Design Optimization of a Linear Actuator

DSE311 Project Presentation
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Motivation

- The linear actuator is a critical component in various mechanical systems, requiring efficient performance under cost and space constraints.
- This project is motivated by the need to optimize its design to minimize manufacturing costs while satisfying mechanical strength and functional requirements using advanced optimization techniques.

Problem Statement

- Minimize the manufacturing cost of a drive screw linear actuator by optimizing its geometric and discrete design variables, subject to mechanical, spatial, and functional constraints.

Challenges

- **Nonlinear Constraints:** Numerous mechanical and operational constraints must be satisfied simultaneously.
- **Sensitivity to Assumptions:** Results heavily depend on added constraints and design assumptions.
- Balancing Robustness and Feasibility.
- **Non-Standard Outputs:** Optimal solutions may yield impractical, non-manufacturable dimensions.

Approach

The report uses three optimization approaches: Sequential Quadratic Programming (SQP) combined with Branch and Bound, Genetic Algorithms, and Differential Evolution. All methods were implemented in python and MATLAB and yielded consistent results.

Assumptions

- The material of the drive screw is stainless steel.
- The shape of the drive screw is pre-defined.
- Threads are standard unified threads.
- Assembly force is concentrated at the midpoint of a lead screw.
- Frictional forces are only assumed to exist between threads and load nut, negligible everywhere else.
- The material cost coefficient Cm to be unity (for simplicity).

Assumptions

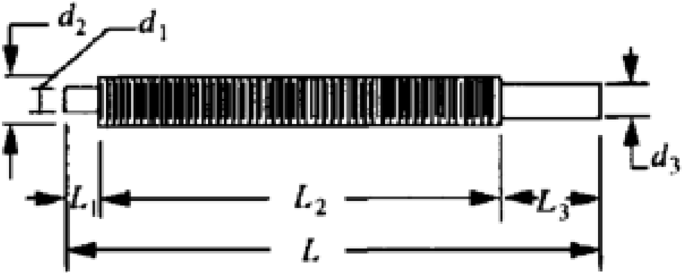

Figure: Schematic of Drive Screw in linear actuator

Design Variables

- d_1 = Diameter of shaft/gear interface.
- d_2 = Diameter of threaded portion.
- d_3 = Diameters of the lower bearing surface.
- N_s = Number of teeth of the pinion gear.
- N_m = Number of teeth on the drive gear.
- N_T = Number of threads per inch.
- L_2, L_3 = Lengths of a screw having diameters as d_2, d_3 respectively.

Design Parameters

- T_m = Motor torque = 2 inch-ounce
- S_m = Motor speed = 300 rpm
- τ_{all} = Maximum allowable shear stress = 22,000 psi
- σ_{all} = Maximum allowable bending stress = 20,000 psi
- F_a = Force required to assembly drive screw into the assembly = 6lb
- K = Stress concentration factor = 3
- W = Drive screw load = 3lb
- μ = friction coefficient = 0.35
- L_1 = Length of gear/drive screw interface = 0.405 inch
- α_n = Thread angle = 60°
- S = Linear cycle rate = 0.0583 inch/sec

- **Constraint for Bending stress due to assembly force F_a:**

$$\frac{M}{I}\frac{d_1}{2} \leq \sigma_{all}$$

Where, $M =$ bending moment due to assembly force $= F_a \times L/2$.
$L =$ Total length of screw $= L_1 + L_2 + L_3$

- **Shear stress due to torsion:**

$$\frac{KT(d_1/2)}{J} \leq \tau_{all}$$

Where;
$T =$ Applied torque $= C_2 \times T_m \times N_s/N_m$
$C_2 = \frac{1}{16}$(lb/ounce) i.e (Conversion factor)
$K =$ Stress concentration factor.
$J =$ Polar moment of inertia $(\pi d_1^4/32)$.

