FORECASTING USING R

Dynamic regression

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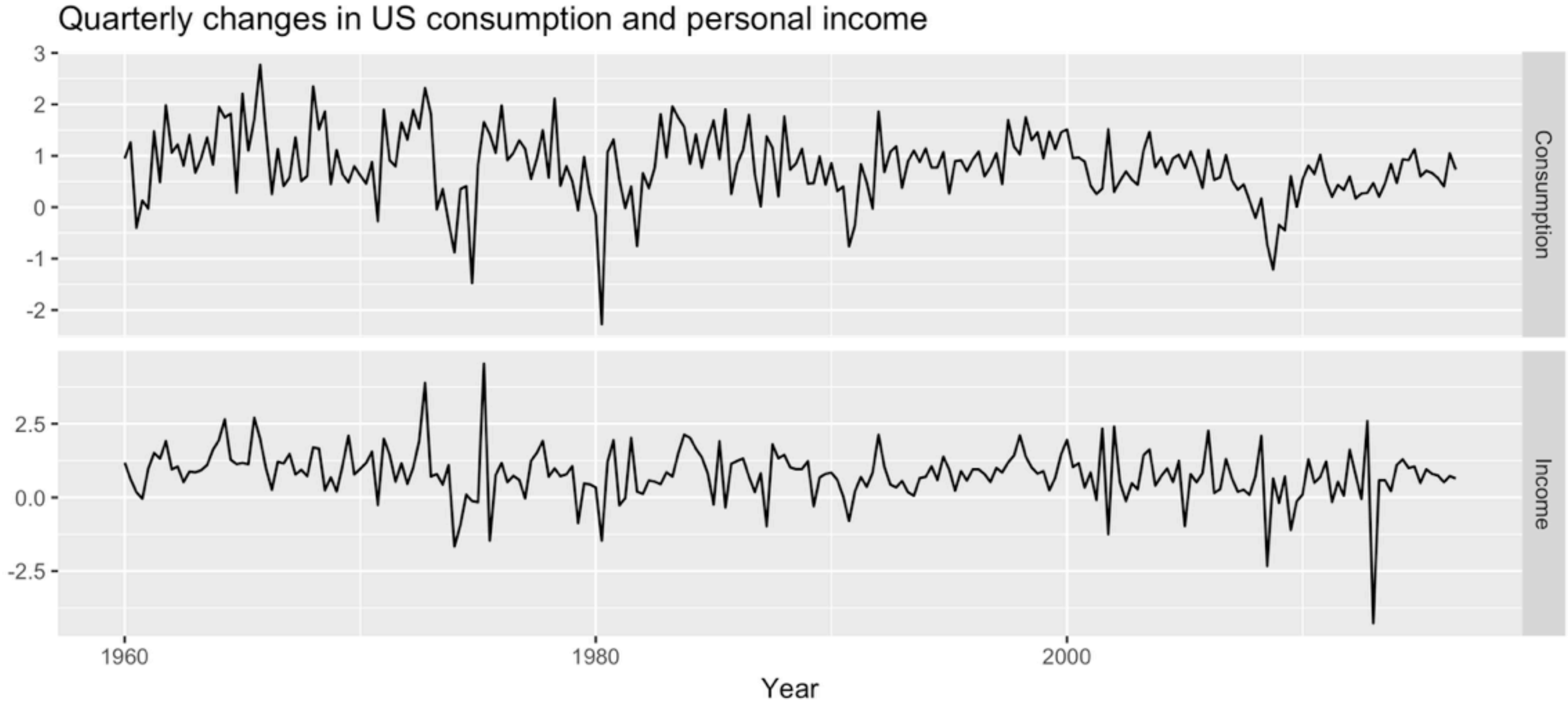
Dynamic regression

Regression model with ARIMA errors
$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_r x_{r,t} + e_t$

- y_t modeled as function of r explanatory variables $x_{1,t}, \ldots, x_{r,t}$
- In dynamic regression, we allow e_t to be an ARIMA process
- In ordinary regression, we assume that e_t is white noise

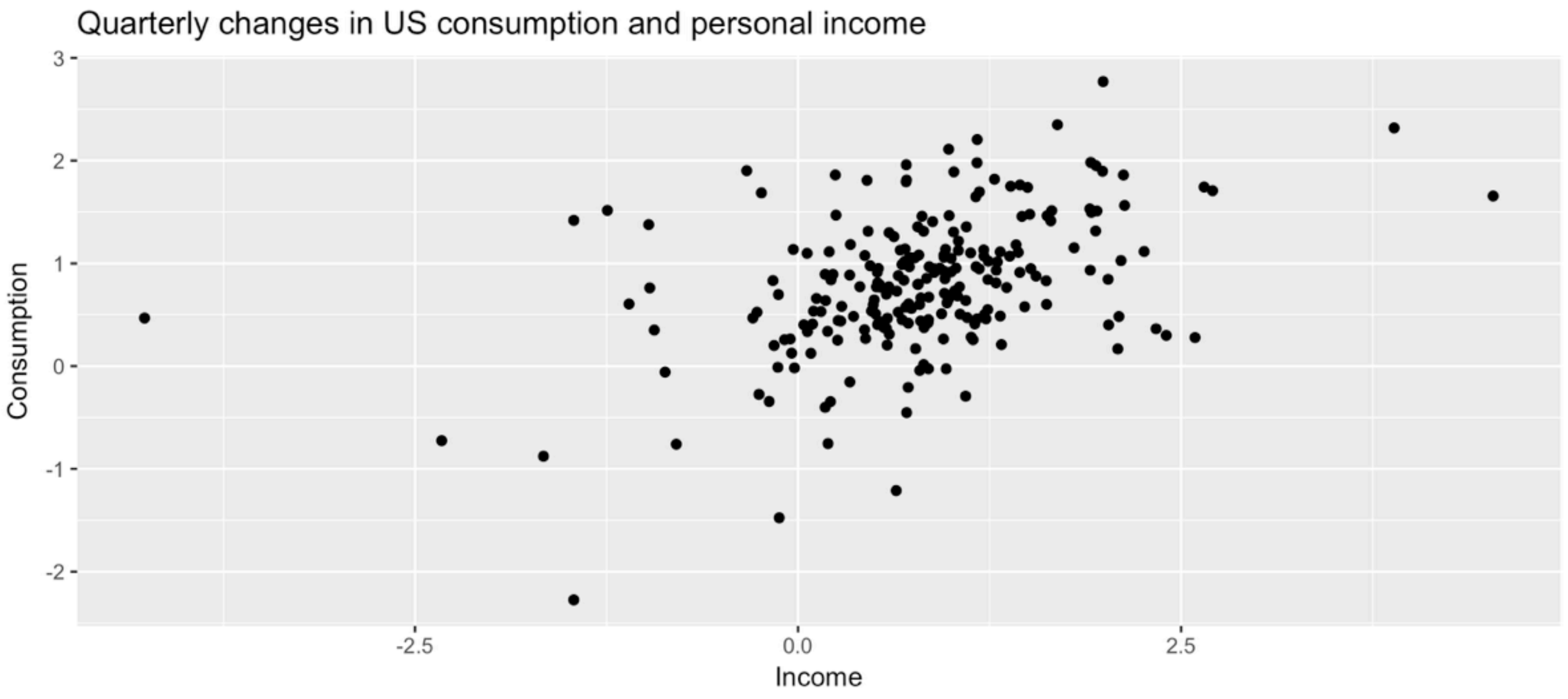
US personal consumption and income

```
> autoplot(uschange[,1:2], facets = TRUE) +
    xlab("Year") + ylab("") +
    ggtitle("Quarterly changes in US consumption
             and personal income")
```



