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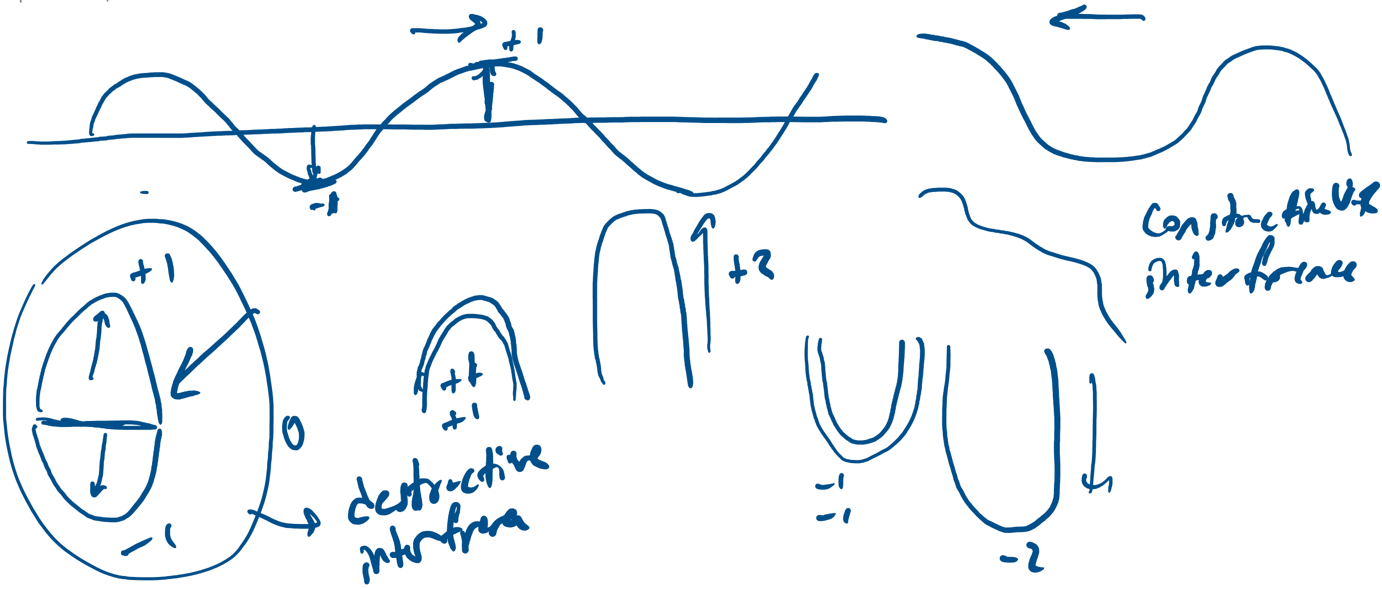

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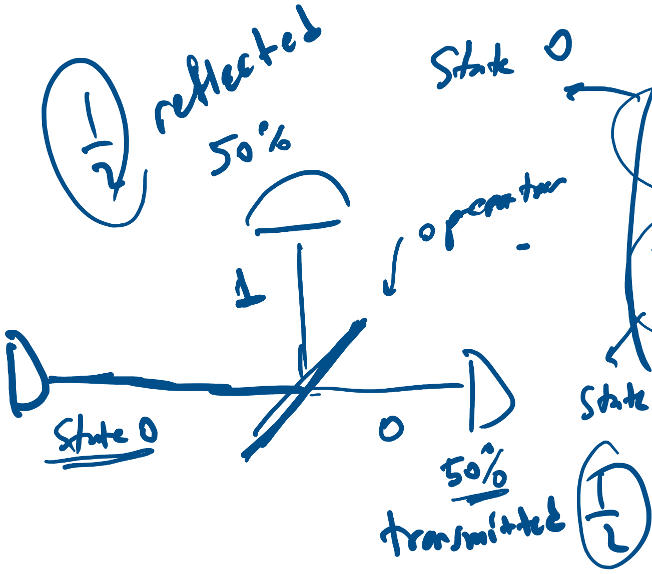


final state

Hadamard

Initial state

State 0

$$\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$$

State 1 amplitudes

$$= \begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$$ → coin flipping

Amplitudes → the value of the system being in its states.

