Chapter 2: Video 4 - Supplementary Slides

To obtain parsimony in a time series model we often assume some form of distributional invariance over time, or stationarity.

For observed time series:

- Fluctuations appear random.
- However, same type of stochastic behavior holds from one time period to the next.

For example, returns on stocks or changes in interest rates:

- Individually, very different from the previous year.
- But mean, standard deviation, and other statistical properties are often similar from one year to the next.

Strict Stationarity

A process is strictly stationary if all aspects of its probabilistic behavior are unchanged by shifts in time.

Mathematically,

- for every m and n,
- $(Y_1, \ldots, Y_n)$ and $(Y_{1+m}, \ldots, Y_{n+m})$ have same distributions;
- the distribution of a sequence of n observations does NOT depend on their time origin (1 or $1 + m$, above).

Strict stationarity is a very strong assumption.

It will often suffice to assume less...

Weak Stationarity

A process is weakly stationary if its mean, variance, and covariance are unchanged by time shifts.

$Y_1, Y_2, \ldots$ is a *weakly stationary process* if

- $E(Y_t) = \mu$ (a finite constant) for all t;
- Var$(Y_t) = \sigma^2$ (a positive finite constant) for all t; and
- Cov$(Y_t, Y_s) = \gamma(|t-s|)$ for all t and s for some function $\gamma(h)$.

Weakly stationary is also referred to as covariance stationary.

- The mean and variance do not change with time
- The covariance between two observations depends only on the lag, the time distance $|t - s|$ between observations, not the indices t or s directly.