INTRODUCTION TO TIME SERIES ANALYSIS

The Autoregressive Model

The Autoregressive Model - I

The Autoregressive (AR) recursion:

$$Today = Constant + Slope * Yesterday + Noise$$

Mean centered version:

$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

The Autoregressive Model - II

$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

More formally:

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

where ϵ_t is mean zero white noise (WN).

Three parameters:

- The mean μ
- The slope ϕ
- The WN variance σ_ϵ^2

AR Processes - I

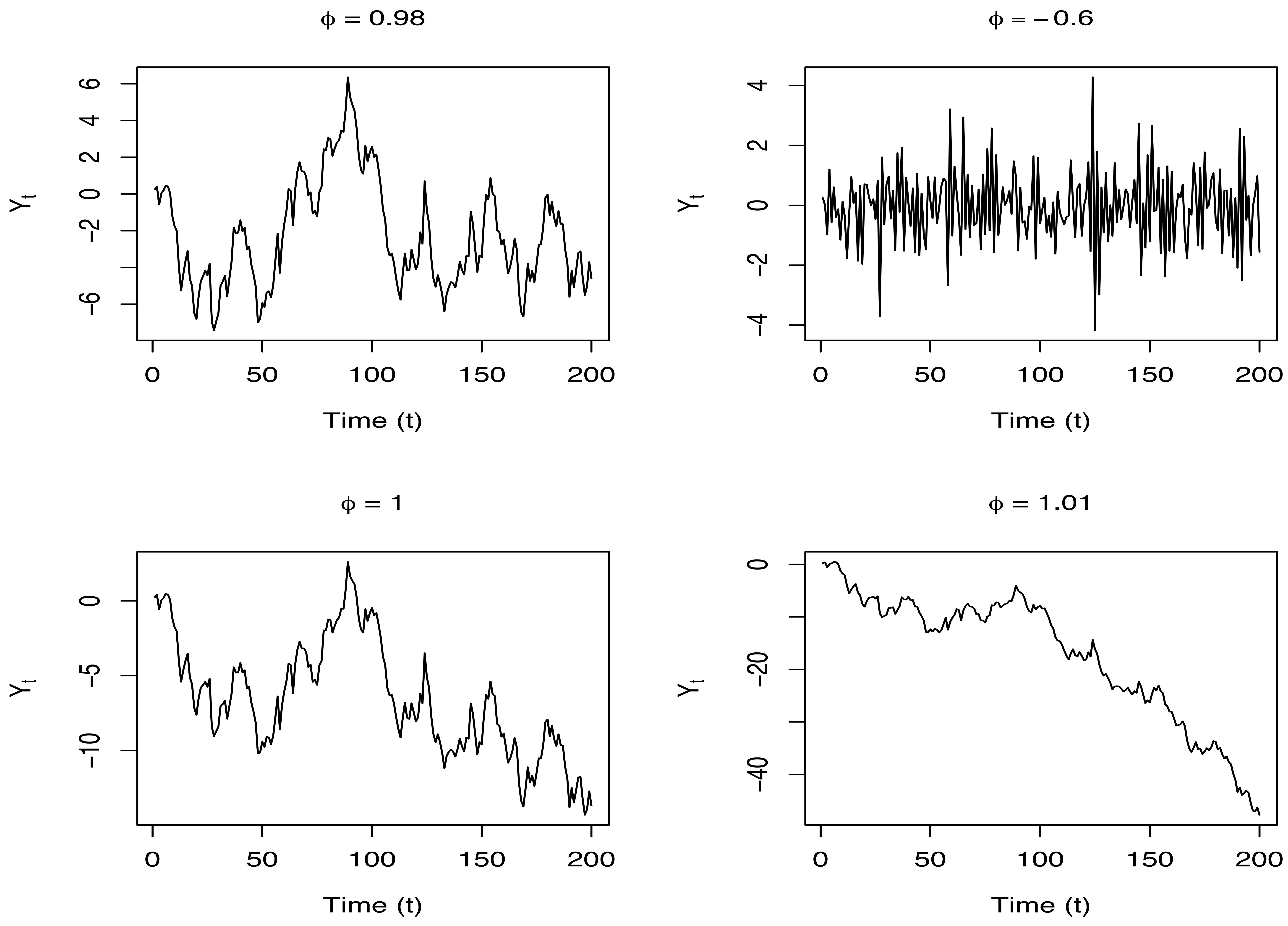
$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

