DSE311
Applied Optimization Project Report

Design Optimization of a Linear Actuator

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April 23, 2025

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Abstract

It is often required to reduce cost or mass or any physical quantity of a product in industrial applications. Hence optimization techniques are required to carry out that analysis. Present project is done to reduce the cost of linear actuator subjected to constraints imposed by mechanical strength, space restrictions, machining limitations, and functional requirements. A mathematical model is formulated to convert requirements in terms of equations. Sequential Quadratic Programming (SQP) with branch and bound and Differential Evolution methods is exlored to optimize the cost. The genetic algorithm is also used to verify results in MATLAB. The results obtained in all three cases are in agreement with each other. Each of the solution methods gives various insightful information about the problem. Constraint Analysis reveals that the d3 diameter is actually unbounded and needs to be well defined. The geometry of standard unified threads provides the required relation between d2 and d3 diameters and lower bound of d3 governed by thrust required to avoid wearing. The value of objective function greatly depends upon the choice of addition constraint, by relaxing the constraint added by bearing thrust requirement ($d_3 \geq 0.1875$) objective value can be further reduced. The SQP analysis provides the Lagrange multiplier values which are critical to sensitivity analysis. For different initial guesses, SQP provides objective function value almost the same as optimum value, but with a non-integer number of teeth. Four iterations of the branch and bound technique are required to converge the solution to integer values. Sensitivity analysis shows that

the objective function value is sensitive for the extra constraints added, which is expected since we have added them to bound d3. The sensitivity of the objective function is also more for L2 and L3.

1 Introduction

The present design optimization problem statement is intended to optimize the design of the drive screw linear actuator to reduce its total cost of production. The drive screw has a wide range of application where rotary motion is to be converted into linear motion. Some of the common applications are Screw jack, lead screw of lathe machine, linear motors. The application ranges from light to heavy-duty applications. Considering the wide variety of application of drive screw, it is important to select the most suitable material and dimensions so as to minimize the cost while ensuring that drive screw meets constraints imposed by mechanical strength, space restrictions, machining limitations, and functional requirements. For the problem under consideration, the stainless steel is selected as material, specifying the material reduces the complications of the problem.

The linear actuator assembly consists of a prime mover, drive gear, pinion gear, and load-carrying nut and chassis which supports the whole assembly in bearing.

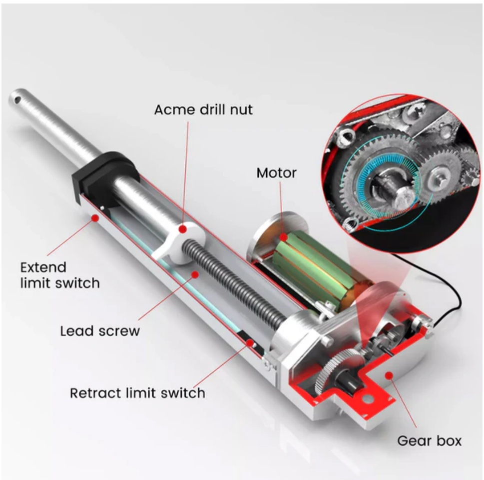

Figure 1: Design of linear actuator

2 Aim

The objective of the report is to minimize the material cost of drive screw linear actuator made out of stainless steel and subjected due to mechanical strength, space restrictions, machining limitations, and functional requirements.

3 Mathematical Formulation

3.1 Objective function

The objective of the optimization problem is to reduce the total manufacturing cost of the drive screw. The total cost of production involves the cost of machining and the cost of the material. The cost of machining is a function of the shape of drive screw, the drive screw is assumed to have definite shape indicated by screw and the only variation in geometric parameters is allowed. Variation of dimensional parameter will not affect the manufacturing process to be used and since the production rate is assumed high, the cost of machining can be assumed to be constant as long as the above two assumptions are valid. The problem statement simplifies to a reduction of cost of drive screw, which is the only function of material cost, effectively we can also model the problem as minimize weight of drive screw.

$$\mathcal{F} = C_m \times \frac{\pi}{4} \times (d_1^2 \times L_1 + d_2^2 \times L_2 + d_3^2 \times L_3)$$

Where,

$$C_m = \text{Material cost } (\$/\text{inch}^3)$$

$$d_1, d_2, d_3 = \text{Diameters of drive screw indicated by figure 2.}$$

$$L_1, L_2, L_3 = \text{Lengths of a screw having diameters as } d_1, d_2, d_3 \text{ respectively.}$$

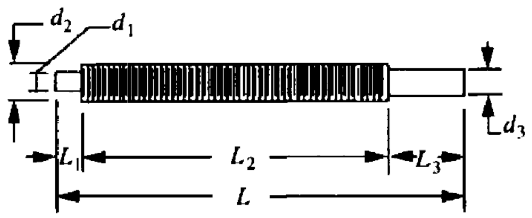

Figure 2: Schematic of Drive Screw in linear actuator

3.2 Assumptions

1. The material of the drive screw is stainless steel.
2. The shape of the drive screw is pre-defined.
3. Threads are standard unified threads.
4. Assembly force is concentrated at the midpoint of a lead screw.
5. Frictional forces are only assumed to exist between threads and load nut, negligible everywhere else.
6. For the sake of simplicity, we assume the material cost coefficient C_m to be unity.

