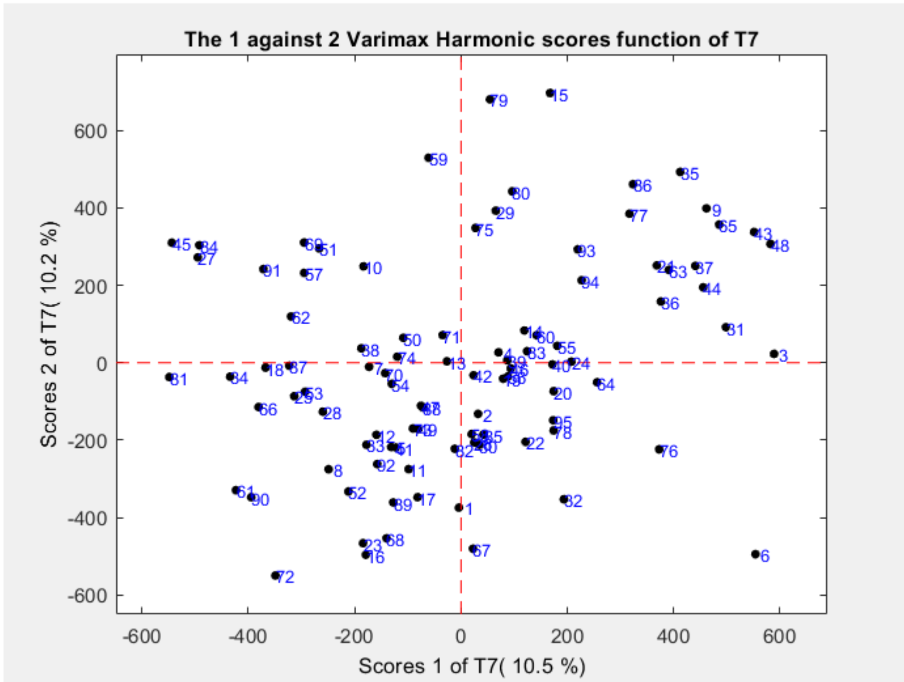


Figure 18– Plot scores of VARIMAX Rotated FPCAs of the first two FPCAs.

The blue numbers on plots are the epochs number. According to these plots, there exist two or three clusters between epochs. They also showed that the number 6 is far from others. But these two eigenfunctions are not cover a large percentage of variations.

ERP Analysis Examples – Descriptive

Example 1: ERP Analysis – Descriptive – Mean and Standard Deviations.

In this analysis, we want to compare the smooth mean and standard deviation of two stimuli (125 standard and 150 targets). In this regard, we do the following steps:

1- Load the ‘a1_1_5_125_ICA_Cleaned.set’
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_125_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → Descriptive.
3- In **Preprocessing (Step 1)** part, we want to smooth **channel 25** ERP curves for each epoch. The **time windows** are set to **-200 to 800** ms, the **basis function type** is **B-Spline**, a **number of basis** is **120**, **order of basis** is **6**, uncheck all other options except **Plot All Smooth Curve**.
4- In the **Descriptive Analysis** part, check Mean and Standard Deviation and uncheck all other options. (Figure 19)
5- Click **Ok**.

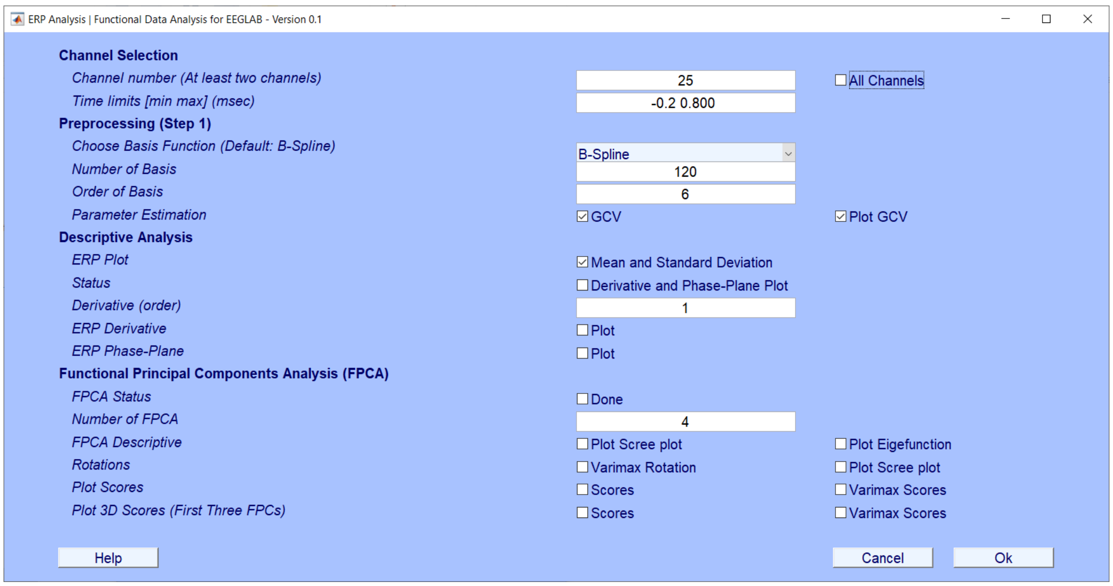


Figure 19– Plot scores of VARIMAX Rotated FPCAs of the first two FPCAs.

The output is in figure 20. (You need to maximize the window to see the same plot.) It has four windows. The left panel is for smoothed ERP and the right panel is for unsmoothed ERP. The top row is functional and point-wise mean, respectively. It seems these two means are similar to each other. The bottom row is a functional and point-wise standard deviation, respectively. They are very similar to each other. The blue line is for channel 14, T7 near the left ear and the red line is for channel 18, T8 near the right ear. We keep this plot open on the page and we repeat the above steps. But in step one, we change the dataset as follows:

1- Load the ‘a1_2_5_150_ICA_Cleaned.set’
   a. in EEGLAB → File → Load Existing Dataset → a1_2_5_150_ICA_Cleaned.set .

This dataset is for target stimuli and has 23 epochs and 33 channels. The same analysis produces the figure 21 plots. These two channels in this trial have different epochs mean between standard and target stimuli, and their two peaks between 100 and 200 ms are higher for the target than standard stimuli. We can see these patterns by comparing Figures 20 and 21. (The means are reverse but their standard deviation is near each other.)

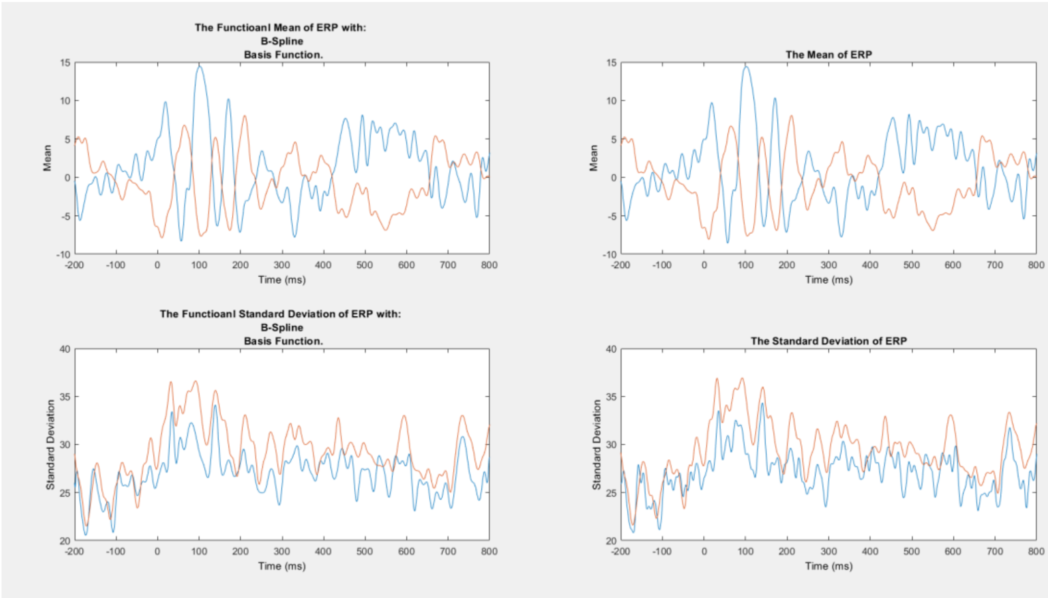


Figure 20– (Left Panel: smoothed data) The functional mean (top) and functional standard deviation (bottom), (Right Panel: unsmoothed data) point-wise mean (top) and point-wise standard deviation (bottom) of channel 14 (Blue) and 18 (Red) of Standard Stimuli (CODE:125).

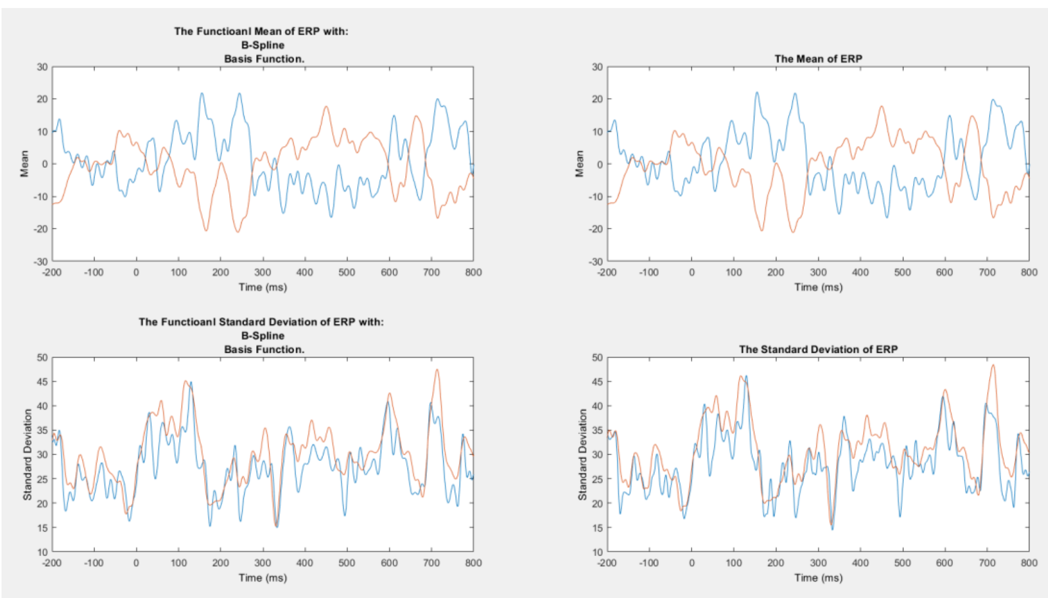


Figure 21– (Left Panel: smoothed data) The functional mean (top) and functional standard deviation (bottom), (Right Panel: unsmoothed data) point-wise mean (top) and point-wise standard deviation (bottom) of channel 14 (Blue) and 18 (Red) of Target Stimuli (CODE-150).

