QWORLD

UNIVERSITY OF LATVIA
FACULTY OF COMPUTING

University Level Quantum Courses

QCLASS24/25

Lecture 1

Today

QWORLD

EVENTS PROJECTS QCOUSINS QEDUCATION QRESEARCH ABOUT

QBook101 Contents

Fundamentals of Quantum Computers with Programming

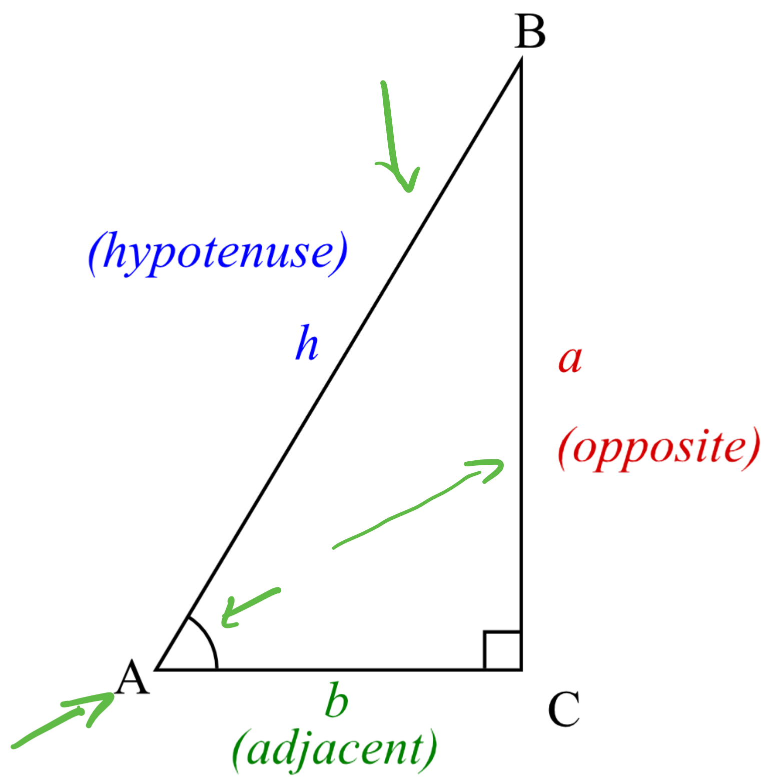
Trigonometry (quick review)

mnemonics

$$\sin A = \frac{a}{h} \qquad \text{SOH}$$

$$\cos A = \frac{b}{h} \qquad \text{CAH}$$

$$\tan A = \frac{a}{b} \qquad \text{TOA}$$



Trigonometry (quick review)

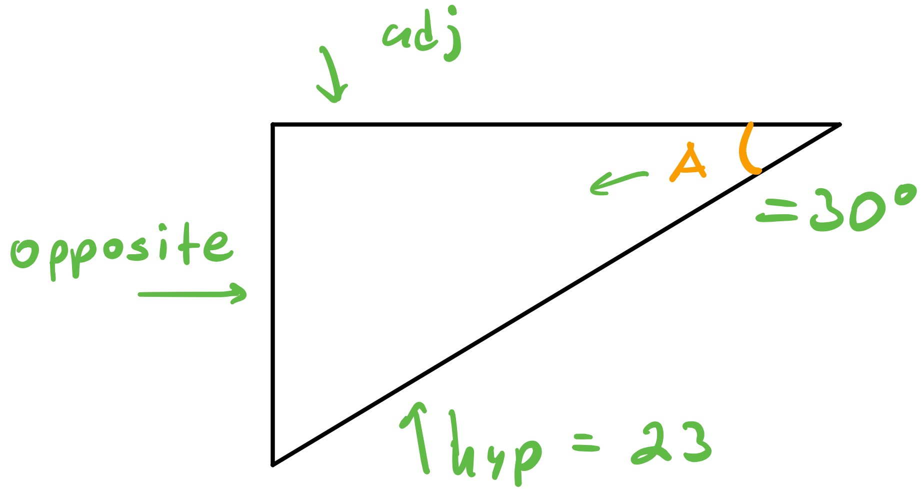
π

```
from math import sin, cos, tan, pi

A = 30                # degrees
hyp = 23

A_rad = A * pi/180    # radians

adj = hyp * cos(A_rad)
```



$$\text{CAH} \rightarrow \cos(A) = \frac{\text{adj}}{\text{hyp}}$$

$$\hookrightarrow \text{adj} = \frac{\text{hyp}}{\cos(A)} = \frac{23}{\cos(30^\circ)} = 19.92$$