3.3 Design variables

- d_1 = Diameter of shaft/gear interface.
- d_2 = Diameter of threaded portion.
- d_3 = Diameters of the lower bearing surface.
- N_s = Number of teeth of the pinion gear.
- N_m = Number of teeth on the drive gear.
- N_T = Number of threads per inch.
- L_2, L_3 = Lengths of a screw having diameters as d_2, d_3 respectively.

3.4 Design parameters considered

- T_m = Motor torque = 2 inch-ounce
- S_m = Motor speed = 300 rpm
- τ_{all} = Maximum allowable shear stress = 22,000 psi
- σ_{all} = Maximum allowable bending stress = 20,000 psi
- F_a = Force required to assembly drive screw into the assembly = 6lb
- K = Stress concentration factor = 3
- W = Drive screw load = 3lb
- μ = friction coefficient = 0.35
- L_1 = Length of gear/drive screw interface = 0.405 inch
- α_n = Thread angle = 60°
- S = Linear cycle rate = 0.0583 inch/sec

3.5 Physics Constraints

- **Constraint for Bending stress due to assembly force F_a:**

$$\frac{M}{I}\frac{d_1}{2} \leq \sigma_{all}$$

Where, M = bending moment due to assembly force = $F_a \times L/2$.
L = Total length of screw = $L_1 + L_2 + L_3$

- **Shear stress due to torsion:** It is a Constraint against fatigue failure in shear. Drive screw is required to sustain a given number of stress cycle under shear stress.

$$\frac{KT(d_1/2)}{J} \leq \tau_{all}$$

Where;
T = Applied torque = $C_2 \times T_m \times N_s/N_m$
$C_2 = \frac{1}{16}$(lb/ounce) i.e (Conversion factor)
K = Stress concentration factor.
J = Polar moment of inertia $(\pi d_1^4/32)$.

- **The specified linear rate of oscillation:** The drive is required to move the load at a specific linear rate.

$$C_4 \times N_m S_m/(N_s N_T) \leq S$$

$$C_4 = 60^{-1}(\text{no. of threads/revolution})(\text{min/S})$$

- **No slip condition:**

$$\left(\frac{N_1 \times d_2}{2}\right)\left[\frac{\pi \times \mu \times d_2 + N_T^{-1} \times \cos(\alpha_n)}{\pi \times d_2 \cos(\alpha_n) - N_T^{-1} \times \mu}\right] \leq T$$

- **Upper bound for threads per inch due to mass production:**

$$N_T \leq 24$$

- **Limit on the number of teeth on gears to avoid interference:**

$$N_m \geq 8 \text{ and } N_s \leq 52$$

- **Space constraints & packaging consideration:**

$$\begin{aligned} 8.75 \leq L_1 + L_2 + L_3 \leq 10.0 \\ 7.023 \leq L_2 \leq 7.523 \\ 1.1525 \leq L_3 \leq 1.6525 \\ d_2 \leq 0.625 \end{aligned}$$

To facilitate assembly and from a manufacturing standpoint, some constraints are necessary and are not taken as constraints explicitly

$$d_1 < d_2 \text{ and } d_3 < d_2$$

4 Optimization problem

The problem can be represented in negative null form by substituting the values of design parameters. Based on the design constraints we can substitute .

Objective function: Minimize $\mathcal{F} = C_m \times \frac{\pi}{4} \times (d_1^2 \times L_1 + d_2^2 \times L_2 + d_3^2 \times L_3)$

Subject to:

$$
\begin{aligned}
g_1 &: 38.88 + 96L_2 + 96L_3 - \pi\sigma_{all}d_1^3 \leq 0 \\
g_2 &: 6(N_s/N_m) - \pi\tau_{all}d_1^3 \leq 0 \\
g_3 &: 8.345 - L_2 - L_3 \leq 0 \\
g_4 &: -9.595 + L_2 + L_3 \leq 0 \\
g_5 &: L_2 - 7.523 \leq 0 \\
g_6 &: 7.023 - L_2 \leq 0 \\
g_7 &: L_3 - 1.6525 \leq 0 \\
g_8 &: 1.1525 - L_3 \leq 0 \\
g_9 &: d_2 - 0.625 \leq 0 \\
g_{10} &: 5(N_m/N_s) - 0.0583N_T \leq 0 \\
g_{11} &: (1.5d_2)\left[\frac{\pi\mu d_2 + 0.5N_T^{-1}}{0.5\pi d_2 - 0.35N_T^{-1}}\right] - 0.125(N_s/N_m) \leq 0 \\
g_{12} &: N_T - 24 \leq 0 \\
g_{13} &: 8 - N_m \leq 0 \\
g_{14} &: N_s - 52 \leq 0
\end{aligned}
$$

The d_1, d_2, d_3 are continuous variables and N_m, N_T, N_s are integer variables.