Example 2: ERP Analysis – Descriptive – Derivative and Phase-Plane Plot.

In this analysis, we want to calculate and plot the derivative and phase-plane plots. We calculate the first, second, and third derivatives and we also see from figure 21 that there exist two peaks between 100ms to 300 ms. We choose these time windows, and calculate the derivatives and plot the phase-plane plot:

Steps are:

1- Load the ‘a1_1_5_150_ICA_Cleaned.set’
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → Descriptive.
3- In **Preprocessing (Step 1)** part, we want to smooth **channel 14** and **18** ERP curves for each epoch. The **time windows** are set to **100 to 300** ms, the **basis function type** is **B-Spline**, the **number of basis** is **120**, **order of basis** is **6**, uncheck all other options.
4- In the **Descriptive Analysis** part, check the **Derivative and Phase-Plane plot**, the **derivative number is 3** that means the first, second, and third derivatives of ERP will calculate. Check **plot** in the front of **ERP Derivative**. Check **plot** in the front of **Phase-Plane Plot**.
5- Uncheck all other options.
6- Click **Ok**.

To get the results, we repeat the above steps twice, one time for channel 14 and another time for channel 18. (figure 22)

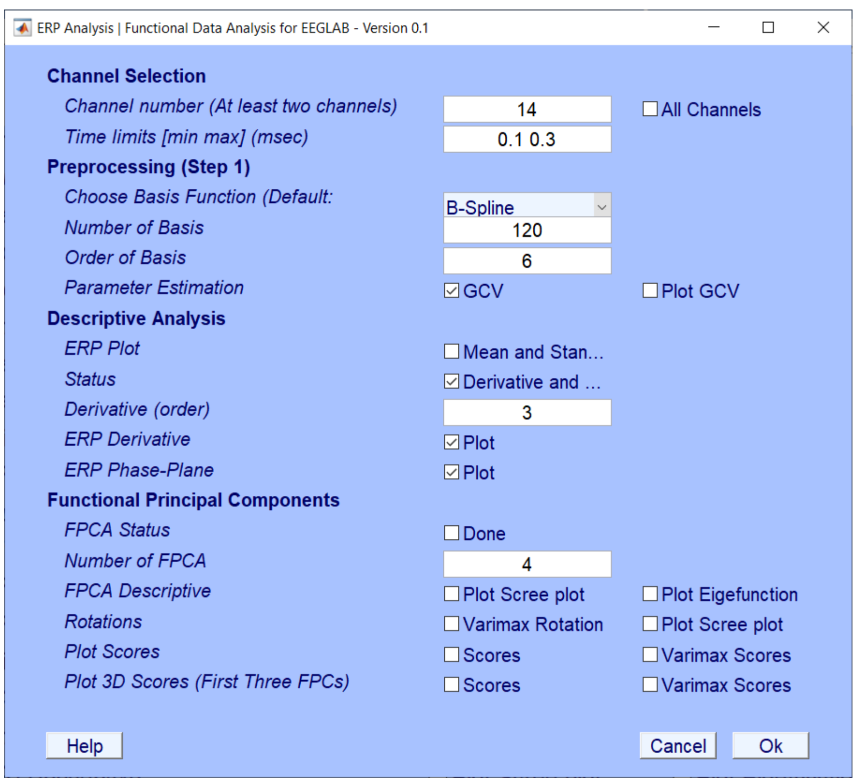


Figure 22- The derivative and phase-plane plot of two channels 14 and 18.

The output shows the following items:

Derivatives **Channel 14** **Channel 18**

First

Second

Third

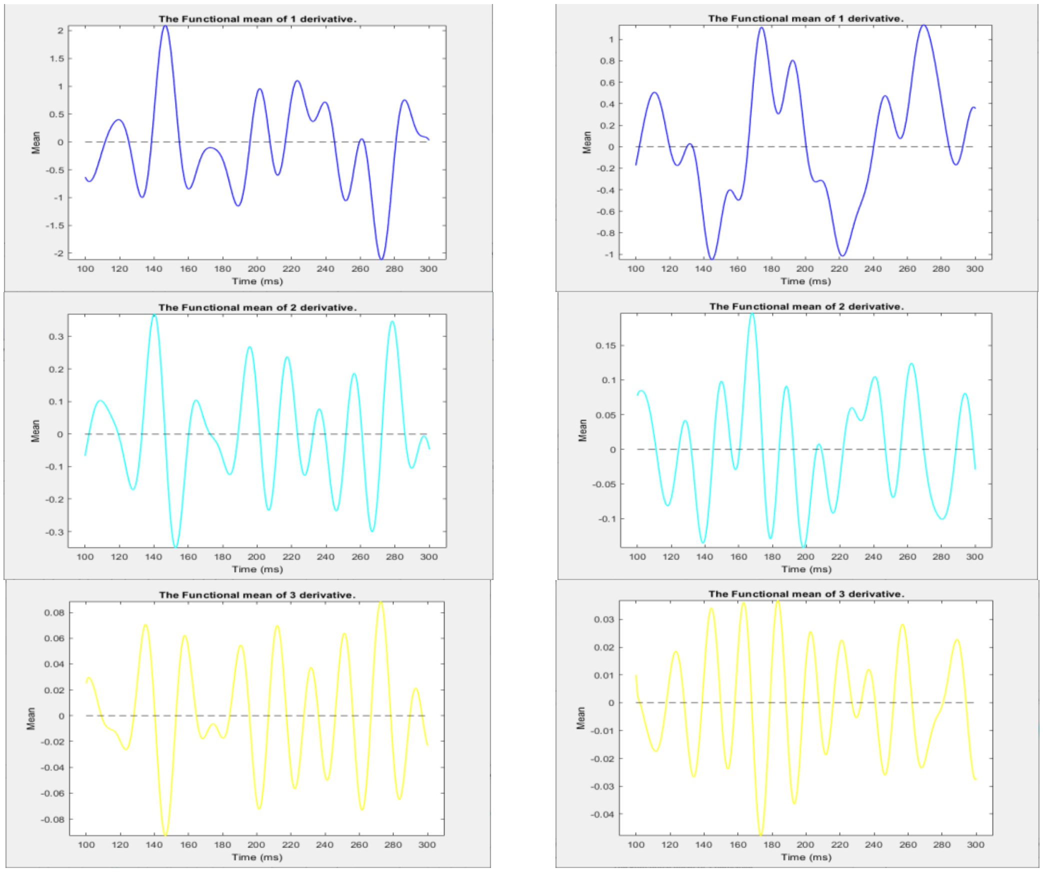


Figure 23 – The first, second, and third derivatives of ERP plots for channels 14 and 18.

The first row is the first derivative of ERP for channels 14 and 18. The x-axis is 100 to 300 ms. It is a velocity plot. The second row is the second derivative of these two channels and they are acceleration plots. We plot the velocity against acceleration and it is the phase-plane plot. They are shown in figure 24. The start and the end of time are characterized by big stars and the number near the lines are the time points. It starts from 100 to 300 ms. The start and the end of the peaks are detectable and it helps to understand the process of the peaks with velocity and acceleration.

Channel | Phase-Plane Plot

14

18

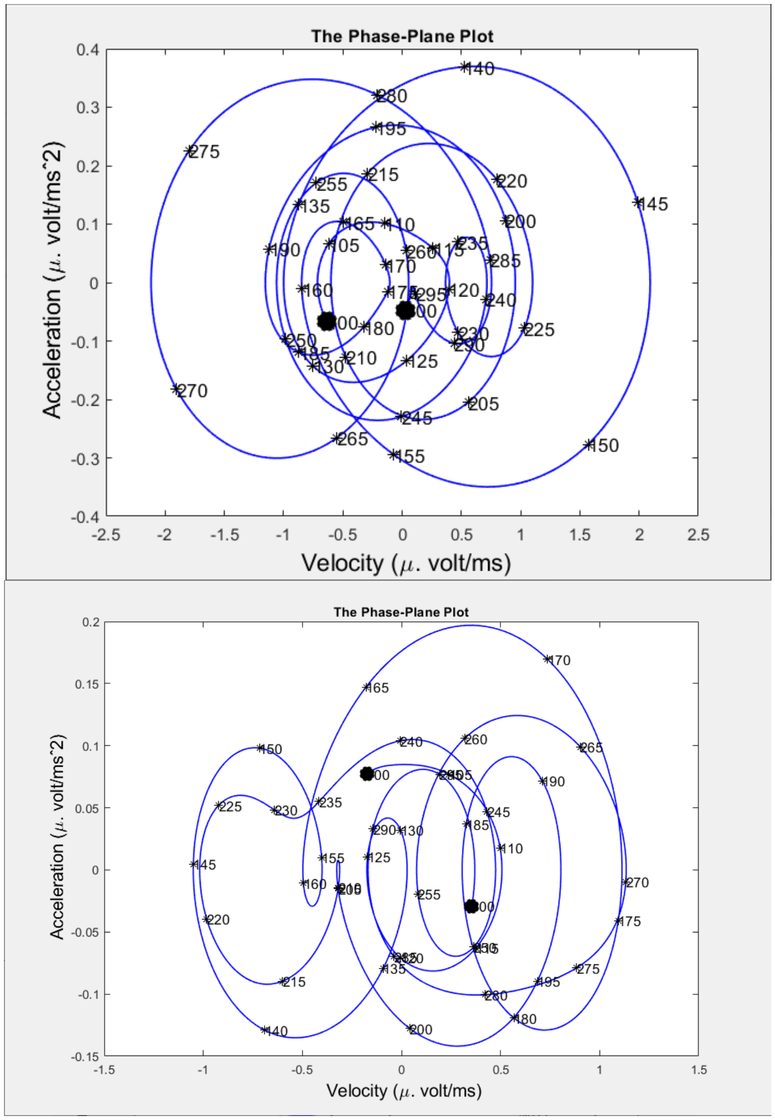


Figure 24 – The Phase-Plane Plot of ERP for channels 14 and 18.

