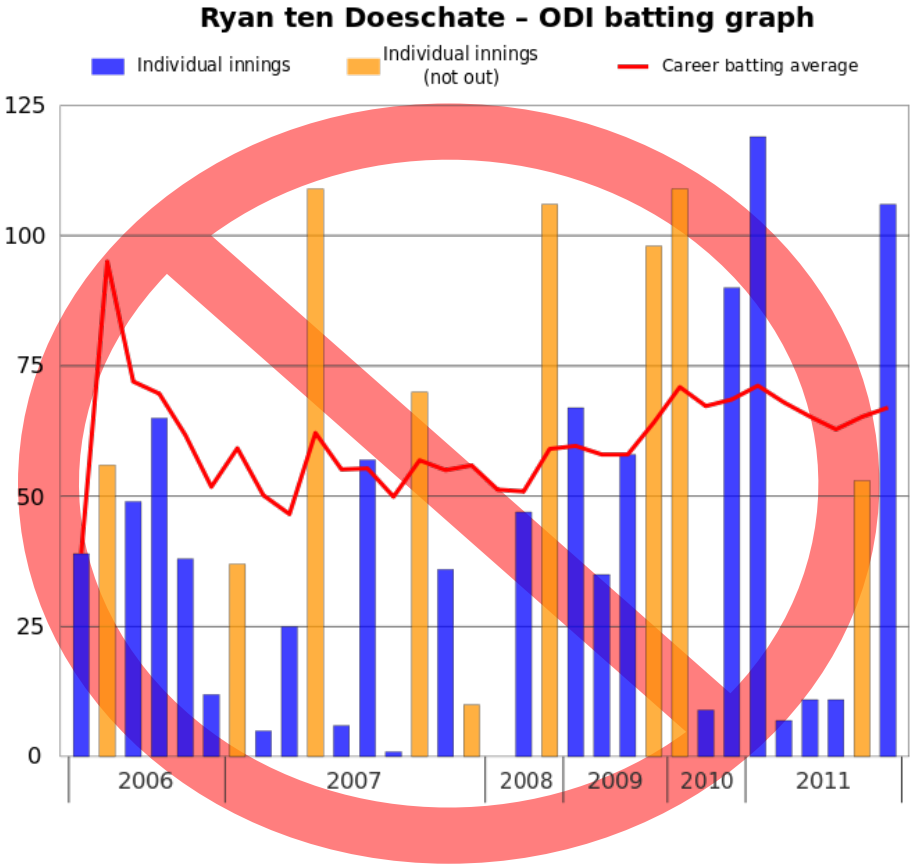
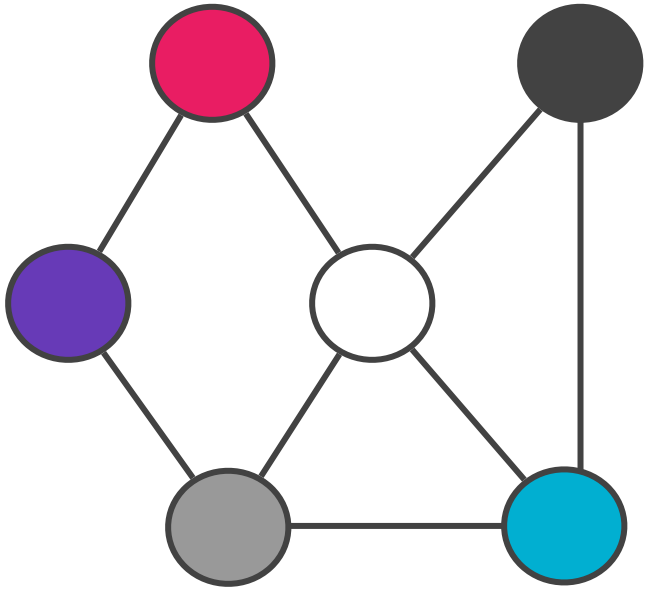
Lecture 10

Graphs

Graphs


Ryan ten Doeschate – ODI batting graph
Individual innings
Individual innings (not out)
Career batting average
125
100
75
50
25
0
2006
2007
2008
2009
2010
2011