Complex Numbers

$$z = \overbrace{a}^{\text{real}} + \overbrace{b}^{\text{imaginary}} i \in \mathbb{C}$$

belongs to

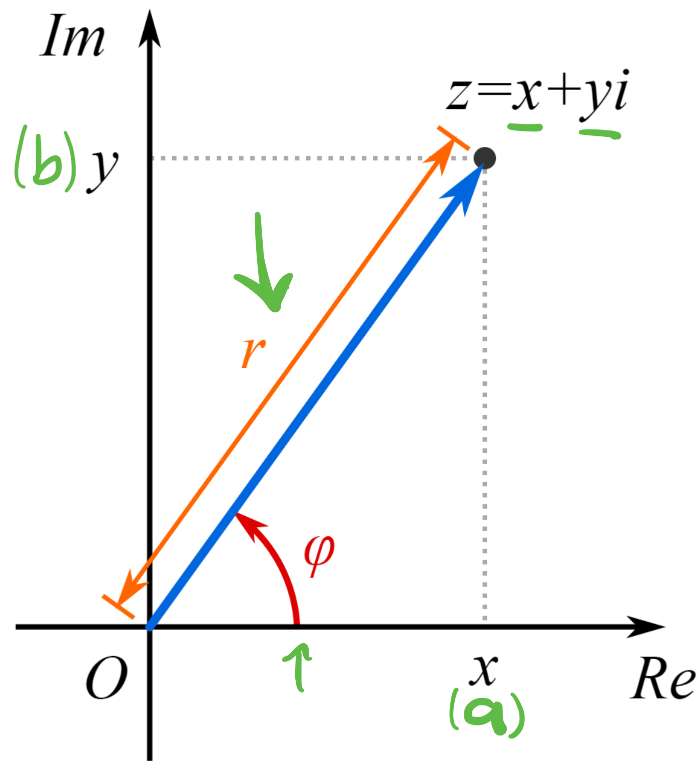
set of complex

$i^2 = -1 \rightarrow i = \sqrt{-1}$

$a, b \in \mathbb{R} \rightarrow$ 2D

complex conjugate: $\bar{z} = z^* = a - bi$

$z = re^{i\varphi}$ ← polar form



Complex Numbers

```
z1 = 3+2j
print(z1)
z2 = complex(3,2)
print(z2)
z3 = 5j
print(z3)
z4 = complex(0,5)
print(z4)
```

$i \xrightarrow{\text{Python}} j$

$z1 = z2$

```
z1 = 3+2j
z2 = 4+5j
print('z1=', z1, 'z2=', z2)
print('z1+z2=', z1+z2)
print('z1-z2=', z1-z2)
print('z1*z2=', z1*z2)
print('z1/z2=', z1/z2)
```

```
z = 3+2j
print('real part of 3+2j:', z.real)
print('imaginary part of 3+2j:', z.imag)
print('conjugate of 3+2j:', z.conjugate())
print('absolute value of 3+2j:', abs(z))
print('(3+2j)^3:', pow(z,3))
```

calculate r

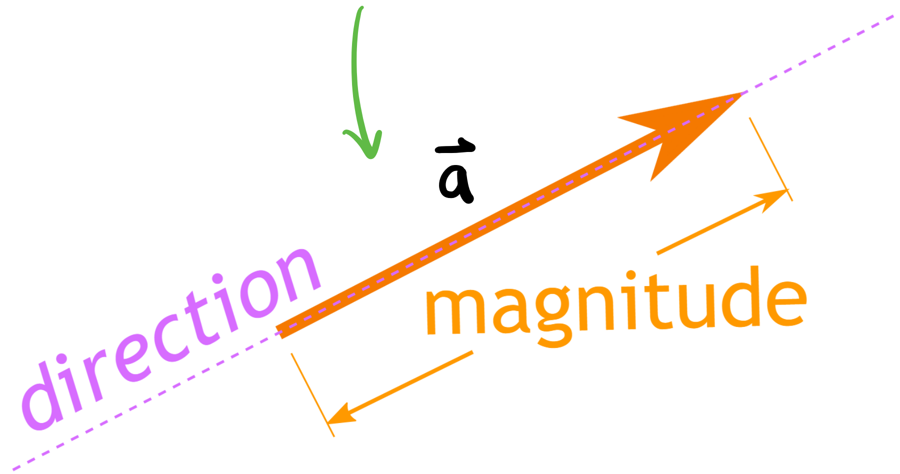
Vectors

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$a_1, a_2 \in \mathbb{R}$

$\vec{a} \in \mathbb{R}$

column vector



$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$b_1, b_2, b_3, b_4 \in \mathbb{C}$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \vdots \\ v_n \end{pmatrix}$$