We can also see the behavior of the two channels together. In this regard, we add two channels 14 and 18 in the GUI (figure 25)

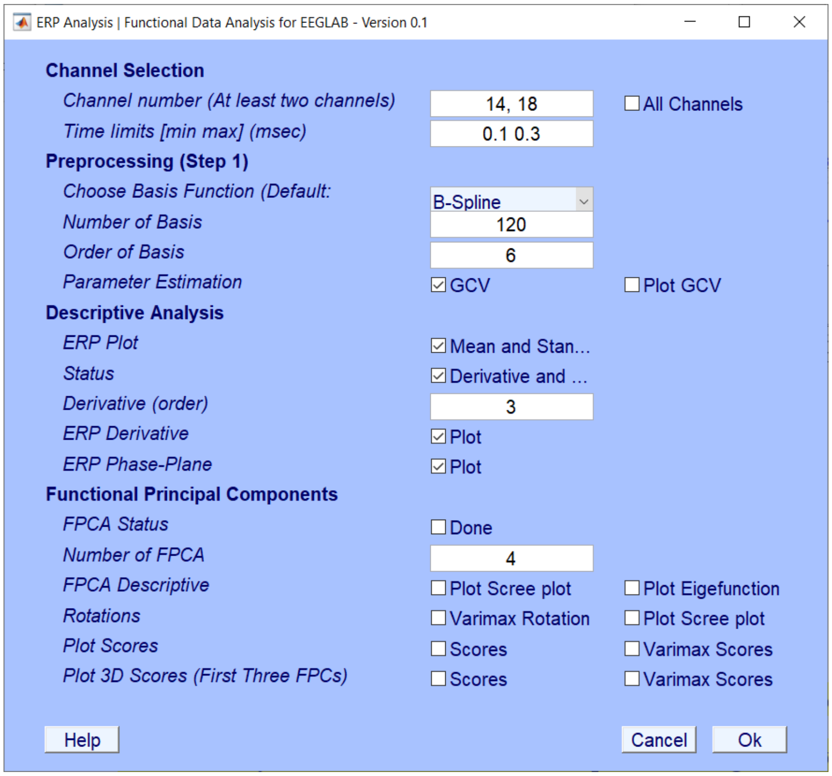


Figure 25 – The Phase-Plane Plot of ERP for both channels 14 and 18.

In the mean plot, two curves have reverse behavior. (Figure 26)

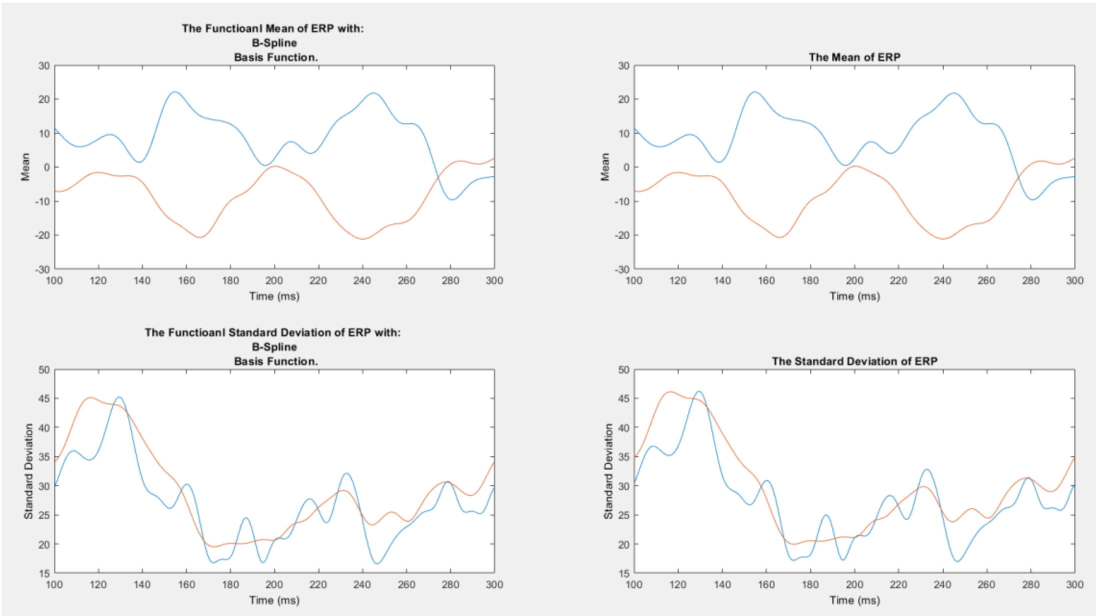


Figure 26 – The functional mean and standard deviation of channel 14 (Blue) and channel 18 (Red).

The derivative functions of smoothed two channels curves together are shown in figure 27.

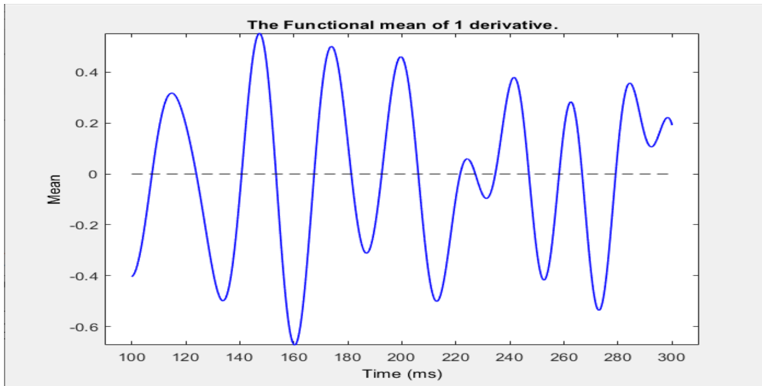
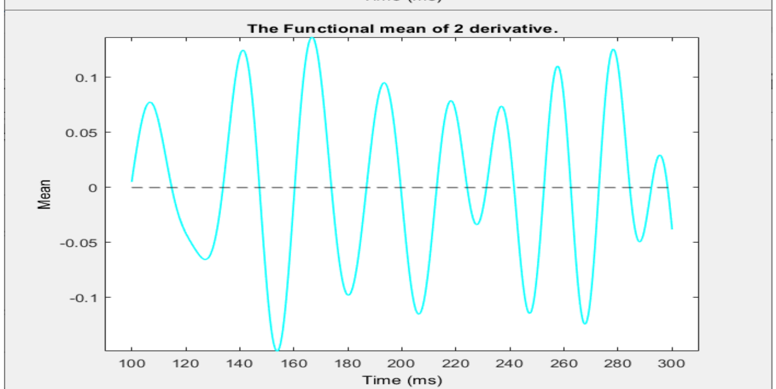
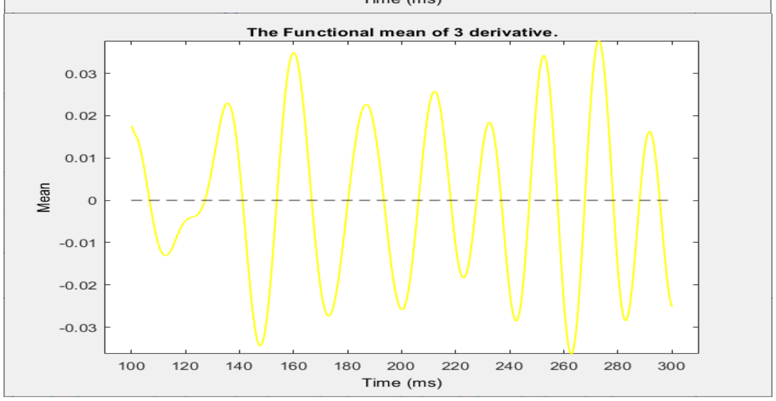
Derivative	Derivative Curve
First	
Second	
Third	

Figure 27 – The first, second and third functional derivative of channel 14 and 18 together.

The phase-plane plot is shown in figure 28:

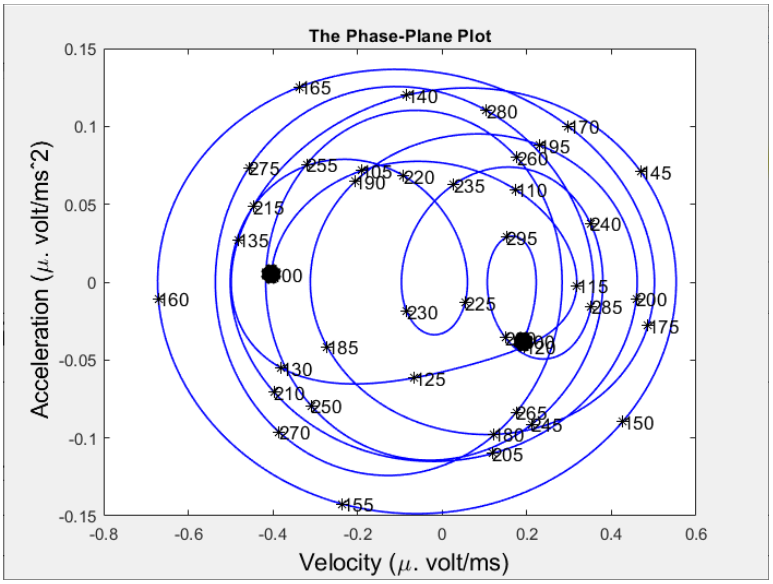


Figure 28 – The phase-plane plot of chapters 14 and 18 together.

