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What is (a) bit?

there is	there is not
true	false
0	1
yes	no
On	off
+	−
heads	tails
alive	dead
up	down
low	high

Any 2-state system

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0 1

Start	End
0	{0, 1}
or	
1	

$\xrightarrow{\text{operator}}$ new state

reversible? — I, NOT

not reversible? — ZERO, ONE

I

0 → 0

1 → 1

NOT

0 → 1

1 → 0

ZERO

0 → 0

1 → 0

ONE

0 → 1

1 → 1

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ZERO	0	1
0	on	on
1	off	off

I	0	1
0	on	off
1	off	on

start

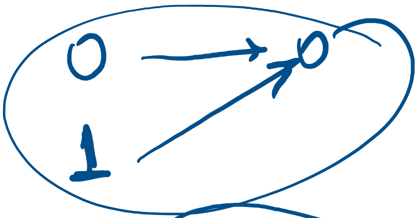
final

NOT	0	1
0	off	on
1	on	off

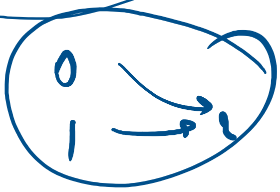
ONE	0	1
0	off	off
1	on	on

Not	0	1
0	0	1
1	1	0

a single bit



ZERO



ONE

$0 \xrightarrow{I} 0 \xrightarrow{\text{NOT}} 1 \xrightarrow{\text{ONE}} 1 \xrightarrow{\text{NOT}} 0$?

$1 \longrightarrow 0$

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2 bits

$\{0, 1\}$ $\{0, 1\}$

00
01
10
11

Nur	00	01	10	11
00	0	1	0	0
01	1	0	0	0
10	0	0	0	0
11	0	0	1	1

→ not reversible

initial state

I	00	01	10	11
00	1	0	0	0
01	0	1	0	0
10	0	0	1	0
11	0	0	0	1

→ reversible

final state

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2 bit. # of operators

	00	01	10	11
00	01			
01	00			
10	01			
11	10			

$\underbrace{4 \cdot 4 \cdot 4 \cdot 4}_{4 \cdot 4 \cdot 4 \cdot 4} = 256$

- 16
- 64
- 2^{16}
- 256
- infinite
- 16!

# of reversible operators	4!
	64

3 bits

8

$$
\begin{array}{c|cccc}
 & 00 & 01 & 10 & 11 \\
\hline
00 & 0 & 0 & 0 & 0 \\
01 & 0 & 1 & 0 & 0 \\
10 & 1 & 0 & 0 & 0 \\
11 & 0 & 0 & 1 & 0
\end{array}
$$

$4 \quad 3 \quad 2 \quad 1 = 4!$

$00 \to 10$

$11 \to 10$

$8 \; 8 \; \ldots \ldots \; 8 = 8^8 \qquad 8!$

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probabilistic bit?

Coin $\nearrow$ Heads $\equiv 0$

Coin $\searrow$ Tails $\equiv 1$

FAIR	0	1
0	0.5	0.5
1	0.5	0.5

0.5

0.5

fair coin:

flipping a fair coin	0	1
0	1/2	1/2
1	1/2	1/2

biased coin!

	0	1
0	0.3	0.3
1	0.7	0.7

sum = 1

Coin(p)	0	1
0	p	p
1	1-p	1-p

H	T
45	55

$\frac{45}{100}$

$p \approx 0.45$

~~know~~ how to find the value of p?

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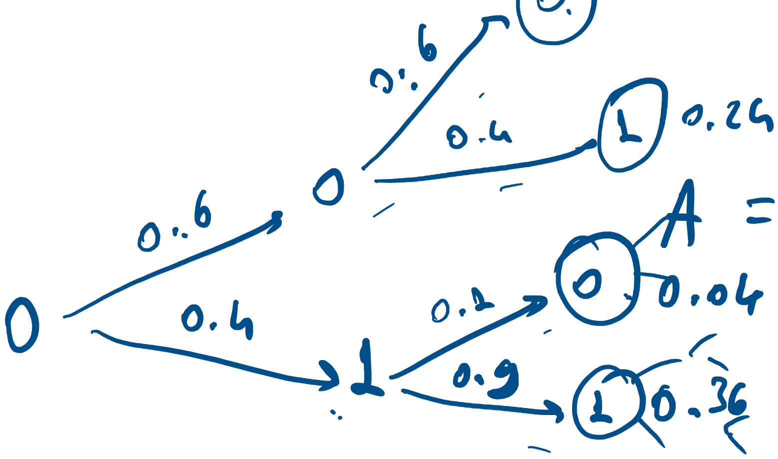
	0	1
0	a	b
1	1-a	1-b