1 column
n rows

Norm

$$\|\vec{x}\| = \sqrt{x_1 x_1 + x_2 x_2 + \cdots + x_n x_n} = r$$

$$= \sqrt{\vec{x} \cdot \vec{x}}$$

dot product

$$= |\vec{x}|$$

$$\vec{u} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \Rightarrow \|\vec{u}\| = \sqrt{3(3) + 2(2) + 4(4)} = \sqrt{29} = 5.38$$

Matrices

$$N = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}$$

2×2

n columns

$$M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

m rows

Operations: Vector & Matrices

$$N\vec{a} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} n_{11}a_1 + n_{12}a_2 \\ n_{21}a_1 + n_{22}a_2 \end{pmatrix}$$

$$2\times 2 \quad 2\times 1 \quad = \quad 2\times 1$$

$$(m \times n)(n \times p) = (m \times p)$$

$$=$$

$$\vec{v}\,\vec{u} = ?$$

$$(1\times n)\,(1\times n)$$

$$\neq$$

Transpose

column vector

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \longrightarrow \vec{a}^T = \begin{pmatrix} a_1 & a_2 \end{pmatrix}$$

row vector

2x1

1x2

$\vec{v}^T \vec{u}$ ✓

$(n \times 1)\,(1 \times n) = (n \times n)$

=

$\vec{v}\,\vec{u}^T$

$(1 \times n)\,(n \times 1) = 1 \times 1$

✓

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \longrightarrow A^T = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

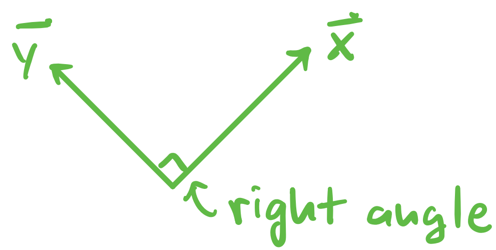
Dot Product

(scalar product or inner product)

$$\vec{x} \cdot \vec{y} = \sum_{k=1}^{n} x_k y_k$$

$$= x_1 y_1 + x_2 y_2 + x_3 y_3 + \cdots + x_n y_n$$

note if $\vec{x} \cdot \vec{y} = \underline{0}$ $\overset{\text{then}}{\Rightarrow}$ $\vec{x}$ and $\vec{y}$ are orthogonal to each other



Tensor Product

$$A \otimes B = \begin{pmatrix} a_{11} B & \cdots & a_{1n} B \\ \vdots & & \\ a_{m1} B & \cdots & a_{mn} B \end{pmatrix}$$

$$\vec{a} \otimes \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ a_2 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}$$

$2n \times 1$

$$C \otimes D = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \otimes \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} & 2 \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \\ 3 \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} & 4 \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 5 & 4 & 10 \\ 1 & 3 & 2 & 6 \\ 6 & 15 & 8 & 20 \\ 3 & 9 & 4 & 12 \end{pmatrix}$$

Vectors & Matrices with Python

```python
a = [3, -1, 5]          # vector

A = [                   # matrix
     [ 1 ,  2 , -3],
     [-6 ,  5 ,  4],
     [ 9 , 10 , 11],
     [ 7 , -8 ,  3],
   ]


# A * a

result_A_a = []

rows_A = len(A)
cols_A = len(A[0])
rows_a = len(a)
for i in range(rows_A):
    result_A_a.append(0)
    for j in range(rows_a):
        result_A_a[i] += A[i][j] * a[j]
```

```python
import numpy as np

a = np.array([3, -1, 5])        # vector

A = np.array([                  # matrix
     [ 1 ,  2 , -3],
     [-6 ,  5 ,  4],
     [ 9 , 10 , 11],
     [ 7 , -8 ,  3],
    ])

# A * a

res_A_a = A.dot(a)
```

Dirac Notation

(bra-ket notation)

shorter notation

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |c\rangle$$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \underline{|a\rangle}$$

$n \times 1$ ket

Paul Dirac

$a_i \in \mathbb{C}$

Take
- transpose
- conjugate

$$\Rightarrow \begin{pmatrix} a_1^* & a_2^* & \cdots & a_n^* \end{pmatrix} = \langle a|$$

bra

$$\vec{b} = |b\rangle \rightarrow \langle b|$$

<u>inner product</u>:

$$\overbrace{\langle b|a\rangle}^{\text{bra-ket}} = C$$

single number

$$(1 \times n)(n \times 1) = 1 \times 1$$

can also do:
$\langle a|a\rangle$

Example

$$\vec{a} = \begin{pmatrix} 1+i \\ 2-3i \\ 3+2i \end{pmatrix} = |a\rangle \qquad \vec{b} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = |b\rangle$$

$$\langle a| = \begin{pmatrix} 1-i & 2+3i & 3-2i \end{pmatrix}$$

$$\langle a|b\rangle = \begin{pmatrix} 1-i & 2+3i & 3-2i \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$= 1-i+4+6i+3-2i$$

$$= 8-3i \quad \in \mathbb{C}$$

(single number)

