$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = ? \quad \begin{pmatrix} a \\ c \end{pmatrix}$$

first column

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \text{second column}$$

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4×4

$$\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

$|00\rangle \to C_1$

$|01\rangle \to C_2$

$|10\rangle = C_3$

$|11\rangle = C_4$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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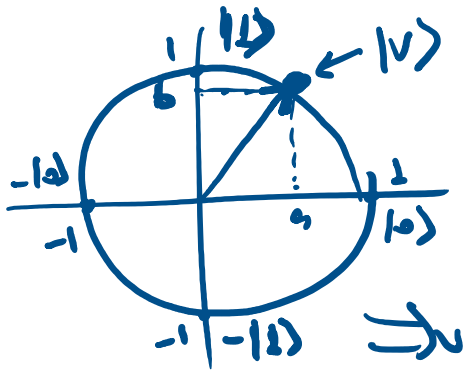
a single qubit

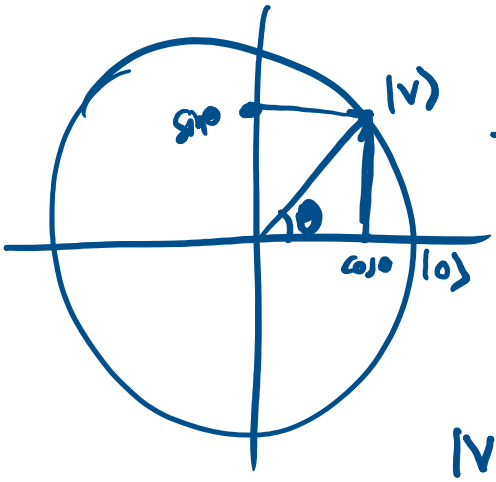
$$|v\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2 = a|0\rangle + b|1\rangle = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