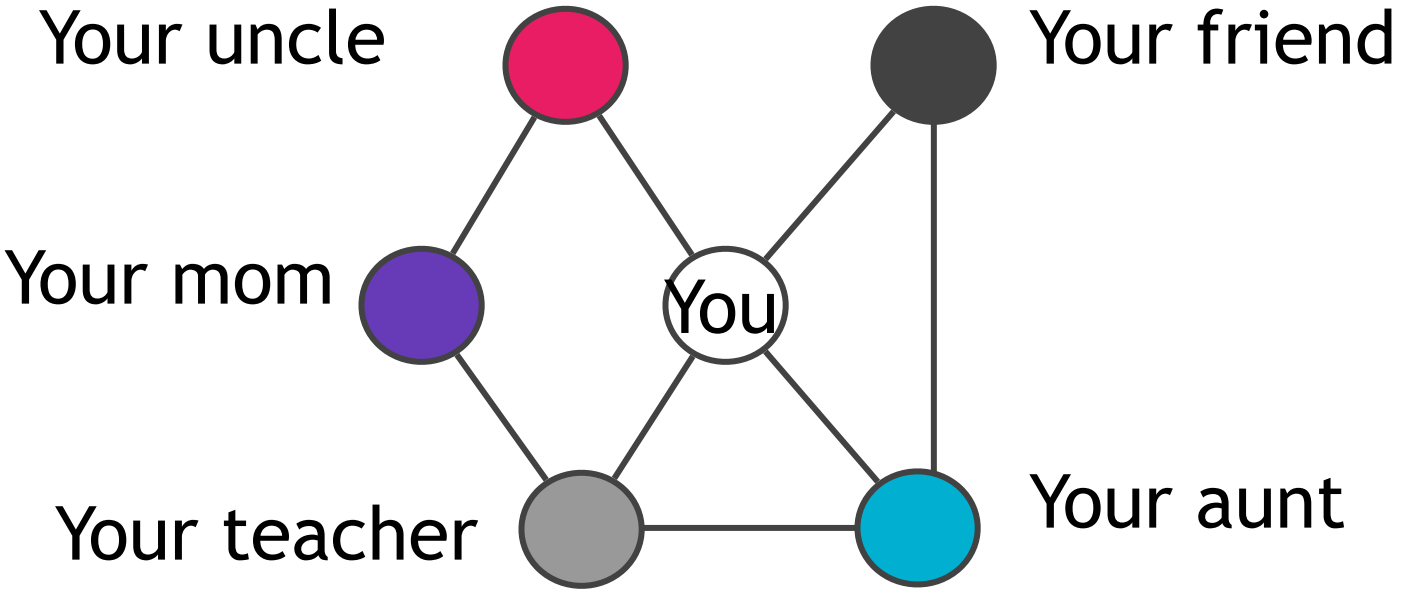
Graphs in Computer Science



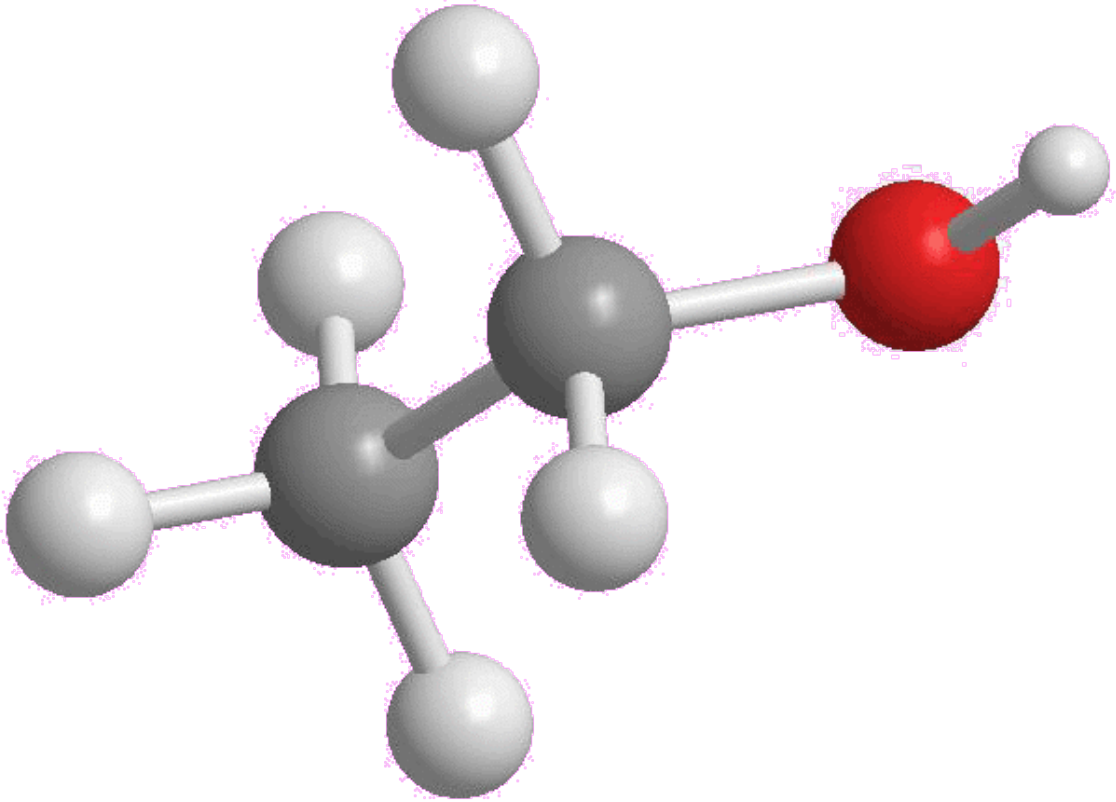
A graph is a mathematical structure for representing relationships