Hilbert Space

$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2$

addition:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

product by scalar:

$$c \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} ca_1 \\ ca_2 \end{pmatrix}$$

also in same vector space

- Vector space :
  - elements ← vectors
  - addition
  - product by scalar

  } comply w/properties

inner product:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

- Hilbert space $\mathcal{H}$: vector space + inner product

→ finite dimension
→ $|a\rangle \in \mathbb{C}^d$
→ $|a\rangle$ normalized
↓
length $= 1$

} for Quantum Computing
↓
$d = 2^n$

$n > 0$

Qubit

→ two-level quantum mechanical system

when $n=1$

$d=2^n$

$$|\psi\rangle \in \mathcal{H}_2$$

$\mathcal{H}_2 = \mathbb{C}^2$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\{|0\rangle, |1\rangle\}$ computational basis

$\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$ normalization

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

if needed, do:

$$\frac{|\psi\rangle}{\| |\psi\rangle \|} = |\psi\rangle_{\text{normalized}}$$

- A qubit is the basic unit of information in quantum computing

Postulates of Quantum Mechanics

P1: How to describe states of quantum systems

The state of a system is described by a unit vector in a Hilbert space $\mathcal{H}$

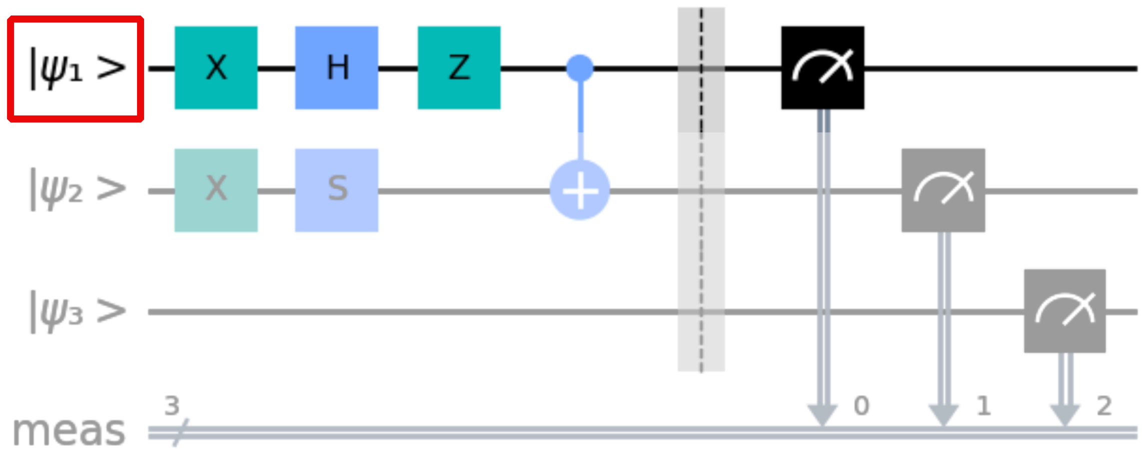
→ definition of qubit

$$|\phi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$|\phi_2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right| = 1$$