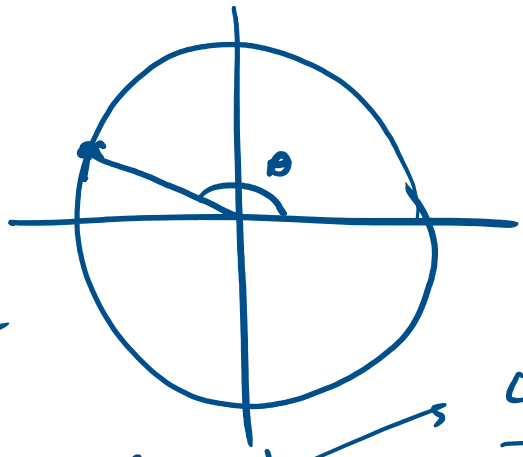
$$a^2 + b^2 = 1$$

$$x^2 + y^2 = 1$$



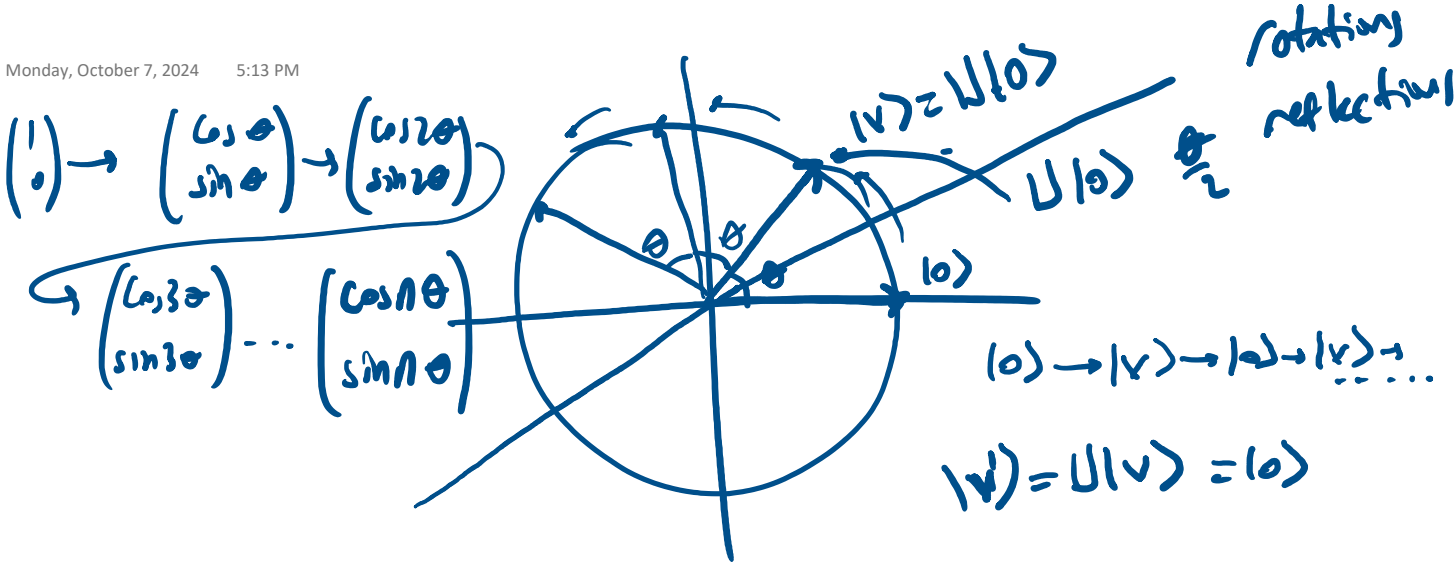
→ unit circle





$$\cos^2\theta + \sin^2\theta = 1$$

$$|v\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = (\cos\theta)|0\rangle + (\sin\theta)|1\rangle$$


rotation
reflection
|v⟩ = U|0⟩
U|0⟩
θ/2
θ
θ
θ
|0⟩
|0⟩ → |v⟩ → |e⟩ → |v⟩ → ...
|v⟩ = U|v⟩ = |0⟩

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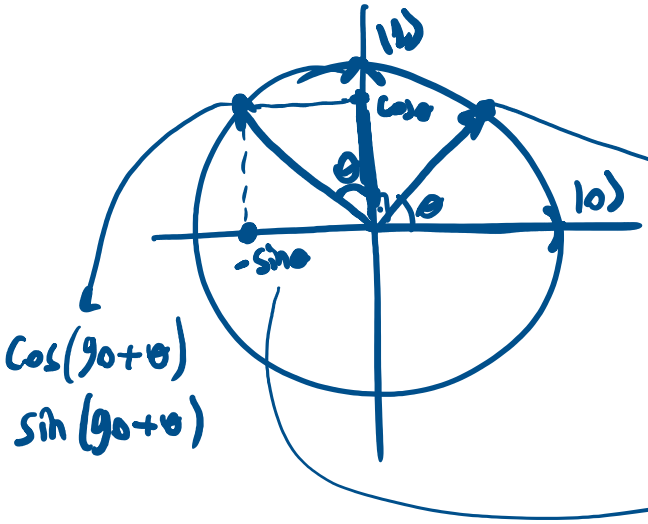
rotation, θ, its matrix form?



$R_\theta|0\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ ← 1st column

$R_\theta|1\rangle = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$ ← 2nd column