US personal consumption and income

```
> ggplot(aes(x = Income, y = Consumption),
         data = as.data.frame(uschange)) +
    geom_point() +
    ggtitle("Quarterly changes in US consumption and
            personal income")
```

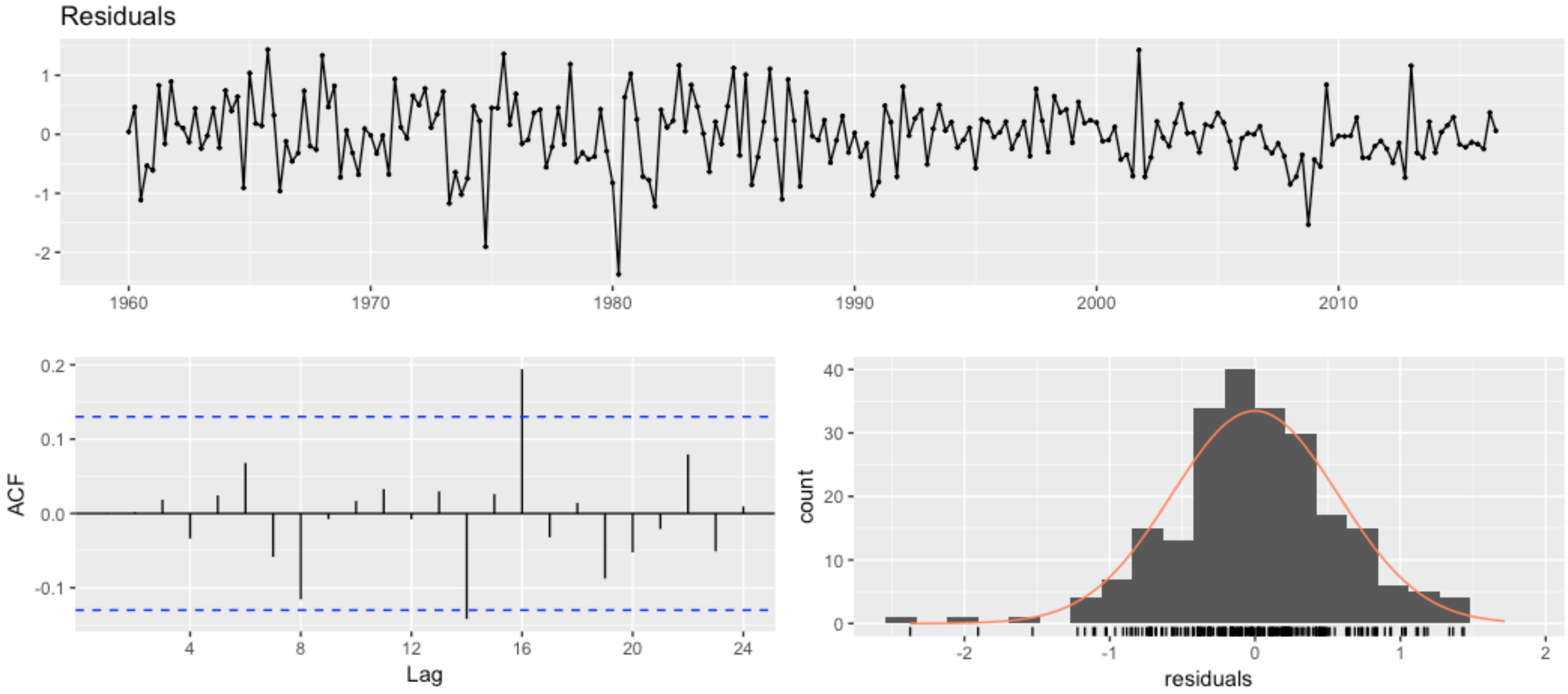


Dynamic regression model for US personal consumption

```
> fit <- auto.arima(uschange[,"Consumption"],
                    xreg = uschange[,"Income"])
> fit

Series: uschange[, "Consumption"]
Regression with ARIMA(1,0,2) errors

Coefficients:
         ar1      ma1     ma2  intercept  origxreg
      0.6191  -0.5424  0.2367     0.6099    0.2492
s.e.  0.1422   0.1475  0.0685     0.0777    0.0459

sigma^2 estimated as 0.334:  log likelihood=-195.22
AIC=402.44   AICc=402.82   BIC=422.99
```

