$|0\rangle$

$\theta|0\rangle$

global phase

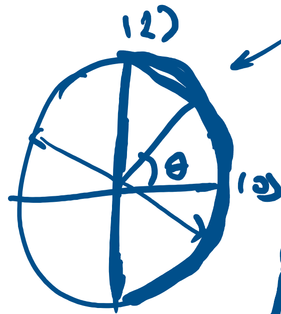
$\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2$

$a^2 + b^2 = 1$

(a, b)

$\theta \rightarrow \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \in \mathbb{R}^2$

Qiskit

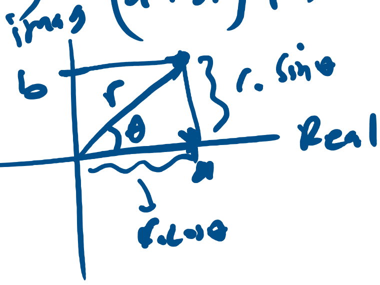


$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \in \mathbb{C}^2 \qquad \underline{\underline{|c_1|^2 + |c_2|^2 = 1}}$$

$$\begin{pmatrix} a+bi \\ c+di \end{pmatrix} \in \mathbb{C}^2 \Rightarrow (a+bi)|0\rangle + (c+di)|1\rangle$$

$$(a, b, c, d)$$

$$(a,b) \longrightarrow (r, \theta) \Rightarrow r \cdot e^{i\theta}$$



$$a+bi$$
$$\equiv$$
$$r.\cos\theta + r.\sin\theta\, i$$
$$r\,(\underbrace{\cos\theta + i\sin\theta}_{r.e^{i\theta}}) \leftarrow \text{Euler form}$$

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$(a+bi)|0\rangle + (c+di)|1\rangle$

$\begin{pmatrix} a+bi \\ c+di \end{pmatrix}$

$r_1 \cdot e^{i\theta_1}|0\rangle + r_2 e^{i\theta_2} \cdot |1\rangle$

$|e^{i\theta}|^2 = 1$,

$\Downarrow$

$\cos\theta + i\sin\theta$

$\boxed{r_1^2 + r_2^2 = 1}$

$0 \leq r_1, r_2 \leq 1$

keep the amplitude of $|0\rangle$ always only real (remove imaginary part).

global phase

$\cancel{e^{i\theta_1}}\left(r_1 \cdot |0\rangle + r_2 \frac{e^{i\theta_2}}{e^{i\theta_1}}|1\rangle\right)$

$e^{i(\theta_2 - \theta_1)} = e^{i\phi}$

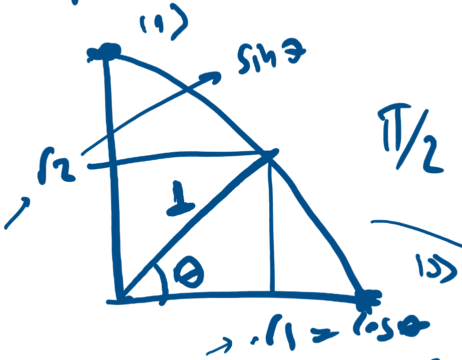
relative phase

$r_1|0\rangle + r_2 e^{i\phi}|1\rangle$ → relative phase

$e^{i\phi}$ → $\cos\phi + i\sin\phi$

$$\boxed{\cos\theta|0\rangle + \sin\theta\, e^{i\phi}|1\rangle}$$

$r_1^2 + r_2^2 = 1$



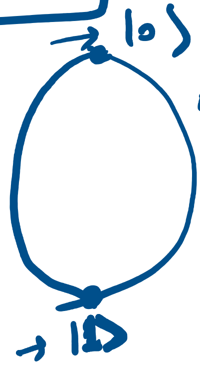
$r_1 = \cos\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{\pi}{2}$

(θ, ϕ)

$\times 2$

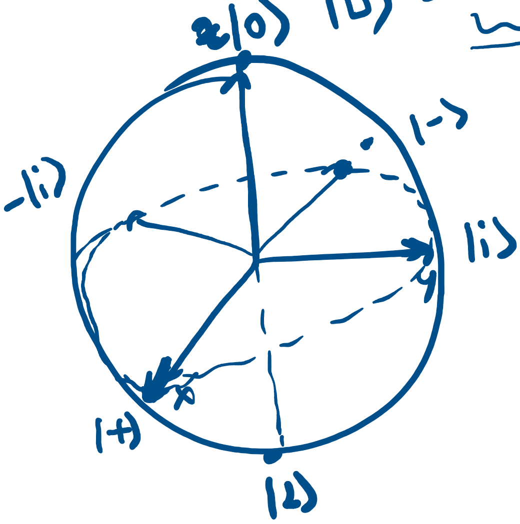


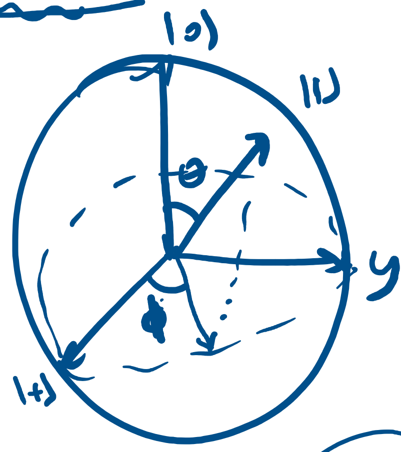
π

$\frac{\theta}{2}$

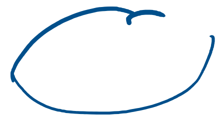
$$|\psi\rangle = \underbrace{\cos\frac{\theta}{2}}|0\rangle + e^{i\phi}\underbrace{\sin\frac{\theta}{2}}|1\rangle, \quad (\theta, \phi)$$

