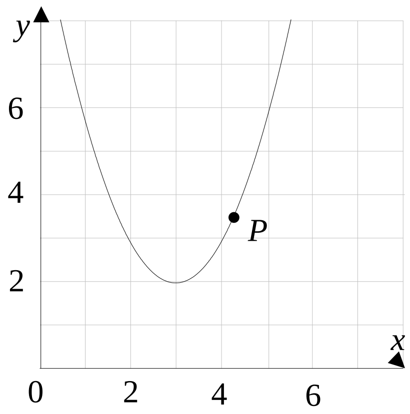
## Chapter 3 The Derived Function

### Section 3.1 The Derivative

(70) **1.** On the coordinate plane below, draw the tangent to the curve at $P$ and estimate the gradient of the curve at $P$.




(71) **2.** Find the gradient of $y = x^5$ at $x = -1$.

(49) **3.** [G1] Given $y = 4x^5 + 11x^2 - 8x + 13$. Find $y'$.

(X23) **4.** Consider the function $y = \frac{x^3}{4} - \frac{1}{3x^2}, x \neq 0$.

a. Complete the table of values for the function $y$. Give your answers correct to one decimal place.

| $x$ | –3 | –2 | –1 | –0.5 | –0.2 | | 0.2 | 0.5 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | –6.8 | –2.1 | –0.6 | | –8.3 | | –8.3 | –1.3 | | | 6.7 |

b. On a grid, draw the graph of $y = \frac{x^3}{4} - \frac{1}{3x^2}$ for $-3 \leq x \leq -0.2$ and $0.2 \leq x \leq 3$.

c. i. Draw a suitable tangent to find the gradient of the curve at $x = -2$. Round your answer to the nearest integer.

ii. Using part i, write down the equation of the tangent to the curve at $x = -2$. Give your answer in the form $y = mx + c$.

d. Use the graph obtained to solve the following equations.

i. $\frac{x^3}{4} - \frac{1}{3x^2} = 0$ ii. $\frac{x^3}{4} - \frac{1}{3x^2} - 2 = 0$

e. The equation $\frac{x^3}{4} - \frac{1}{3x^2} - 2 = 0$ can be written in the form $ax^n + bx^{n-3} - 4 = 0$. Find the values of $a$, $b$, and $n$.

(X24) **5.** A table of values for $y = \frac{4}{x^2}$ is given below. (The values for $y$ are correct to 1 decimal place).

| $x$ | –4 | –3 | –2 | –1 | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.3 | 0.4 | 1.0 | 4.0 | | 4.0 | 1.0 | 0.4 | 0.3 |

a. Using a scale of 1 cm to represent 1 unit on the $x$-axis and 1 cm to represent 1 unit on the $y$-axis, draw the graph of $y = \frac{4}{x^2}$.

b. Express $x^3 + 4x^2 - 8 = 0$ in the form $\frac{4}{x^2} = ax + b$.

c. Using the values of $a$ and $b$ obtained in the previous part, draw the line $y = ax + b$ on your graph.

d. Using your graph, estimate the roots of $x^3 + 4x^2 - 8 = 0$.

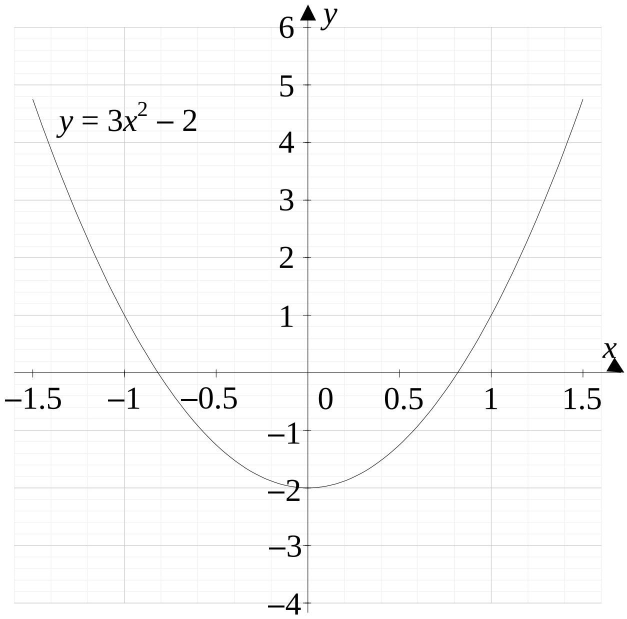
e. By drawing a suitable tangent to your curve, estimate the gradient of the curve at $x = 2$.

(X25) **6.** [G1] Consider the curve ($C$) with equation $y = x^3 + 4x^2 + 10$.

What is the value of $b$ if $(10, b)$ is a point on the tangent to ($C$) at $x = -2$?

[Hint: Find the equation of the tangent to ($C$) at $x = -2$ and substitute $x = 10$ in the equation to obtain the y-coordinate.]