- If slope $\phi = 0$ then: $Y_t = \mu + \epsilon_t$

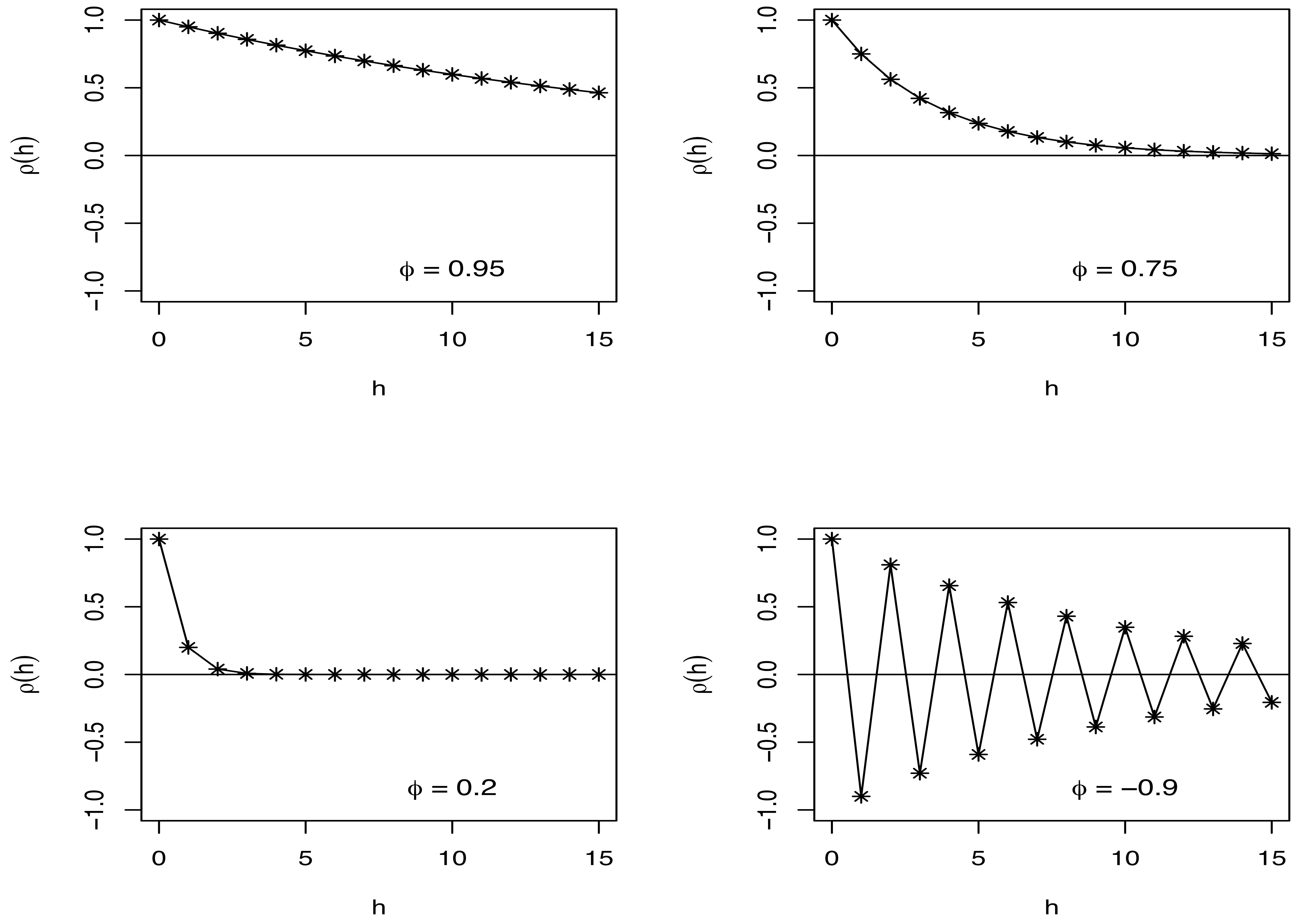
  And Y_t is White Noise (μ, σ^2_ϵ)
- If slope $\phi \neq 0$ then Y_t depends on both ϵ_t and Y_{t-1}

