Chapter 11

Exercise Solutions

E11.1

$V_d = V_1 - V_2$

$= 2 + 0.005 \sin \omega t - (0.5 - 0.005 \sin \omega t)$

$\Rightarrow \underline{V_d = 1.5 + 0.010 \sin \omega t \text{ (V)}}$

$V_{cm} = \dfrac{V_1 + V_2}{2}$

$= \dfrac{2 + 0.005 \sin \omega t + 0.5 - 0.005 \sin \omega t}{2}$

$\Rightarrow \underline{V_{cm} = 1.25 \text{ V}}$

E11.2

$v_E = -V_{BE}(\text{on}) \Rightarrow \underline{v_E = -0.7 \text{ V}}$

$I_{C1} = I_{C2} = 0.5 \text{ mA}$

$v_{C1} = v_{C2} = 10 - (0.5)(10)$

$\Rightarrow \underline{v_{C1} = v_{C2} = 5 \text{ V}}$

E11.3

For $v_1 = v_2 = +4$ V

$\Rightarrow$ Minimum $v_{C1} = v_{C2} = 4$ V

$I_{C1} = I_{C2} = \dfrac{I_Q}{2} = 1 \text{ mA}$

$R_C = \dfrac{10 - 4}{1} \Rightarrow \underline{R_C = 6 \text{ k}\Omega}$

E11.4

$\dfrac{i_{C2}}{I_Q} = \dfrac{1}{1 + \exp\left(\dfrac{v_d}{V_T}\right)} = 0.99$

$1 + \exp\left(\dfrac{v_d}{V_T}\right) = \dfrac{1}{0.99}$

$\exp\left(\dfrac{v_d}{V_T}\right) = \dfrac{1}{0.99} - 1$

$v_d = V_T \ln\left[\dfrac{1}{0.99} - 1\right]$

$\Rightarrow \underline{v_d = -119.5 \text{ mV}}$

E11.6

a. $v_1 = v_2 = 0 \Rightarrow \underline{v_E = 0.7 \text{ V}}$

$\Delta V_{RC} = (0.25)(8) = 2 \text{ V}$

$\Rightarrow v_{C1} = v_{C2} = -3 \text{ V}$

$\Rightarrow \underline{v_{EC1} = 3.7 \text{ V}}$

b. $v_1 = v_2 = 2.5 \text{ V} \Rightarrow \underline{v_E = 3.2 \text{ V}}$

$\Rightarrow \underline{v_{EC1} = 6.2 \text{ V}}$

c. $v_1 = v_2 = -2.5 \text{ V} \Rightarrow \underline{v_E = -1.8 \text{ V}}$

$\Rightarrow \underline{v_{EC1} = 1.2 \text{ V}}$

E11.7

Let $I_Q = 1\,mA$, then $I_{CQ1} = I_{CQ2} = 0.5\,mA$

$g_{m1} = g_{m2} = \dfrac{0.5}{0.026} = 19.23\,mA/V$

At v_{C2},

$A_d = \dfrac{v_{c2}}{v_d} = \dfrac{1}{2} g_m R_{C2}$

So, $150 = \dfrac{1}{2}(19.23) R_{C2} \Rightarrow \underline{R_{C2} = 15.6\,k\Omega}$

At v_{C1},

$A_d = \dfrac{v_{c1}}{v_d} = -\dfrac{1}{2} g_m R_{C1}$

So, $-100 = -\dfrac{1}{2}(19.23) R_{C1} \Rightarrow \underline{R_{C1} = 10.4\,k\Omega}$

If $V^+ = +10\,V$ and $V^- = -10\,V$, dc biasing is OK.

E11.8

a. Diff. Gain $A_d = \dfrac{I_Q R_C}{4V_T}$

For $v_1 = v_2 = 5$ V $\Rightarrow$ Minimum collector voltage

$v_{C2} = 5$ V

$\Rightarrow \dfrac{I_Q}{2} \cdot R_C = 15 - 5 = 10 \text{ V}$

or $\underline{I_Q R_C = 20 \text{ V}}$ for max. A_d

Then

$A_d = \dfrac{20}{2(0.026)} \Rightarrow \underline{A_d(\max) = 192}$

b. If $I_Q = 0.5$ mA, $R_C = 40$ kΩ

$A_{cm} = \dfrac{-\left(\dfrac{I_Q R_C}{2V_T}\right)}{\left[1 + \dfrac{(1+\beta) I_Q R_o}{V_T \beta}\right]}$

Then $A_{cm} = \dfrac{-\left(\dfrac{20}{2(0.026)}\right)}{\left[1 + \dfrac{(201)(0.5)(100)}{(0.026)(200)}\right]}$

$\Rightarrow \underline{A_{cm} = -0.199}$

and $CMRR_{dB} = 20 \log_{10}\left(\dfrac{192}{0.199}\right)$

$\Rightarrow \underline{CMRR_{dB} = 59.7 \text{ dB}}$

E11.9

For $v_1 = v_2 = 5$ V $\Rightarrow$ min $v_{C1} = v_{C2} = 5$ V

So $I_{C1}R_C = 10 - 5 = 0.25R_C \Rightarrow \underline{R_C = 20\text{ k}\Omega}$

$$A_d = \frac{I_Q R_C}{4V_T} = \frac{(0.5)(20)}{4(0.026)} \Rightarrow A_d = 96.2 \quad \text{Let } I_Q = 0.5\,mA$$

$CMRR_{dB} = 95$ db $\Rightarrow CMRR = 5.62 \times 10^4$

$$\Rightarrow A_{cm} = \frac{96.2}{5.62 \times 10^4} \Rightarrow |A_{cm}| = 1.71 \times 10^{-3}$$

$$|A_{cm}| = \frac{\left(\dfrac{I_Q R_C}{2V_T}\right)}{\left[1 + \dfrac{(1+\beta)I_Q R_0}{V_T\beta}\right]} = 1.71 \times 10^{-3}$$

$$\frac{\left[\dfrac{(0.5)(20)}{2(0.026)}\right]}{\left[1 + \dfrac{(201)(0.5)R_0}{(0.026)(200)}\right]} = 1.71 \times 10^{-3}$$

$1 + 19.3R_0 = 1.12 \times 10^5 \Rightarrow R_0 = 5.83 \times 10^3$ kΩ

$= 5.83$ MΩ

We have $R_0 = r_{04}[1 + g_{m2}(R_2 \| r_{\pi 2})]$

$$r_{04} = \frac{V_A}{I_Q} = \frac{125}{0.5} = 250\text{ k}\Omega$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.5}{0.026} = 19.2\text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_Q} = \frac{(200)(0.026)}{0.5} = 10.4\text{ k}\Omega$$

$5830 = 250[1 + g_{m2}(R_2 \| r_{\pi 2})]$

$19.2(R_2 \| r_{\pi 2}) = 22.3$

$(R_2 \| r_{\pi 2}) = 1.16$ kΩ

$$\frac{R_2(10.4)}{R_2 + 10.4} = 1.16$$

$R_2(10.4 - 1.16) = (1.16)(10.4)$

$\Rightarrow \underline{R_2 = 1.31\text{ k}\Omega}$

$$I_Q R_2 - I_1 R_3 = V_T \ln\left(\frac{I_1}{I_Q}\right) \qquad \text{Let } I_1 = 1\,mA$$

$$(0.5)(1.31) - (1)R_3 = (0.026)\ln\left(\frac{1}{0.5}\right)$$

$\Rightarrow \underline{R_3 = 0.637\text{ k}\Omega}$

If $V_{BE}(Q_3) \cong 0.7$ V

$$R_1 + R_3 = \frac{10 - 0.7 - (-10)}{1} = 19.3$$

$\Rightarrow \underline{R_1 \cong 18.7\text{ k}\Omega}$

E11.10

a. $v_0 = A_d v_d + A_{cm} v_{cm}$

$v_d = v_1 - v_2 = 0.505 \sin\omega t - 0.495 \sin\omega t$

$= 0.01 \sin\omega t$

$$v_{cm} = \frac{v_1 + v_2}{2} = \frac{0.505 \sin\omega t + 0.495 \sin\omega t}{2}$$

$= 0.50 \sin\omega t$

$v_0 = (60)(0.01 \sin\omega t) + (0.5)(0.5 \sin\omega t)$

$\Rightarrow \underline{v_0 = 0.85 \sin\omega t\ (\text{V})}$

b.