(73) **7.** [T] Consider the graph of the function $y = 3x^2 - 2$ for $-1.5 \le x \le 1.5$ as shown below.




a. Write down the equation of the line of symmetry of the graph.
b. Draw the tangent to the curve at the point where $x = -0.5$. Find the gradient of this tangent.
c. The table below shows some values for $y = -x^3 + 3x + 1$.

| $x$ | –1.5 | –1 | –0.5 | 0 | 0.5 | 1 | 1.5 |
|---|---|---|---|---|---|---|---|
| $y$ | –0.125 | | | | | 3.0 | |

i. Complete the table.
ii. Draw the graph of $y = -x^3 + 3x + 1$ for $-1.5 \le x \le 1.5$.

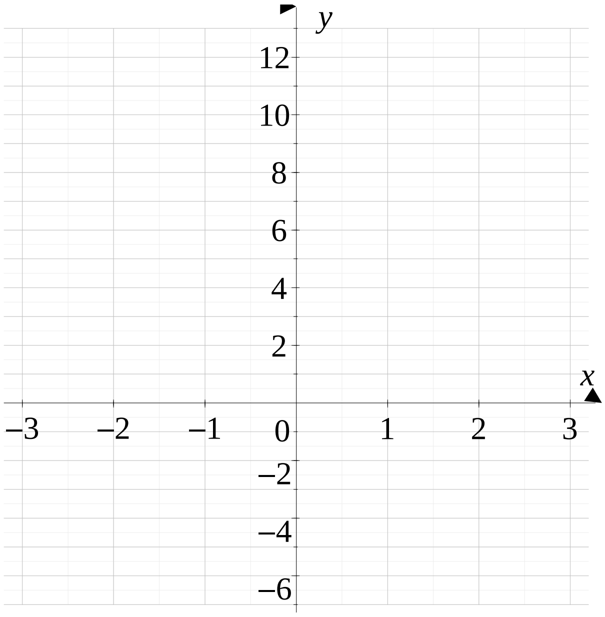
d. Show that the values of $x$ where the two curves intersect are the solutions of the equation $-x^3 - 3x^2 + 3x + 3 = 0$.
e. By drawing a suitable straight line, solve the equation $-x^3 + 5x + 1 = 0$ for $-1.5 \le x \le 1.5$.

(74) **8.** [T] Consider the function $y = \dfrac{x^3}{4} + \dfrac{1}{2x^2}, x \ne 0$.

a. Complete the table of values for the function $y$. Give your answer correct to one decimal place.

| $x$ | –3 | –2 | –1 | –0.5 | –0.2 | | 0.3 | 0.5 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | –6.7 | | | 2.0 | 12.5 | | 5.6 | | 0.8 | 2.1 | 6.8 |

b. On the grid, draw the graph of $y = \dfrac{x^3}{4} + \dfrac{1}{2x^2}$ for $-3 \le x \le -0.2$ and $0.2 \le x \le 3$.




c. i. By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = 2$.

ii. Write down the equation of the tangent to the curve at $x = 2$. Give your answer in the form $y = mx + c$.

d. Use the graph obtained to solve the following equations.

i. $\frac{x^3}{4} + \frac{1}{2x^2} = 0$

ii. $\frac{x^3}{4} + \frac{1}{2x^2} - 4 = 0$

e. The equation $\frac{x^3}{4} + \frac{1}{2x^2} - 4 = 0$ can be written in the form $ax^n - bx^{n-3} + 2 = 0$.

Find the values of $a$, $b$, and $n$.

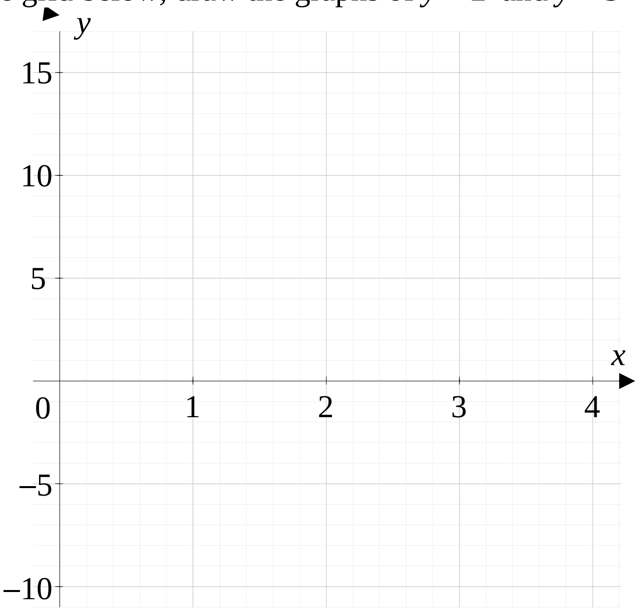
(75) **9.** [T] a. i. Consider the function $y = 2^x$ and fill in the blanks to complete the table below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | 2 | 4 | | 16 |

ii. Consider the function $y = 5 - x^2$ and fill in the blanks to complete the table below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 5 | | 1 | | −11 |

b. On the grid below, draw the graphs of $y = 2^x$ and $y = 5 - x^2$ for $0 \le x \le 4$.




c. Use the graphs to solve the equations.