Example 3: ERP Analysis – Descriptive – Functional Principal Component Analysis (FPCA).

In this analysis, we continue the example 2 settings and we want to extract the functional principal component analysis of 100 to 300 ms. It shows the most important components of these two peaks. In this regard, we do the following steps:

1- Load the 'a1_1_5_150_ICA_Cleaned.set'
    a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → Descriptive.
3- In **Preprocessing (Step 1)** part, we want to smooth **channel 14** and **18** ERP curves for each epoch. The **time windows** are set to **100 to 300** ms, the **basis function type** is **B-Spline**, a **number of basis** is **120**, **order of basis** is **6**, uncheck all other options.
4- In the **Descriptive Analysis** part, check the **Derivative and Phase-Plane plot**, the **derivative number is 3** that means the first, second, and third derivatives of ERP will calculate. Check **plot** in the front of **ERP Derivative**. Check **plot** in the front of **Phase-Plane Plot**.
5- Uncheck all other options.
6- Click **Ok**. (figure 22)

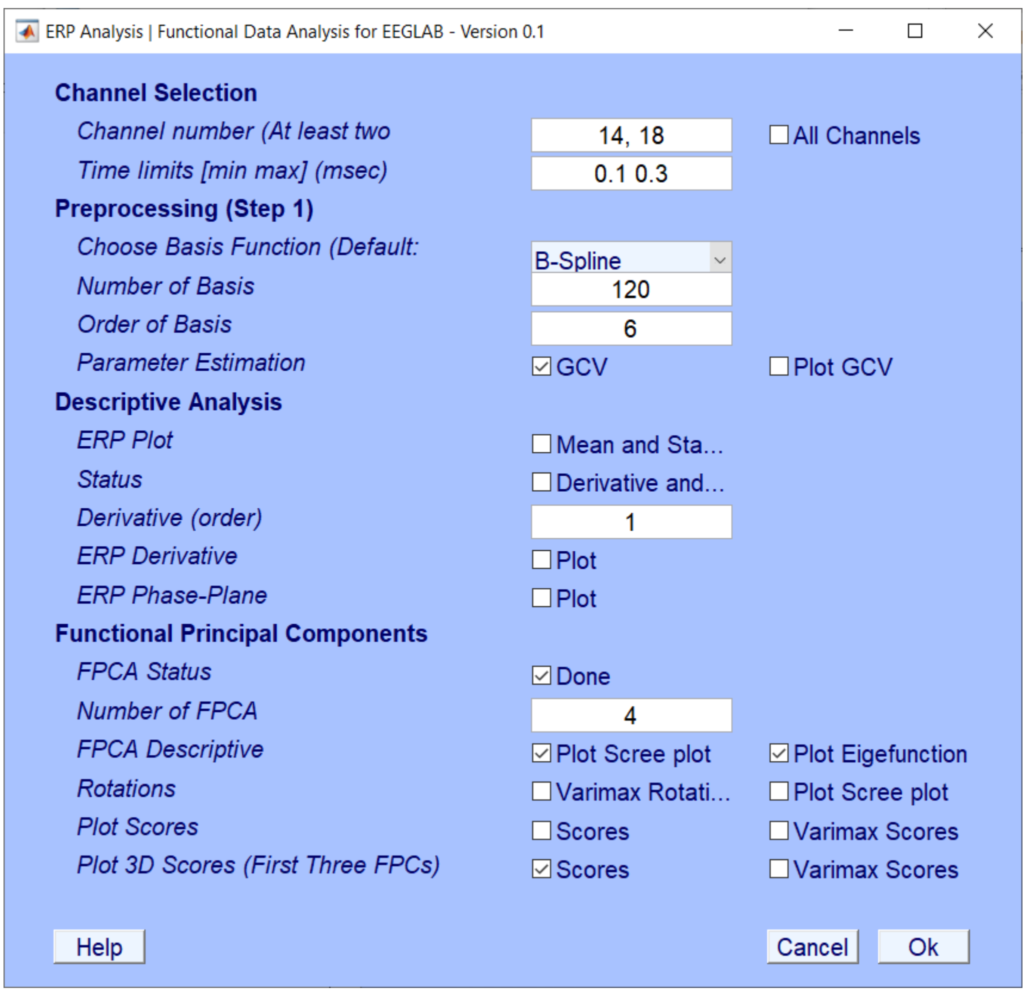


Figure 29 – The functional principal components analysis (FPCA) of Channels 14 and 18.

The scree plot (figure 30) shows that the first FPCA captures almost 100% of variations. The first eigenfunction shows two peaks (figure 31), the 3D plot of functional scores shows the relations between T7 and T8 that are far from each other (figure 32):

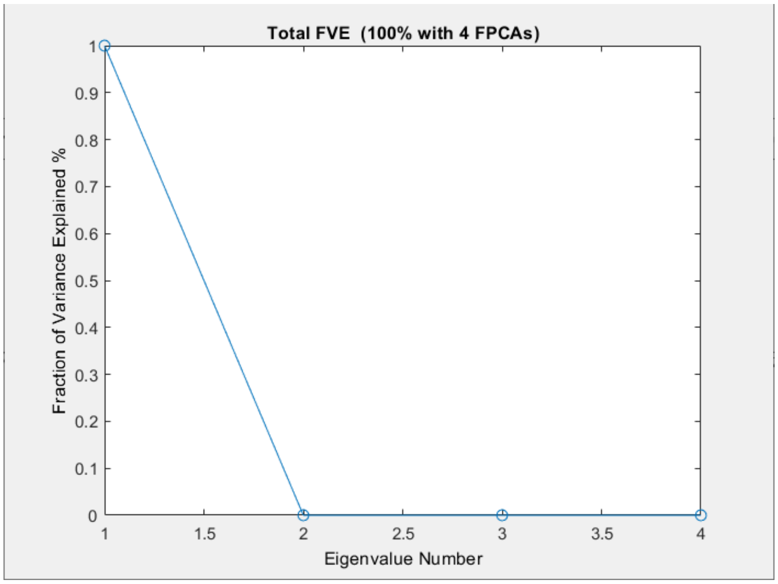


Figure 30 –The Scree plot

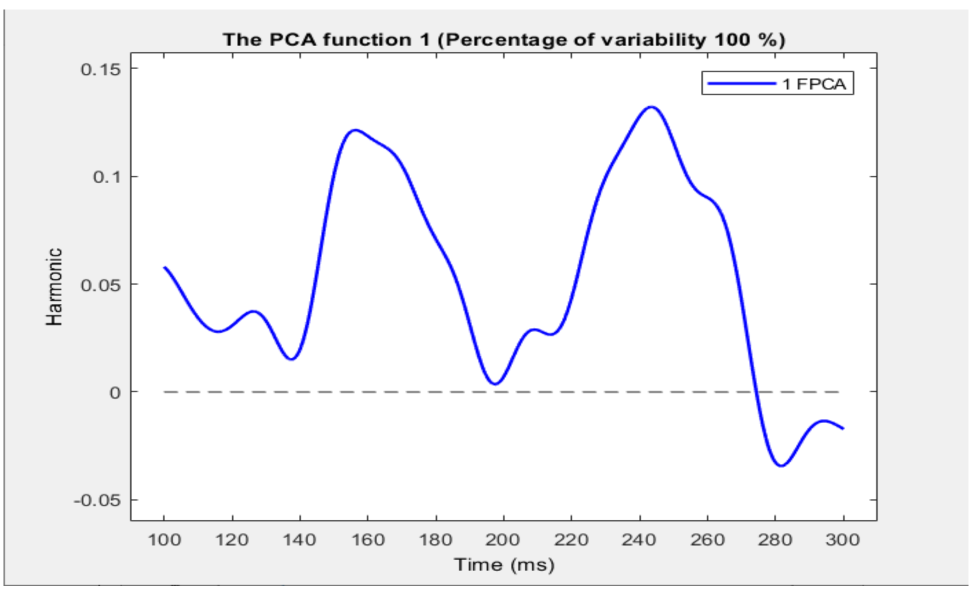


Figure 31 –First functional eigenfunction

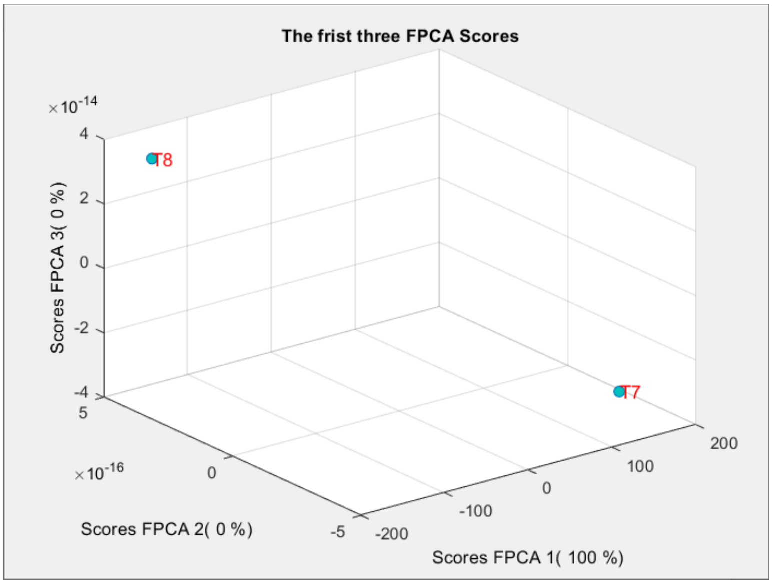


Figure 32 –The 3D plot of the first three scores of FPCA of T7 and T8.