5 Optimization Methods

5.1 SQP with the help of Branch and Bound method

Sequential Quadratic Programming (SQP) tackles the continuous portion of the mixed-integer problem by iteratively solving a quadratic approximation of the nonlinear objective subject to linearized constraints. At each iteration, SQP builds and solves a Quadratic Programming (QP) subproblem whose solution direction guides the update of the design variables. To handle integer variables (such as thread count or gear teeth), a Branch and Bound framework systematically fixes discrete choices, creating subproblems that SQP solves independently. Branch and Bound then uses the solution bounds to prune branches that cannot improve upon the current best, yielding a globally valid, integer-feasible optimum while retaining the fast local convergence properties of SQP

```
Optimal solution: [ 0.06934764  0.241625    0.1875      7.023       1.322       8.00055506
 30.73522582 24.        ]
Objective function value at optimal solution: 0.3600622780169267
```

Figure 3: Optimization results of SQP with Branch & Bound

5.2 Genetic Algorithm

Genetic Algorithms are population-based metaheuristics inspired by natural selection. A population of candidate designs (chromosomes) evolves over generations via operators such as selection (choosing higher-fitness individuals), crossover (recombining design genes), and mutation (random perturbations). By encoding both continuous variables (diameters, lengths) and discrete variables (numbers of threads or teeth). We used the GA toolbox in MATLAB for solving this problem.

```
x =

    0.0680    0.2419    0.1888    7.0862    1.2578    8.0000   29.0000   24.0000


fval =

    0.3623
```

Figure 4: Optimization results of Genetic Algorithm

5.3 Differential Evolution

Differential Evolution is a evolutionary strategy for continuous optimization. Starting with a population of real-valued vectors, DE generates trial vectors by adding the weighted difference between two population members to a third, then uses crossover to mix trial and target vectors.

```
Optimal x: [ 0.06770641  0.24162528  0.18750009  7.0230145   1.32199503 12.52656558
 44.77499911 23.99995686]
Objective function value: 0.3599920416533173
```

Figure 5: Optimization results of Differential Evolution

6 Results

Problem is solved using 3 Mixed Integer techniques:
1) SQP with branch and bound algorithm
2) Genetic Algorithm
3) Differential Evolution.

Methods	d1	d2	d3	L1	L2	Ns	Nm	Nt	Obj Function Value
SQP	0.06934764	0.241625	0.1875	7.023	1.322	8	30	24	0.3600623
Genetic Algorithm	0.068	0.2419	0.1888	7.023	1.322	8	29	24	0.3623
Differential Evolution	0.0677	0.241625	0.1875	7.023	1.32199	8	29	24	0.35999

Figure 6: Comparison of Results

These values of Objective function depend on the assumption made for well boundedness of variable d_3. For our case, we have taken

$$d_3 \geq 0.1875(\text{reference from literature})$$

and used a geometric constraint written below which needs to be satisfied

$$d_3 = d_2 - \frac{1.299}{N_T}$$

Objective value can be further reduced by changing the lower bound on d_3 depending upon the assumption made earlier **(lower bound=0.3200**$\times C_m$**)** .

The problem solution converges, and the results of all three techniques used agree well with each other.

7 Conclusion

- Any optimization problem can be solved using various methods and one should use discretion to choose the best method for the application depending on the formulation of the problem and which aspect of the problem is to be studied. This project is solved by Differential Evolution, SQP with branch and bound, genetic algorithm.

- SQP with branch and bound method is better for the current project where sensitivity study can be easily done. The Genetic algorithm is also used to solve this problem.

- Based on Solution Obtained the resulting diameters are in positive fraction up to three digits. In actual case making such Diameters is not possible. Those have to be rounded off at the cost of some increase in the objective function. The decrease in cost function will determine whether to go with such precision diameters or not. More constraints required to be added to make more realistic values of the design variable.

8 Future Work

- Design Variable complexity can be increased. Packaging and manufacturing cost parameters can be incorporated in objective function.
- More Manufacturing Constraints and Physics Constraints need to be incorporated.
- We have considered the material of drive screw as stainless steel, however the linear actuator is an assembly of various parts of various other materials (plastics, aluminium, etc) whose design parameters needs to be included in objective function.

9 References

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