i. $2^x = 10$

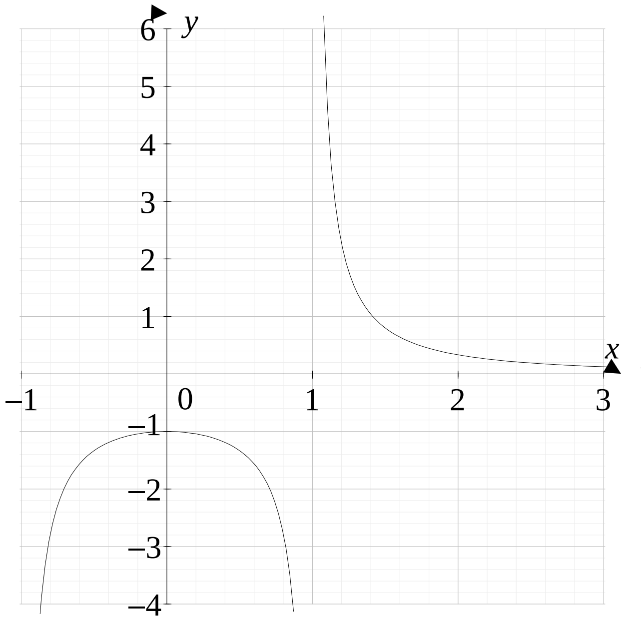
ii. $2^x = 7 - x^2$, for $0 \le x \le 4$.

d. i. On the grid, draw the line from the point (3, 0) that has a gradient of −2.

ii. Complete the statement below.

This straight line is a _______________ to the graph of $y = 5 - x^2$ at the point (1, 4).

(76) **10.** [T] Consider the diagram below that shows the graph of $y = f(x)$ for $-1 \le x \le 3$.




a. Find $f(0.8)$.

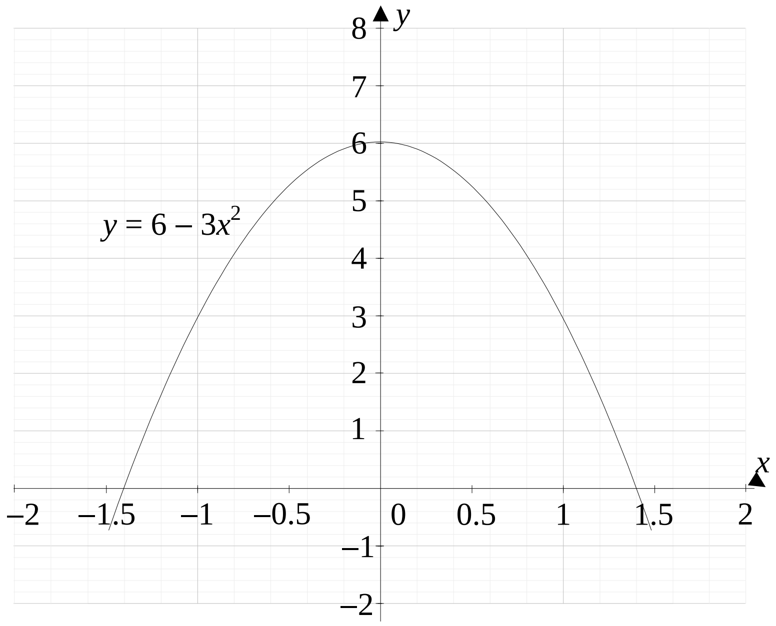
b. Solve $f(x) = 2.2$.

c. The equation $f(x) = k$ has only one solution for $-1 \le x \le 3$. Find the range of values of $k$ in this case.

d. By drawing a suitable straight line, solve the equation $f(x) = 2x - 1$.

e. Draw a tangent to the graph of $y = f(x)$ at the point where $x = 2$ and estimate the gradient.

(77) **11.** [T] Consider the graph of the function $f(x) = 6 - 3x^2$ given below for $-1.5 \le x \le 1.5$.




a. Use the graph to solve the equation $f(x) = 3$.

b. i. Draw the tangent to the graph $y = f(x)$ at the point (1, 3).

ii. Using the tangent line, estimate the gradient of $y = f(x)$ when $x = 1$.

c. Consider the function $g(x) = 3^x$.

i. Complete the table for $g(x) = 3^x$.

| $x$ | $-1.5$ | $-1$ | 0 | 1 | 1.5 |
|---|---|---|---|---|---|
| $y$ | 0.2 | $\frac{1}{3}$ | | | 5.2 |

ii. On the same grid, draw the graph of $y = g(x)$ for $-1.5 \le x \le 1.5$.

d. Use the graph obtained to solve the following.