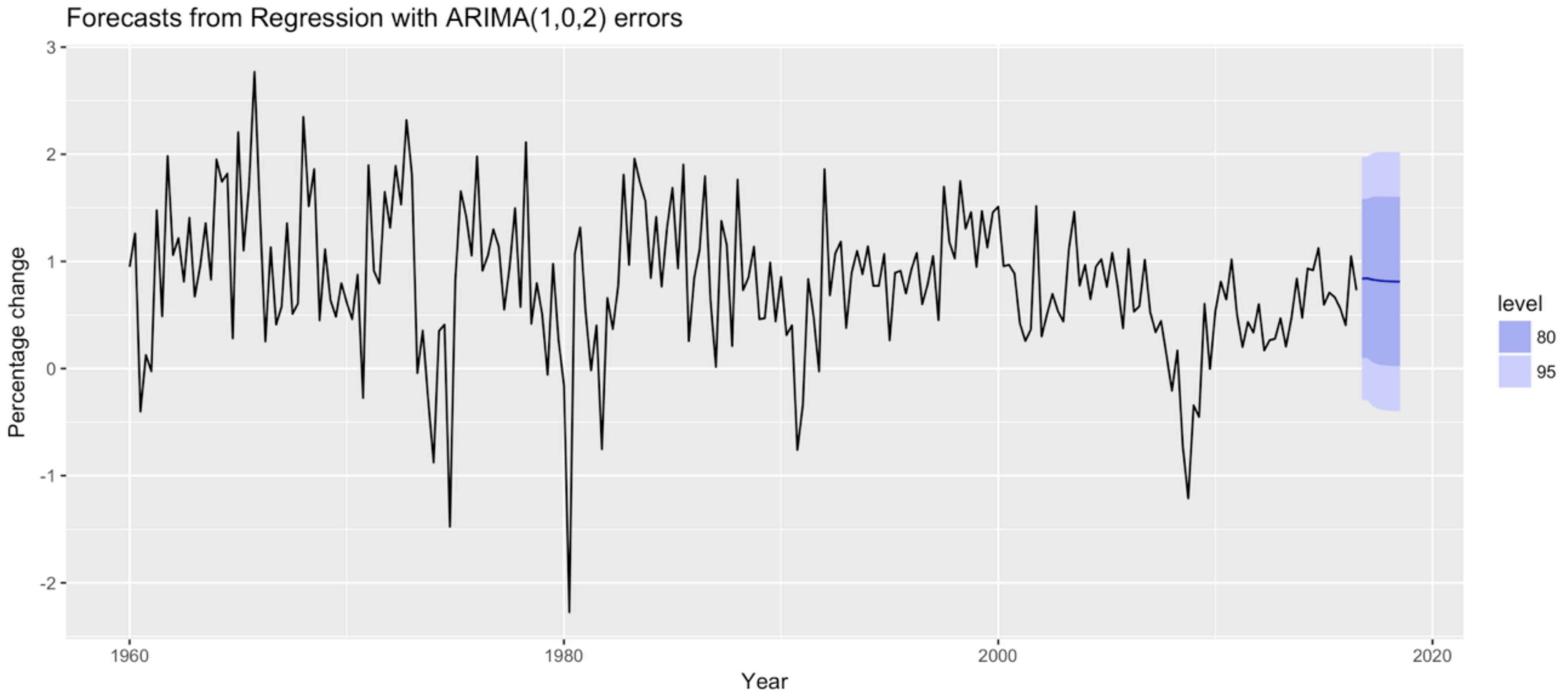
Residuals from dynamic regression model

```
> checkresiduals(fit)
  Ljung-Box test
data:  residuals
Q* = 5.5543, df = 3, p-value = 0.1354
Model df: 5.   Total lags used: 8
```



Forecasts from dynamic regression model

```
> fcast <- forecast(fit, xreg = rep(0.8, 8))
> autoplot(fcast) +
    xlab("Year") + ylab("Percentage change")
```



Let's practice!

Dynamic harmonic regression

Dynamic harmonic regression

Periodic seasonality can be handled using pairs of Fourier terms:

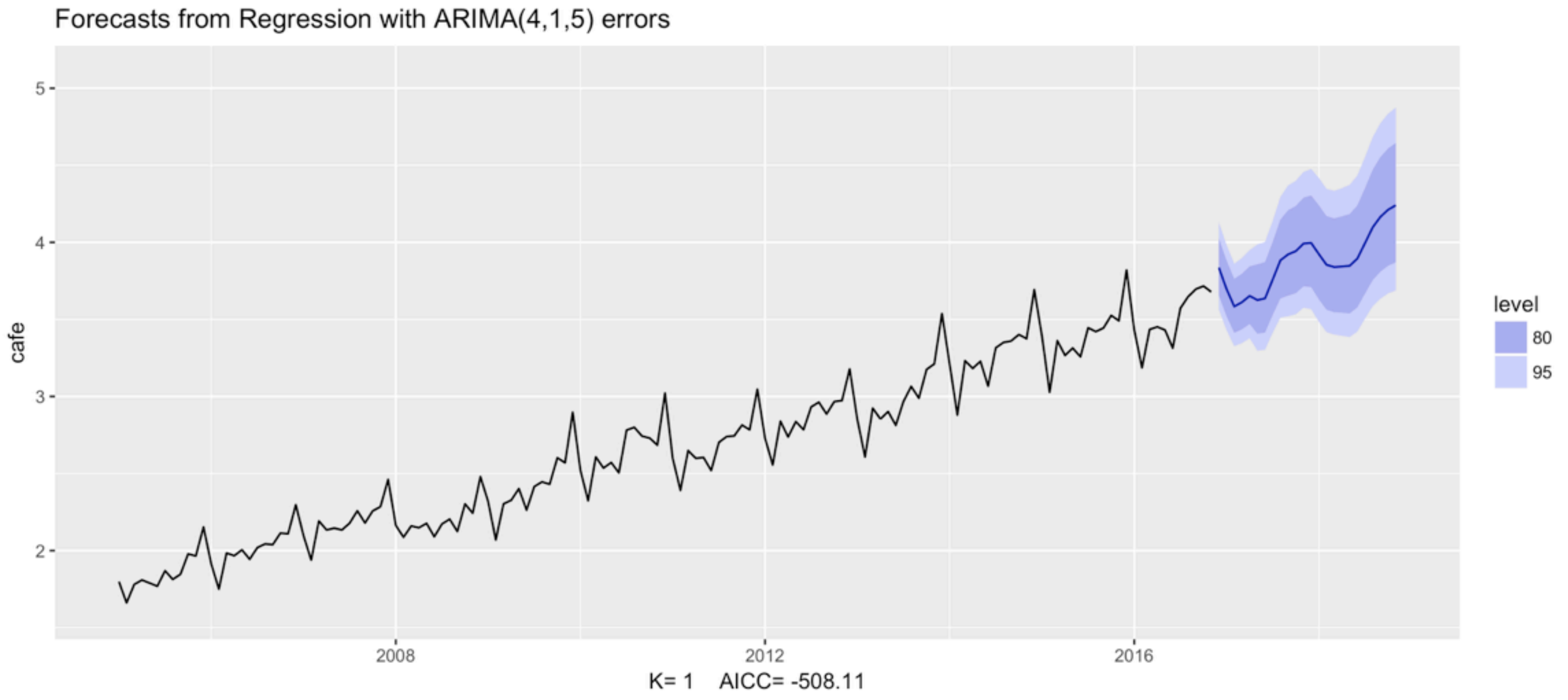
$$s_k(t) = \sin\left(\frac{2\pi kt}{m}\right) \qquad c_k(t) = \cos\left(\frac{2\pi kt}{m}\right)$$

$$\boxed{y_t = \beta_0 + \sum_{k=1}^{K}\left[\alpha_k s_k(t) + \gamma_k c_k(t)\right] + e_t}$$

- $m =$ seasonal period
- Every periodic function can be approximated by sums of sin and cos terms for large enough K
- Regression coefficients: α_k and γ_k
- e_t can be modeled as a non-seasonal ARIMA process
- Assumes seasonal pattern is unchanging

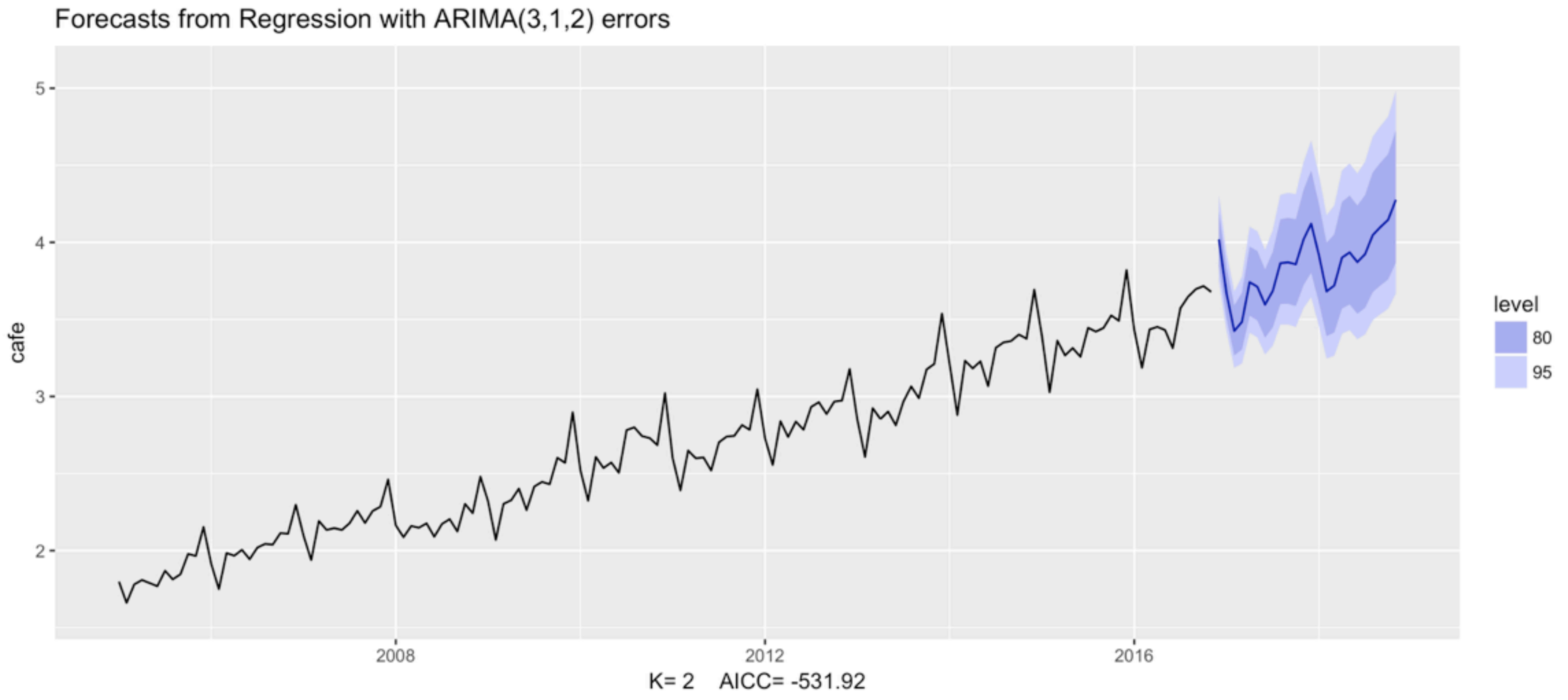
Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 1),
                    seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 1, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