$\cos(\varphi_0+\theta)$

$\sin(\varphi_0+\theta)$

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

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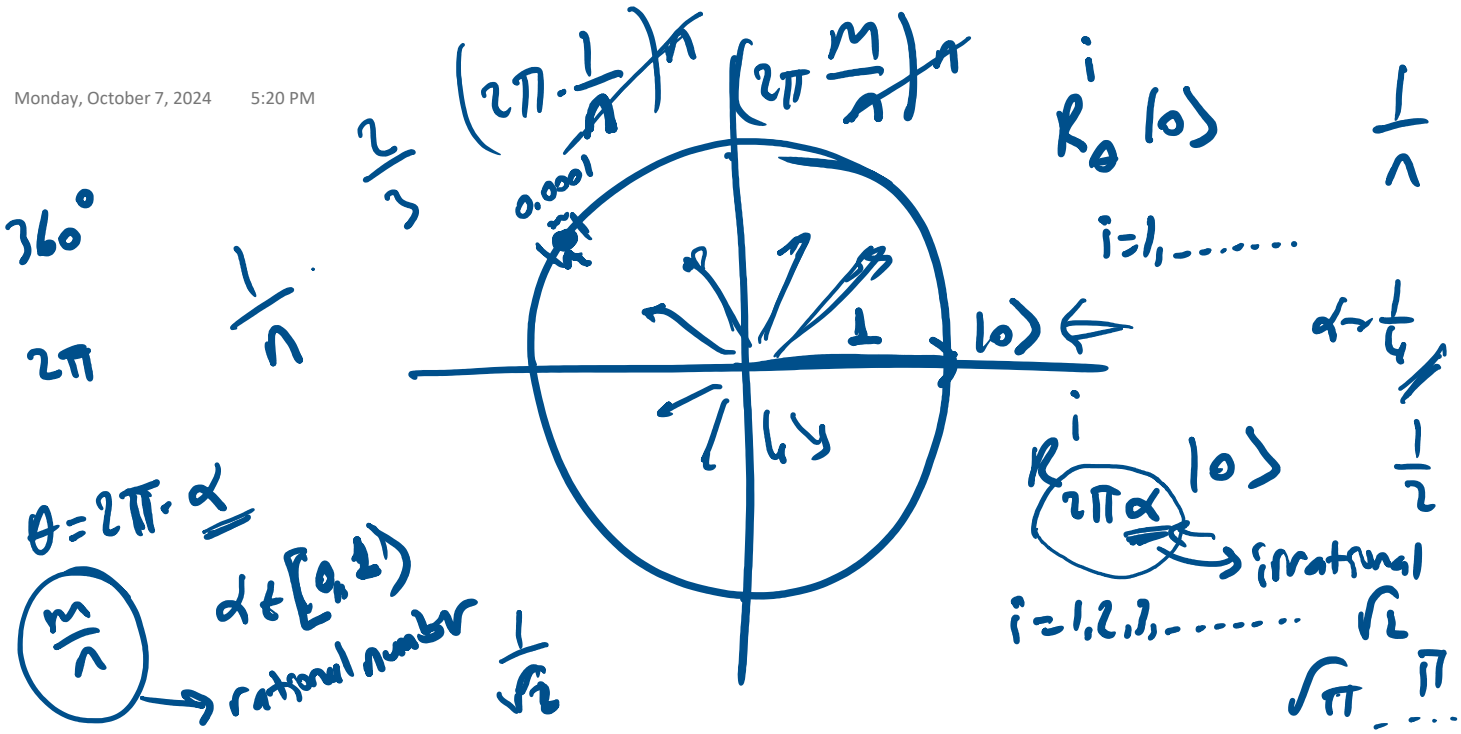