i. $f(x) = g(x)$

ii. $f(x) > g(x)$

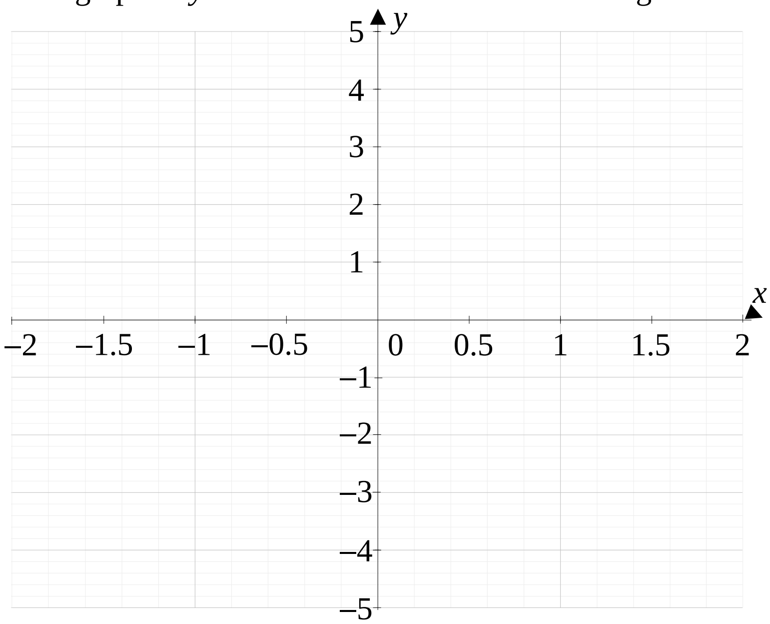
e. What value does $g(x)$ approach as $x$ decreases?

[78] **12.** [T] Consider the table below that shows some values for $y = 2x^3 - 3x^2$.

| $x$ | $-1$ | $-0.6$ | $-0.5$ | $0$ | $0.5$ | $1$ | $1.5$ | $2$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | | $-1.5$ | | $0$ | | $-1$ | $0$ | |

a. Complete the given table.

b. Draw the graph of $y = 2x^3 - 3x^2$ for $-1 \le x \le 2$ on the grid below.




c. Find the number of solutions to the equation $2x^3 - 3x^2 = -1$.

d. i. The equation $2x^3 - 3x^2 - x = -1$ can be solved by drawing a straight line on the grid. Find the equation of this line.

ii. Use the graph obtained to solve the equation $2x^3 - 3x^2 - x = -1$.

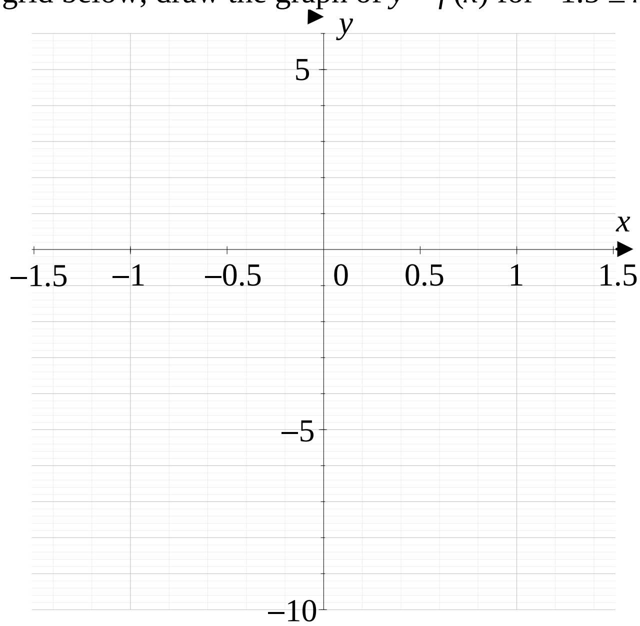
e. The tangent to the graph of $y = 2x^3 - 3x^2$ has a negative gradient when $x = k$. Give the range of $k$ as an inequality.

(79) **13.** [T] Consider the function $f(x) = x^3 - 4x^2 + 3$.

a. Complete the table of input-output values for $y = f(x)$.

| $x$ | −1.5 | −1 | −0.5 | 0 | 0.5 | 1 | 1.5 |
|---|---|---|---|---|---|---|---|
| $y$ | −9.4 | | 1.9 | | 2.1 | | −2.6 |

b. On the grid below, draw the graph of $y = f(x)$ for $-1.5 \le x \le 1.5$.




c. Use the graph to solve the equation $f(x) = 0$ for $-1.5 \le x \le 1.5$.

d. By drawing a suitable tangent, estimate the gradient of the graph of $y = f(x)$ when $x = 1$.

e. By drawing a suitable straight line on the grid, solve the equation $x^3 - 4x^2 + x + 6 = 0$ for $-1.5 \le x \le 1.5$.

(80) **14.** [T] A table of values for $y = -\dfrac{2}{x^2}$ is given below. (The values for $y$ are correct to 1 decimal place).

| $x$ | −4 | −3 | −2 | −1 | −0.6 | | 0.6 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | −0.1 | −0.2 | −0.5 | −2 | −5.6 | | −5.6 | −2 | −0.5 | −0.2 | −0.1 |

a. Draw the graph of $y = -\dfrac{2}{x^2}$.

b. Draw the line $y = -2x - 4$ on your graph.

c. Using your graph, solve $-\dfrac{2}{x^2} = -2x - 4$.

d. By drawing a suitable tangent to your curve, estimate the gradient of the curve at $x = 1$.