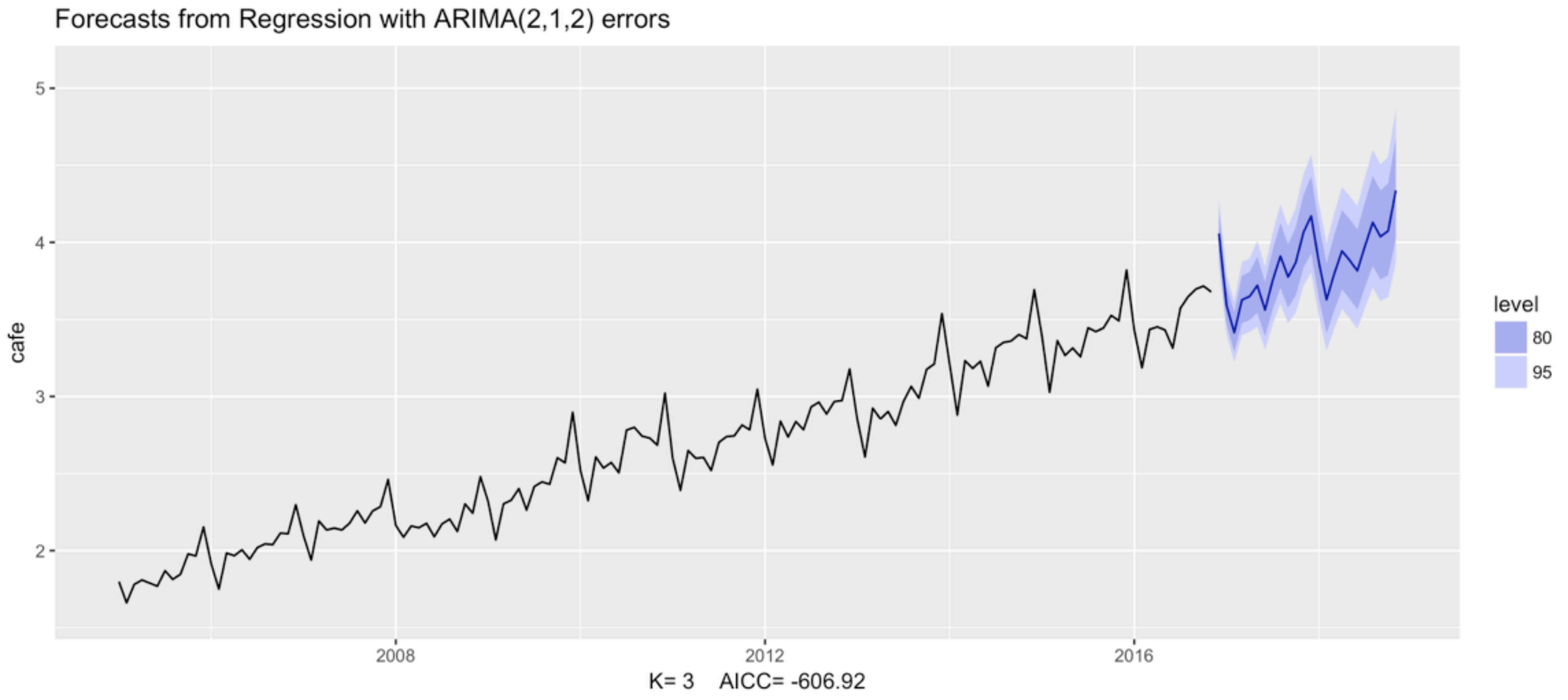
Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 2),
                    seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 2, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



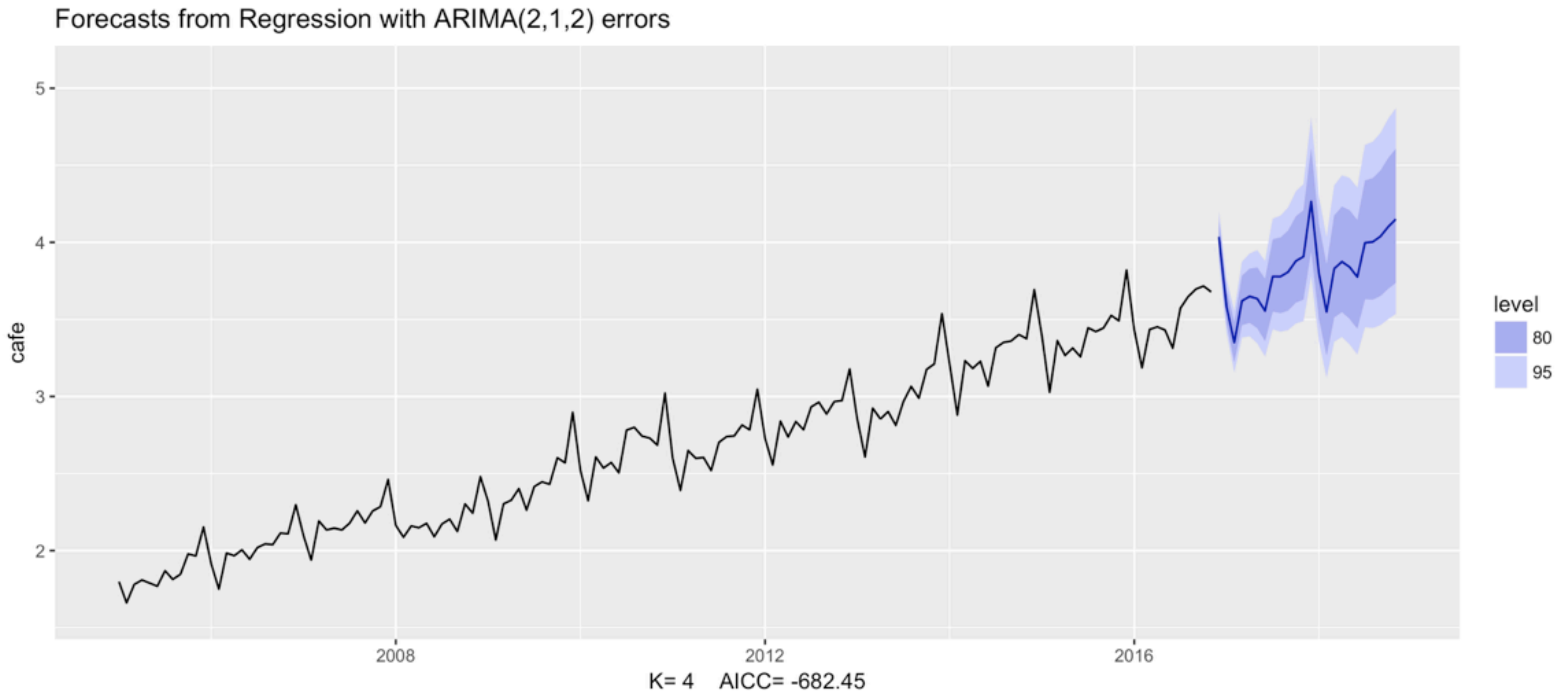
Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 3),
                     seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 3, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