$v_d = v_1 - v_2$

$= 0.5 + 0.005 \sin\omega t - (0.5 - 0.005 \sin\omega t)$

$= 0.01 \sin\omega t$

$$v_{cm} = \frac{v_1 + v_2}{2}$$

$$= \frac{0.5 + 0.005 \sin\omega t + 0.5 - 0.005 \sin\omega t}{2}$$

$= 0.5$

$v_0 = (60)(0.01 \sin\omega t) + (0.5)(0.5)$

$\Rightarrow \underline{v_0 = 0.25 + 0.6 \sin\omega t\ (\text{V})}$

E11.11

a. $$I_{B1} = I_{B2} = \frac{I_Q/2}{(1+\beta)} = \frac{1}{151}$$

$\Rightarrow \underline{I_{B1} = I_{B2} = 6.62\ \mu\text{A}}$

b. $$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{1} = 3.9\text{ k}\Omega$$

$R_{id} = 2r_\pi = 2(3.9) = 7.8$ kΩ

$$I_b = \frac{V_d}{R_{id}} = \frac{10 \sin\omega t\ (\text{mV})}{7.8\text{ k}\Omega}$$

$\Rightarrow \underline{I_b = 1.28 \sin\omega t\ (\mu\text{A})}$

c. $R_{icm} \cong 2(1+\beta)R_0 = 2(151)(50) \Rightarrow 15.1$ MΩ

$$I_b = \frac{V_{cm}}{R_{icm}} = \frac{3 \sin\omega t}{15.1\text{ M}\Omega} \Rightarrow \underline{I_b = 0.199 \sin\omega t\ (\mu\text{A})}$$

E11.12

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right)v_d^2}$$

Using the parameters in Example 11.11,
$K_n = 0.5\,mA/V^2$, $I_Q = 1\,mA$, then

$$\frac{i_{D1}}{I_Q} = 0.90 = \frac{1}{2} + \sqrt{\frac{0.5}{2(1)}} \cdot v_d \sqrt{1 - \left(\frac{0.5}{2(1)}\right)v_d^2}$$

By trial and error,
$\underline{v_d = 0.894\,V}$

E11.13

$$I_1 = \frac{10 - V_{GS4}}{R_1} = K_{n3}(V_{GS4} - V_{TN})^2$$

$10 - V_{GS4} = (0.1)(80)(V_{GS4} - 0.8)^2$

$10 - V_{GS4} = 8(V_{GS4}^2 - 1.6V_{GS4} + 0.64)$

$8V_{GS4}^2 - 11.8V_{GS4} - 4.88 = 0$

$$V_{GS4} = \frac{11.8 \pm \sqrt{(11.8)^2 + 4(8)(4.88)}}{2(8)} = 1.81\text{ V}$$

$$I_1 = I_Q = \frac{10 - 1.81}{80} = 0.102 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{0.102}{2} = 0.051 \text{ mA}$$

$$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_o} = \frac{-(0.342)(16)}{1 + 2(0.342)(85)}$$

$$A_{cm} = -0.0925$$

$$CMRR_{dB} = 29.4 \text{ dB}$$

$$v_{o1} = v_{o2} = 5 - (0.051)(40) = 2.96 \text{ V}$$

$$CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$$

E11.19

a. $I_{B5} = \dfrac{I_Q}{\beta(1+\beta)} = \dfrac{0.5}{(180)(181)} \Rightarrow 15.3 \text{ nA}$

So $\underline{I_0 = 15.3 \text{ nA}}$

b. For a balanced condition

$V_{EC4} = V_{EC3} = V_{EB3} \Rightarrow \underline{V_{EC4} = 0.7 \text{ V}}$

$V_{CE2} = V_{C2} - V_{E2} = (10 - 0.7) - (-0.7)$

$\Rightarrow \underline{V_{CE2} = 10 \text{ V}}$

E11.20

a. $g_f = \dfrac{I_Q}{4V_T} = \dfrac{0.5}{4(0.026)} = 4.81 \text{ mA/V}$

$r_{o2} = \dfrac{V_{A2}}{I_{C2}} = \dfrac{125}{0.25} = 500 \text{ k}\Omega$

$r_{o4} = \dfrac{V_{A4}}{I_{C4}} = \dfrac{85}{0.25} = 340 \text{ k}\Omega$

$A_d = 2g_f(r_{o2}\|r_{o4}) = 2(4.81)(500\|340)$

$\Rightarrow \underline{A_d = 1947}$

b. $A_d = 2g_f(r_{o2}\|r_{o4}\|R_L)$

$A_d = 2(4.81)[500\|340\|100]$

$\Rightarrow \underline{A_d = 644}$

c. $r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(150)(0.026)}{0.25} = 15.6 \text{ k}\Omega$

$R_{id} = 2r_\pi \Rightarrow \underline{R_{id} = 31.2 \text{ k}\Omega}$

d. $R_0 = r_{o2}\|r_{o4} = 500\|340 \Rightarrow \underline{R_0 = 202 \text{ k}\Omega}$

E11.21

$A_d = 2g_f(r_{o2}\|r_{o4})$

$g_f = \dfrac{I_Q}{4V_T} = \dfrac{0.2}{4(0.026)} = 1.92 \text{ mA/V}$

$r_{o2} = \dfrac{V_{A2}}{I_{C2}} = \dfrac{120}{0.1} = 1200 \text{ k}\Omega$

$r_{o4} = \dfrac{V_{A4}}{I_{C4}} = \dfrac{80}{0.1} = 800 \text{ k}\Omega$

$A_d = 2(1.92)(1200\|800) \Rightarrow \underline{A_d = 1843}$

E11.22

$P = (I_Q + I_{REF})(5-(-5))$

$10 = (0.1 + I_{REF})(10) \Rightarrow \quad I_{REF} = 0.9\, mA$

$R_1 = \dfrac{5-0.7-(-5)}{I_{REF}} = \dfrac{9.3}{0.9} \Rightarrow \quad \underline{R_1 = 10.3\, k\Omega}$

$I_Q R_E = V_T \ln\left(\dfrac{I_{REF}}{I_Q}\right)$

$R_E = \dfrac{0.026}{0.1}\ln\left(\dfrac{0.9}{0.1}\right) \Rightarrow \quad \underline{R_E = 0.571\, k\Omega}$

