



Intro to Statistics with R: Student's T-test

Statistical tests to compare means

Tests to compare means

- z-test
- t-test (single sample)
- t-test (dependent)
- t-test (independent)

Statistical tests

- Compare observed to expected
- Relative to standard error (SE)
- Important to know when each test applies

$$z = (\text{Observed} - \text{Expected}) / \text{Standard Error}$$

$$t = (\text{Observed} - \text{Expected}) / \text{Standard Error}$$

When to use z- and t-tests?

- Uncommon to know population parameters
- Won't focus on z-test or single sample t-test

	Compares sample mean to population mean?	SD of Population Known?
z-test	YES	YES
Single sample t-test	YES	NO

Which t-test do you use?

- Dependent t-test
 - Difference between two related samples
 - E.g. same people measured twice
- Independent t-test
 - Difference between two independent samples
 - E.g. men vs women, drug vs. placebo

Observed, Expected, and SE

	Observed	Expected	SE
z-test	Sample mean	Population mean	SE of the mean
t-test (single sample)	Sample mean	Population mean	SE of the mean
t-test (dependent)	Sample mean of difference scores	Population mean of difference scores	SE of the mean difference
t-test (independent)	Difference between two sample means	Difference between two population means	SE of the difference between means



Intro to Statistics with R: Student's T-test

Sampling distribution (1/2)

Sampling distribution

- Hypothetical distribution of summary statistic
- Multiple samples of the same size
- Body temperature example
 - Multiple samples, 10 people each
 - Sampling error leads to slightly different means
 - Distribution of sample means will have a mean close to population mean

Sampling distribution

In the case of a z-test...

- Mean of sampling distribution approximately equal to mean of population
- Standard deviation of sampling distribution (SD) is the Standard Error (SE)
- As sample size increases, standard error decreases

Central Limit Theorem



Intro to Statistics with R: Student's T-test

Sampling distribution (2/2)

Significance tests

Null Hypothesis Significance Testing (NHST)

- Null hypothesis = "no effect"
- No difference between sample mean and population mean **Expected value = 0**

$$\downarrow \text{Standard Error} = \frac{\text{Population SD}}{\text{Sq. root of sample size}} \uparrow$$

Probability values

- Probability value, or p-value
- Conditional probability (assumes null hypothesis true)
- As or more extreme than observed value

	Expected	Actual	p-value
z-value	0	2	< 0.05

p-values with t-tests

- Family of sampling distributions
- For large samples:
 - Very similar to sampling distribution of z-test
 - t-value of 2 yields statistically significant result
- For small samples:
 - Sampling distribution wider
 - t-value slightly greater than 2 required

Summary

- z-tests and t-tests designed to compare means
- $(\text{Observed} - \text{Expected}) / \text{Standard Error}$
- Standard Error = difference expected due to chance
- Numerator at least 2x SE to claim significance



Intro to Statistics with R: Student's T-test

Dependent t-test

Dependent t-test

- Also known as paired samples t-test
- Appropriate when same subjects being compared (i.e. measured on same variable twice)
- Calculate difference score for each subject
- Mean of difference scores

Dependent t-test

Thorough analysis will include...

- t-value
- p-value
- Cohen's d (effect size)

Dependent t-value

$$t = (\text{Observed} - \text{Expected}) / SE$$



**Mean of
difference
scores**



**Zero
("no effect")**

$$t = (\text{Mean of difference scores}) / SE$$

*Observed difference relative to
difference expected by chance*

Effect size

- Significance tests biased by sample size
 - When sample size is large, SE is very small
 - Small observed difference may be "statistically significant"
- Effect size estimates magnitude of the effect

Cohen's d

- $d = (\text{Mean of difference scores}) / SD$
- Divide by standard deviation, not standard error
- Mean difference in terms of standard deviation units
- Cohen's d of 1 means people changed by one SD