

# The Fast Fourier Transform (FFT)

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What is FFT?

To get started, a series of concepts should be briefly introduced the discrete Fourier transform or the DFT.

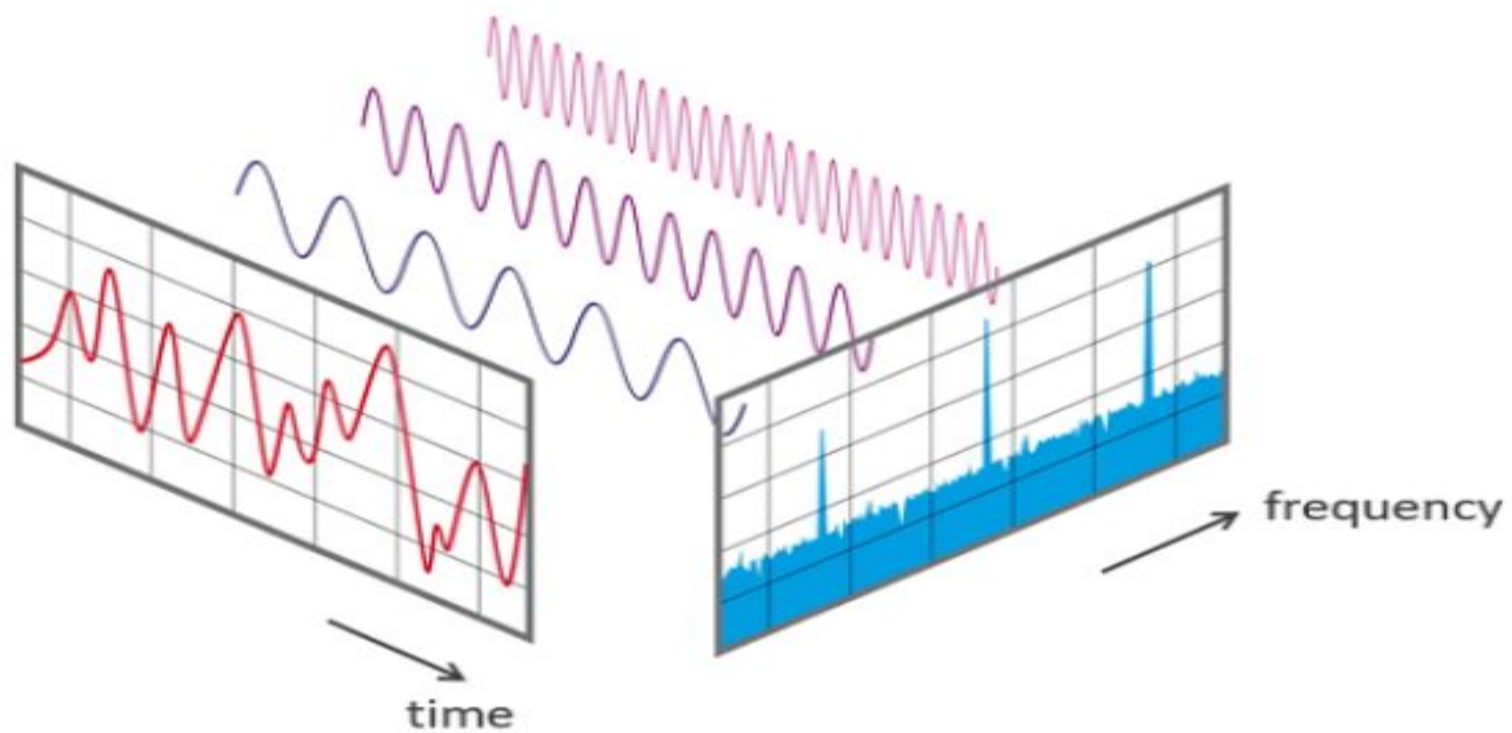
What is DFT?

# What is DFT?

<When N=4>

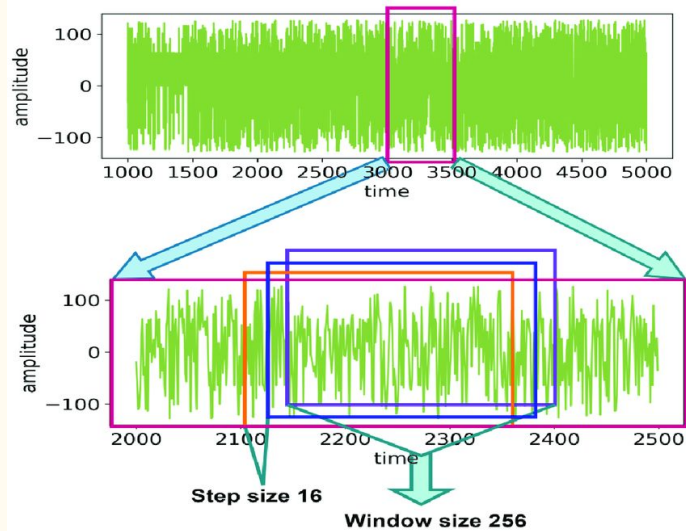
$$F(t) = \sum_{x=0}^{N-1} f(x) e^{-i \frac{2\pi t x}{N}}$$
$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \begin{bmatrix} e^{-i \frac{0}{4}} & e^{-i \frac{0}{4}} & e^{-i \frac{0}{4}} & e^{-i \frac{0}{4}} \\ e^{-i \frac{0}{4}} & e^{-i \frac{2\pi}{4}} & e^{-i \frac{4\pi}{4}} & e^{-i \frac{6\pi}{4}} \\ e^{-i \frac{0}{4}} & e^{-i \frac{4\pi}{4}} & e^{-i \frac{8\pi}{4}} & e^{-i \frac{12\pi}{4}} \\ e^{-i \frac{0}{4}} & e^{-i \frac{6\pi}{4}} & e^{-i \frac{12\pi}{4}} & e^{-i \frac{18\pi}{4}} \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \end{bmatrix}$$

Diagram illustrating the Discrete Fourier Transform (DFT) for N=4. The equation shows the DFT matrix (a 4x4 matrix of complex exponentials) multiplied by the input vector  $f(x)$  to produce the output vector  $F(t)$ . The matrix is labeled with  $t=0$  (top row) and  $x=0$  (left column).



*View of a signal in the time and frequency domain*

# FFT



What is FFT?

Why we need FFT?

Who invented FFT?

Uses of FFT?

What is FFT?



Why we need FFT?

# Order of time.

DFT :  $O(n^2)$

FFT :  $O(n \log(n))$

Example:  $n = 60 \cdot 30 \cdot 10^3 = 1.8 \cdot 10^6$

DFT :  $3.24 \cdot 10^{12} = 1 \text{ day}$

FFT :  $2.59 \cdot 10^7 = \text{less than 1 sec.}$

Who invented FFT?

Uses of FFT?

# Uses of FFT?

- to compute derivatives
- to solve complicated PDE s that describe real-world.
- denoise data
- data analysis
- audio and images compression.