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|v\rangle = \begin{pmatrix} a_1 \to |00\rangle \\ a_2 \to |01\rangle \\ a_3 \to |10\rangle \\ a_4 \to |11\rangle \end{pmatrix}$ ← 4 states. / 2 quantum bits qubits

$|\rangle$ ← "ket"

$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

complex valued amplitudes

$p_1 + p_2 + p_3 + p_4 = 1$ ← classical

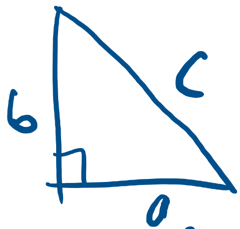
$$|v\rangle = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \in \mathbb{C}^4$$

unit vector

$$|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1$$

probabilities being in states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$a^2 + b^2 + c^2 + d^2 = 1$$



$a^2 + b^2 = c^2$

ℓ_2 - norm

ℓ_1 - norm

$$v = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

ℓ_1-norm of v is

$$|a| + |b| + |c| + |d| = |v|_1$$

$$\sqrt{|a|^2 + |b|^2 + |c|^2 + |d|^2} = ||v||_2$$

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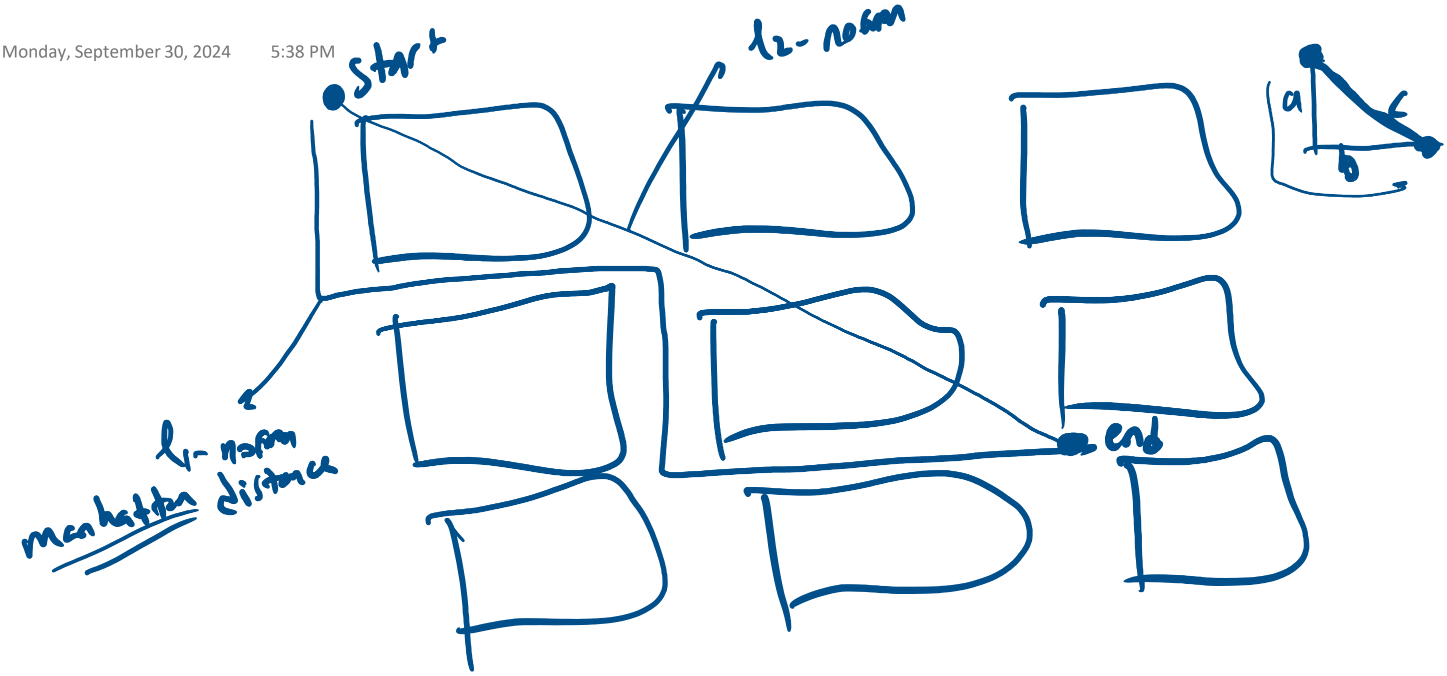


quantum state $\longleftrightarrow$ unit vector

length preserving operators!!!