Example 4: ERP Analysis – Descriptive – Comparing FPCA scores of all Channels for two Stimuli.

In this analysis, we use all channels options and do FPCA on them for both standard and target stimuli. In this regard, we do the following steps twice:

1- Load the ‘a1_1_5_150_ICA_Cleaned.set’
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → Descriptive.
3- In **Preprocessing (Step 1)** part, we want to smooth **all channels** ERP curves for each epoch. The **time windows** are set to **100 to 300** ms, the **basis function type** is **B-Spline**, a **number of basis** is **120**, **order of basis** is **6**, uncheck all other options.
4- In the **Descriptive Analysis** part, check **ERP Plot, Status, and ERP Phase-Plane plot**, the **derivative number is 1** that means the first derivatives of ERP will calculate. Check **plot** in the front of **Phase-Plane Plot**.
5- In **Functional Principal Component Analysis (FPCA)**, check **FPCA Status**, type **number of FPCA** to **10**, check **FPCA Descriptive** (**Plot Scree plot** and **Plot Eigenfunction**), in **Plot 3D Scores (First Three FPCs)**, check **Scores**.
6- Uncheck all other options.
7- Click **Ok**. (figure 33)

Repeat the above steps with standard dataset:

1- Load the ‘a1_1_5_125_ICA_Cleaned.set’
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_125_ICA_Cleaned.set .

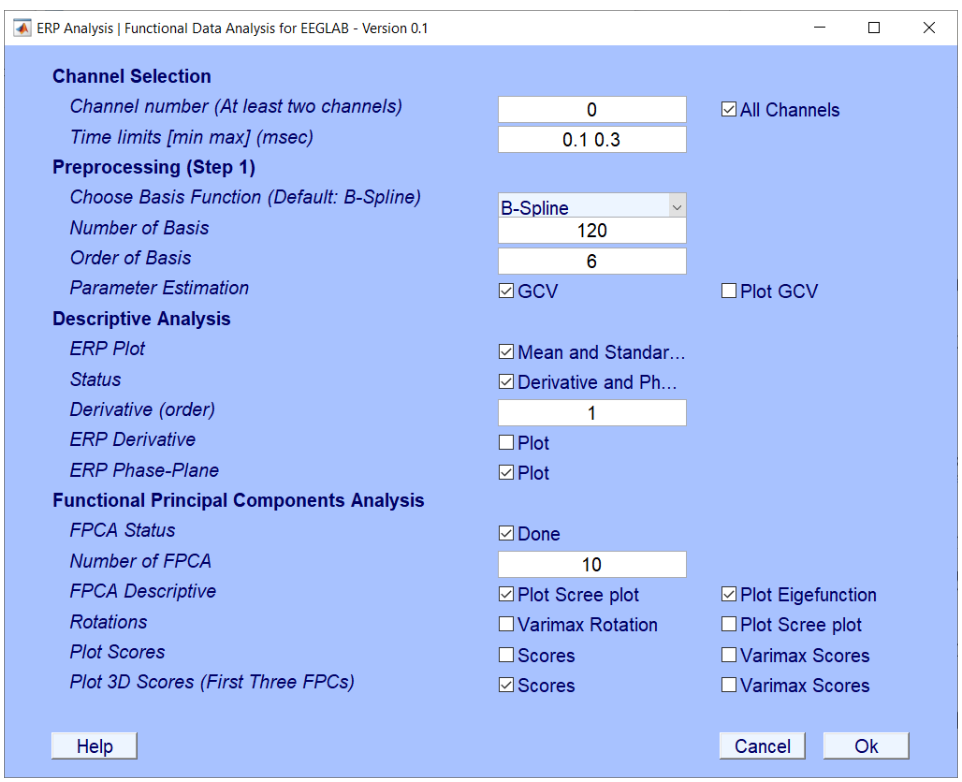


Figure 33 –The GUI for FPCA on all Channels for Target Stimuli (similar to Standard Stimuli).

Figures 34(A) and 34(B) are functional mean and standard deviation for standard and target stimuli, simultaneously. Their behaviors are different and in the target stimuli, they are two peaks at 160 and 240 ms.

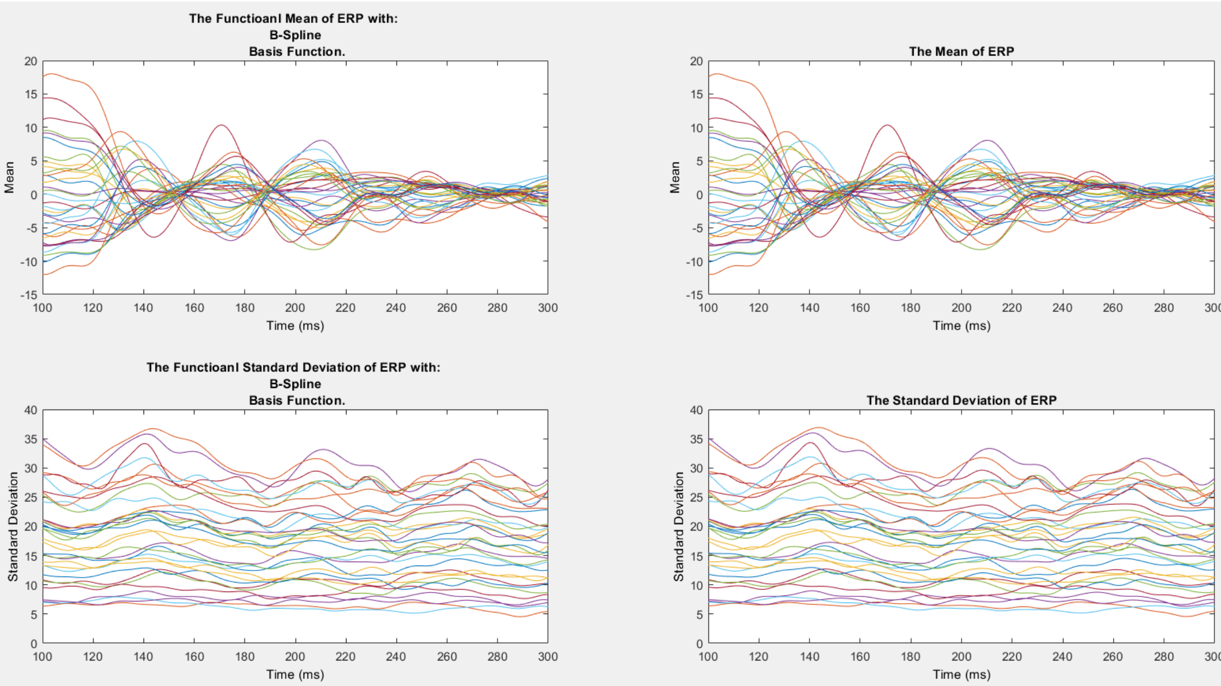


Figure 34 (A) –The functional mean and standard deviation of all channels (each color) of standard Stimuli

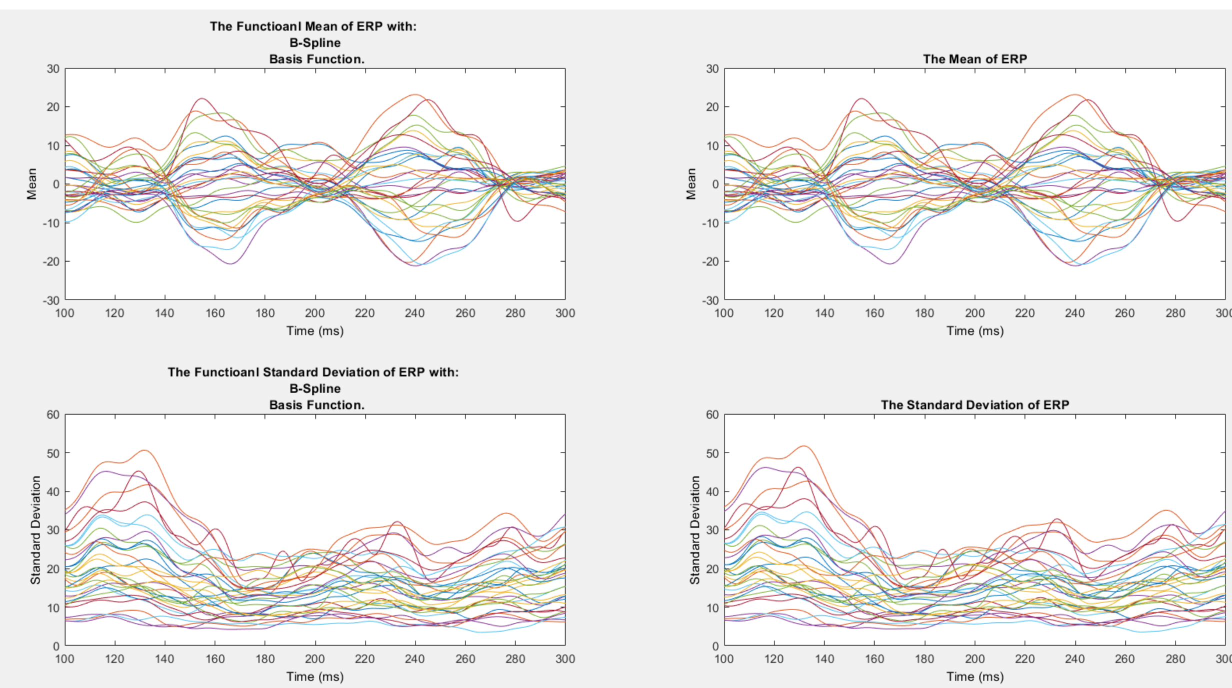


Figure 34 (B) –The functional mean and standard deviation of all channels (each color) of Target Stimuli

Figures 35 are phase-plane plots for two stimuli.