(81) **15.** [T]

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-21$ | $0$ | $6$ | $3$ | $-3$ | $-6$ | $0$ | $21$ |

a. Draw the curve $y = f(x)$ for $-4 \le x \le 3$ and $-21 \le y \le 21$.

b. Using your graph, find the roots of $f(x)$.

c. On the same grid, draw $h(x) = -x - 2$ for $-4 \le x \le 3$.

d. Write down the values of

i. $h(1)$

ii. $f\,h(1)$

iii. $h^{-1}(1.5)$

iv. the positive solution of $f(x) = h(x)$

e. By drawing a suitable tangent, estimate the gradient of the curve at $x = -2$.

(82) **16.** [T]

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $6$ | $-11.25$ | $-15$ | $-9.75$ | $0$ | $9.75$ | $15$ | $11.25$ | $-6$ |

a. Using a scale of 1.6 cm to represent 1 unit on the $x$-axis and 0.65 cm to represent 2 units on the $y$-axis, draw axes for $-4 \le x \le 4$ and $-16 \le y \le 16$. Draw the curve $y = f(x)$.

b. Use your graph to find the roots of $f(x)$.

c. On the same grid, draw $h(x) = 5x - 1$ for $-4 \le x \le 4$.

d. Write down the value of

i. $h(0.6)$

ii. $f\,h(0.6)$

iii. $h^{-1}(2)$

iv. the positive solution of $f(x) = h(x)$.

e. By drawing a suitable tangent, estimate the gradient of the curve at $x = 3$.

(83) **17.** [T] A table of values for $y = -\frac{5}{x^3}$ is given below.

(The values for $y$ are correct to 1 decimal place.)

| $x$ | –4 | –3 | –2 | –1 | –0.8 | | 0.8 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.1 | 0.2 | 0.6 | 5 | 9.8 | | –9.8 | –5 | –0.6 | –0.2 | –0.1 |

a. Using a scale of 1.3 cm to represent 1 unit on the $x$-axis and 0.65 cm to represent 1 unit on the $y$-axis, draw the graph of $y = -\frac{5}{x^3}$.

b. Draw the line $y = -0.75x + 1.5$ on your graph.

c. Use your graph to solve $-\frac{5}{x^3} = -0.75x + 1.5$.

d. By drawing a suitable tangent to your curve, estimate the gradient of the curve at $x = -2$.

**Section 3.2 Stationary Points**

(84) **18.** Consider the function given by $y = -3x^2 + 12x + 8$.

a. Find $y'$.
b. Study the sign of $y'$ and construct the table of variation for $y$.
c. Deduce the local extreme point(s) of the curve representing the given function and determine the nature of the point(s).

(85) **19.** Find all local stationary points of the curve given by $y = x^3 - 3x^2$ and identify each as a local maximum, a local minimum, or a saddle point.

(86) **20.** [G1] Consider the function given by $y = -2x^3 + 12x + 20$. Find the local extreme values of this function and the values of $x$ where they occur.

(X26) **21.** Find the coordinates of the stationary points of the curve ($C$) whose equation is $y = x^4 - 6x^2 + 2$.

(X27) **22.** [G1] Determine the nature of the stationary points of the curve ($C$) whose equation is $y = x^4 - 6x^2 + 2$.

(87) **23.** [T] Given the curve ($C$) with equation $y = x^3 - 27x + 4$.

a. Find the coordinates of each of the stationary points and determine their nature.
b. The point $M$, on the curve ($C$), has $x$-coordinate 1.
Find the equation of the tangent to ($C$) at $M$.

**Section 3.3 Rates of Change**

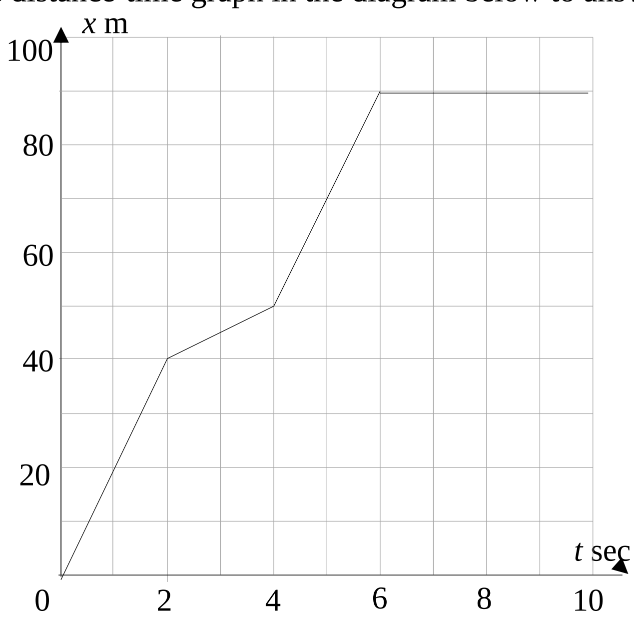
(88) **24.** A particle moves along the $x$-axis so that at $t$ seconds, its displacement from the origin $O$ is $x$ meters, where $x = -2t^3 - 4t^2 - 5t$.

a. When is the particle at $O$?

b. What is the acceleration of the particle at any time $t$?

c. What is the acceleration of the particle at $t$ = 4 sec?