- A set V of **vertices** (or *nodes*)
- A set E of **edges** (or *arcs*) connecting a pair of vertices

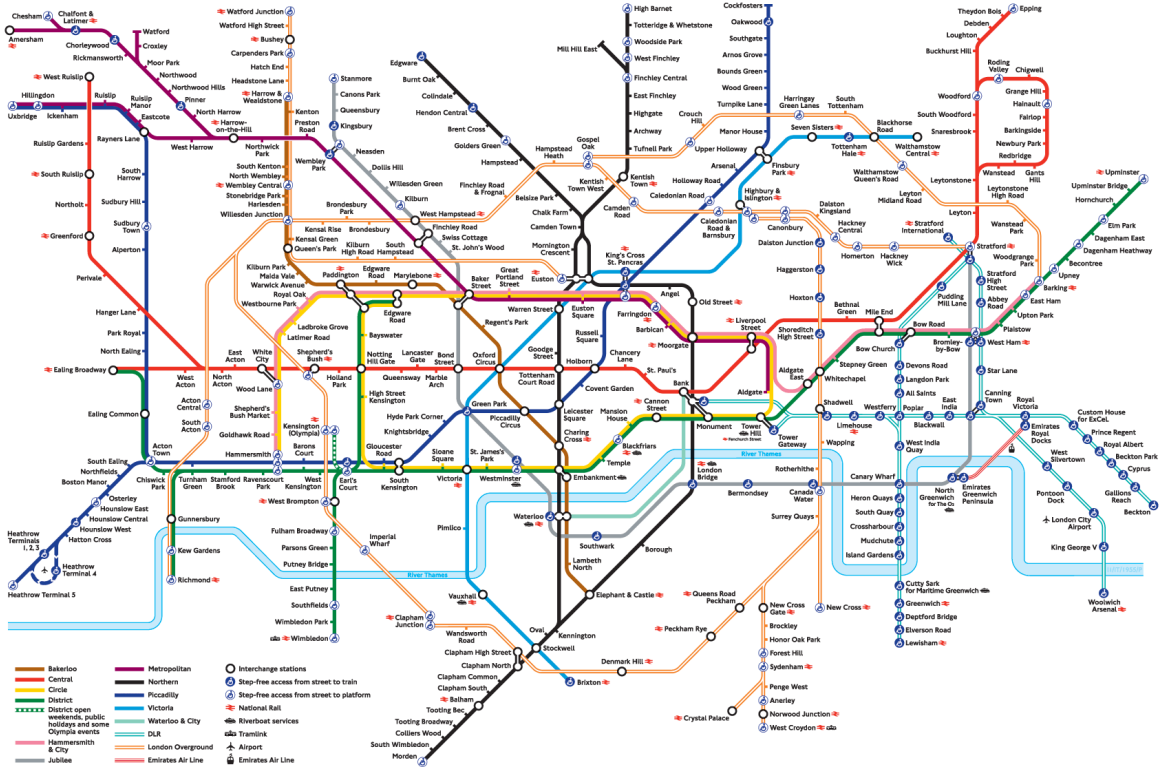
A Social Network



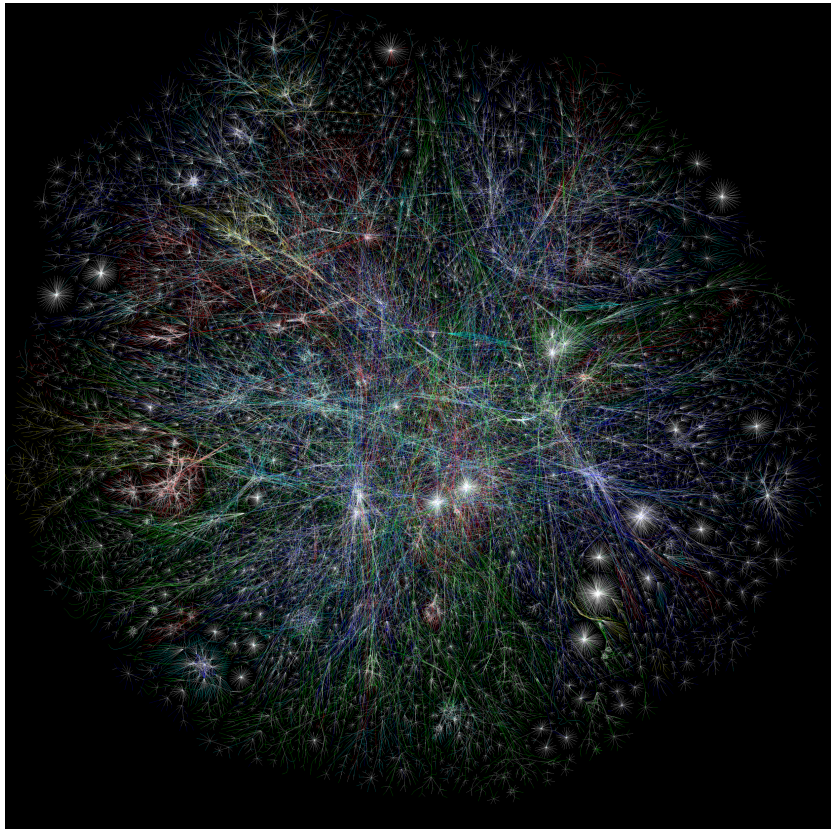
Chemical Bonds



London Underground

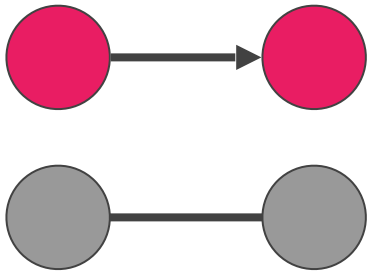


Internet



Graph terminology

- A set V of **vertices** (or *nodes*)
  - Often have an associated label
- A set E of **edges** (or *arcs*) connecting a pair of vertices
  - Often have an associated cost or weight
- A graph may be **directed** (an edge from A to B only allow you to go from A to B, not B to A)
- or **undirected** (an edge between A and B allows travel in both directions)
- Number of vertices or edges is denoted using the notation: **|V|** and **|E|**



Graph terminology : Paths

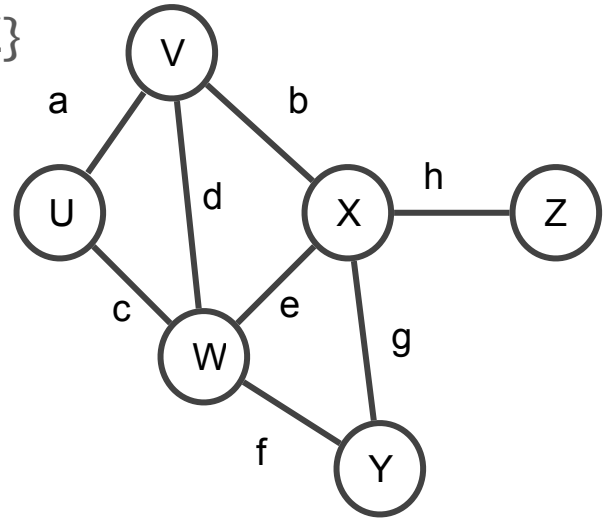