(a,b)

	0	1
0	0.6	0.1
1	0.4	0.9

1 1



$$A = \begin{array}{cc} 0 & 1 \\ \hline \end{array} \text{ initial}$$

$$A = \begin{pmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{pmatrix}$$

$0.36 + 0.04 = 0.4$

$0.24 + 0.36 = 0.6$

0.6

Starting in state 0, Apply A twice, what are the probabilities of being in 0 and 1.

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$$\begin{pmatrix} 0.36+0.04 \\ 0.24+0.36 \end{pmatrix} = \begin{pmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{pmatrix} \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} = \underbrace{\begin{pmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{pmatrix}}_{\text{operator}} \begin{pmatrix} .1 \\ .0 \end{pmatrix}$$

matrix

vector

new state

probabilistic state

$$\text{State } 0 \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\text{State } 1 \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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<u>n-state</u> probabilistic system

probabilistic state

n-dimensional vector

$$\begin{pmatrix} \text{prob. of being in the 1st state} \\ \text{" " " 2nd state} \\ \text{" 3rd state} \\ \vdots \\ \text{prob of being in the } n^{th} \text{ state} \end{pmatrix}$$

non-negative

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{pmatrix} \geq 0$$

$+$ ―――― 1

$$\cancel{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}} \quad 1$$

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3 - state

$$\left\{ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} \right\}$$

stochastic vector

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

basis states

$$\begin{pmatrix} 0.2 \\ 0.3 \\ 0.5 \end{pmatrix} = 0.2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0.3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0.5 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

constraints on scalar values.

→ nonnegative

→ $\Sigma = 1$

→ linear combination of the basis states.

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5-state probabilistic operator

(left) stochastic matrix

$$\begin{array}{c|ccccc} & s_1 & s_2 & s_3 & s_4 & s_5 \\ \hline s_1 & p_{11} & - & - & - & p_{15} \\ s_2 & p_{21} & & & & \vdots \\ s_3 & p_{31} & & & & \vdots \\ s_4 & p_{41} & & & & \vdots \\ s_5 & p_{51} & - & - & - & p_{55} \\ & 1 & 1 & & & 1 \end{array} \geq 0$$

$p_{ij} \leftarrow$ from state j to state i

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linear system

$$v' = A \cdot v$$

new probabilistic state

probabilistic operator

= stochastic matrices

→ non-negative

→ Σ columns = 1

probabilistic state

linear combination of the basis states

→ non-negative

→ Σ = 1

two probability bits

$$\begin{array}{c} 0 \\ 1 \end{array}\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix} \otimes \begin{array}{c} 0 \\ 1 \end{array}\begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 0.3 \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix} \\ 0.7 \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix} \end{pmatrix} =$$

$$\begin{array}{c} (0.3)(0.4) = \\ (0.3)(0.6) = \\ (0.7)\cdot(0.4) \\ (0.7)(0.6) \end{array} \begin{pmatrix} 0.12 \\ 0.18 \\ 0.28 \\ 0.42 \end{pmatrix} \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array}$$

$+$

$1.$

3 bits

$101 \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \vdots \\ 1 \\ \vdots \end{pmatrix}$

states = 8

$$\underline{\begin{pmatrix} a \\ b \end{pmatrix}} \otimes \underline{\begin{pmatrix} c \\ d \end{pmatrix}} \stackrel{?}{=} \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} \qquad \frac{1}{2}\,00 \qquad \frac{1}{2}\,11$$

Not possible to separate $\Rightarrow$ two subsystems are correlated!