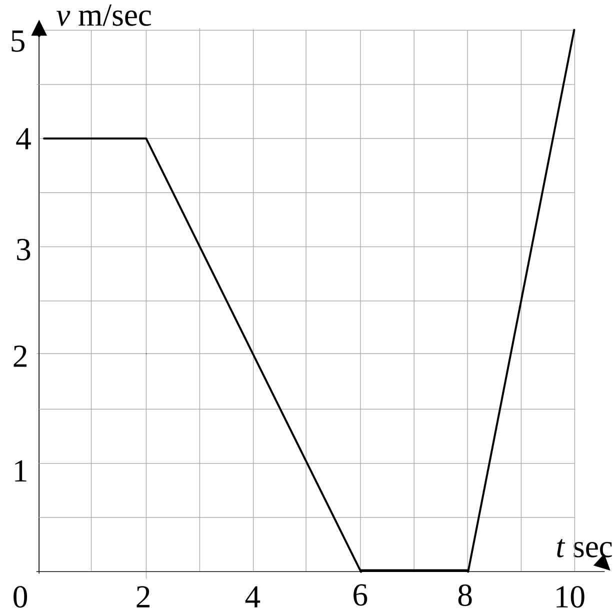
(89) **25.** Refer to the distance-time graph in the diagram below to answer the question.




Find the average speed during each of the following time intervals.

a. [0, 2]    b. [4, 8]    c. [0, 10]

(90) **26.** Consider the speed-time graph in the diagram below. It describes the journey of an object during a time interval of length 10 seconds.



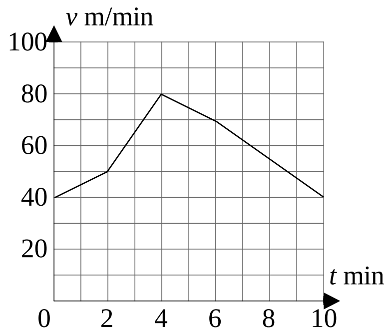

a. Find the distance traveled during the time intervals [0, 2] and [0, 10].
b. When is the object at rest?
c. What is the furthest point reached during the trip?

(91) **27.** [G1] A particle is moving along the $x$-axis so that at any time $t$ seconds, its position from the origin $O$ is given in meters by $x(t) = 0.5t^2 + 4t - 2$.
What is the average speed of this particle between $t = 2$ seconds and $t = 8$ seconds?

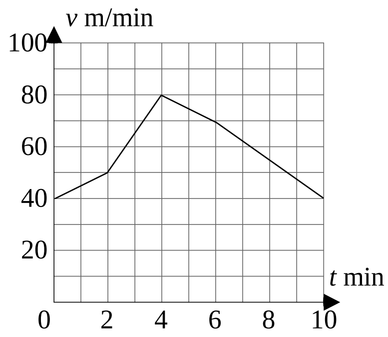
(92) **28.** [G1] A particle is moving along the $x$-axis so that at any time $t$ seconds, its position from the origin $O$ is given in meters by $x(t) = t^2 + 4t - 6$.
What is the average acceleration of this particle between $t = 2$ seconds and $t = 8$ seconds?

(93) **29.** [G1] A particle is moving along the $x$-axis so that at any time $t$ seconds, its position from the origin $O$ is given in meters by $x(t) = 0.5t^2 + 4t - 2$.
What is the speed of this particle at $t = 2$ seconds?

(94) **30.** [G1] Consider the speed-time ($v$-$t$) graph in the figure below. The graph describes a particle moving along the $x$-axis.
What is the average acceleration of this particle during the time interval [2 min, 4 min]?




(95) **31.** [G1] Consider the speed-time ($v$-$t$) graph in the figure below. The graph describes a particle moving along the $x$-axis.
What is the distance traveled by this particle during the time interval [2 min, 4 min]?




(X28) **32.** A particle moves in a straight line so that its displacement $s$, in meters, from the fixed point $O$ after $t$ seconds is given by $s = t^3 - 2t^2 + 9t + 3$, where $t \geq 0$.

What whole number must be used to fill in the blank?

The distance the particle travels during the third second is ________ meters.

(X29) **33.** A particle moves in a straight line so that its displacement $s$, in meters, from the fixed point $O$ after $t$ seconds is given by $s = t^3 - 2t^2 + 9t + 3$.

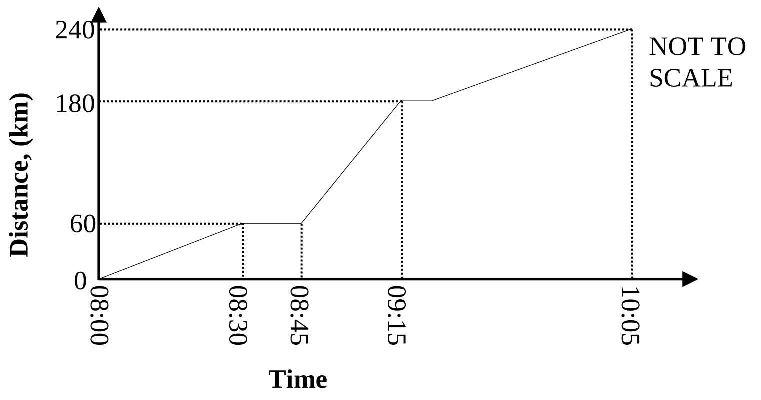
What whole number must be used to fill in the blank?