$r_{o2} = \dfrac{V_{A2}}{I_{C2}} = \dfrac{125}{0.05} \Rightarrow 2.5\, M\Omega$

$r_{o4} = \dfrac{V_{A4}}{I_{C4}} = \dfrac{85}{0.05} \Rightarrow 1.7\, M\Omega$

$g_m = \dfrac{0.05}{0.026} = 1.923\, mA/V$

$A_d = g_m(r_{o2}\|r_{o4}\|R_L) = (1.923)(2500\|1700\|90) \Rightarrow$

$\underline{A_d = 159}$

E11.23

a. $R_0 = r_{o2}\|r_{o4}$

$r_{o2} = \dfrac{120}{0.1} = 1.2 \text{ M}\Omega$

$r_{o4} = \dfrac{80}{0.1} = 0.8 \text{ M}\Omega$

$R_0 = 1.2\|0.8 \Rightarrow \underline{R_0 = 0.48 \text{ M}\Omega}$

b. $A_d(\text{open circuit}) = 2g_f(r_{o2}\|r_{o4})$

$A_d(\text{with load}) = 2g_f(r_{o2}\|r_{o4}\|R_L)$

For $A_d(\text{with load}) = \dfrac{1}{2}A_d(\text{open circuit})$

$\Rightarrow R_L = (r_{o2}\|r_{o4}) \Rightarrow \underline{R_L = 0.48 \text{ M}\Omega}$

E11.24

$A_d = 2\sqrt{\dfrac{2K_n}{I_Q}}\cdot\dfrac{1}{(\lambda_2+\lambda_4)}$

$= 2\sqrt{\dfrac{2(0.1)}{0.1}}\cdot\dfrac{1}{(0.01+0.015)} \Rightarrow$

$\underline{A_d = 113}$

E11.25

For the MOSFET, $I_D = 25\ \mu\text{A}$

$25 = 20(V_{GS}-1)^2 \Rightarrow V_{GS} = 2.12 \text{ V}$

$g_{m1} = 2K_{n1}(V_{GS} - V_{TN}) = 2(20)(2.12-1)$

$\Rightarrow \underline{g_{m1} - 44.8\ \mu\text{A/V}}, \quad \underline{r_0 = \infty}$

For the Bipolar, $I_Q = 100 - 25 = 75\ \mu\text{A}$

$r_{\pi 2} = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{0.075} \Rightarrow \underline{r_{\pi 2} = 34.7 \text{ k}\Omega}$

$g_{m2} = \dfrac{I_{CQ}}{V_T} = \dfrac{75}{0.026}$

$\Rightarrow \underline{g_{m2} = 2.88 \text{ mA/V}}, \quad \underline{r_{o2} = \infty}$

$g_m^C = \dfrac{g_{m1}(1+g_{m2}r_\pi)}{(1+g_{m1}r_\pi)}$

$= \dfrac{(44.8)[1+(2.88)(34.7)]}{1+(0.0448)(34.7)}$

$= \dfrac{(44.8)(100.9)}{2.55}$

$\Rightarrow \underline{g_m^C = 1.77 \text{ mA/V}}$

E11.26

From Figure 11.41

$$r_{o4} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

$$R_o \cong \beta r_{o4} = (150)(160) \text{ k}\Omega \Rightarrow \underline{R_o = 24 \text{ M}\Omega}$$

From Figure 11.42

$$r_{o6} = \frac{1}{\lambda I_D} = \frac{1}{(0.0125)(0.5)} \Rightarrow \underline{r_{o6} = 160 \text{ k}\Omega}$$

$$0.5 = 0.5(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2 \text{ V}$$

$$g_{m6} = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2-1) = 1 \, mA/V$$

$$r_{o4} = 160 \text{ k}\Omega$$

$$R_o = (g_{m6})(r_{o6})(\beta r_{o4}) = (1)(160)(150)(160)$$

$$\Rightarrow \underline{R_o = 3,840 \text{ M}\Omega}$$

E11.27

From Equation (11.103)

$$R_i = \frac{2(1+\beta)\beta V_T}{I_Q} = \frac{2(121)(120)(0.026)}{0.5}$$

$$\Rightarrow \underline{R_i = 1.51 \text{ M}\Omega}$$

$$r_{\pi 11} = \frac{\beta V_T}{I_Q} = \frac{(120)(0.026)}{0.5} = 6.24 \text{ k}\Omega$$

$$R'_E = r_{\pi 11} \| R_3 = 6.24 \| 0.1 = 0.0984 \text{ k}\Omega$$

$$g_{m11} = \frac{I_Q}{V_T} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{o11} = \frac{V_A}{I_Q} = \frac{120}{0.5} = 240 \text{ k}\Omega$$

Then

$$R_{C11} = r_{o11}(1 + g_{m11}R'_E)$$

$$= 240[1 + (19.23)(0.0984)]$$

$$= 694 \text{ k}\Omega$$

$$r_{\pi 8} = \frac{\beta V_T}{I_{C8}} = \frac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$$

$$R_{b8} = r_{\pi 8} + (1+\beta)R_4 = 1.56 + (121)(5)$$

$$= 607 \text{ k}\Omega$$

Then

$$R_{L7} = R_{C11} \| R_{b8} = 694 \| 607 = 324 \text{ k}\Omega$$

Then

$$A_v = \left(\frac{I_Q}{2V_T}\right) R_{L7} = \left[\frac{0.5}{2(0.026)}\right](324)$$

$$\Rightarrow \underline{A_v = 3115}$$

$$R_o = R_4 \left\| \left(\frac{r_{\pi 8} + Z}{1+\beta}\right)\right.$$

$$Z = R_{C11} \| R_{C7}$$

$$R_{C7} = \frac{V_A}{I_Q} = \frac{120}{0.5} = 240 \text{ k}\Omega$$

$$Z = 694 \| 240 = 178 \text{ k}\Omega$$

$$R_o = 5 \left\| \left(\frac{1.56 + 178}{121}\right)\right. = 5 \| 1.48$$

$$\Rightarrow \underline{R_o = 1.14 \text{ k}\Omega}$$

E11.28

$$A_v = \left(\frac{I_Q}{2V_T}\right) R_{L7}$$

$$10^3 = \left(\frac{0.5}{2(0.026)}\right) R_{L7}$$

$$\Rightarrow \underline{R_{L7} = 104 \text{ k}\Omega}$$

E11.29

a. $R_1 = \frac{10 - 0.7 - (-10)}{I_1} = \frac{19.3}{0.6} \Rightarrow \underline{R_1 = 32.2\ \text{k}\Omega}$

$I_{C1} = I_{C2} = 0.1\ \text{mA} \Rightarrow I_Q \cong 0.2\ \text{mA}$

$R_2 = \frac{V_T}{I_Q} \cdot \ln\left(\frac{I_1}{I_Q}\right) = \frac{0.026}{0.2} \cdot \ln\left(\frac{0.6}{0.2}\right)$

$\Rightarrow \underline{R_2 = 143\ \Omega}$

$I_{R6} = I_1 = 0.6\ \text{mA} \Rightarrow \underline{R_3 = 0}$

$V_{02} = V_{CE2} + V_E = 4 - 0.7 = 3.3\ \text{V}$

$R_C = \frac{10 - 3.3}{I_{C2}} = \frac{6.7}{0.1} \Rightarrow \underline{R_C = 67\ \text{k}\Omega}$

$V_{E4} = V_{02} - 2V_{BE} = 3.3 - 2(0.7) = 1.9\ \text{V}$

$R_4 = \frac{1.9}{I_{R4}} = \frac{1.9}{0.6} \Rightarrow \underline{R_4 = 3.17\ \text{k}\Omega}$

$V_{03} = V_{CE4} + V_{E4} = 3 + 1.9 = 4.9$

$R_5 = \frac{10 - 4.9}{I_{R4}} = \frac{5.1}{0.6} \Rightarrow \underline{R_5 = 8.5\ \text{k}\Omega}$

