Chapter 2: Video 2 - Supplementary Slides

White Noise

White noise is the simplest example of a stationary process.

The sequence $Y_1, Y_2, \ldots$ is a weak white noise process with mean μ and variance σ^2, i.e., "weak WN(μ, σ^2)," if

- $E(Y_t) = \mu$ (a finite constant) for all t;
- Var$(Y_t) = \sigma^2$ (a positive finite constant) for all t; and
- Cov$(Y_t, Y_s) = 0$ for all $t \neq s$.

If the mean is not specified, then it is assumed that $\mu = 0$.

A weak white noise process is weakly stationary with

$$\begin{aligned} \gamma(0) &= \sigma^2, \\ \gamma(h) &= 0 \;\text{ if }\; h \neq 0, \end{aligned}$$

so that

$$\begin{aligned} \rho(0) &= 1, \\ \rho(h) &= 0 \;\text{ if }\; h \neq 0. \end{aligned}$$

If $Y_1, Y_2, \ldots$ is an i.i.d. process, call it an i.i.d. white noise process:

i.i.d. WN(μ, σ^2).

- Weak WN is weakly stationary,
- However, i.i.d. WN is strictly stationary.
- An i.i.d. WN process with σ^2 finite is also a weak WN process, but not vice versa.

If, in addition, $Y_1, Y_2 \ldots$ is an i.i.d. process with a specific marginal distribution, then this might be noted.

For example, if $Y_1, Y_2 \ldots$ are i.i.d. normal random variables, then the process is called a Gaussian white noise process.

Similarly, if $Y_1, Y_2 \ldots$ are i.i.d. t random variables with ν degrees of freedom, then it is called a t_ν WN process.

With no dependence, past values of a WN process contain no information that can be used to predict future values.

If $Y_1, Y_2, \ldots$ is an i.i.d. WN(μ, σ^2) process. Then

$$E(Y_{t+h}|Y_1, \ldots, Y_t) = \mu \text{ for all } h \geq 1.$$

- Cannot predict future deviations of WN process from its mean.
- Future is independent of its past and present.
- Best predictor of any future value is simply the mean μ

For weak WN this may not be true, but the best linear predictor of Y_{t+h} given $Y_1, \ldots, Y_t$ is still μ.