Chapter 4: Video 1 - Supplemental slides

Autoregressive (AR) processes

Let $\epsilon_1, \epsilon_2, \ldots$ be White Noise(0,σ_ϵ^2) innovations, with variance σ_ϵ^2

Then, $Y_1, Y_2, \ldots$ is an AR process if for some constants μ and ϕ,

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

- We focus on 1st order case, the simplest AR process

Autoregressive (AR) processes

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

- μ is the mean of the $\{Y_t\}$ process
- If $\phi = 0$, then $Y_t = \mu + \epsilon_t$, such that Y_t is White Noise(μ, σ_ϵ^2)
- If $\phi \neq 0$, then observations Y_t depend on both ϵ_t and Y_{t-1}
- And the process $\{Y_t\}$ is autocorrelated
- If $\phi \neq 0$, then $(Y_{t-1} - \mu)$ is fed forward into Y_t
- ϕ determines the amount of feedback
- Larger values of $|\phi|$ result in more feedback

If $|\phi| < 1$, then

$$\begin{aligned} E(Y_t) &= \mu \\ \text{Var}(Y_t) &= \sigma_Y^2 = \frac{\sigma_\epsilon^2}{1-\phi^2} \\ \text{Corr}(Y_t, Y_{t-h}) &= \rho(h) = \phi^{|h|} \quad for\ all\ h \end{aligned}$$

- If $\mu = 0$ and $\phi = 1$, then

$$Y_t = Y_{t-1} + \epsilon_t$$

which is a random walk process, and $\{Y_t\}$ is NOT stationary