$V_{E5} = V_{03} - V_{BE} = 4.9 - 0.7 = 4.2$

$R_6 = \frac{4.2 - 0.7}{I_{R6}} = \frac{3.5}{0.6} \Rightarrow \underline{R_6 = 5.83\ \text{k}\Omega}$

$R_7 = \frac{0 - (-10)}{I_{R7}} = \frac{10}{5} \Rightarrow \underline{R_7 = 2\ \text{k}\Omega}$

b. $R_{i2} = r_{\pi3} + (1 + \beta) r_{\pi4}$

$r_{\pi4} = \frac{\beta V_T}{I_{R4}} = \frac{(100)(0.026)}{0.6} = 4.33\ \text{k}\Omega$

$r_{\pi3} \cong \frac{\beta^2 V_T}{I_{R4}} = \frac{(100)^2(0.026)}{0.6} = 433\ \text{k}\Omega$

$R_{i2} = 433 + (101)(4.33) \Rightarrow \underline{R_{i2} = 870\ \text{k}\Omega}$

$R_{i3} = r_{\pi5} + (1 + \beta)[R_6 + r_{\pi6} + (1 + \beta) R_7]$

$r_{\pi5} = \frac{\beta V_T}{I_{R6}} = \frac{(100)(0.026)}{0.6} = 4.33\ \text{k}\Omega$

$r_{\pi6} = \frac{\beta V_T}{I_{R7}} = \frac{(100)(0.026)}{5} = 0.52\ \text{k}\Omega$

$R_{i3} = 4.33 + (101)[5.83 + 0.52 + (101)(2)]$

$\Rightarrow \underline{R_{i3} = 21.0\ \text{M}\Omega}$

c. $A_d = A_{d1} \cdot A_2 \cdot A_3$

$A_{d1} = g_f (R_C \| R_{i2})$

$g_f = \frac{I_Q}{4V_T} = \frac{0.2}{4(0.026)} = 1.92\ \text{mA/V}$

$A_{d1} = (1.92)(67 \| 870) = 119$

$A_2 = \left(\frac{I_{R4}}{2V_T}\right) R_5 = \frac{0.6}{2(0.026)}(8.5) = 98.1$

$A_3 \cong 1$

$A_d = (119)(98.1)(1)$

$\Rightarrow \underline{A_d = 11,674}$

Chapter 11

Problem Solutions

11.1

a. $I_E = I_{C1} + I_{C2} = 4\text{ mA} = \dfrac{-0.7-(-8.7)}{R_E}$

So $R_E = \dfrac{8.0}{4} = \underline{2\text{ k}\Omega = R_E}$

Then

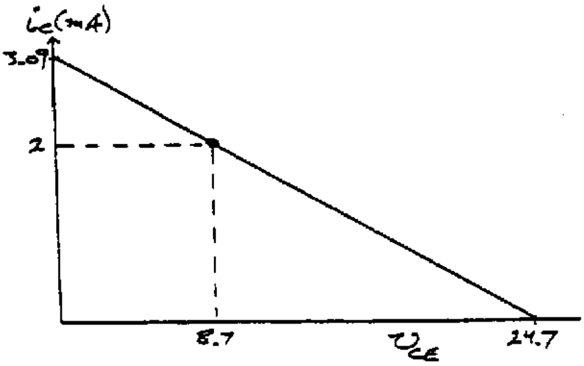
$R_C = \dfrac{16 - V_{02}}{I_{C2}} = \dfrac{16-8}{2} \Rightarrow \underline{R_C = 4\text{ k}\Omega}$

b. Neglecting base currents

$16 = I_C R_C + V_{CE2} + 2I_C R_E - 8.7$

$V_{CE2} = 24.7 - I_C(R_C + 2R_E) = 24.7 - I_C(8)$

For $I_C = 2$ mA, $V_{CEQ} = 8.7$ V



c. $v_{cm}(\max)$ for $V_{CB} = 0 \Rightarrow \underline{v_{cm}(\max) = 8\text{ V}}$

$v_{cm}(\min)$ for Q_1 and Q_2 at edge of cutoff

$\Rightarrow \underline{v_{cm}(\min) = -8\text{ V}}$

11.2

$P = (I_1 + I_{C4})(V^+ - V^-)$

$I_1 \cong I_{C4}$ so

$1.2 = 2I_1(6) \Rightarrow \underline{I_1 = I_{C4} = 0.1\,mA}$

$R_1 = \dfrac{3-0.7-(-3)}{0.1} \Rightarrow \underline{R_1 = 53\,k\Omega}$

For $v_{CM} = +1V \Rightarrow V_{C1} = V_{C2} = 1V \Rightarrow$

$R_C = \dfrac{3-1}{0.05} \Rightarrow \underline{R_C = 40\,k\Omega}$

One-sided output

$A_d = \dfrac{1}{2}g_m R_C$ where $g_m = \dfrac{0.05}{0.026} = 1.923\,mA/V$

Then

$A_d = \dfrac{1}{2}(1.923)(40) \Rightarrow \underline{A_d = 38.5}$

11.3

a. $I_1 = \dfrac{10-2(0.7)}{8.5} \Rightarrow \underline{I_1 = 1.01\text{ mA}}$

$I_{C2} = \dfrac{I_1}{1+\dfrac{2}{\beta(1+\beta)}} = \dfrac{1.01}{1+\dfrac{2}{(100)(101)}}$

$\Rightarrow \underline{I_{C2} \cong 1.01\text{ mA}}$

$I_{C4} = \left(\dfrac{100}{101}\right)\left(\dfrac{1.01}{2}\right) \Rightarrow \underline{I_{C4} \cong 0.50\text{ mA}}$

$V_{CE2} = (0-0.7)-(-5) \Rightarrow \underline{V_{CE2} = 4.3\text{ V}}$

$V_{CE4} = [5-(0.5)(2)] - (-0.7) \Rightarrow \underline{V_{CE4} = 4.7\text{ V}}$

b. For $V_{CE4} = 2.5\text{ V} \Rightarrow V_{C4} = -0.7 + 2.5 = 1.8$ V

$I_{C4} = \dfrac{5-1.8}{2} \Rightarrow \underline{I_{C4} = 1.6\text{ mA}}$

$I_{C2} + \left(\dfrac{1+\beta}{\beta}\right)(2I_{C4}) = \left(\dfrac{101}{100}\right)(2)(1.6)$

$\Rightarrow \underline{I_{C2} = 3.23\text{ mA}}$

$\underline{I_1 \approx I_{C2} = 3.23\text{ mA}}$

$R_1 = \dfrac{10-2(0.7)}{3.23} \Rightarrow \underline{R_1 = 2.66\text{ k}\Omega}$

11.4

a. $0 = 0.7 + \dfrac{I_E}{2}(2) + I_E(85) - 5$

$I_E = \dfrac{5-0.7}{85+1} \Rightarrow \underline{I_E = 0.050\text{ mA}}$

$I_{C1} = I_{C2} = \left(\dfrac{\beta}{1+\beta}\right)\left(\dfrac{I_E}{2}\right) = \left(\dfrac{100}{101}\right)\left(\dfrac{0.050}{2}\right)$