path: A path from vertex *a* to *b* is a sequence of edges that can be followed starting from *a* to reach *b*.

- can be represented as vertices visited, or edges taken
- Example: one path from *V* to *Z*: {b, h} or {V, X, Z}

path length: Number of vertices or edges contained in the path.

neighbor or **adjacent**: Two vertices connected directly by an edge.

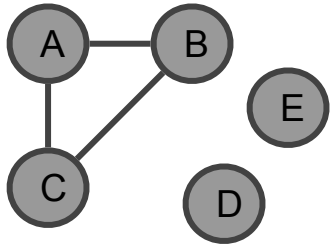
- example: V and X



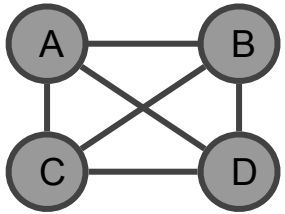
Graph terminology: Reachability, Connectedness

reachable: Vertex *a* is *reachable* from *b* if a path exists from *a* to *b*.

connected: A graph is *connected* if every vertex is reachable from every other.



complete: If every vertex has a direct edge to every other.



Graph terminology: Loops and Cycles

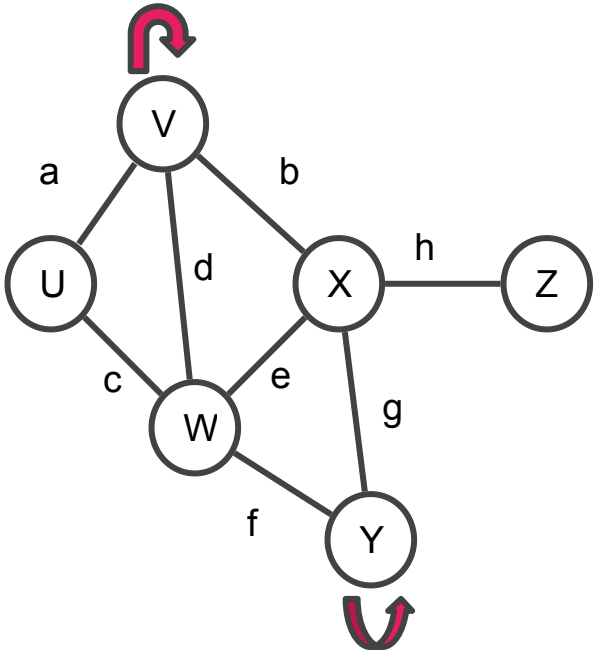
cycle: A path that begins and ends at the same node.