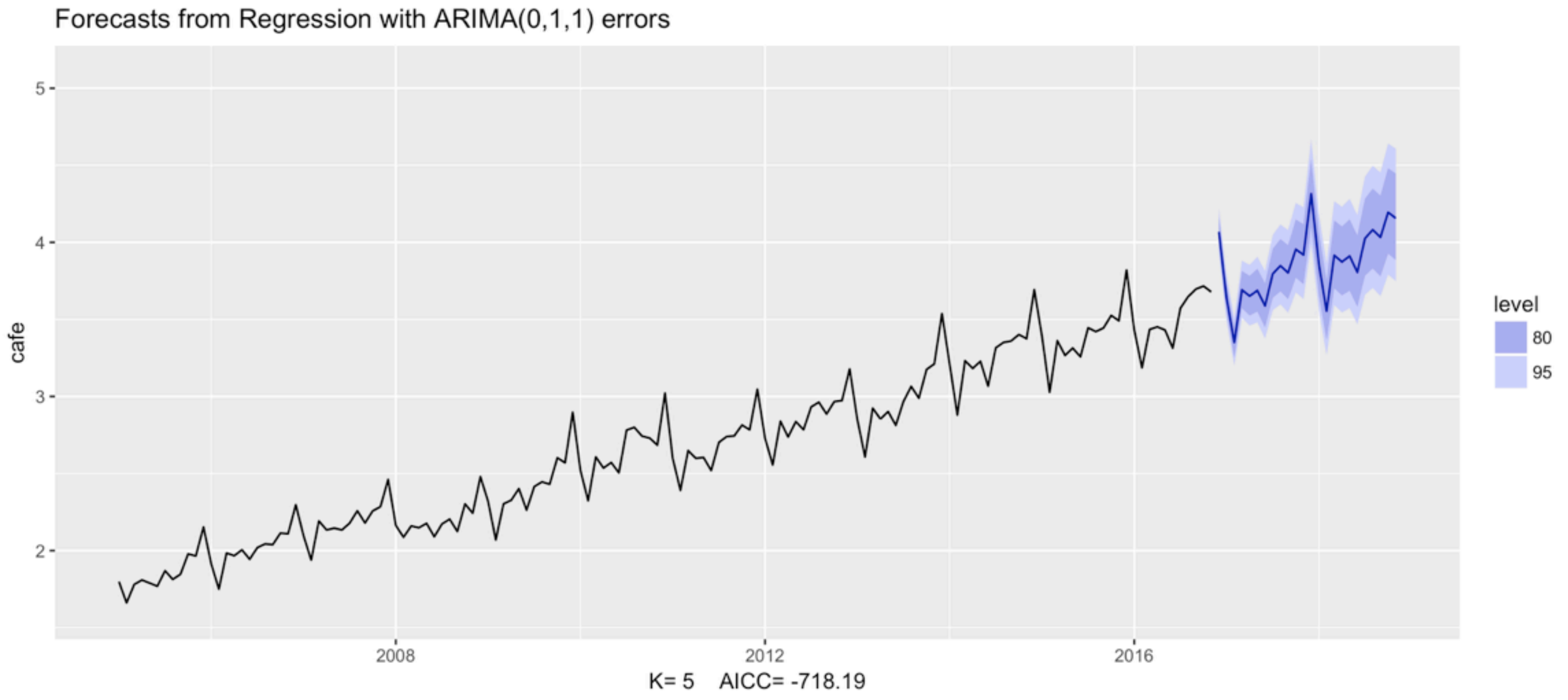
Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 4),
                    seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 4, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