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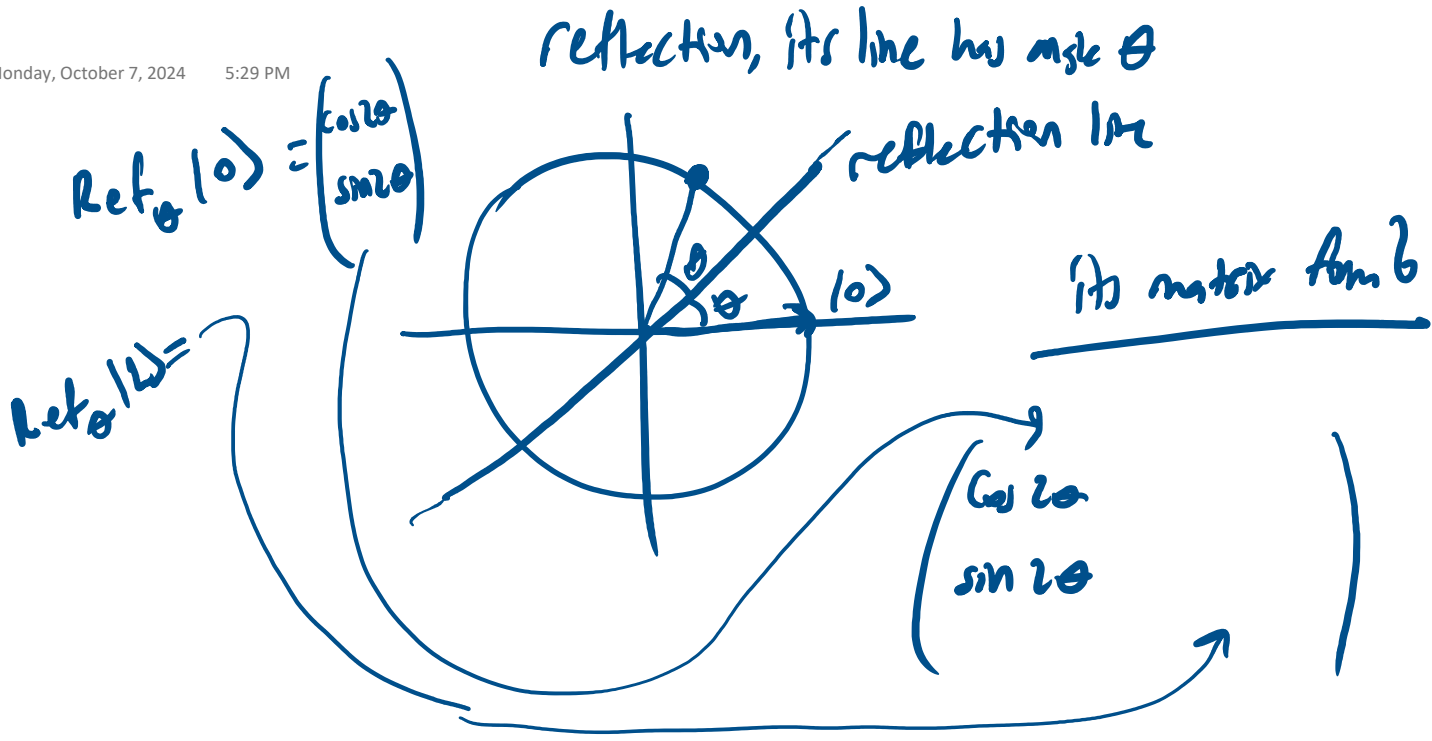

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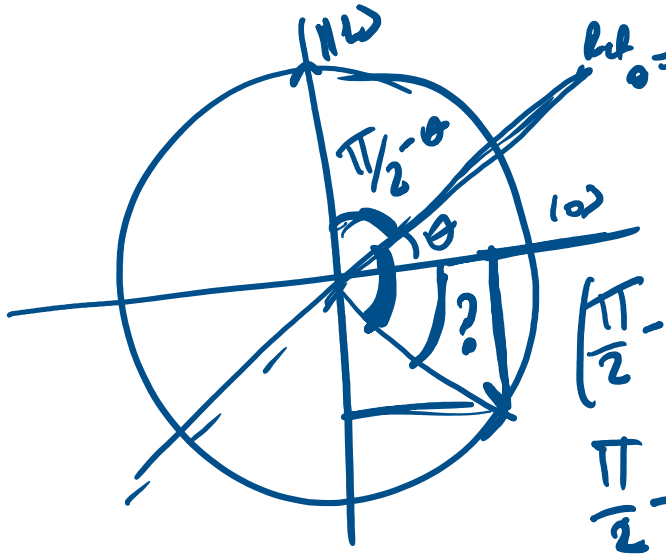

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

$\left(\frac{\pi}{2} - \theta - \theta\right)$

$\frac{\pi}{2} - 2\theta$ $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot |0\rangle$