- example: {V, X, Y, W, U, V}.
- example: {U, W, V, U}.

- **acyclic graph**: One that does not contain any cycles.

loop: An edge directly from a node to itself.

- Many graphs don't allow loops.



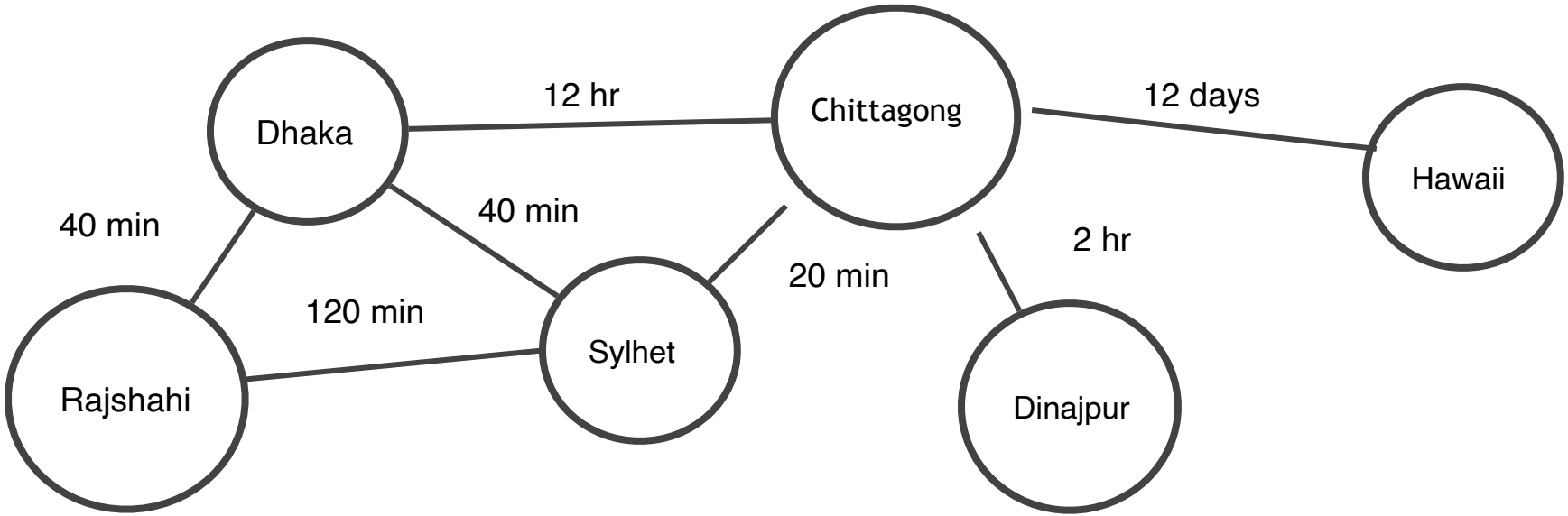
Graph terminology: Weighted Graphs

weight: Cost associated with a given edge.

