سوال 3:

الف)

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k + \ldots\ldots + n^k \le n.n^k \xrightarrow{c=n} 0 \le 1^k + 2^k + 3^k + \ldots\ldots + n^k \le n^{k+1}$$

$$\to \sum_{i=1}^{n} i^k = O(n^{k+1}) \qquad I$$

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k + \ldots\ldots + n^k \ge n^k + (n-1)^k + (n-2)^k + \ldots\ldots + (\frac{n}{2})^k \ge (\frac{n}{2})^k + (\frac{n}{2})^k + (\frac{n}{2})^k + \ldots\ldots + (\frac{n}{2})^k = \frac{n}{2}.(\frac{n}{2})^k = (\frac{n}{2})^{k+1}$$

$$\longrightarrow 0 \le (\frac{n}{2})^{k+1} \le 1^k + 2^k + 3^k + \ldots\ldots + n^k \to \sum_{i=1}^{n} i^k = \Omega((\frac{n}{2})^{k+1}) = \Omega(n^{k+1}) \qquad II$$

$$\xrightarrow{II \wedge I} \sum_{i=1}^{n} i^k = \Theta(n^{k+1})$$

ب)

$$\sum_{i=1}^{n} \log(i) = \log 1 + \log 2 + \log 3 + \ldots\ldots + \log n \ge \log(n) + \log(n-1) + \log(n-2) + \ldots\ldots + \log\frac{n}{2}$$

$$\ge \log\frac{n}{2} + \log\frac{n}{2} + \log\frac{n}{2} + \log\frac{n}{2} \ldots\ldots + \log\frac{n}{2} = \frac{n}{2}.\log\frac{n}{2} = \frac{n}{2}(\log n - 2) = (\frac{n}{2}.\log n - n) \to$$

$$\sum_{i=1}^{n} \log(i) = \Omega((\frac{n}{2}.\log n - n)) = \Omega(n.\log n) \qquad I$$

$$\sum_{i=1}^{n} \log(i) = \log 1 + \log 2 + \log 3 + \ldots\ldots + \log n \le \log n + \log n + \log n + \ldots\ldots + \log n = n.\log n \to$$

$$\sum_{i=1}^{n} \log(i) = O(n.\log n) \qquad II$$

$$\xrightarrow{I \wedge II} \sum_{i=1}^{n} \log(i) = \Theta(n.\log n)$$