Or $\underline{I_{C1} = I_{C2} = 0.0248\text{ mA}}$

$V_{CE1} = V_{CE2} = [5 - I_{C1}(100)] - (-0.7)$

So $\underline{V_{CE1} = V_{CE2} = 3.22\text{ V}}$

b. $v_{cm}(\max)$ for $V_{CB} = 0$ and

$V_C = 5 - I_{C1}(100) = 2.52$ V

So $\underline{v_{cm}(\max) = 2.52\text{ V}}$

$v_{cm}(\min)$ for Q_1 and Q_2 at the edge of cutoff

$\Rightarrow \underline{v_{cm}(\min) = -4.3\text{ V}}$

(c) Differential-mode half circuits

$-\dfrac{v_d}{2} = V_\pi + \left(\dfrac{V_\pi}{r_\pi} + g_m V_\pi\right)\cdot R_E$

$= V_\pi\left[1 + \dfrac{(1+\beta)}{r_\pi}R_E\right]$

Then

$$V_\pi = \frac{-(v_d/2)}{\left[1+\frac{(1+\beta)}{r_\pi}R_E\right]}$$

$$v_o = -g_m V_\pi R_C \Rightarrow A_d = \frac{1}{2}\cdot\frac{\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105\ k\Omega$$

Then

$$A_d = \frac{1}{2}\cdot\frac{(100)(100)}{105+(101)(2)} \Rightarrow \underline{A_d = 16.3}$$

11.5

a. i. $\underline{(v_{01} - v_{02}) = 0}$

ii. $I_{C1} = I_{C2} = 1\text{ mA}$

$$v_{01} - v_{02} = [V^+ - I_{C1}R_{C1}] - [V^+ - I_{C2}R_{C2}]$$

$$= I_C(R_{C2} - R_{C1}) = (1)(7.9 - 8)$$

$$\Rightarrow \underline{v_{01} - v_{02} = -0.1\text{ V}}$$

b. $I_0 = (I_{S1} + I_{S2})\exp\left(\frac{v_{BE}}{V_T}\right)$

$$\text{So } \exp\left(\frac{v_{BE}}{V_T}\right) = \frac{2\times 10^{-3}}{10^{-13} + 1.1\times 10^{-13}}$$

$$= 9.524\times 10^9$$

$$I_{C1} = I_{S1}\exp\left(\frac{v_{BE}}{V_T}\right) = (10^{-13})(9.524\times 10^9)$$

$$\Rightarrow I_{C1} = 0.952\text{ mA}$$

$$I_{C2} = (1.1\times 10^{-13})(9.524\times 10^9)$$

$$\Rightarrow I_{C2} = 1.048\text{ mA}$$

i. $v_{01} - v_{02} = I_{C2}R_{C2} - I_{C1}R_{C1}$

$$\Rightarrow v_{01} - v_{02} = (1.048 - 0.952)(8)$$

$$\Rightarrow \underline{v_{01} - v_{02} = 0.768\text{ V}}$$

ii. $v_{01} - v_{02} = (1.048)(7.9) - (0.952)(8)$

$$v_{01} - v_{02} = 8.279 - 7.616$$

$$\Rightarrow \underline{v_{01} - v_{02} = 0.663\text{ V}}$$

11.6

From Equation (11.12(b))

$$i_{C2} = \frac{I_Q}{1+e^{v_d/V_T}}$$

$$0.90 = \frac{1}{1+e^{v_d/V_T}}$$

$$\text{So } e^{v_d/V_T} = \frac{1}{0.90} - 1 = 0.111$$

$$v_d = V_T\ln(0.111) = (0.026)\ln(0.111)$$

$$\Rightarrow \underline{v_d = -0.0571\text{ V}}$$

11.7

For $v_{CM} = 3.5\,V$ and a maximum peak-to-peak swing in the output voltage of $2\,V$, we need the quiescent collector voltage to be

$V_C = 3.5 + 1 = 4.5\,V$

Assume the bias is $\pm 10\,V$, and $I_Q = 0.5\,mA$.

Then $I_C = 0.25\,mA$

$$\text{Now } R_C = \frac{10-4.5}{0.25} \Rightarrow \underline{R_C = 22\ k\Omega}$$

$$\text{In this case, } r_\pi = \frac{(100)(0.026)}{0.25} = 10.4\ k\Omega$$

Then

$A_d = \dfrac{(100)(22)}{2(10.4+0.5)} = 101$ So gain specification is met.

For $CMRR_{dB} = 80\ dB \Rightarrow$

$$CMRR = 10^4 = \frac{1}{2}\left[1+\frac{(1+\beta)I_Q R_o}{V_T\beta}\right]$$

$$= \frac{1}{2}\left[1+\frac{(101)(0.5)R_o}{(0.026)(100)}\right] \Rightarrow$$

$$\underline{R_o = 1.03\ M\Omega}$$

Need to use a Modified Widlar current source.

$$R_o = r_o\left[1+g_m(R_{E1}\|r_\pi)\right]$$

$$\text{If } V_A = 100\,V\text{, then } r_o = \frac{100}{0.5} = 200\ k\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$$

$$g_m = \frac{0.5}{0.026} = 19.23\ mA/V$$

Then

$$1030 = 200\left[1+(19.23)(R_{E1}\|r_\pi)\right] \Rightarrow$$

$$R_{E1}\|r_\pi = 0.216\ k\Omega = R_{E1}\|5.2 \Rightarrow$$

$$\underline{R_{E1} = 225\ \Omega}$$

Also let $R_{E2} = 225\ \Omega$ and $I_{REF} \cong 0.5\ mA$

11.8

a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7-(-10)}{0.15} \Rightarrow \underline{R_E = 62\ \text{k}\Omega}$$

b. Using the small-signal equivalent circuit shown in Figure 11.8, but including the R_B resistors, we have

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = \frac{V_e}{R_E}$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1+\beta}{r_\pi}\right) = \frac{V_e}{R_E}$$

Now $\frac{v_1 - V_e}{R_B + r_\pi} = \frac{V_{\pi 1}}{r_\pi}$ and $\frac{v_2 - V_e}{R_B + r_\pi} = \frac{V_{\pi 2}}{r_\pi}$

Then

$$V_{\pi 1} = \left(\frac{r_\pi}{R_B + r_\pi}\right)(v_1 - V_e)$$

and $V_{\pi 2} = \left(\frac{r_\pi}{R_B + r_\pi}\right)(v_2 - V_e)$

Substituting, we find

$$(v_1 + v_2 - 2V_e)\left(\frac{r_\pi}{R_B + r_\pi}\right)\left(\frac{1+\beta}{r_\pi}\right) = \frac{V_e}{R_E}$$

or

$$(v_1 + v_2 - 2V_e)\left(\frac{1+\beta}{R_B + r_\pi}\right) = \frac{V_e}{R_E}$$

Solving for V_e,

$$V_e = \frac{v_1 + v_2}{2 + \frac{R_B + r_\pi}{(1+\beta)R_E}}$$

Now $v_{02} = -g_m V_{\pi 2} R_C$

$$= -g_m R_C \left(\frac{r_\pi}{R_B + r_\pi}\right)(v_2 - V_e)$$

Substituting for V_e,

$$v_{02} = \frac{-\beta R_C}{R_B + r_\pi}\left[v_2 - \frac{v_1 + v_2}{2 + \frac{R_B + r_\pi}{(1+\beta)R_E}}\right]$$

$$= \frac{-\beta R_C}{R_B + r_\pi}\left[\frac{v_2\left(1 + \frac{R_B + r_\pi}{(1+\beta)R_E}\right) - v_1}{2 + \frac{R_B + r_\pi}{(1+\beta)R_E}}\right]$$