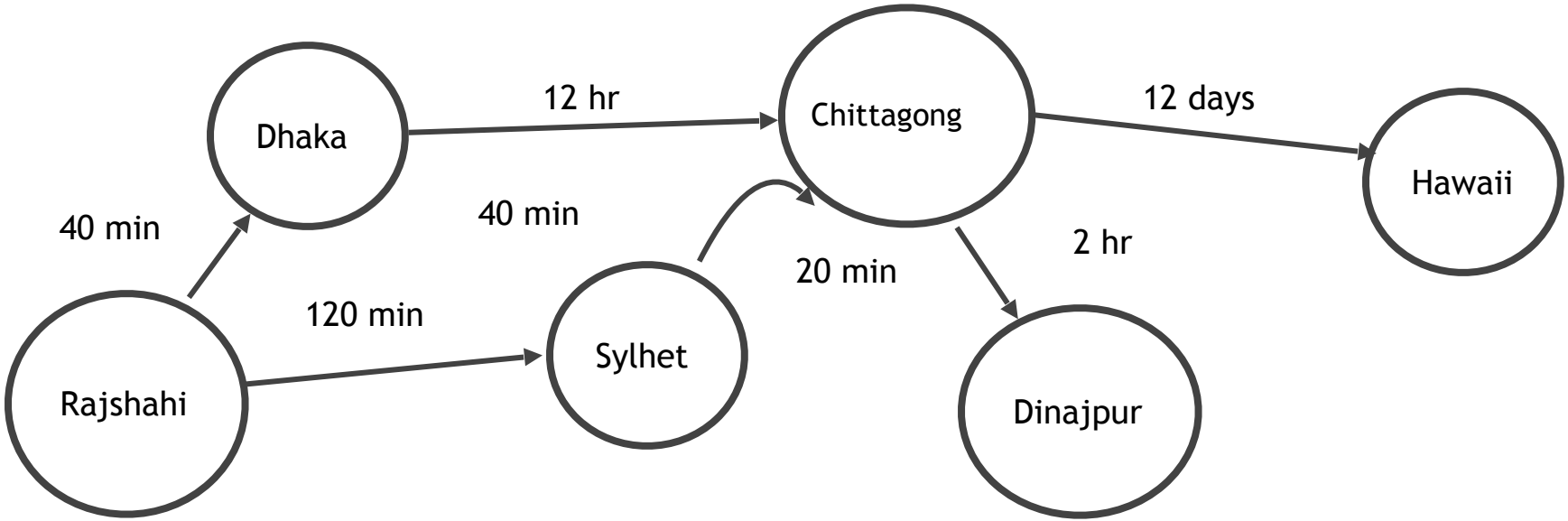
- Some graphs have weighted edges, and some are unweighted.
- Edges in an unweighted graph can be thought of as having equal weight (e.g. all 0, or all 1, etc.)
- Most graphs do not allow negative weights.

example: graph of airline flights, weighted by miles between cities:

Graph terminology: Weighted Graphs



Graph terminology: Weighted & Directed Graphs



Examples of directed and undirected graphs

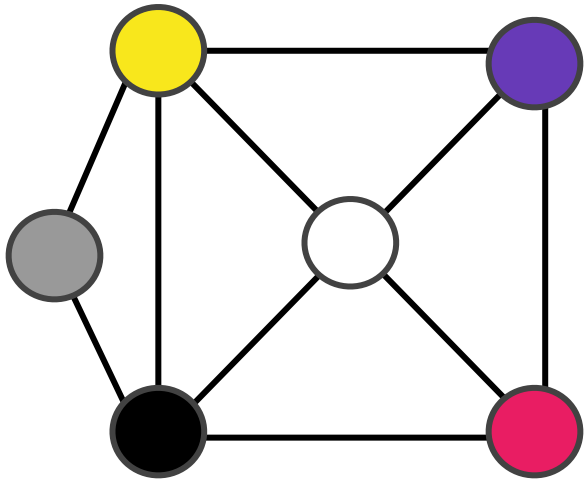
a. One way street

b. Chemical Bonds

c. River flow from one city to another.