$\cos\left(\frac{\pi}{2} - 2\theta\right)$

$-\sin\left(\frac{\pi}{2} - 2\theta\right)$

$\sin 2\theta$

$-\cos 2\theta$

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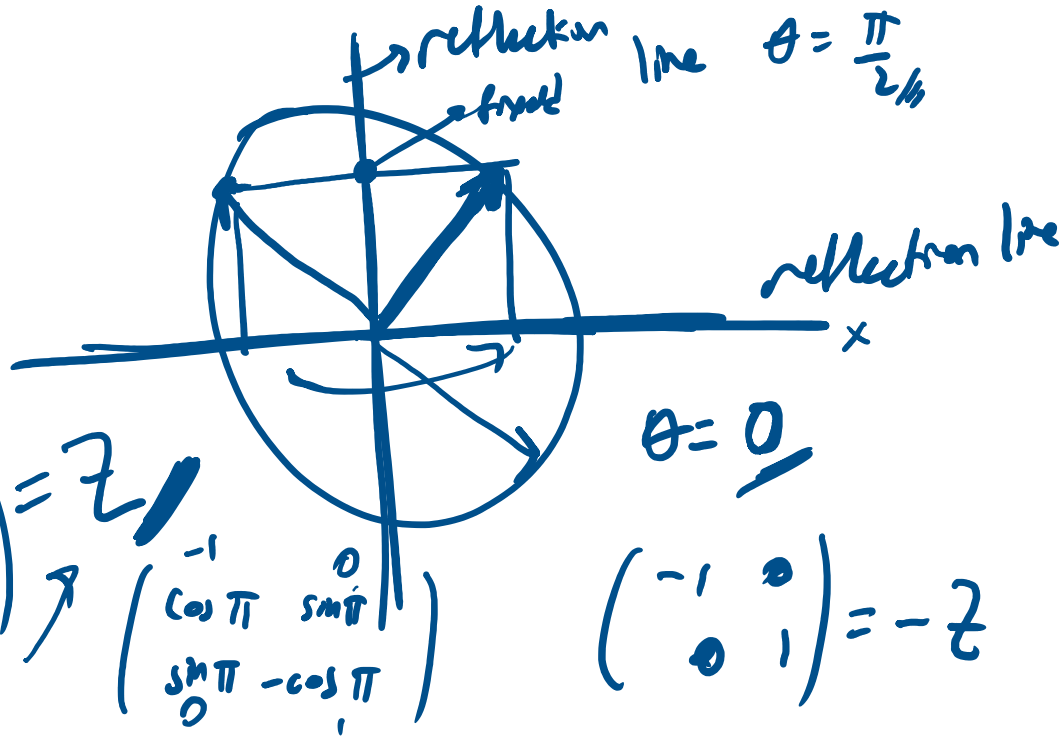

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

$$\begin{pmatrix} \cos\pi & \sin\pi \\ \sin\pi & -\cos\pi \end{pmatrix} \quad \begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -Z$$

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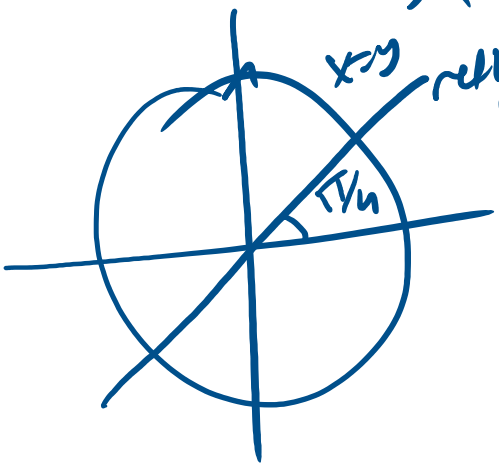
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ not? ref? $\theta = ?$

- Rotation by $0/\pi/2\pi$
  - 1
  - $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

ref ! θ ?
not ?