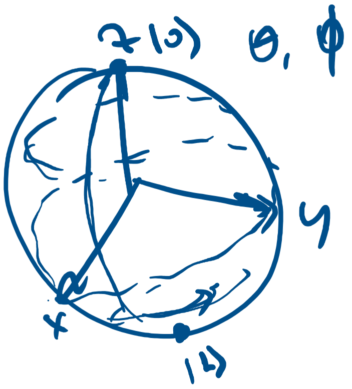
magnitudes





$$|0\rangle \xrightarrow{R_y(\theta)} \xrightarrow[R_z(\phi)]{} |\psi\rangle$$





Pauli matrices $= \{X, Y, Z, I\}$

rotations with angle θ

$RY(\theta)$

$RX(\theta)$

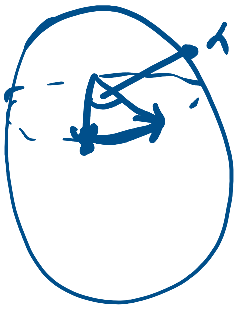
$RZ(\theta)$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

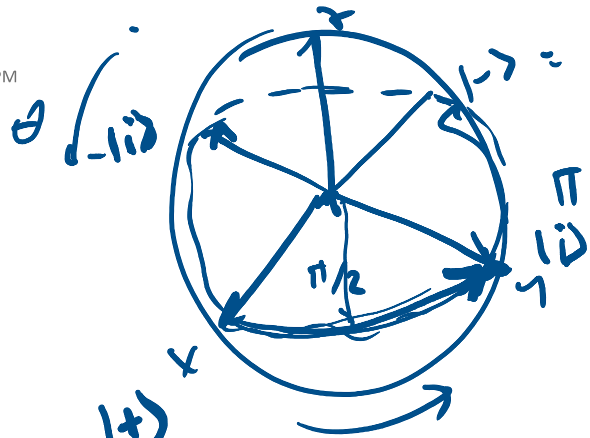
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_z(\alpha) = \begin{pmatrix} e^{-i\frac{\alpha}{2}} & 0 \\ 0 & e^{i\frac{\alpha}{2}} \end{pmatrix} = e^{-i\frac{\alpha}{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix} \quad |0\rangle$$



$$Z = R_z(\pi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} \qquad \begin{pmatrix} \cos\theta \\ e^{i(\phi+\alpha)}\sin\theta \end{pmatrix}$$

$$e^{i\pi} = \underbrace{\cos\pi}_{-1} + i\underbrace{\sin\pi}_{0} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

$Z^2 = I$

$R_Z(\phi)$

$S^\dagger \uparrow \pi/2$

$T^\dagger \uparrow \pi/4 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = |+\rangle$

$Z|+\rangle = |-\rangle$

π $\quad S^2 = Z$

$\frac{\pi}{2}$ $\quad T^2 = S$

$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$ $\quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

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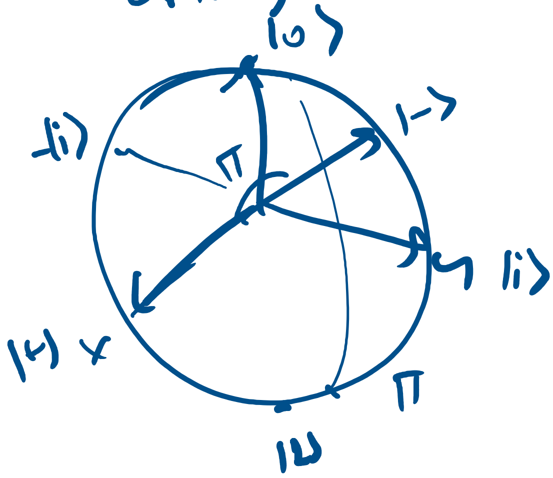
Orthogonality on the Bloch sphere



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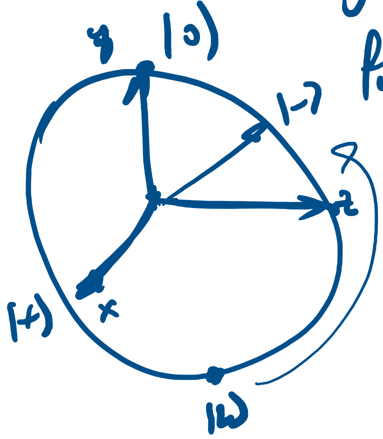
Hadamard ≡ H

90° rotation around y-axis

followed by a 180° rotation around the x-axis



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$$U_3(\theta, \phi, \lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i\lambda+i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

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SWAP

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$|00\rangle \to |00\rangle$

$|01\rangle \to |10\rangle$

$|10\rangle \to |01\rangle$

$|11\rangle \to |11\rangle$