P2: How quantum states evolve

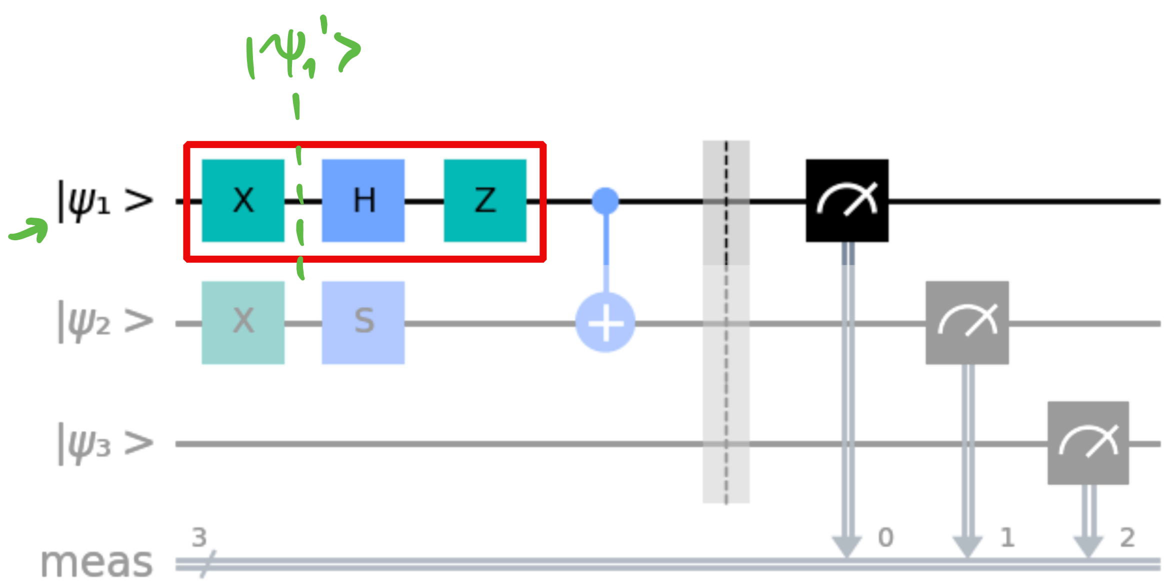
matrix or operator

$M|a\rangle = |b\rangle$

The time-evolution of the state of a closed quantum system is described by a unitary operator U ← quantum gate

→ a quantum system changes over time

$X|\psi_1\rangle = |\psi_1'\rangle$



P3: How to extract information from quantum systems

The measurements of quantum states are probabilistic

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

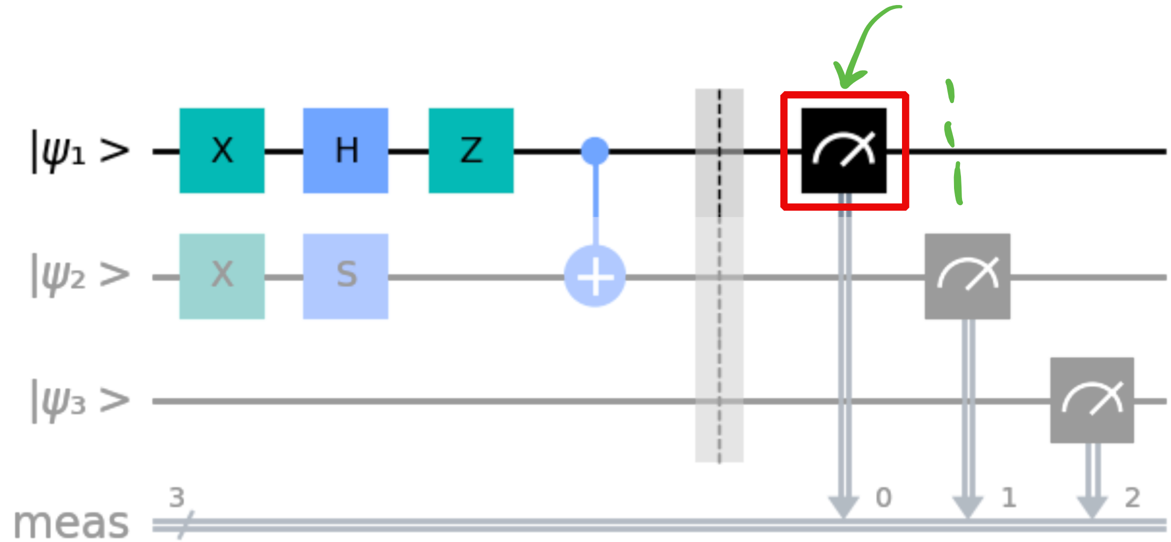
$|\alpha|^2 + |\beta|^2 = 1$

$\Rightarrow$

$$\left.\begin{aligned} p(0) &= |\alpha|^2 \\ p(1) &= |\beta|^2 \end{aligned}\right\}$$

$$= 1 - |\alpha|^2$$

α, β : probability amplitudes



P4: How to combine quantum systems

The tensor product defines the resulting system from the combination of other quantum systems

$$|\phi_1\rangle \in \mathbb{C}^2$$
$$|\phi_2\rangle \in \mathbb{C}^2$$
$$\Rightarrow \quad |\phi_1\rangle \otimes |\phi_2\rangle \in \mathbb{C}^4 = \mathbb{C}^2 \cdot \mathbb{C}^2 = \mathbb{C}^{2^2}$$

$$|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \in \mathbb{C}^{2^3} = \mathbb{C}^8$$

$2^n \leftarrow$ #qubits