

سوال 1

(الف)

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{100n + \log(n)}{n + (\log(n))^2} = \lim_{n \rightarrow \infty} \frac{n + \log(n)}{n + (\log(n))^2} = \lim_{n \rightarrow \infty} \frac{n}{n + (\log(n))^2} + \lim_{n \rightarrow \infty} \frac{\log(n)}{n + (\log(n))^2} = 0 + 0 = 0$$

$$\rightarrow f(n) = O(g(n))$$

(ب)

از آنجا که داریم $\log(n^2) = 2\log(n)$ پس طبق تعریف O داریم که:

$$0 \leq \log(n) \leq 2c \log(n) \xrightarrow{\div \log n} 0 \leq 1 \leq 2c \rightarrow c \geq \frac{1}{2} \wedge \forall n \rightarrow \log(n^2) = O(\log n)$$

(ج)

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{(\log(n))^5} = \infty \rightarrow \sqrt{n} = \Omega((\log(n))^5)$$

(د)

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n2^n}{3^n} = 0 \rightarrow \frac{n2^n}{3^n} = O(3^n)$$