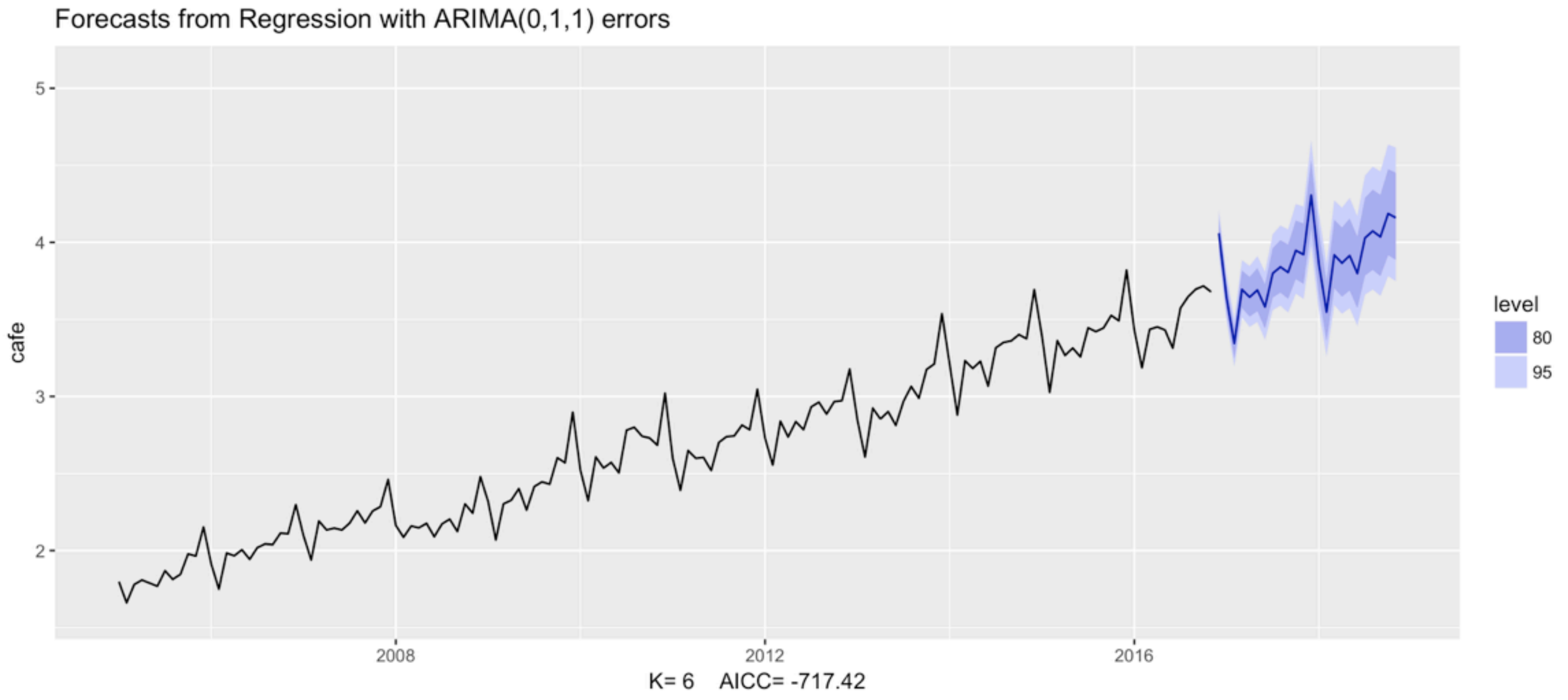
Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 5),
                     seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 5, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



Example: Australian cafe expenditure

```
> fit <- auto.arima(cafe, xreg = fourier(cafe, K = 6),
                    seasonal = FALSE, lambda = 0)
> fit %>% forecast(xreg = fourier(cafe, K = 6, h = 24)) %>%
    autoplot() + ylim(1.6, 5.1)
```



Dynamic harmonic regression

$$\boxed{y_t = \beta_0 + \beta_1 x_{t,1} + \cdots + \beta_{t,r} x_{t,r} + \sum_{k=1}^{K} [\alpha_k s_k(t) + \gamma_k c_k(t)] + e_t}$$

- Other predictor variables can be added as well: $x_{t,1}, \ldots, x_{t,r}$
- Choose K to minimize the AICc
- K can not be more than $m/2$
- This is particularly useful for weekly data, daily data and sub-daily data

Let’s practice!