Now $v_1 = v_{cm} + \frac{v_d}{2}$

$v_2 = v_{cm} - \frac{v_d}{2}$

Substituting and rearranging terms, we obtain

$$v_{02} = \frac{-\beta R_C}{R_B + r_\pi}\left\{\frac{v_d}{2} - v_{cm}\left[\frac{1}{1 + \frac{2R_E(1+\beta)}{R_B + r_\pi}}\right]\right\}$$

$$A_d = \frac{v_{02}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7\ \text{k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow \underline{A_d = 71.0}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left[\frac{1}{1 + \frac{2R_E(1+\beta)}{r_\pi + R_B}}\right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5}\left[\frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}}\right]$$

$$\Rightarrow \underline{A_{cm} = -0.398}$$

$$CMRR_{dB} = 20\log_{10}\left|\frac{71.0}{0.398}\right| \Rightarrow \underline{CMRR_{dB} = 45.0\ \text{dB}}$$

c. $R_{id} = 2(r_\pi + R_B)$

$$R_{id} = 2(34.7 + 0.5) \Rightarrow \underline{R_{id} = 70.4\ \text{k}\Omega}$$

Common-mode input resistance

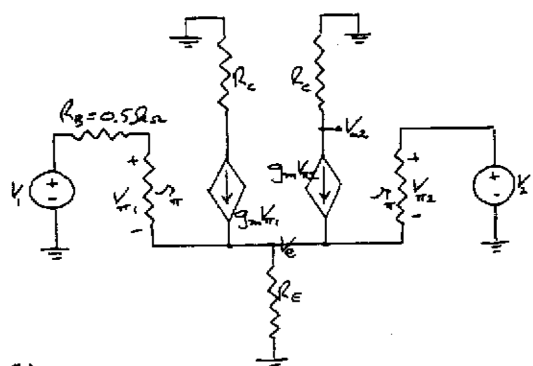
$$R_{icm} = \frac{1}{2}[r_\pi + R_B + 2(1+\beta)R_E]$$

$$= \frac{1}{2}[34.7 + 0.5 + 2(101)(62)]$$

$$\Rightarrow \underline{R_{icm} = 6.28\ \text{M}\Omega}$$

11.9

(a) $R_E = \dfrac{-0.7-(-10)}{0.25} \Rightarrow \underline{R_E = 37.2\ k\Omega}$



(b)

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = \frac{V_e}{R_E}$$

or (1) $\left(\dfrac{1+\beta}{r_\pi}\right)(V_{\pi 1} + V_{\pi 2}) = \dfrac{V_e}{R_E}$

$$\frac{V_{\pi 1}}{r_\pi} = \frac{V_1 - V_e}{R_B + r_\pi} \Rightarrow V_{\pi 1} = \left(\frac{r_\pi}{r_\pi + R_B}\right)(V_1 - V_e)$$

$V_{\pi 2} = V_2 - V_e$

Then

$$(1)\ \left(\frac{1+\beta}{r_\pi}\right)\left[\frac{r_\pi}{r_\pi + R_B}(V_1 - V_e) + (V_2 - V_e)\right] = \frac{V_e}{R_E}$$

From this, we find

$$V_e = \frac{V_1 + \frac{r_\pi + R_B}{r_\pi}\cdot V_2}{\left[\frac{r_\pi + R_B}{R_E(1+\beta)} + 1 + \frac{r_\pi + R_B}{r_\pi}\right]}$$

Now

$V_o = -g_m V_{\pi 2} R_C = -g_m R_C (V_2 - V_e)$

We have

$$r_\pi = \frac{(120)(0.026)}{0.125} \cong 25\,k\Omega\ ,\ g_m = \frac{0.125}{0.026} = 4.81\,mA/V$$

(i) Set $V_1 = \frac{V_d}{2}$ and $V_2 = -\frac{V_d}{2}$

Then

$$V_e = \frac{\frac{V_d}{2}\left(1 - \left(\frac{25+0.5}{25}\right)\right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}\right]} = \frac{\frac{V_d}{2}(-0.02)}{2.026}$$

So

$V_e = -0.00494 V_d$

Now

$$V_o = -(4.81)(50)\left(-\frac{V_d}{2} - (-0.00494)V_d\right) \Rightarrow$$

$$\underline{A_d = \frac{V_o}{V_d} = 119}$$

(ii) Set $V_1 = V_2 = V_{cm}$

Then

$$V_e = \frac{V_{cm}\left(1 + \frac{25+0.5}{25}\right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}\right]} = \frac{V_{cm}(2.02)}{2.02567}$$

$V_e = V_{cm}(0.9972)$

Then

$V_o = -(4.81)(50)[V_{cm} - V_{cm}(0.9972)]$

or

$$\underline{A_{cm} = \frac{V_o}{V_{cm}} = -0.673}$$

11.10

a. Neglecting base currents

$$I_1 = I_3 = 400\ \mu A \Rightarrow R_1 = \frac{30 - 0.7}{0.4}$$

$\Rightarrow \underline{R_1 = 73.25\ k\Omega}$

$V_{CE1} = 10\ V \Rightarrow V_{C1} = 9.3\ V$

$$R_C = \frac{15 - 9.3}{0.2} \Rightarrow \underline{R_C = 28.5\ k\Omega}$$

b. $r_\pi = \frac{(100)(0.026)}{0.2} = 13\ k\Omega$

$$r_0(Q_3) = \frac{50}{0.4} = 125\ k\Omega$$

Using the results from problem 11.9, we have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13+10)} \Rightarrow \underline{A_d = 62}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left\{\frac{1}{1 + \frac{2r_0(1+\beta)}{r_\pi + R_B}}\right\}$$

$$= -\frac{(100)(28.5)}{13+10}\left\{\frac{1}{1 + \frac{2(125)(101)}{13+10}}\right\}$$

$\Rightarrow \underline{A_{cm} = -0.113}$

$$CMRR_{dB} = 20\log_{10}\left(\frac{62}{0.113}\right)$$

$\Rightarrow \underline{CMRR_{dB} = 54.8\ \text{dB}}$

c. $R_{id} = 2(r_\pi + R_B) = 2(13+10) \Rightarrow \underline{R_{id} = 46\ k\Omega}$

$$R_{icm} = \frac{1}{2}[r_\pi + R_B + 2(1+\beta)r_0]$$

$$= \frac{1}{2}[13 + 10 + 2(101)(125)]$$

$\Rightarrow \underline{R_{icm} = 12.6\ M\Omega}$

11.11

From Equation (11.18)

$v_0 = v_{C2} - v_{C1} = g_m R_C v_d$

$$g_m = \frac{I_{CQ}}{V_T}$$

For $I_Q = 2$ mA, $I_{CQ} = 1$ mA

Then $g_m = \frac{1}{0.026} = 38.46$ mA/V

Now

$2 = (38.46)R_C(0.015)$

So $\underline{R_C = 3.47\ k\Omega}$

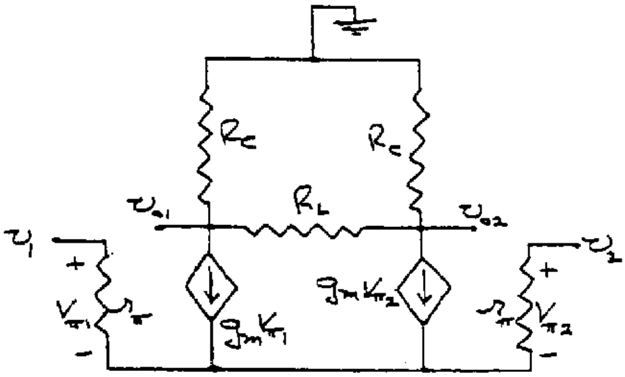
Now $V_C = V^+ - I_C R_C = 10 - (1)(3.47)$