Standard Stimuli **Target Stimuli**

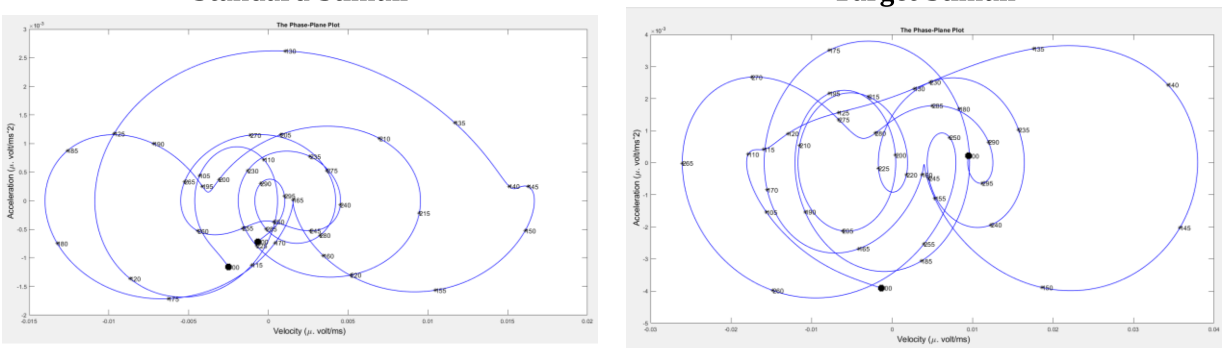


Figure 35 –the Phase-Plane plot for standard and target stimuli of all channels.

Figures 36 are scree plots with 10 first FPCAs for two stimuli.

Standard Stimuli Target Stimuli

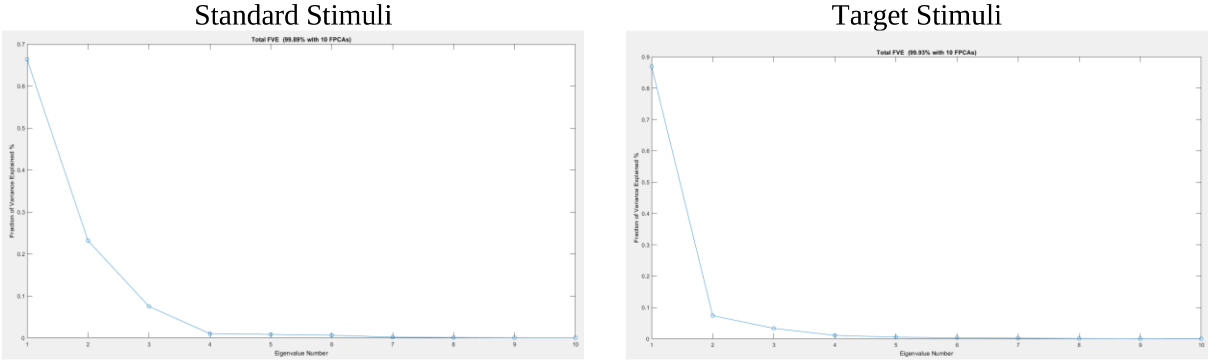


Figure 36 –The scree plot of standard and target stimuli of all channels.

Figures 37 are the first three functional eigenfunctions group by stimuli.

Figure 37 –The first three eigenfunction of standard and target stimuli of all channels.

Figures 38 (A) and 38 (B) are 3D plots of the first three functional eigenfunctions group by stimuli. The location of channels is labeled on the plot.

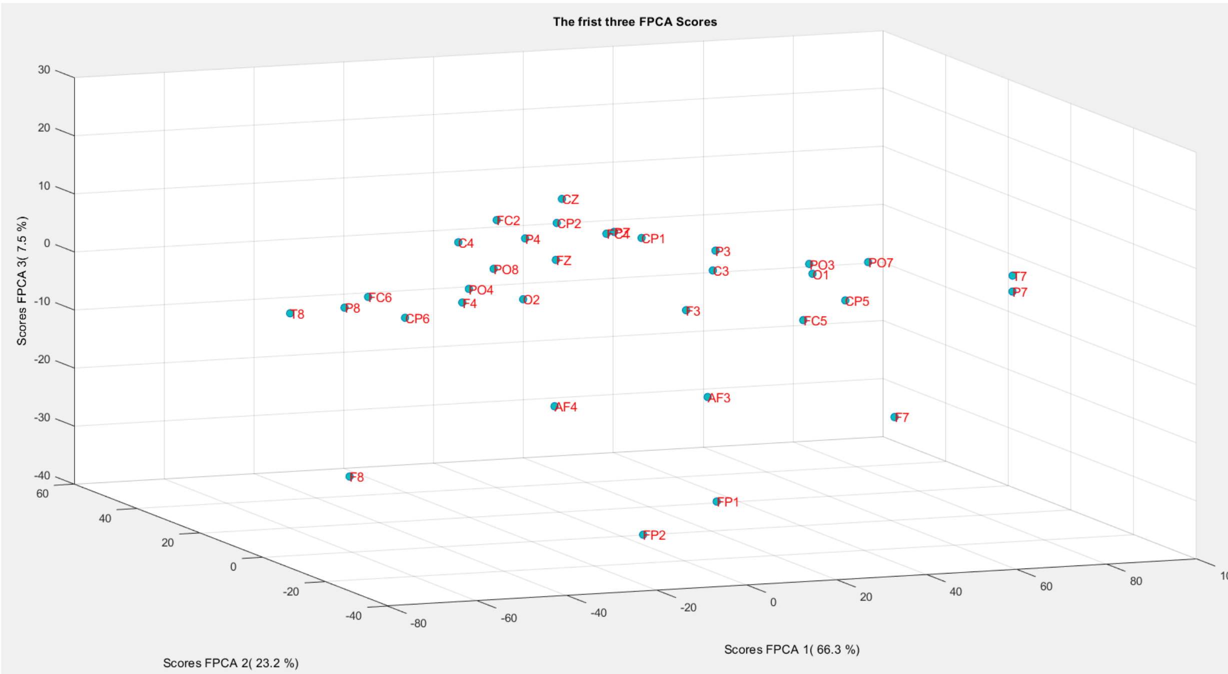


Figure 38 (A) –The 3D of first three fpca scores of all channels for Standard Stimuli.

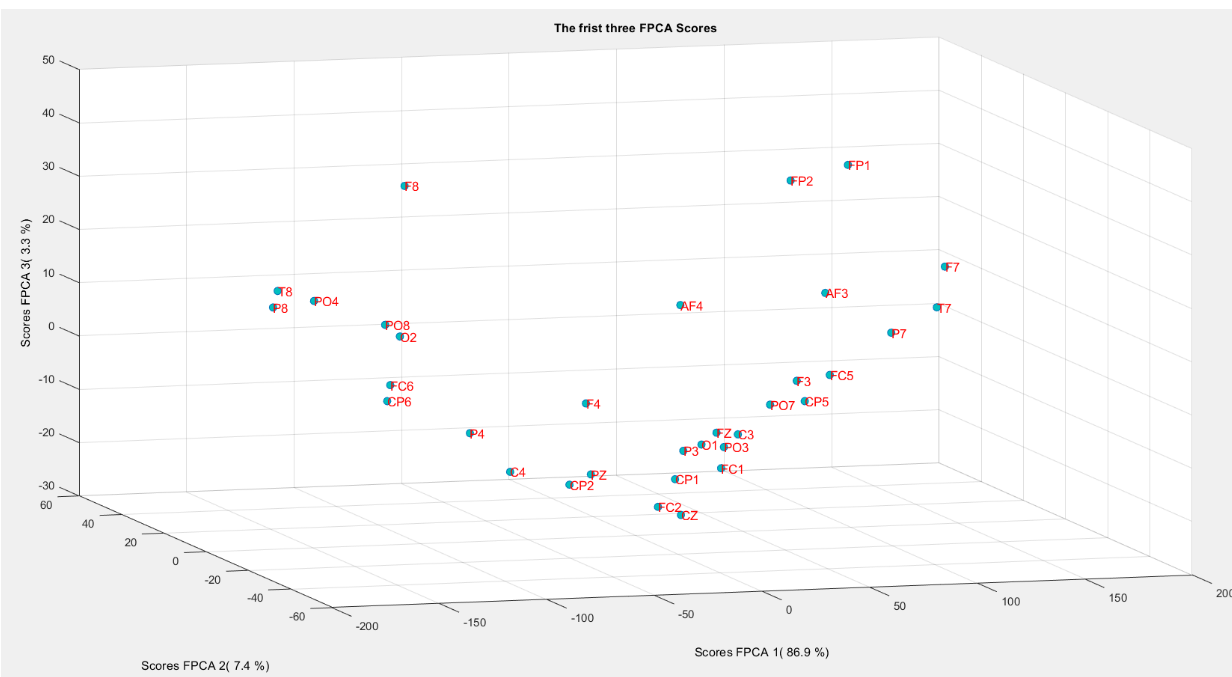


Figure 38 (B) –The 3D of first three fpca scores of all channels for Target Stimuli.

ERP Analysis Examples –Functional Canonical Correlation Analysis Examples (FCCA)

Example 1: ERP Analysis – FCCA – Cross Covariance and Cross-Correlation.

