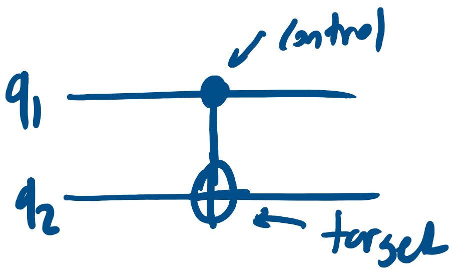
two qubits $\begin{matrix} 0 & 0 \\ 1 & 1 \end{matrix}$

$$\begin{matrix} 00 \to \\ 01 \to \\ 10 \to \\ 11 \to \end{matrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad \begin{pmatrix} 4 \times 4 \end{pmatrix}$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Monday, October 21, 2024 5:03 PM



$CNOT = \underline{CX}$

$CNOT|00\rangle = |00\rangle$

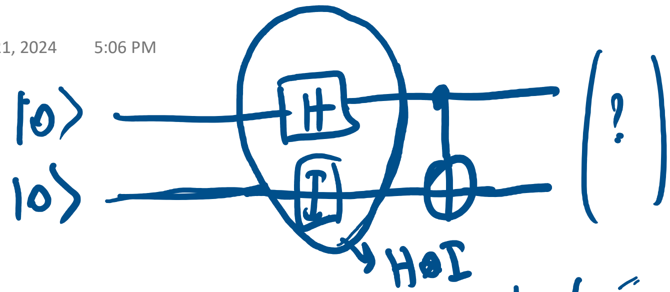
$CNOT|01\rangle = |01\rangle$

$CNOT|10\rangle = |11\rangle$

$CNOT|11\rangle = |10\rangle$

reversible deterministic operator

$$
\begin{array}{c|c}
 & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} &
\left(\begin{array}{cc|cc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{array}\right)
\end{array}
= \left(\begin{array}{c|c} I & 0 \\ \hline 0 & X \end{array}\right)
$$



$|0\rangle|0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |00\rangle$

$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

first qubit

second qubit

$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \cdot \left(|0\rangle\right)$

50% 50%

$\frac{1}{\sqrt{2}}\left(\underline{|00\rangle} + \underline{|10\rangle}\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + \underline{|11\rangle}\right)$

$|0|0\rangle$ $|1|1\rangle$

Monday, October 21, 2024 5:18 PM

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \leftarrow$$

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \leftarrow$$

$$\begin{array}{l} 00\to \\ 01\to \\ 10\to \\ 11\to \end{array} \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}} \begin{array}{l} = a\cdot c \\ a\cdot d \\ b\cdot c \\ = bd \end{array} \neq \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}} \otimes \underbrace{\begin{pmatrix} c \\ d \end{pmatrix}}$$

$a, b, c, d \neq 0$

if this isn't possible,

$|U\rangle = |U_1\rangle \otimes |U_2\rangle$ then two subsystems are correlated!

$|V_1\rangle \otimes |V_2\rangle = |V\rangle$

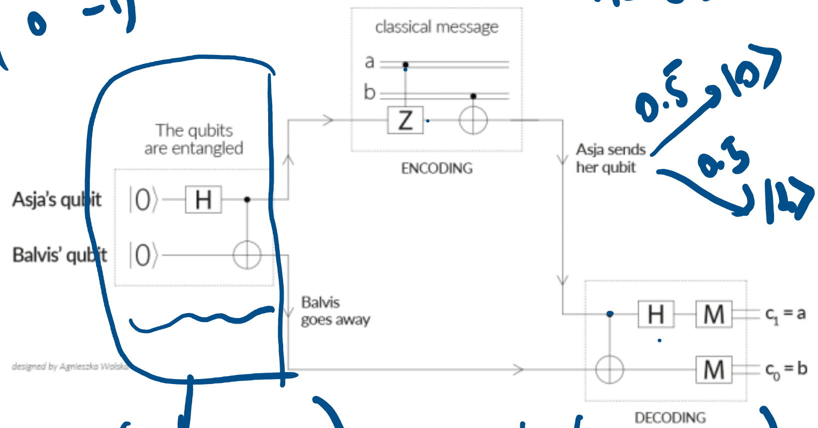
Superdense coding

Monday, October 21, 2024 15:35

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$a, b \in \{0, 1\}$

two bits of information



$0.5 \to |0\rangle$

$0.5 \to |1\rangle$

$\underline{00}$ $\quad \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$|10\rangle$