  And the process $\{Y_t\}$ is autocorrelated
- Large values of ϕ lead to greater autocorrelation
- Negative values of ϕ result in oscillatory time series

AR Examples


ϕ = 0.98
Y_t
6
4
2
0
−2
−6
0
50
100
150
200
Time (t)
ϕ = −0.6
Y_t
4
2
0
−2
−4
0
50
100
150
200
Time (t)
ϕ = 1
Y_t
0
−5
−10
0
50
100
150
200
Time (t)
ϕ = 1.01
Y_t
0
−20
−40
0
50
100
150
200
Time (t)

Autocorrelations



Random Walk

If $\mu = 0$ and slope $\phi = 1$, then:

$$Y_t = Y_{t-1} + \epsilon_t$$

Which is:

$$Today = Yesterday + Noise$$

But this is a **random walk.**

And $\{Y_t\}$ is **not** stationary in this case.

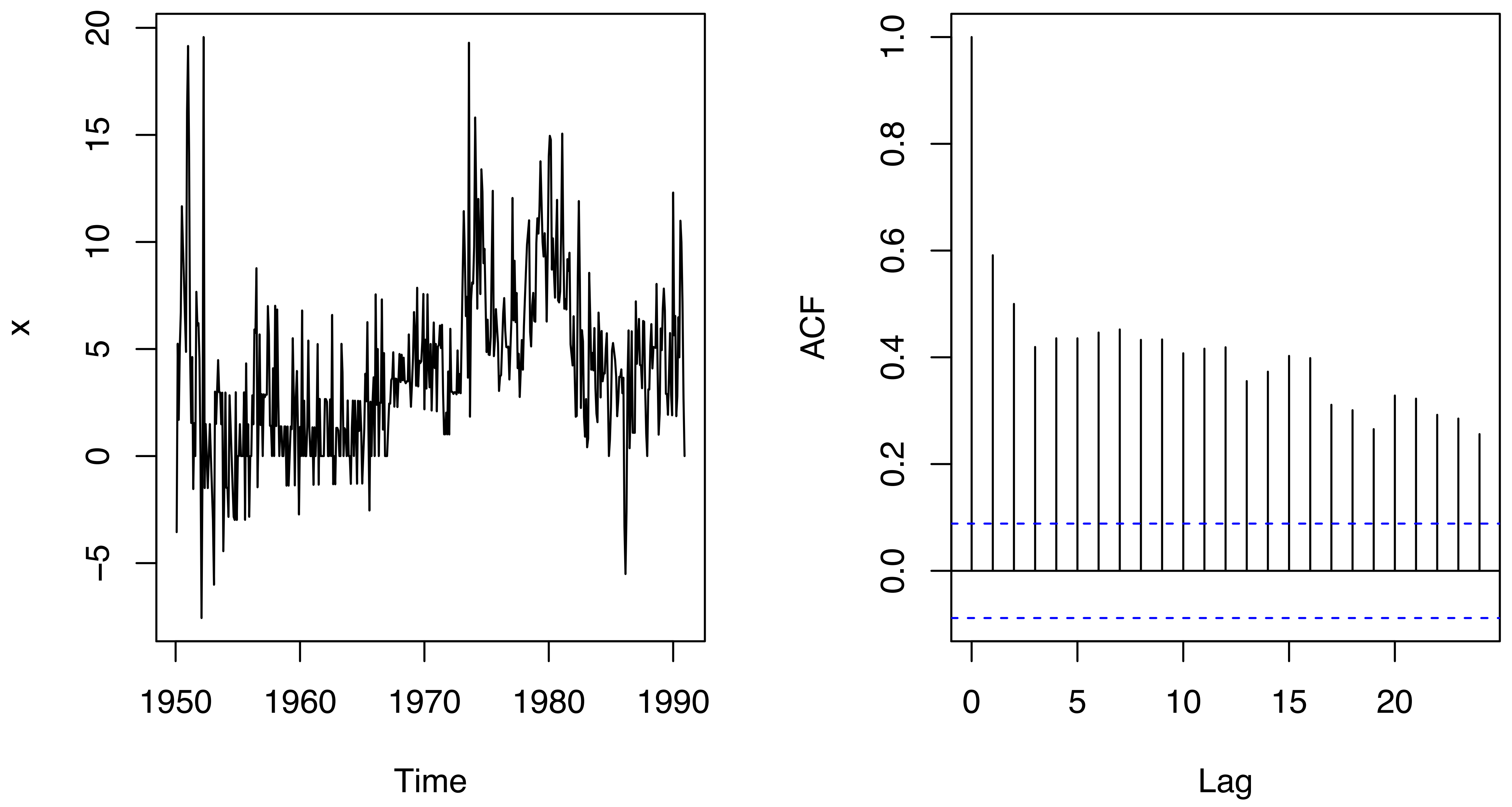
Let’s practice!