In this analysis, we want to compare the smooth mean and standard deviation of two stimuli (125 standard and 150 targets). In this regard, we do the following steps:

1- Load the 'a1_1_5_150_ICA_Cleaned.set'
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → FCCA.
3- **Chanel numbers** 1 and 2 are **14 and 18** with time limits between **100 and 300 ms**. Both of them are smoothed with a **B-spline**, **120 basis functions,** and an **order of 6**. The parameter estimations are with **GCV**.
4- Check the **Cross Covariance and Cross-Correlation.**
5- Click OK (figure 39)

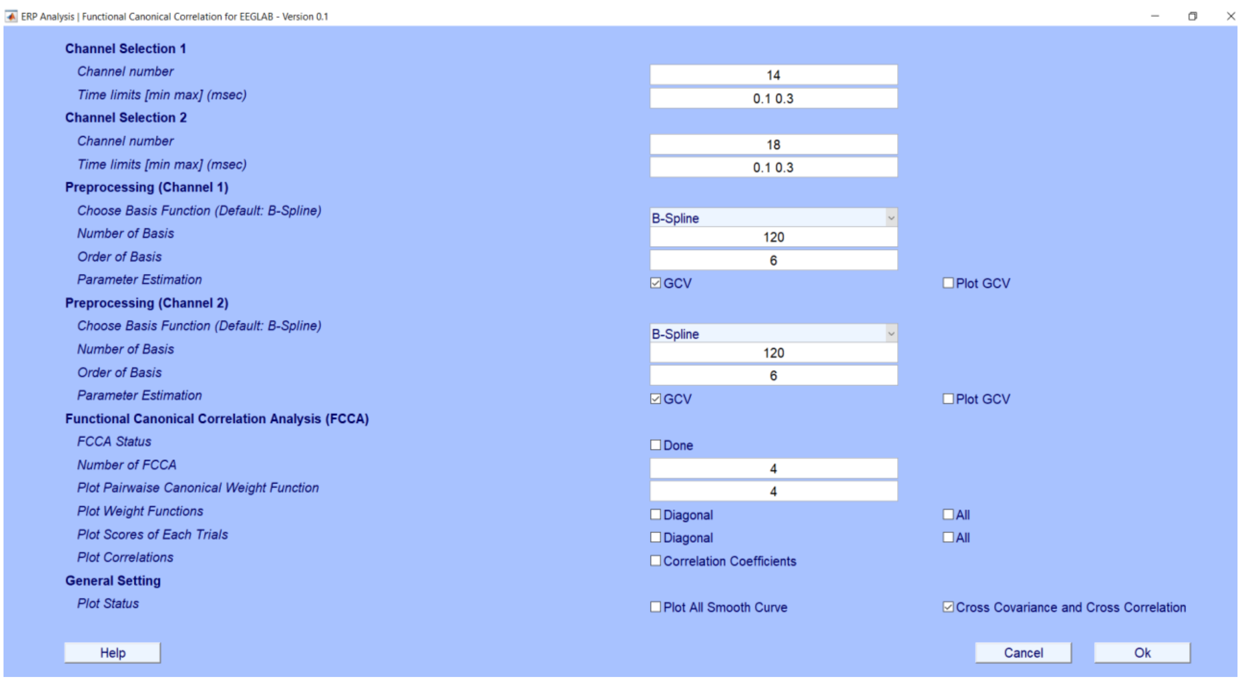


Figure 39 – The GUI for cross-covariance and correlation between channels 14 and 18.

The output are in figure 40: the cross-covariance surface and image between channel 14 and 18 are in the first row and the cross-correlation with values between -1 and 1 are in the second row.

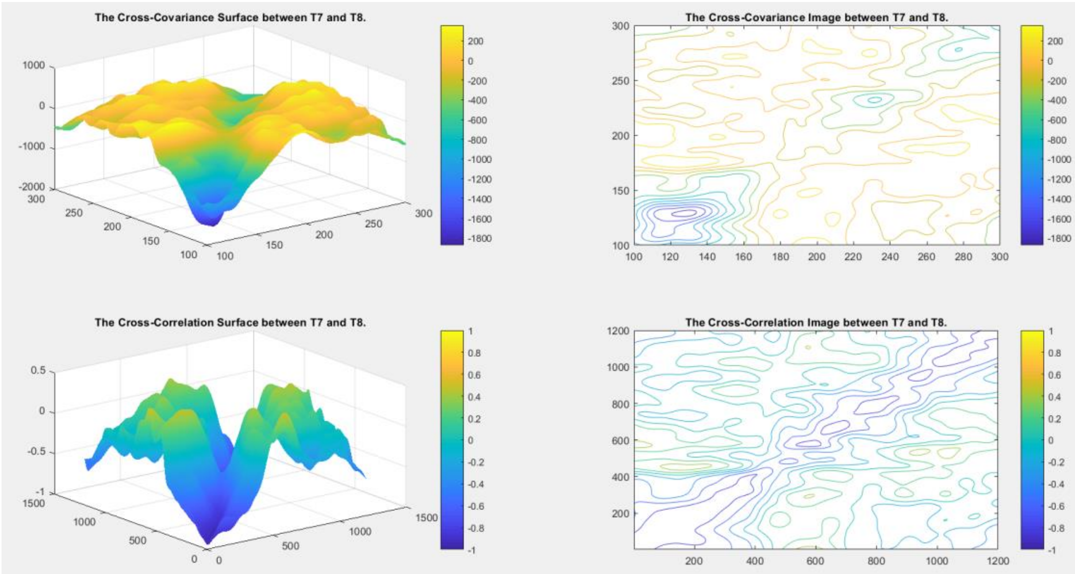


Figure 40 – The cross-covariance and cross-correlation surface and image between channels 14 and 18.

Example 2: ERP Analysis – FCCA – Functional Canonical Correlation Analysis.

In this analysis, we continue with the previous dataset and we calculate the functional canonical correlation analysis (FCCA) with 4 FCCA:

1- Load the 'a1_1_5_150_ICA_Cleaned.set'
   a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → FCCA.
3- **Chanel numbers** 1 and 2 are **14 and 18** with time limits between **100 and 300 ms**. Both of them are smoothed with a **B-spline**, **120 basis functions,** and an **order of 6**. The parameter estimations are with **GCV**.
4- In the **Functional Canonical Correlation Analysis (FCCA)** part, check the **FCCA Status**, the **number of FCCA** is **4**, check the **Diagonal** of **Plot Weight Functions**.
5- Click Ok.

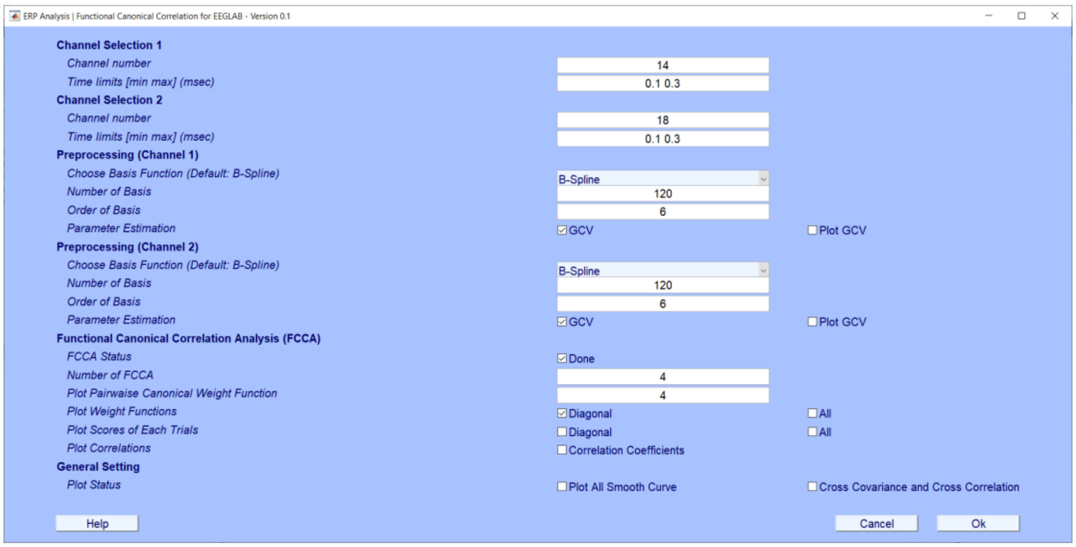


Figure 41 – The GUI for Functional Canonical Correlation between channels 14 and 18.

Figure 42 show the first four canonical weight functions between channel 14 and 18. It shows that they are inverse of each other over time.

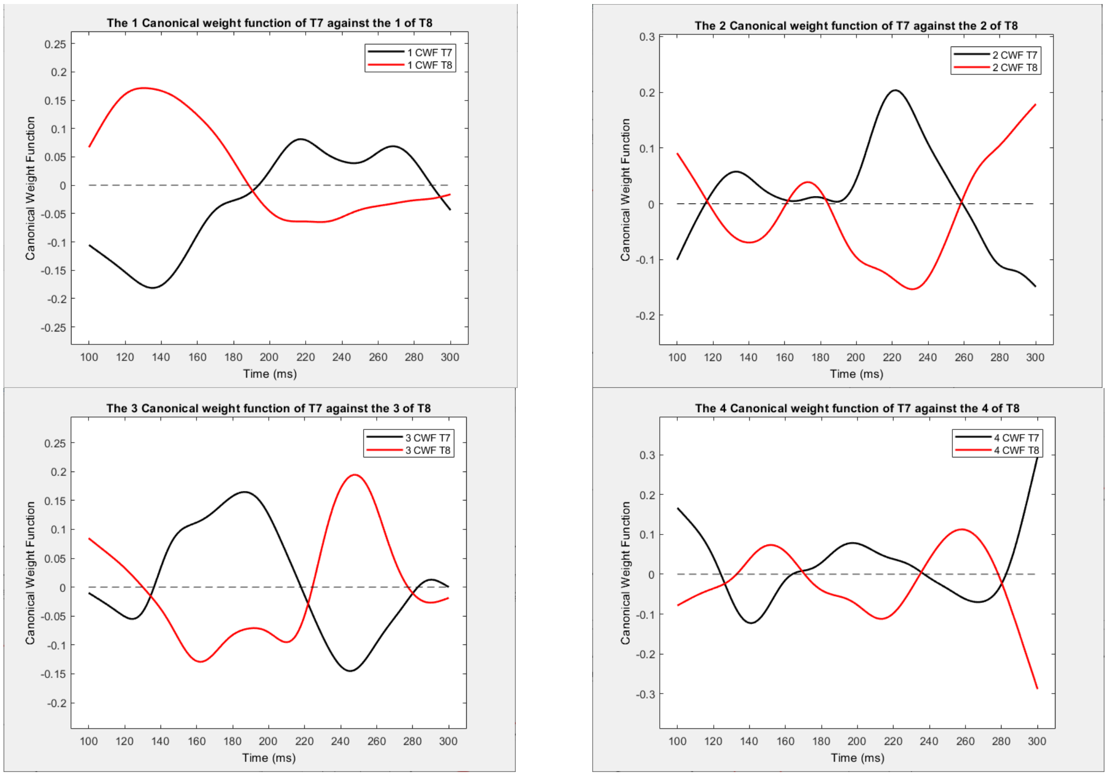


Figure 42 – The first four canonical weight functions between channel 14 (black) and 18 (red).