The time when the acceleration is 38 m/s$^2$ is $t$ = _______ seconds.

(X30) **34.** a) A particle moves in a straight line so that its displacement *s*, in meters, from the fixed point *O* after *t* seconds is given by $s = t^3 + 24t - 8$.
Is there a time when the particle is at rest?

b) A particle moves in a straight line so that its displacement *s*, in meters, from the fixed point *O* after *t* seconds is given by $s = t^3 - 24t^2 - 8$.
Is there a time when the particle is at rest?

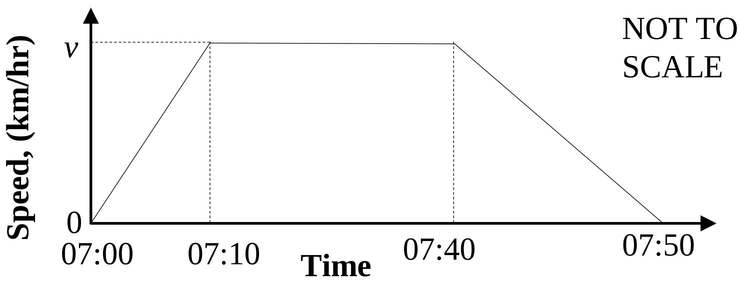
(X31) **35.** [G1] The distance-time graph below shows the journey of a train.




a) Find the average speed of the train, in km/hr, between 08:00 and 08:30.

b) Find the average speed of the train, in km/hr, for the whole journey.

(X32) **36.** [G1] A train started from rest at 07:00. The train accelerated for 10 minutes to reach a speed of *v* km/hr. The train maintained this speed for 30 minutes then decelerated for 10 minutes to reach a speed of 0 km/hr. The distance travelled during the entire 50 minute period is 90 km. The speed-time graph below displays the trip graphically.




a) Find the value of *v*.

b) Compute the average acceleration of the train, in meters per second squared, during the first 10 minutes.

(96) **37.** [T] The displacement, *s* meters, of a car from a fixed point at time *t* seconds is given by $s = t^3 - 6t^2 + 24t + 7$.

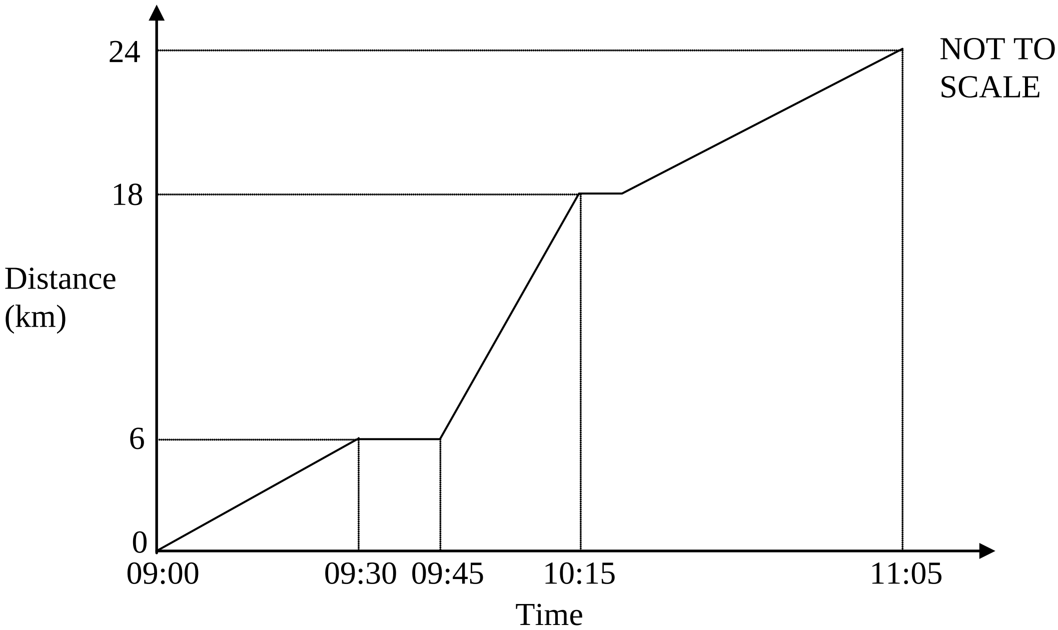
a. Calculate the value of *s* when $t = 3$.

b. Find the distance traveled in the 3rd second.

c. Calculate the value of *t* when the acceleration is 12 $m/s^2$.

d. Is there any time *t* when the particle is at rest? Justify.