$$|v'\rangle = U \cdot |v\rangle$$

final quantum state

quantum operator??

initial quantum state

length preserving operators

$\mathbb{R}$	$\mathbb{C}$
orthogonal matrices	unitary matrices

$|\rangle$

$\langle|$

not only transpose but also conjugate each value

ket

$$|a\rangle = \begin{pmatrix} 1-i \\ 2+3i \end{pmatrix}$$

$\langle v| \cdot |v\rangle$

$\langle v|v\rangle$ ← braket

inner product

bra

$$\langle a| = \begin{pmatrix} 1+i & 2-3i \end{pmatrix} \cdot \begin{pmatrix} 1-i \\ 2+3i \end{pmatrix}$$

$$= (1^2 - i^2) + (2^2 - 9i^2)$$

$$= 1+1+4+9$$

U $U^\dagger$ ← dagger

← conjugate transpose

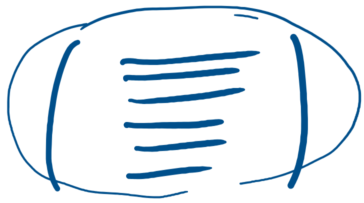
$\cdot A \cdot A^T$

length preserving

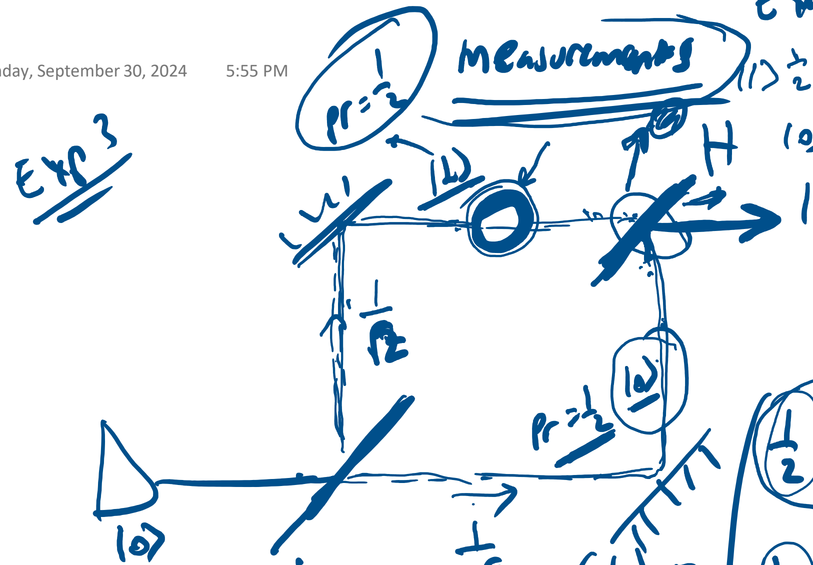
unitary matrices

$UU^\dagger = I$ $\equiv$ columns (rows) form an orthonormal set

$U^{-1} = U^\dagger$



Exp 3



Exp 2

Superposition

$H|0\rangle$

$$|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$H|+\rangle$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \frac{1}{2} \\ \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = H|0\rangle$

$|-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = H|1\rangle$

$H|+\rangle = H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}H|0\rangle + \frac{1}{\sqrt{2}}H|1\rangle = \frac{1}{2}|0\rangle + \cancel{\frac{1}{2}|1\rangle} + \frac{1}{2}|0\rangle - \cancel{\frac{1}{2}|1\rangle}$

$= 1 \cdot |0\rangle + \cancel{0 \cdot |1\rangle}$

observe only state $|0\rangle$

$|+\rangle \perp |-\rangle$!