Example 3: ERP Analysis – FCCA – Functional Canonical Correlation Coefficient.

In this analysis, we continue with the previous dataset and we calculate the functional canonical correlation Coefficient from FCCA with 4 FCCA:

1- Load the ‘a1_1_5_150_ICA_Cleaned.set’
    a. in EEGLAB → File → Load Existing Dataset → a1_1_5_150_ICA_Cleaned.set .
2- Click Tools → FDA → ERP Analysis → FCCA.
3- **Chanel numbers** 1 and 2 are **14 and 18** with time limits between **100 and 300 ms**. Both of them are smoothed with a **B-spline**, **120 basis functions,** and an **order of 6**. The parameter estimations are with **GCV**.
4- In the **Functional Canonical Correlation Analysis (FCCA)** part, check the **FCCA Status**, the **number of FCCA** is **2**, check the **Diagonal** of **Plot Scores of Each Trial,** and check the **Correlation Coefficients** in the **Plot Correlations**.
5- Click Ok. (figure 43)

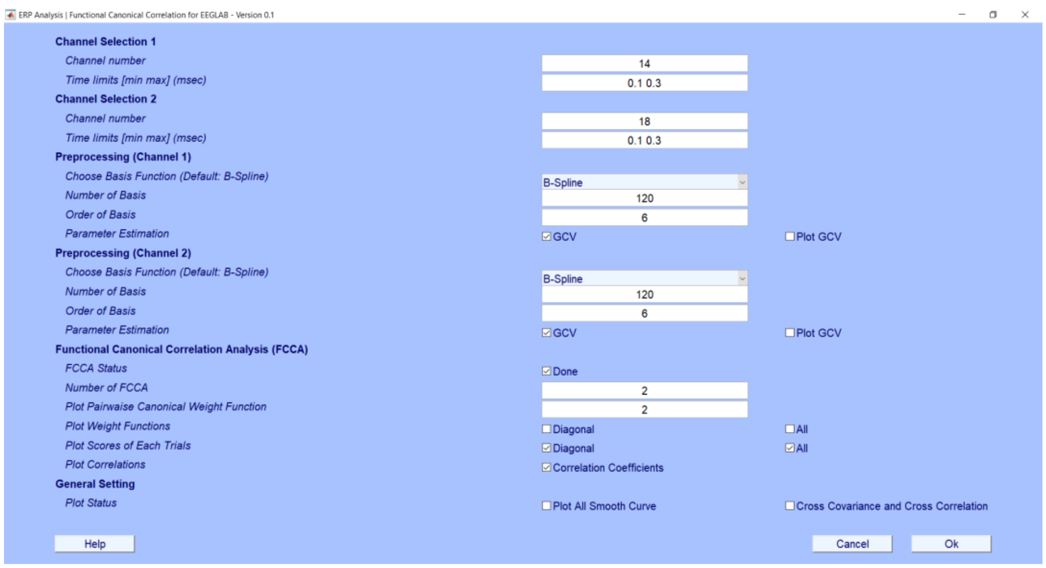


Figure 43 – The GUI for Canonical Correlation Coefficient between channels 14 and 18.

Figure 44 shows the scatterplot between canonical score functions of 14 and 18 two-by-two.

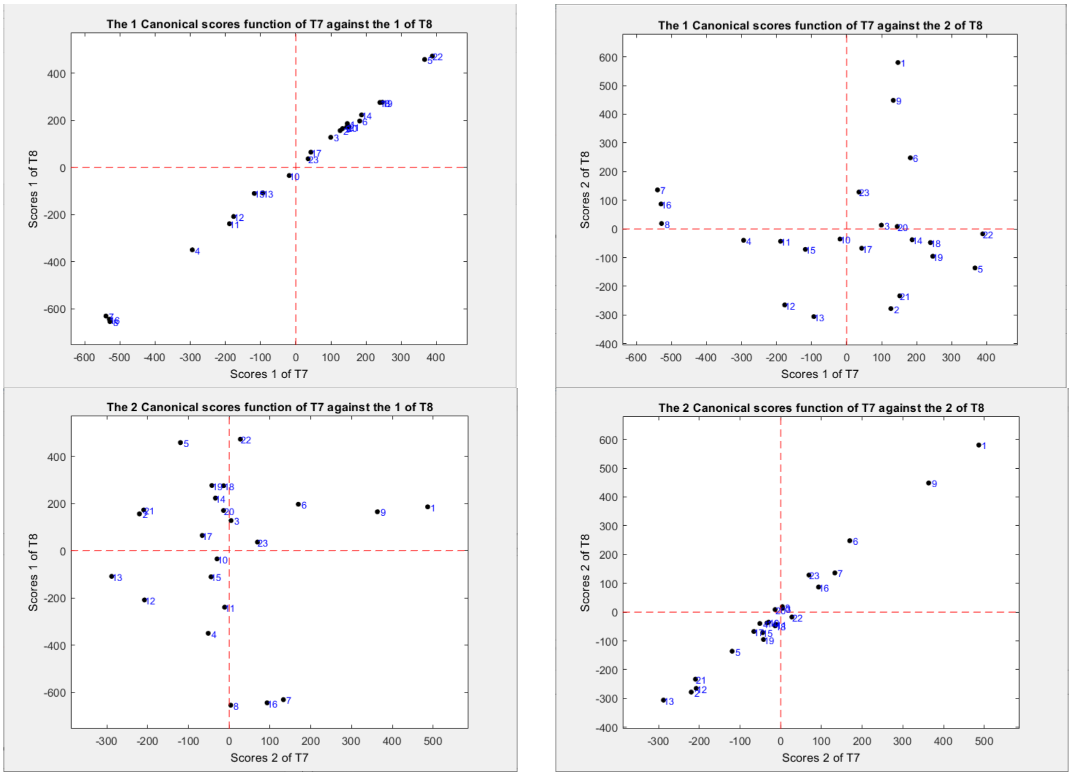


Figure 44 –Scatterplot of all canonical scores functions between channels 14 and 18.

Figure 44 shows The canonical correlation coefficients between channels 14 and 18 against a number of basis functions.

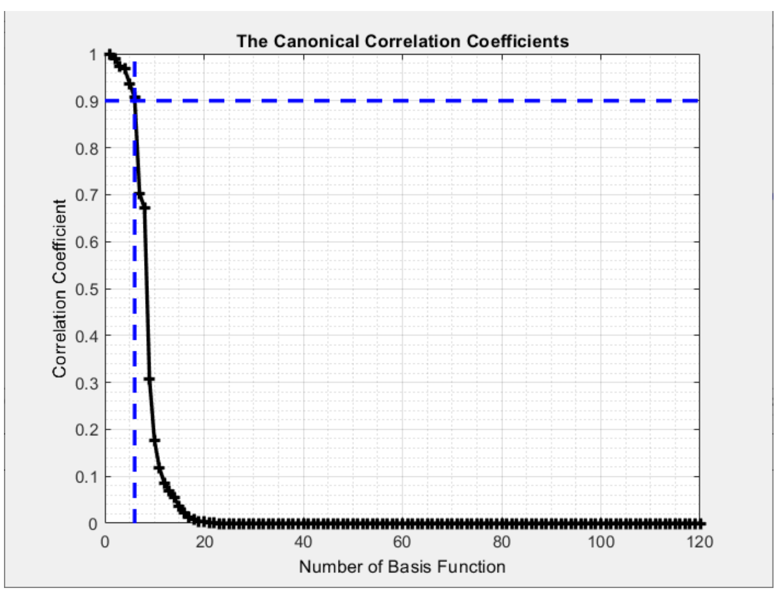


Figure 45 –The canonical correlation coefficients between channels 14 and 18 against a number of basis functions.

Permissions

Version 0.1

fdaM MATLAB Code

Feel free to use the fda Matlab code as you like. Spencer, Giles Hooker and I have never made any attempt to protect the code, and are happy to see it put to work.

(Dr. James Owen Ramsay)

May I suggest the GNU GPL license, used by R?

I think it's better to specify a license, because US copyright law legally requires a user to get permission or face the possibility of being sued for much more than everything they've got, while making it virtually impossible for someone to know who owns the material unless the copyright status and owner are clearly identified on it.

The US has done many great things. However, since the mid-1970s, US copyright law has become a poster child for political corruption. I'm not an attorney, but I have attended multiple presentations and workshops dealing with copyright issues in the US. My favorite reference on this is Lawrence Lessig (2004) Free Culture - How Big Media Uses Technology and the Law to Lock Down Culture and Control Creativity (Penguin), available for free online:
https://web.archive.org/web/20160128023748/http://www.jus.uio.no/sisu/free_culture.lawrence_lessig/sisu_manifest.html

OR the summary available on Wikipedia:
https://en.wikipedia.org/wiki/Free_Culture_(book)

(Dr. Spencer Graves)

The fda package in R is listed as copyrighted under GPL 3. I think we should regard the Matlab functions as having the same copyright.

This allows the free use and adaptation of the functionality, but bars you from directly charging for the original code or claiming ownership of it.

(Dr. Giles Hooker)

Reference

[1] Walz, Jennifer M., et al. "Simultaneous EEG-fMRI reveals temporal evolution of coupling between supramodal cortical attention networks and the brainstem." Journal of Neuroscience 33.49 (2013): 19212-19222.

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[3] Johanna Wagner, “ICA Decomposition of EEG Data”, Virtual EEGLAB workshop 2021, University of California San Diego

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