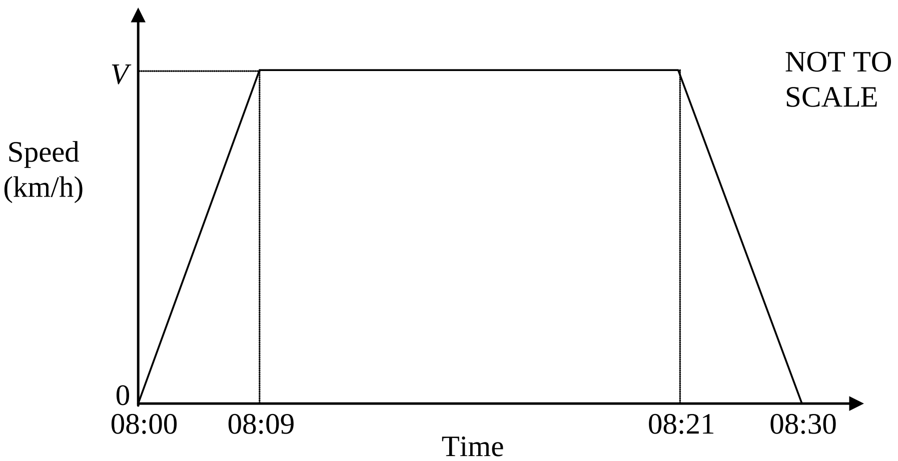
(97) **38.** [T] a. The distance-time graph below shows the journey of a cyclist.




i. Find the speed of the cyclist between 09:00 and 09:30.

ii. Find the average speed of the cyclist for the whole journey.

b. The speed-time graph below shows the first 30 minutes of another cyclist's journey.

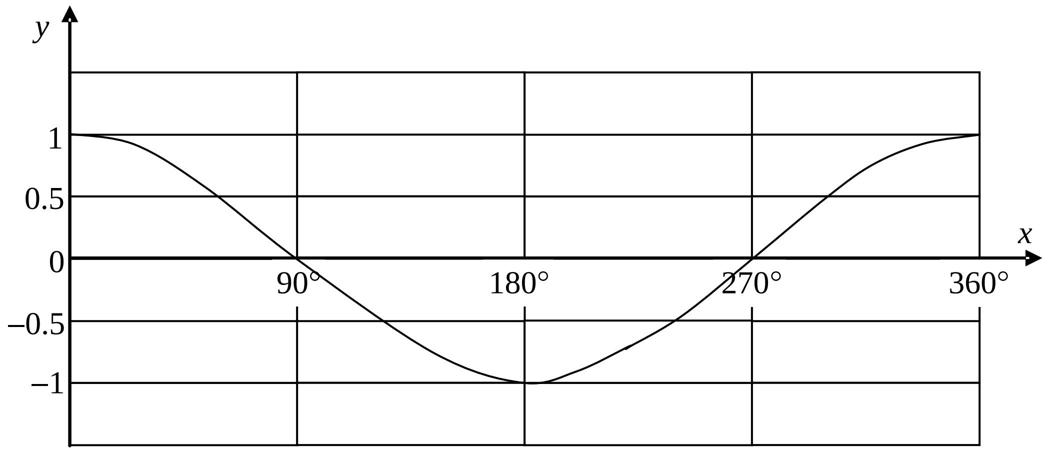



The distance travelled is 7 km. The maximum speed of the cyclist is *V* km/h.

i. Find the value of *V*.

ii. Compute the acceleration, in $m/s^2$, of the cyclist during the first 9 minutes.

(98) **39.** [T] Consider the graph of function $y = \cos x$ for $0° \le x \le 360°$ given below.




a. Solve the equation $5\cos x = 1$ for $0° \le x \le 360°$. Give your answer correct to 1 decimal place.
b. On the same grid, sketch the graph of $y = \sin x$ for $0° \le x \le 360°$.

**SAT**

(S223) **40.** [G2]

$$5(x + 3)(x + 4) = 20$$

The given equation has roots $x = \alpha$ and $x = \beta$. Find the value of $\alpha\beta$.

(S224) **41.** [G2]

$$16x^2 - 8x - 47=0$$

The given equation has roots $\frac{1}{4}+\sqrt{b}$ $\frac{1}{4}+\sqrt{b}$ and $\frac{1}{4}-\sqrt{b}$, $\frac{1}{4}-\sqrt{b}$, where $b > 0$. Find the value of $b$.

(S225) **42.** [G2]

$$y = 3x - 1$$

$$(x + y)^2 = 9$$

The given system of equations has a solution $(a, b)$, where $a$ is positive. Find the value of $a$.

(S226) **43.** [G2]

$$y = 2x^2 + 8x + 20$$
$$y = 2 - 4x$$

In the $xy$-plane, the graphs of the two equations in the system given above, intersect at exactly one point $(x, y)$. What is the value of $x$?

(S232) **44. [G2]** $P(x) = (x + a)(x + 5)(x + 4)(x + 2)$

Given that $x = 2$ is a zero of the polynomial $P$, find the product of all of its zeros.

(S234) **45.** [G2]

$$5x^2 - 10(2a - b)x + 3b = 0$$

The sum of the roots of the given equation is 4 and their product is $-6$. Find the value of $a$?