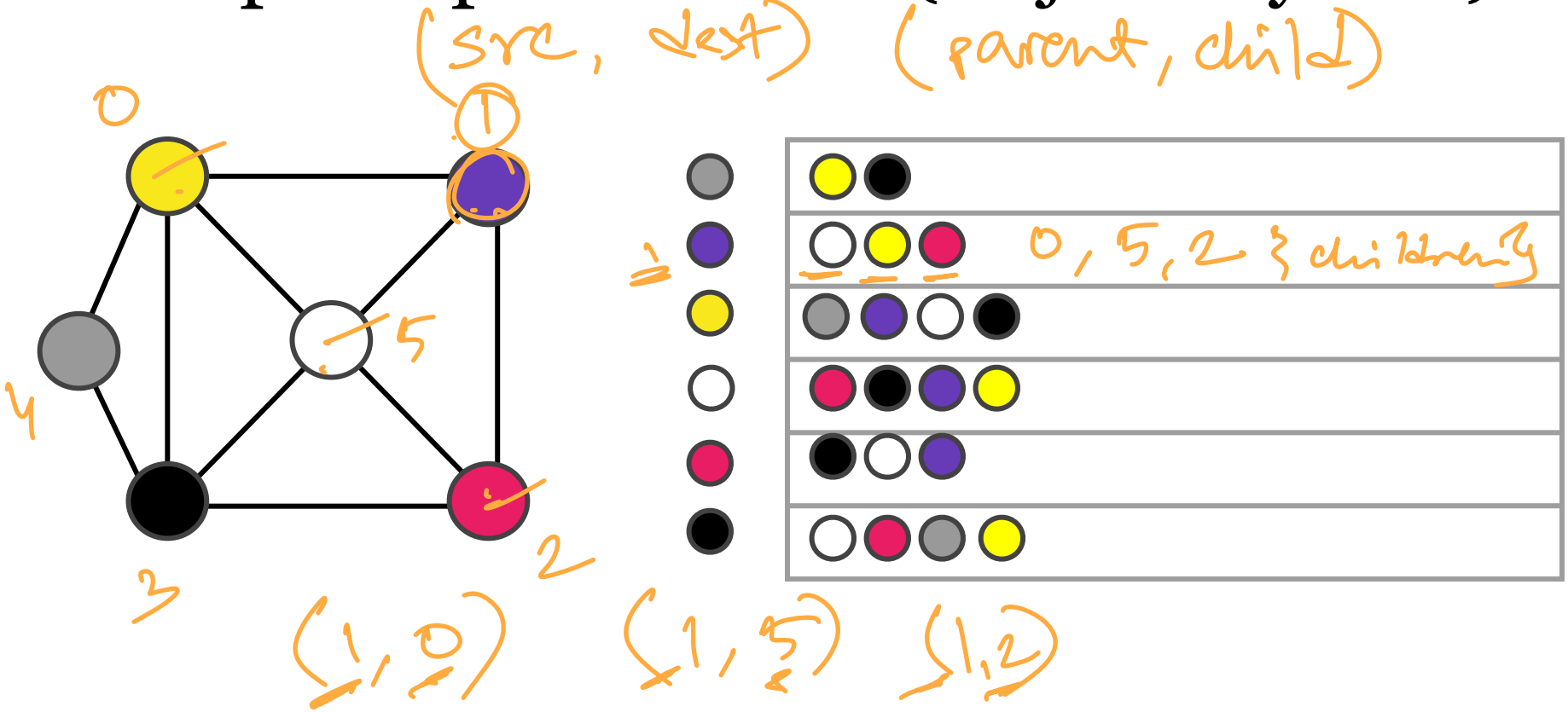
d. Telephone communication

Graph Representation(Adjacency Matrix)

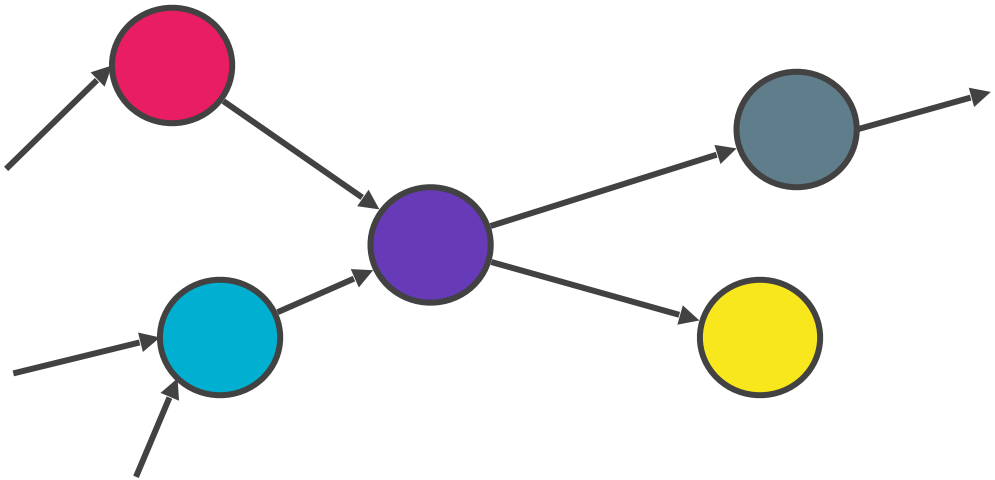


0	0	1	0	0	1
0	0	1	1	1	0
1	1	0	1	0	1
0	1	1	0	1	1
0	1	0	1	0	1
1	0	1	1	1	0

Graph Representation(Adjacency List)



Directed Graph(indegree and outdegree)



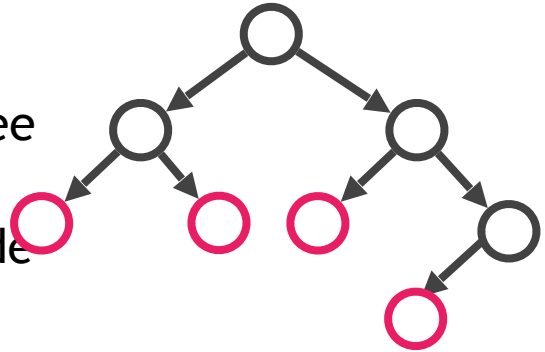
Linked List, Trees , Graphs

A *binary tree* is a graph with some restrictions:

Tree is unweighted directed acyclic graph (DAG)

Each node in-degree is at most 1 , and out-degree is at most 2

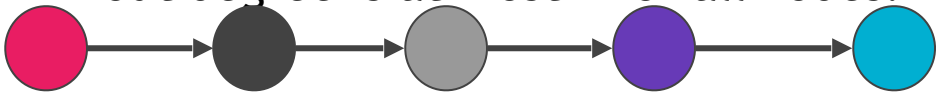
There is exactly one path from root to every node