$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos \frac{\pi}{2} & \sin \frac{\pi}{2} \\ \sin \frac{\pi}{2} & -\cos \frac{\pi}{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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H =?

full! not! θ =?

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$H|+\rangle = |0\rangle$

$H|-\rangle = |1\rangle$

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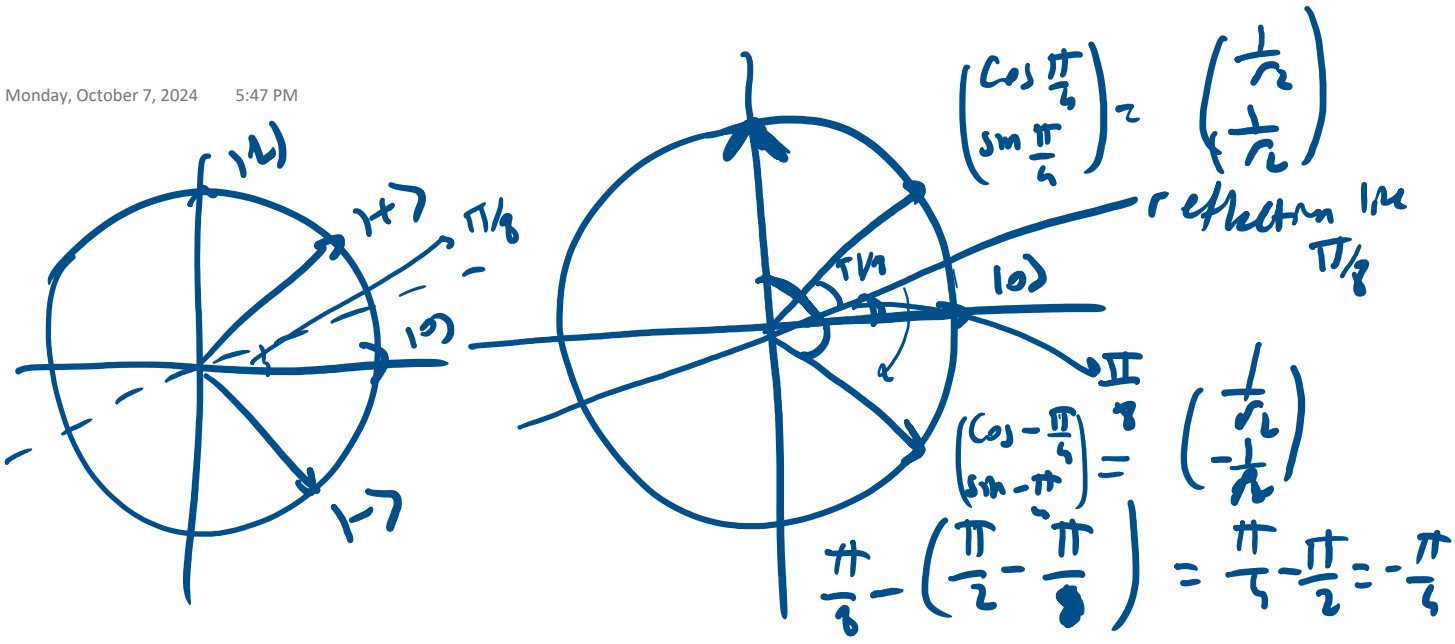


$$\begin{pmatrix}\cos\frac{\pi}{4}\\ \sin\frac{\pi}{4}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\begin{pmatrix}\cos-\frac{\pi}{4}\\ \sin-\pi\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\frac{\pi}{8} - \left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$$

100 copies of the same state

$$|\psi\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$\cos^2\theta = ?$

$\sim$

$\cos^2\theta = \frac{25}{100} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4} \sim \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$ $\theta = ?$

how to find θ?