- **No slip condition:**

$$\left(\frac{N_1 \times d_2}{2}\right)\left[\frac{\pi \times \mu \times d_2 + N_T^{-1} \times \cos(\alpha_n)}{\pi \times d_2 \cos(\alpha_n) - N_T^{-1} \times \mu}\right] \leq T$$

- **The specified linear rate of oscillation:**

$$C_4 \times N_m S_m / (N_s N_T) \leq S$$

$$C_4 = 60^{-1}(\text{no. of threads/revolution})(\text{min/S})$$

Physics Constraints

- **Upper bound for threads per inch due to mass production:**

$$N_T \leq 24$$

- **Limit on the number of teeth on gears to avoid interference:**

$$N_m \geq 8 \text{ and } N_s \leq 52$$

- **Space constraints & packaging consideration:**

$$\begin{aligned} 8.75 \leq L_1 + L_2 + L_3 &\leq 10.0 \\ 7.023 \leq L_2 &\leq 7.523 \\ 1.1525 \leq L_3 &\leq 1.6525 \\ d_2 &\leq 0.625 \end{aligned}$$

Optimization Problem

Minimize $\mathcal{F} = C_m \times \frac{\pi}{4} \times (d_1^2 \times L_1 + d_2^2 \times L_2 + d_3^2 \times L_3)$

Optimization Problem

Subject to:

$g_1 : 38.88 + 96L_2 + 96L_3 - \pi\sigma_{all} d_1^3 \leq 0$

$g_2 : 6(N_s/N_m) - \pi\tau_{all} d_1^3 \leq 0$

$g_3 : 8.345 - L_2 - L_3 \leq 0$

$g_4 : -9.595 + L_2 + L_3 \leq 0$

$g_5 : L_2 - 7.523 \leq 0$

$g_6 : 7.023 - L_2 \leq 0$

$g_7 : L_3 - 1.6525 \leq 0$

$g_8 : 1.1525 - L_3 \leq 0$

$g_9 : d_2 - 0.625 \leq 0$

$g_{10} : 5(N_m/N_s) - 0.0583N_T \leq 0$

Optimization Problem

$$g_{11} : (1.5d_2)\left[\frac{\pi\mu d_2 + 0.5N_T^{-1}}{0.5\pi d_2 - 0.35N_T^{-1}}\right] - 0.125(N_s/N_m) \le 0$$

$$g_{12} : N_T - 24 \le 0$$

$$g_{13} : 8 - N_m \le 0$$

$$g_{14} : N_s - 52 \le 0$$

- SQP + Branch and Bound
  - Efficiently solve nonlinear problems with both continuous and integer variables.
  - SQP solves a local Quadratic Programming (QP) approximation of the problem at each step.
  - Integer constraints (like gear teeth count) are handled by Branch and Bound.
  - This splits the problem into subproblems, solves them, and eliminates unpromising branches.
  - Implemented using Python.

- Genetic Algorithm
  - A population of candidate solutions evolves over generations.
  - Key operations:
    - Selection – Chooses best performers.
    - Crossover – Combines features of two parents.
    - Mutation – Introduces random changes for diversity.
  - Good for mixed-variable optimization (continuous + discrete).
  - Implemented using MATLAB GA Toolbox.

1. **Initialize Population**
   Create a set of random candidate solutions (vectors).
2. **Mutation**
   For each target vector, generate a mutant vector using:

$$\text{Mutant} = \mathbf{x}_a + F \cdot (\mathbf{x}_b - \mathbf{x}_c)$$

3. **Crossover**
   Combine target and mutant vectors to form a *trial vector*.
4. **Selection**
   If the trial vector performs better than the target vector, replace it.
5. **Repeat**
   Iterate through generations until convergence.

Results

Methods	d1	d2	d3	L1	L2	Ns	Nm	Nt	Obj Function Value
SQP	0.06934764	0.241625	0.1875	7.023	1.322	8	30	24	0.3600623
Genetic Algorithm	0.068	0.2419	0.1888	7.023	1.322	8	29	24	0.3623
Differential Evolution	0.0677	0.241625	0.1875	7.023	1.32199	8	29	24	0.35999

Figure: Optimal Parameters and Result Comparisons

- The problem solution converges, and the results of all three techniques used agree well with each other.

Conclusion

- Multiple optimization methods (Differential Evolution, SQP+Branch Bound, Genetic Algorithm) were applied; the choice depends on problem formulation and study goals.
- The optimized diameters are reported to three decimal places; practical manufacturing will require rounding, which may slightly increase the objective value.
- Introducing additional realistic constraints (e.g. standard machining tolerances) can ensure design variables remain manufacturable without compromising cost too heavily.