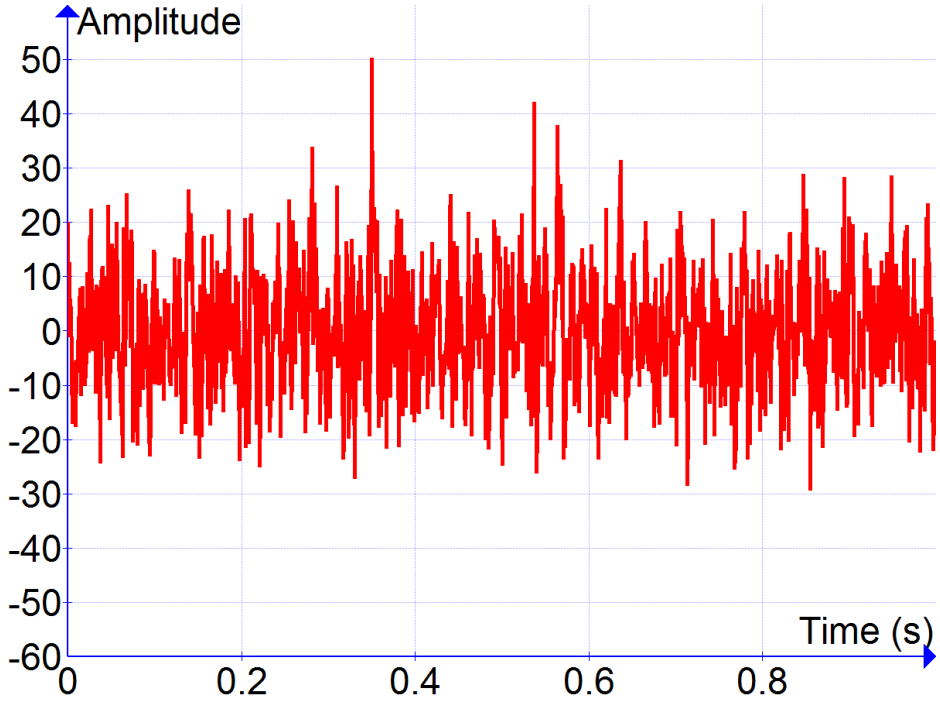
Lab Manual 2
Random Signals

Random Signals:

Signals can be divided into two main categories - deterministic and random. The term random signal is used primarily to denote signals, which have a randomness in its nature source.
As an example we can mention the thermal noise, which is created by the random movement of electrons in an electric conductor.



Apart from this, the term random signal is used also for signals falling into other categories, such as periodic signals, which have one or several parameters that have appropriate random behavior. An example is a periodic sinusoidal signal with a random phase or amplitude.

Signals can be treated either as deterministic or random, depending on the application. Speech, for example, can be considered as a deterministic signal, if one specific speech waveform is considered. It can also be viewed as a random process if one considers the ensemble of all possible speech waveforms in order to design a system that will optimally process speech signals, in general.

Mean and Variance of random signals:

Mean:

The ***mean*** of a discrete random variable *X* is a weighted average of the possible values that the random variable can take. Unlike the sample mean of a group of observations, which gives each observation equal weight, the mean of a random variable weights each outcome x_i according to its probability, p_i. The common symbol for the mean (also known as the ***expected value*** of *X*) is μ, formally defined by

$$\mu_x = x_1p_1 + x_2p_2 + \cdots + x_kp_k$$
$$= \sum x_ip_i$$

The mean of a random variable provides the *long-run average* of the variable, or the expected average outcome over many observations.

For a continuous random variable, the mean is defined by the density curve of the distribution. For a symmetric density curve, such as the normal density, the mean lies at the centre of the curve.

Properties of Means:

If a random variable *X* is adjusted by multiplying by the value *b* and adding the value *a*, then the mean is affected as follows:

$$\mu_{a+bX} = a + b\mu_X$$

Variance:

The ***variance*** of a discrete random variable *X* measures the spread, or variability, of the distribution, and is defined by

$$\sigma_X^2 = \sum (x_i - \mu_x)^2 p_i$$

The standard deviation σ is the square root of the variance.

Properties of Variances:

If a random variable *X* is adjusted by multiplying by the value *b* and adding the value *a*, then the variance is affected as follows:

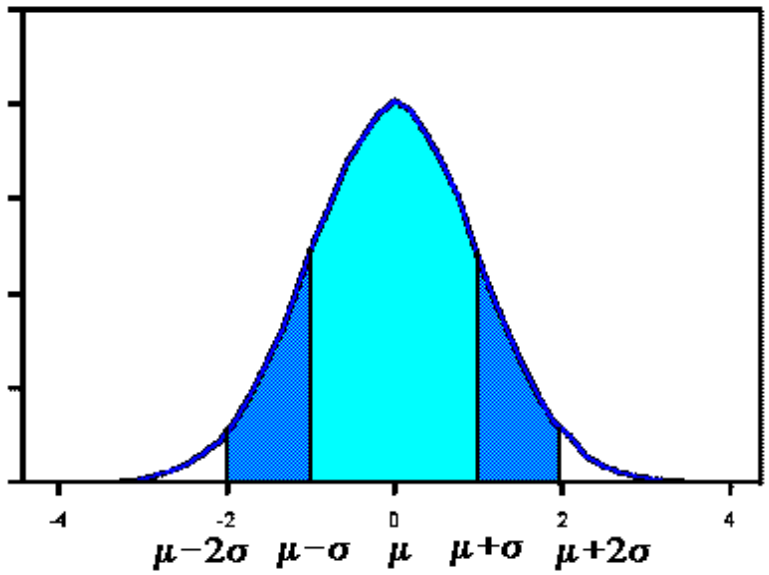
$$\sigma^2_{a+bX} = b^2\sigma^2$$

Since the spread of the distribution is not affected by adding or subtracting a constant, the value *a* is not considered. And, since the variance is a sum of squared terms, any multiplier value *b* must also be squared when adjusting the variance.

PDF of Random Signal:

The Normal Distribution:

A normal distribution has a bell-shaped density curve described by its mean μ and standard deviation σ . The density curve is symmetrical, centred about its mean, with its spread determined by its standard deviation. The height of a normal density curve at a given point x is given by



The **Standard Normal** curve, shown here, has mean 0 and standard deviation 1. If a dataset follows a normal distribution, then about 68% of the observations will fall within σ of the mean μ, which in this case is with the interval (-1,1). About 95% of the observations will fall within 2 standard deviations of the mean, which is the interval

(-2,2) for the standard normal, and about 99.7% of the observations will fall within 3 standard deviations of the mean, which corresponds to the interval (-3,3) in this case.

TASKS:

1) Use randn(1, 20) to generate 10 random values and randn(1, 2000) to generate 2000 random values, and observe the pdf of both for mean and variance.
2) Use randn(1, 2000) to generate 2000 random values. Perform following:
   - Calculate mean and variance using equations.
   - Plot PDF of both signals and compare mean and variance with the once you have collected.
   - Change mean to 300 and -300 than plot the pdfs.
   - Change variance of the signals by 100 and 500 and then display the pdf.