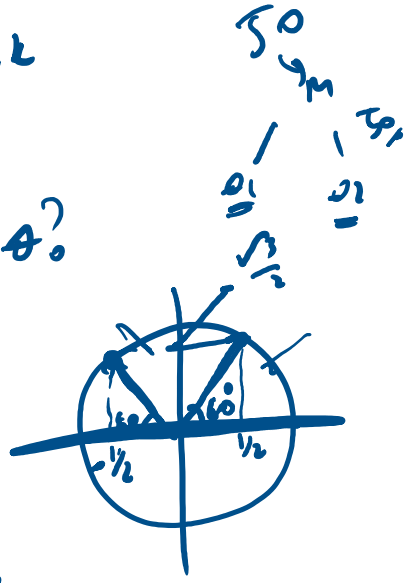
guess

across

$30°$

cos

no



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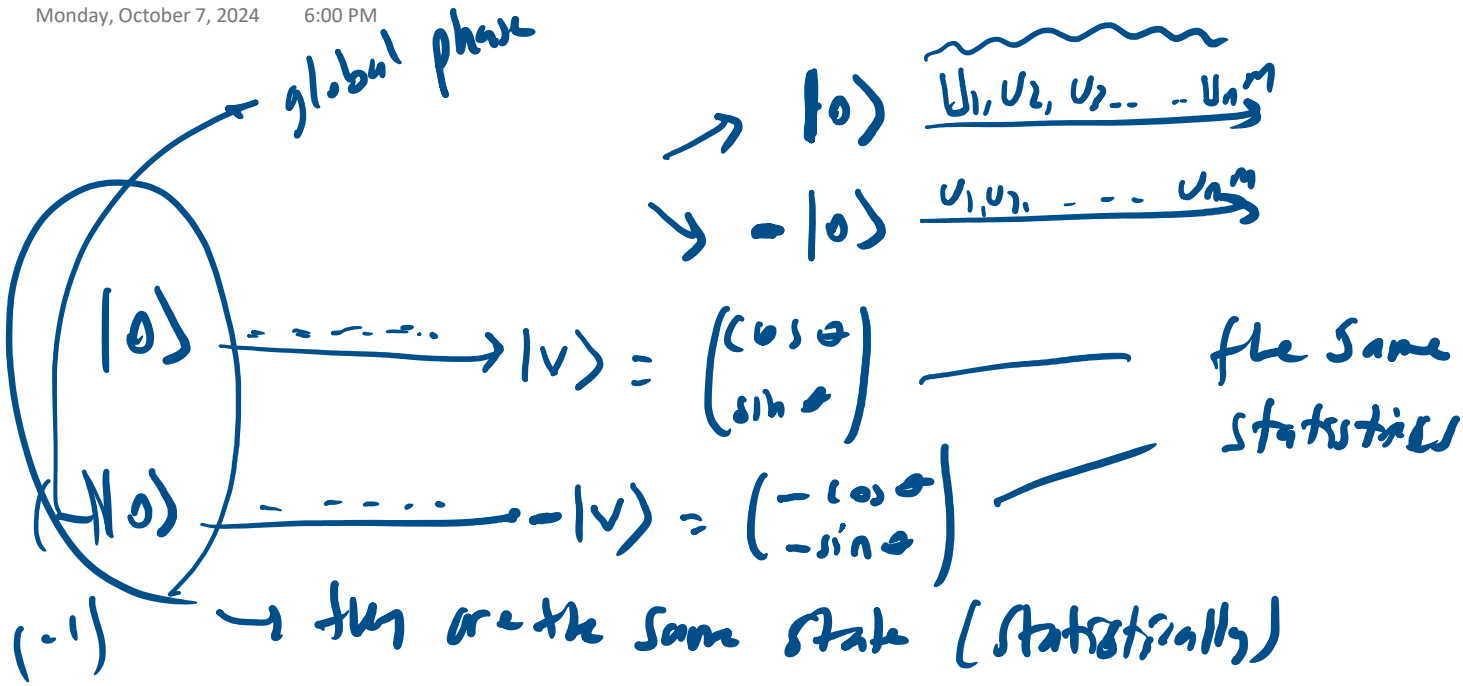

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