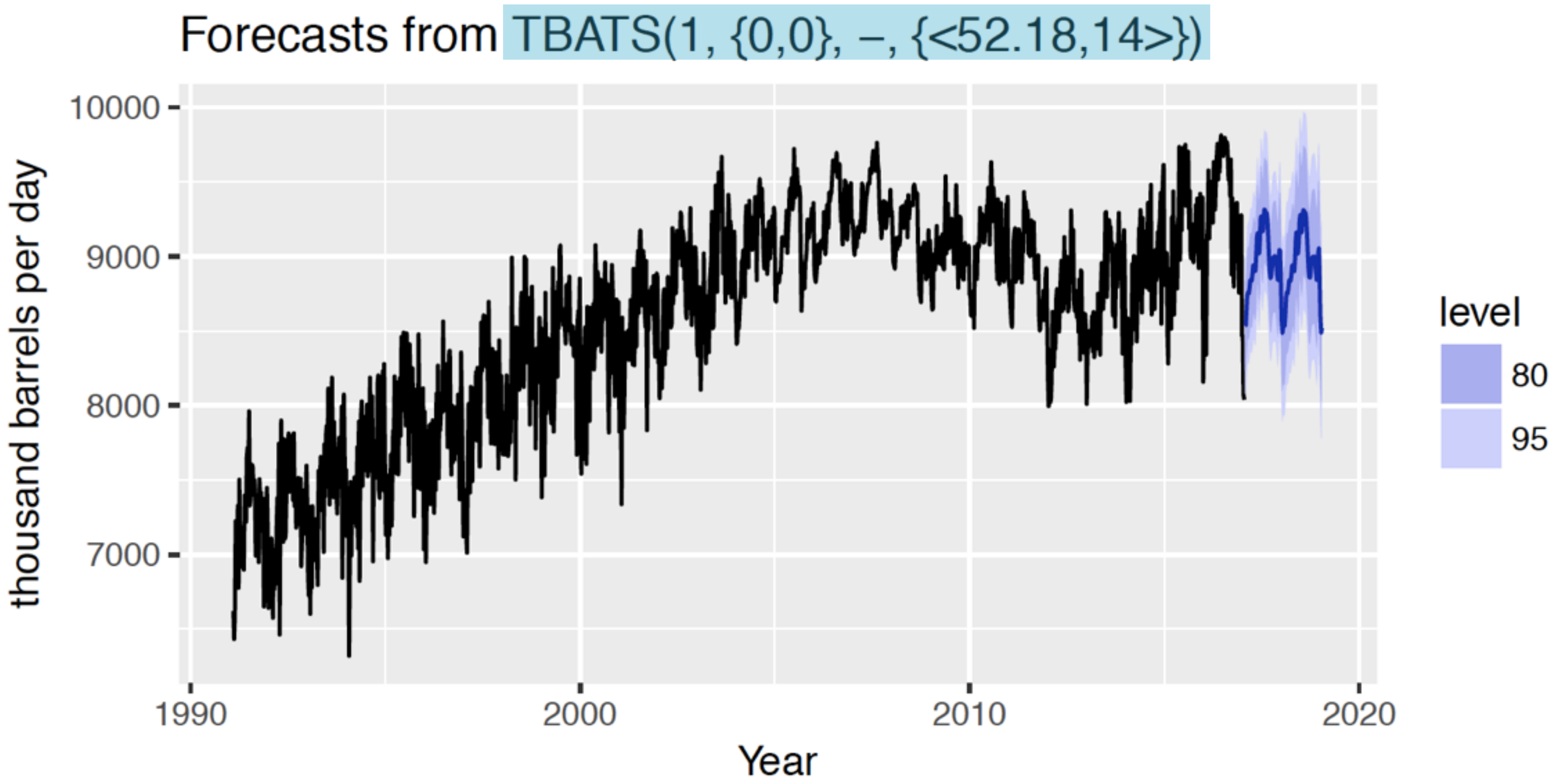
TBATS models

TBATS model

- Trigonometric terms for seasonality
- Box-Cox transformations for heterogeneity
- ARMA errors for short-term dynamics
- Trend (possibly damped)
- Seasonal (including multiple and non-integer periods)

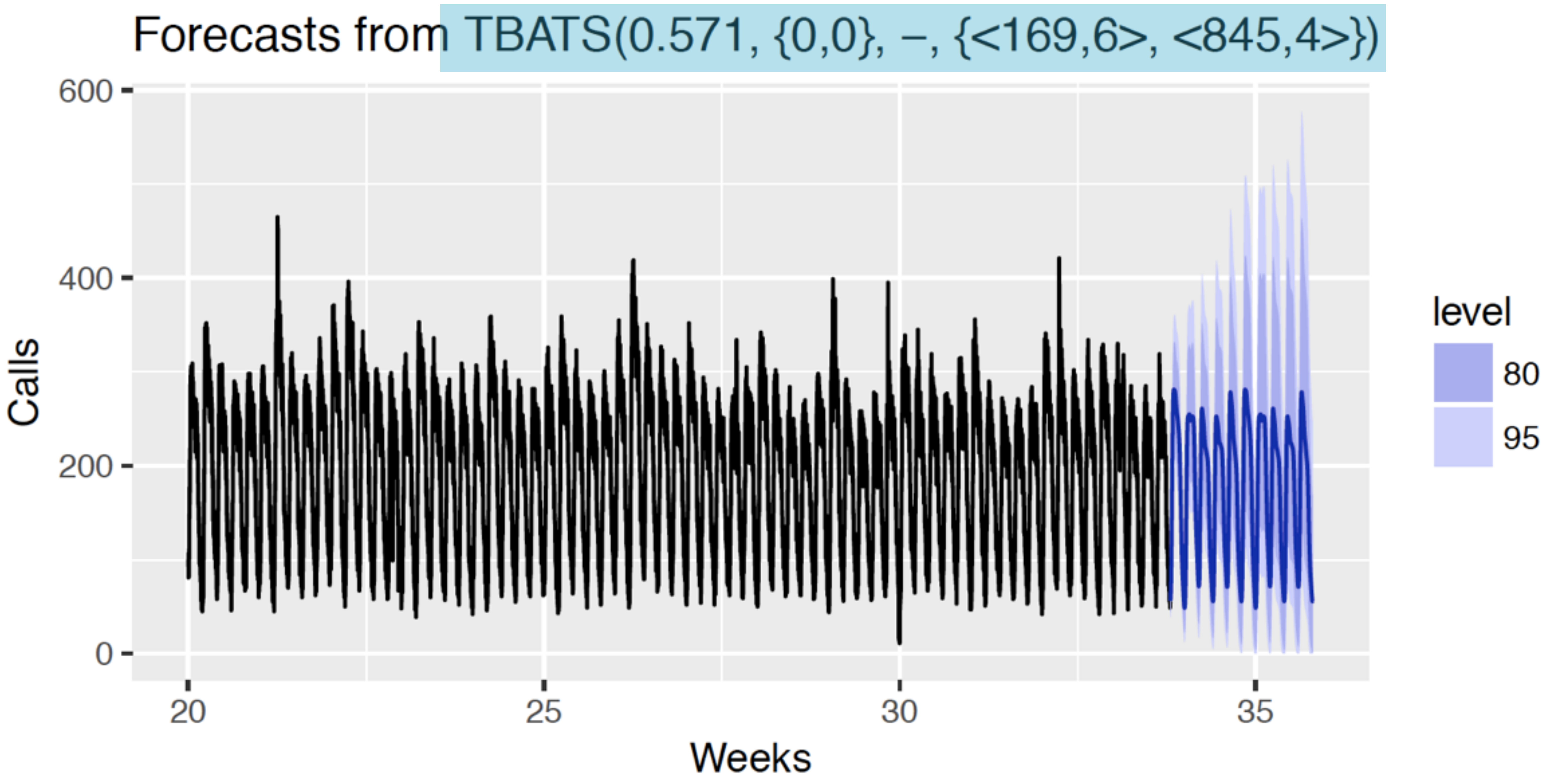
US Gasoline data

```
> gasoline %>% tbats() %>% forecast() %>%
    autoplot() +
    xlab("Year") + ylab("thousand barrels per day")
```



Call center data

```
> calls %>% window(start = 20) %>%
    tbats() %>% forecast() %>%
    autoplot() + xlab("Weeks") + ylab("Calls")
```



TBATS model

- Trigonometric terms for seasonality
- Box-Cox transformations for heterogeneity
- ARMA errors for short-term dynamics
- Trend (possibly damped)
- Seasonal (including multiple and non-integer periods)

  - Handles non-integer seasonality, multiple seasonal periods
  - Entirely automated
  - Prediction intervals often too wide
  - Very slow on long series

Let's practice!