$= 6.53$ V

For $V_{CB} = 0 \Rightarrow \underline{v_{cm}(\text{max}) = 6.53\ V}$

11.12

The small-signal equivalent circuit is



A KVL equation: $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

A KCL equation

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then $v_1 - v_2 = 2V_{\pi 1} \Rightarrow V_{\pi 1} = \frac{1}{2}(v_1 - v_2)$

and $V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$

At the v_{01} node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi 1} = 0$$

$$v_{01}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{02}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_2 - v_1) \qquad (1)$$

At the v_{02} node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi 2} = 0$$

$$v_{02}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{01}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_1 - v_2) \qquad (2)$$

From (1):

$$v_{02} = v_{01}\left(1 + \frac{R_L}{R_C}\right) - \frac{1}{2}g_m R_L(v_2 - v_1)$$

Substituting into (2)

$$v_{01}\left(1 + \frac{R_L}{R_C}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - \frac{1}{2}g_m R_L(v_2 - v_1)\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{01}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_1 - v_2)$$

$$v_{01}\left(\frac{1}{R_C} + \frac{R_L}{R_C^2} + \frac{1}{R_C}\right) = \frac{1}{2}g_m(v_1 - v_2)\left[1 - \left(\frac{R_L}{R_C} + 1\right)\right]$$

$$\frac{v_{01}}{R_C}\left(2 + \frac{R_L}{R_C}\right) = -\frac{1}{2}g_m\left(\frac{R_L}{R_C}\right)(v_1 - v_2)$$

For $v_1 - v_2 = v_d$

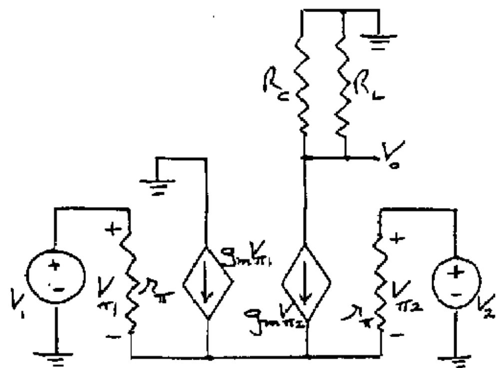
$$A_{v1} = \frac{v_{01}}{v_d} = \frac{-\frac{1}{2}g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$$

From symmetry: $A_{v2} = \dfrac{v_{02}}{v_d} = \dfrac{\frac{1}{2}g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$

Then $A_v = \dfrac{v_{02} - v_{01}}{v_d} = \dfrac{g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$

11.13

The small-signal equivalent circuit is



KVL equation: $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$ or

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

KCL equation:

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then $v_1 - v_2 = -2V_{\pi 2}$ or $V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$

Now $v_0 = -g_m V_{\pi 2}(R_C \| R_L)$

$$= \frac{1}{2}g_m(R_C \| R_L)(v_1 - v_2)$$

For $v_1 - v_2 \equiv v_d$

$$\Rightarrow A_d = \frac{v_0}{v_d} = \frac{1}{2}g_m(R_C \| R_L)$$

11.14

We have

$V_{C2} = -g_m V_{\pi 2} R_C = -g_m (V_{b2} - V_e) R_C$

and

$V_{C1} = -g_m V_{\pi 1} R_C = -g_m (V_{b1} - V_e) R_C$

Then

$V_0 = V_{C2} - V_{C1}$

$= -g_m (V_{b2} - V_e) R_C - [-g_m (V_{b1} - V_e) R_C]$

$= g_m R_C (V_{b1} - V_{b2})$

Differential gain

$$A_d = \frac{V_0}{V_{b1} - V_{b2}} = g_m R_C$$

Common-mode gain

$\underline{A_{cm} = 0}$

11.15

(a) $v_{cm} = 3\,V \Rightarrow V_{C1} = V_{C2} = 3\,V$

Then $R_C = \frac{10-3}{0.1} \Rightarrow \underline{R_C = 70\,k\Omega}$

(b) $CMRR_{dB} = 75\,dB \Rightarrow CMRR = 5623$

Now

$$CMRR = \frac{1}{2}\left[1 + \frac{(1+\beta) I_Q R_o}{\beta V_T}\right]$$

$$5623 = \frac{1}{2}\left[1 + \frac{(151)(0.2)R_o}{(150)(0.026)}\right] \Rightarrow R_o = 1.45\,M\Omega$$

Use a Widlar current source.

$R_o = r_o[1 + g_m R_E']$

Let V_A of current source transistor be $100\,V$.

Then

$r_o = \frac{100}{0.2} = 500\,k\Omega, \; g_m = \frac{0.2}{0.026} = 7.69\,mA/V$

$r_\pi = \frac{(150)(0.026)}{0.2} = 19.5\,k\Omega$

So

$1450 = 500[1 + (7.69)R_E'] \Rightarrow R_E' = 0.247\,k\Omega$

Now

$R_E' = R_E \| r_\pi \Rightarrow 0.247 = R_E \| 19.5 \Rightarrow \underline{R_E = 250\,\Omega}$

Then

$$I_Q R_E = V_T \ln\left(\frac{I_{REF}}{I_Q}\right)$$

$$(0.2)(0.250) = (0.026)\ln\left(\frac{I_{REF}}{(0.2)}\right) \Rightarrow I_{REF} = 1.37\,mA$$

Then

$$R_1 = \frac{10 - 0.7 - (-10)}{1.37} \Rightarrow \underline{R_1 = 14.1\,k\Omega}$$

11.16

$A_d = 180, \; CMRR_{dB} = 85$ dB

$CMRR = 17,783 = \left|\frac{A_d}{A_{cm}}\right| = \frac{180}{A_{cm}}$

$\Rightarrow |A_{cm}| = 0.01012$

Assume the common-mode gain is negative.

$v_0 = A_d v_d + A_{cm} v_{cm}$

$= 180 v_d - 0.01012 v_{cm}$

$v_0 = 180(2\sin\omega t)$ mV $- (0.01012)(2\sin\omega t)$ V

$v_0 = 0.36\sin\omega t - 0.02024\sin\omega t$

Ideal Output: $v_0 = 0.360\sin\omega t$ (V)

Actual Output: $v_0 = 0.340\sin\omega t$ (V)

11.17

At terminal A.

$$R_{THA} = R_A \| R = \frac{R(1+\delta)\cdot R}{R(1+\delta)+R} = \frac{R(1+\delta)}{2+\delta} \cong \frac{R}{2} = 5\,k\Omega$$

Variation in R_{TH} is not significant

$$V_{THA} = \left(\frac{R_A}{R_A+R}\right)V^+ = \frac{R(1+\delta)(5)}{R(1+\delta)+R} = \frac{5(1+\delta)}{2+\delta}$$

At terminal B.

$$R_{THB} = R\|R = \frac{R}{2} = 5\,k\Omega$$

$$V_{THB} = \left(\frac{R}{R+R}\right)V^+ = 2.5\,V$$

From Eq. (11.27)

$$V_O = \frac{-\beta R_C(V_2 - V_1)}{2(r_\pi + R_B)} \quad \text{where } V_2 = V_{THB} \text{ and } V_1 = V_{THA}$$