AR Model Estimation and Forecasting

AR Processes: Inflation Rate

- One-month US inflation rate (in percent, annual rate)
- Monthly observations from 1950 through 1990

```
> data(Mishkin, package = "Ecdat")
> inflation <- as.ts(Mishkin[, 1])
> ts.plot(inflation) ; acf(inflation)
```



AR Model: Inflation Rate

$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

$$\epsilon_t \sim WhiteNoise(0, \sigma_\epsilon^2)$$

```
> AR_inflation <- arima(inflation, order = c(1, 0, 0))
> print(AR_inflation)

Coefficients:
         ar1  intercept
      0.5960     3.9745
s.e.  0.0364     0.3471
sigma^2 estimated as 9.713
```

ar1 $= \hat{\phi}$, intercept $= \hat{\mu}$, sigma^2 $= \hat{\sigma}_\epsilon^2$

AR Processes: Fitted Values - I

- AR fitted values: $\widehat{Today} = \widehat{Mean} + \widehat{Slope} * (Yesterday - \widehat{Mean})$

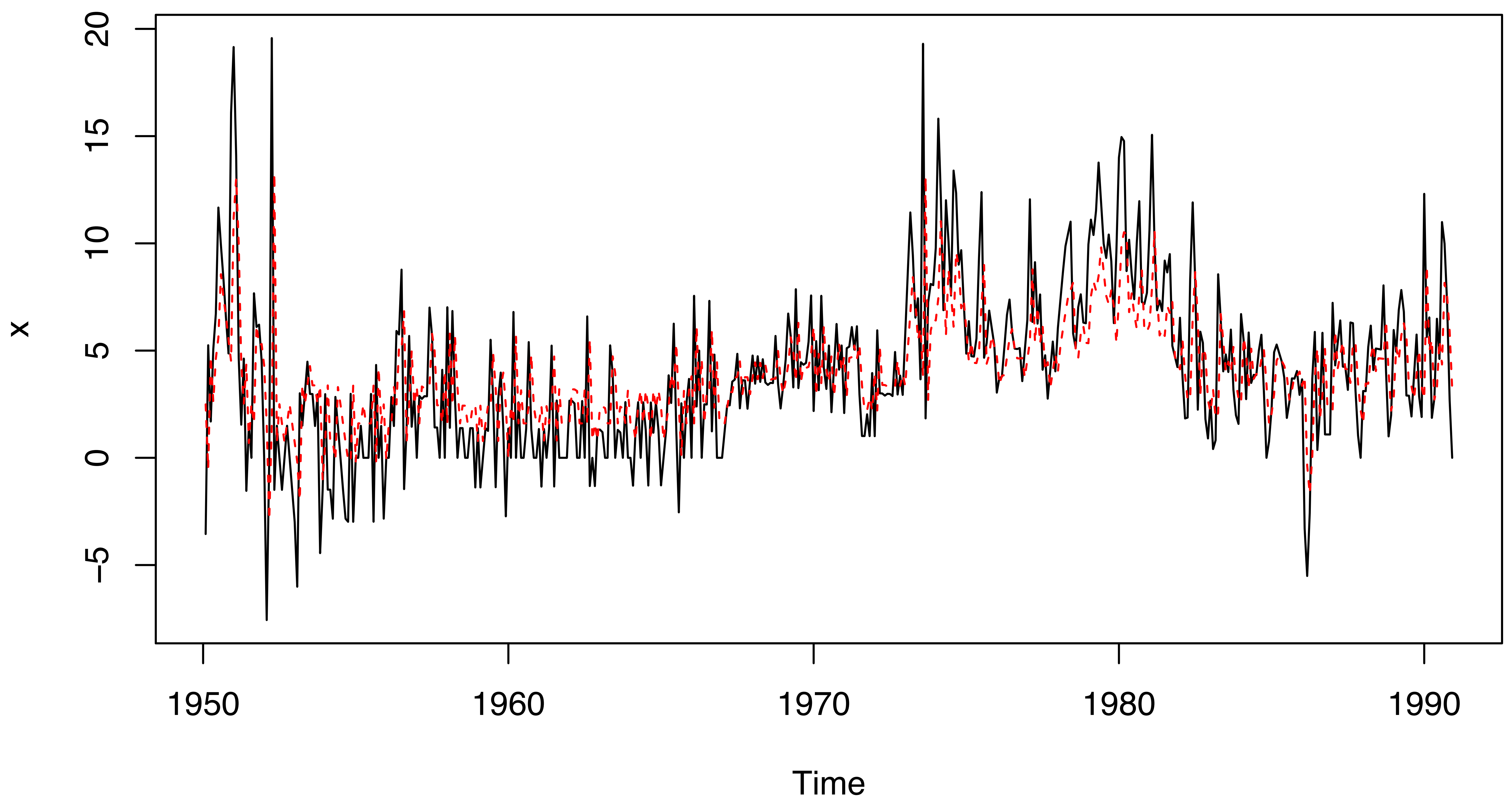
$$\hat{Y}_t = \hat{\mu} + \hat{\phi}(Y_{t-1} - \hat{\mu})$$

- Residuals = $Today - \widehat{Today}$

$$\hat{\epsilon}_t = Y_t - \hat{Y}_t$$

AR Processes: Fitted Values - II

```
> ts.plot(inflation)
> AR_inflation_fitted <- inflation - residuals(AR_inflation)
> points(AR_inflation_fitted, type = "l", col = "red", lty = 2)
```



Forecasting

- 1-step ahead forecasts

```
> predict(AR_inflation)
$pred
          Jan
1991 1.605797

$se
          Jan
1991 3.116526
```

- h-step ahead forecasts

```
> predict(AR_inflation, n.ahead = 6)
$pred
           Jan       Feb       Mar       Apr       May       Jun
1991  1.605797  2.562810  3.133165  3.473082  3.675664  3.796398

$se
           Jan       Feb       Mar       Apr       May       Jun
1991  3.116526  3.628023  3.793136  3.850077  3.870101  3.877188
```

Let's practice!