$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |0\rangle$

$|0\rangle \otimes |0\rangle \to 00$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{Z} \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \xrightarrow{X} \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$

$\xrightarrow{X} \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

$01 \to$

$\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

$\frac{1}{\sqrt{2}}(|11\rangle + |01\rangle)$

$\frac{1}{\sqrt{2}}(|1\rangle + |0\rangle) \otimes |1\rangle$

$|0\rangle \otimes |1\rangle \to 01$

$10 \to \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$

$\underbrace{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}_{H} \otimes |0\rangle$

$|1\rangle \otimes |0\rangle = 10$

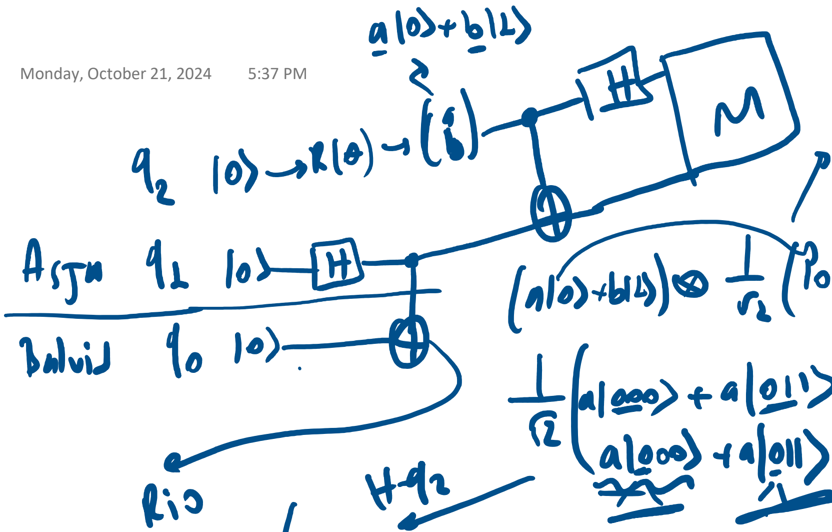
$11 \to$

$\frac{1}{\sqrt{2}}(|11\rangle - |01\rangle)$

$(|1\rangle - |0\rangle)|1\rangle$

$-|1\rangle|1\rangle$

11



$(a|0\rangle + b|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)$ CNOT $q_2 \to q_1$

$a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle$

H q_2

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a|000\rangle + \frac{1}{\sqrt{2}}a|100\rangle + \frac{a}{\sqrt{2}}|011\rangle + \frac{a}{\sqrt{2}}|111\rangle + \frac{b}{\sqrt{2}}|010\rangle - \frac{b}{\sqrt{2}}|110\rangle + \frac{b}{\sqrt{2}}|001\rangle - \frac{b}{\sqrt{2}}|101\rangle\right)$$

$$\frac{1}{2}\Big[|00\rangle\left(a|0\rangle + b|1\rangle\right) + |01\rangle\left(a|1\rangle + b|0\rangle\right) + |10\rangle\left(a|0\rangle - b|1\rangle\right) + |11\rangle\left(a|1\rangle - b|0\rangle\right)\Big]$$

Bob's

00 pr = 1/4 01 10 11

$\begin{pmatrix} b \\ a \end{pmatrix}$ $\begin{pmatrix} a \\ -b \end{pmatrix}$

00 → do nothing

01 → apply X $\begin{pmatrix} a \\ b \end{pmatrix}$

10 → apply Z ⇒ $\begin{pmatrix} a \\ b \end{pmatrix}$

11 → $\begin{pmatrix} -b \\ a \end{pmatrix}$ X, Z $\begin{pmatrix} a \\ -b \end{pmatrix}$ $\begin{pmatrix} a \\ b \end{pmatrix}$