The Fast Fourier Transform (FFT)

Writer: Mohammad Hossein Hajierahimi

What is FFT?

The "Fast Fourier Transform" (FFT) is an important measurement method in the science of audio and acoustics measurement. It converts a signal into individual spectral components and thereby provides frequency information about the signal. FFTs are used for fault analysis, quality control, and condition monitoring of machines or systems.

What is DFT?

the discrete Fourier transform or the DFT which is a way of taking the Fourier transform of vectors of data so instead of having an analytically defined function I just have data defined at endpoints in some interval and if I have that data stacked into a vector I can compute the discrete Fourier transform or the DFT by matrix multiplication with this matrix ,the DFT matrix we write here. this is very useful to break down your data into frequency components sums of sines and cosines but the DFT is very slow to calculate the FT.

so we have to introduce a new algorithm which is called FFT.

The "Fast Fourier Transform" (FFT) is an important measurement method in the science of audio and acoustics measurement. It converts a signal into individual spectral components and thereby provides frequency information about the signal. FFTs are used for fault analysis, quality control, and condition monitoring of machines or systems.

FFT has changed the world and it is enabling us to in most of digital communications audio compression image compression and many other things.

the FFT is an optimized algorithm for the implementation of the "Discrete Fourier Transformation" (DFT). A signal is sampled over a period of time and divided into its frequency components.

so the FFT is kind of the algorithm that you use to compute the discrete Fourier transform .we don't actually multiply this matrix by this vector. we use FFt that's much faster to finds DFT.

in the last few minute we show what is fft is.

why we need fft?

the real reason we need the FFT is speed or efficiency

the DFT calculate the things we need with order of n squared.

which means if we multiplication the dft matrix into vector of data. that's pretty expensive as n gets bigger and bigger so when we think about audio signals or images this int n might be quite large and this would be anexpensive.

so we use FFt to calculate this calculation with order of n log n it's almost linear in n.log n becomes less and less important the bigger that n gets.

notice that the output of FFT and DFt is exact same val.

for example if i want to compress 1 minute of an audio signal sampled at 30 kilohertz with DFT and FFT

so in DFT it takes about one day to calculate it.

in the FFT it takes less then one sec for the exact same 60-second audio you can see FFT faster and more efficient.

if we don’t have fft that we have today we

wouldn't be able to send pictures with high speed across media.

Who invented FFT?

the FFT was invented by Cooley and Toki in last century.

This algorithm, including its recursive application, was invented around 1805 by Gauss but he don’t publish it because he think it is worse less there weren't

computers at the time so it was just kind of a mental math algorithm for him.

The fact that Gauss had described the same algorithm (albeit without analyzing its asymptotic cost) was not realized until several years after Cooley and Tukey's 1965 paper.

you can read a little bit more about the history of FFT in the Wikipedia to find out how FFt is invented.

Uses of FFT?

- to compute derivatives
- to solve complicated PDE s that describe real-world.
- denoise data
- data analysis
- audio and images compression.

Project with FFT in python:

https://github.com/mohammad-hajiebrahimi/FFT_Tutorial