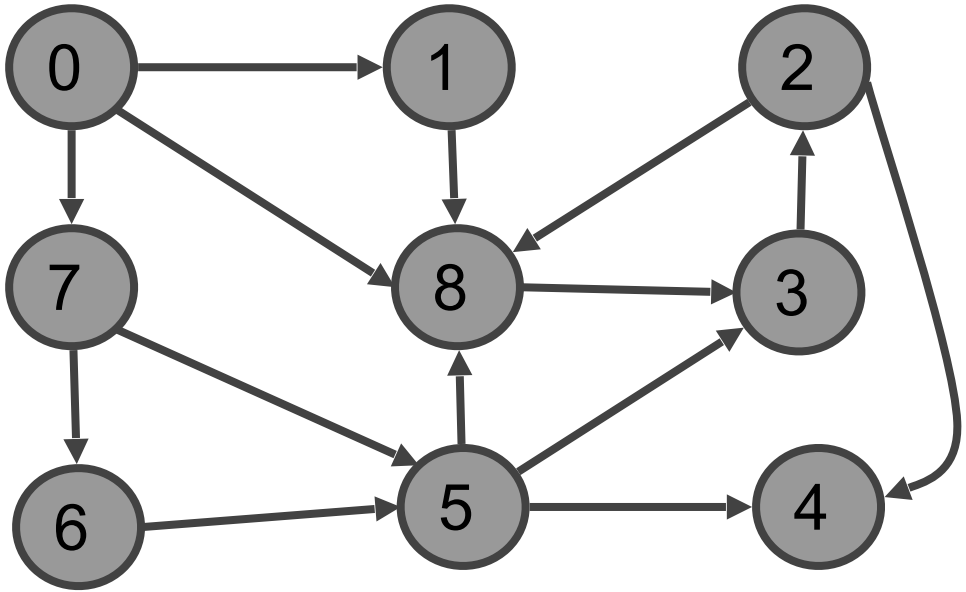
A *linked list* is also a graph:

Unweighted DAG.

In / out degree is at most 1 for all nodes.



Depth first search

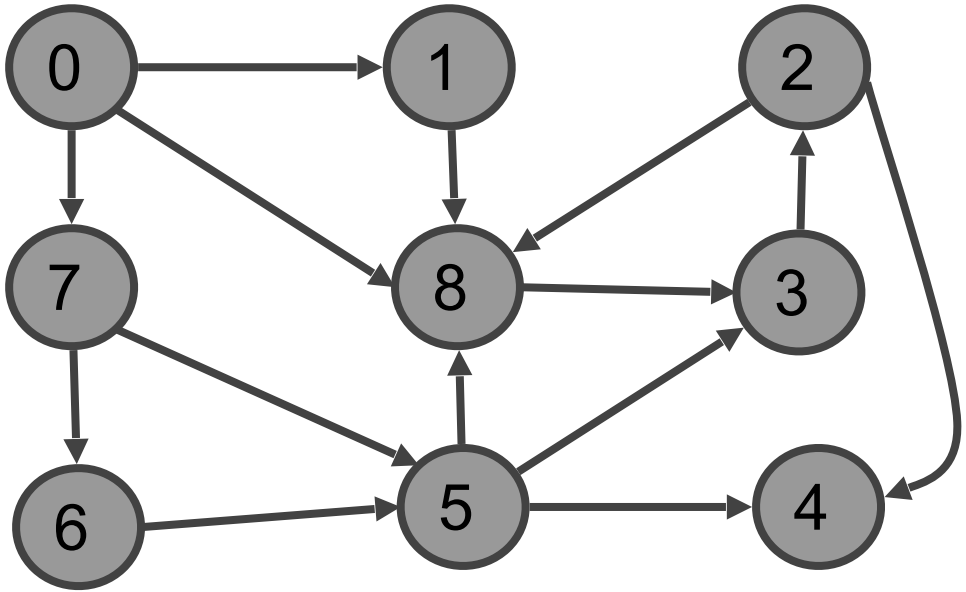


Depth first search pseudocode

```
DFS(G, u)
    u.visited = true
    for each v ∈ G.Adj[u]
        if v.visited == false
            DFS(G,v)

init() {
    For each u ∈ G
        u.visited = false
     For each u ∈ G
       DFS(G, u)
}
```

Breadth first search



BFS pseudocode

```
BFS (G, s)
  let Q be queue.
      Q.enqueue( s )

      mark s as visited.
      while (Q is not empty)
        v  =  Q.dequeue( )

        for all neighbours w of v in Graph G
          if w is not visited
            Q.enqueue( w )
           mark w as visited.
```