$R_B = 5\,k\Omega$, $r_\pi = \frac{(120)(0.026)}{0.25} = 12.5\,k\Omega$

So

$$V_O = \frac{-(120)(3)(V_2 - V_1)}{2(12.5+5)} = -10.3(V_2 - V_1)$$

We can find $V_2 - V_1 = V_{THB} - V_{THA}$

$$V_{THB} - V_{THA} = 2.5 - \left[\frac{5(1+\delta)}{2+\delta}\right]$$

$$= \frac{2.5(2+\delta) - 5(1+\delta)}{2+\delta} = \frac{2.5\delta - 5\delta}{2+\delta}$$

$$\cong \frac{-2.5\delta}{2} = -1.25\delta$$

Then

$V_O = -(10.3)(-1.25)\delta = 12.9\delta$

So for $-0.01 \le \delta \le 0.01$

We have

$\underline{-0.129 \le V_{O2} \le 0.129\,V}$

11.18

a. $R_{id} = 2r_\pi$

$$r_\pi = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

So

$\underline{R_{id} = 46.8 \text{ k}\Omega}$

b. Assuming $r_\mu \to \infty$, then

$$R_{icm} \cong [(1+\beta)R_0] \,\|\, \left[(1+\beta)\left(\frac{r_0}{2}\right)\right]$$

$$r_0 = \frac{125}{0.2} = 625 \text{ k}\Omega$$

$$R_{icm} = [(181)(1)] \| [(181)(0.3125)]$$

$$= 181 \| 56.56$$

$\Rightarrow \underline{R_{icm} = 43.1 \text{ M}\Omega}$

11.19

a. For $I_1 = 1$ mA, $V_{BE3} = 0.7$ V

$$R_1 = \frac{20 - 0.7}{1} \Rightarrow \underline{R_1 = 19.3 \text{ k}\Omega}$$

$$R_2 = \frac{V_T}{I_Q} \cdot \ln\left(\frac{I_1}{I_Q}\right) = \frac{0.026}{0.1} \cdot \ln\left(\frac{1}{0.1}\right)$$

$\Rightarrow \underline{R_2 = 0.599 \text{ k}\Omega}$

b. $$r_{\pi 4} = \frac{(180)(0.026)}{0.1} = 46.8 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{04} = \frac{100}{0.1} \Rightarrow 1 \text{ M}\Omega$$

From Chapter 10

$$R_0 = r_{04}[1 + g_m(R_E \| r_{\pi 4})]$$

$$R_E \| r_{\pi 4} = 0.599 \| 46.8 = 0.591$$

$$R_0 = (1)[1 + (3.846)(0.591)] = 3.27 \text{ M}\Omega$$

$$r_{01} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega$$

$$R_{icm} \cong [(1+\beta)R_0] \,\|\, \left[(1+\beta)\left(\frac{r_{01}}{2}\right)\right]$$

$$= [(181)(3.27)] \| [(181)(1)]$$

$$= 592 \| 181$$

$\Rightarrow \underline{R_{icm} = 139 \text{ M}\Omega}$

(c) From Eq. (11.32(b))

$$A_{cm} = \frac{-g_m R_C}{1 + \dfrac{2(1+\beta)R_o}{r_\pi + R_B}}$$

$$g_m = \frac{0.05}{0.026} = 1.923 \; mA/V$$

$$r_\pi = \frac{(180)(0.026)}{0.05} = 93.6 \; k\Omega$$

$R_B = 0$

Then

$$A_{cm} = \frac{-(1.923)(50)}{1 + \dfrac{2(181)(3270)}{93.6}} \Rightarrow \underline{A_{cm} = -0.00760}$$

11.20

$$A_{d1} = g_{m1}(R_1 \| r_{\pi 3})$$

$$g_{m1} = \frac{I_{Q1}/2}{V_T} = 19.23 I_{Q1}$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{Q2}/2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3}R_2}{2}, \quad g_{m3} = \frac{I_{Q2}/2}{V_T} = 19.23 I_{Q2}$$

Then

$$30 = \frac{(19.23)I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2}R_2 = 3.12 \; V$$

Maximum $v_{o2} - v_{o1} = \pm 18 \; mV$ for linearity

$v_{o3}(\max) = (\pm 18)(30) \; mV \Rightarrow \pm 0.54 \; V$

so $I_{Q2}R_2 = 3.12 \; V$ is OK.

From A_{d1}:

$$20 = 19.23 I_{Q1}(R_1 \| r_{\pi 3})$$

$$= 19.23 I_{Q1} \left(\frac{R_1\left(\dfrac{5.2}{I_{Q2}}\right)}{R_1 + \left(\dfrac{5.2}{I_{Q2}}\right)} \right)$$

$$20 = \frac{19.23 I_{Q1} R_1 (5.2)}{I_{Q2}R_1 + 5.2}$$

Let $\dfrac{I_{Q1}}{2} \cdot R_1 = 5V \Rightarrow I_{Q1}R_1 = 10V$

Then

$$20 = \frac{19.23(10)(5.2)}{I_{Q2}R_1 + 5.2} \Rightarrow I_{Q2}R_1 = 44.8 \; V$$

Now

$$I_{Q1}R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$$

So

$$I_{Q2}\left(\frac{10}{I_{Q1}}\right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$$

Let $\underline{I_{Q1} = 100 \; \mu A, \; I_{Q2} = 448 \; \mu A}$

Then

$I_{Q2}R_2 = 3.12 \Rightarrow \underline{R_2 = 6.96 \; k\Omega}$

$I_{Q1}R_1 = 10 \Rightarrow \underline{R_1 = 100 \; k\Omega}$

11.21

a. $I_1 = \dfrac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$

$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$

$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$

$V_{GS3} = \dfrac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)}$

$\Rightarrow \underline{V_{GS3} = 3.16 \text{ V}}$

$I_1 = \dfrac{20 - 3.16}{50} \Rightarrow \underline{I_1 = I_Q = 0.337 \text{ mA}}$

$I_{D1} = \dfrac{I_Q}{2} \Rightarrow \underline{I_{D1} = 0.168 \text{ mA}}$

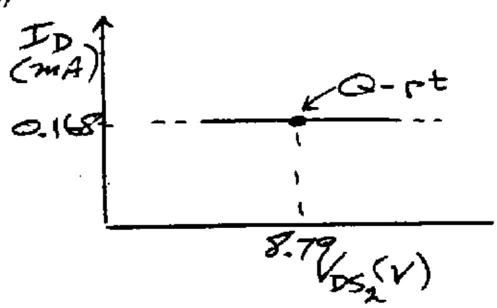
$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 2.82 \text{ V}$

$V_{DS4} = -2.82 - (-10) \Rightarrow \underline{V_{DS4} = 7.18 \text{ V}}$

$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$

$V_{DS1} = 5.97 - (-2.82) \Rightarrow \underline{V_{DS1} = 8.79 \text{ V}}$

(b)



(c)Max $v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(sat) = V_{GS1} - V_{TN}$
$2.82 - 2 = 0.82 V$

Now $V_{D1} = 10 - (0.168)(24) = 5.97 V$

$V_S(\max) = 5.97 - V_{DS1}(sat) = 5.97 - 0.82$

$V_S(\max) = 5.15 V$

$v_{CM}(\max) = V_S(\max) + V_{GS1} = 5.15 + 2.82$

$\underline{v_{CM}(\max) = 7.97 V}$

$v_{CM}(\min) = V^- + V_{DS4}(sat) + V_{GS1}$

$V_{DS4}(sat) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16 V$

Then

$v_{CM}(\min) = -10 + 1.16 + 2.82 \Rightarrow$

$\underline{v_{CM}(\min) = -6.02 V}$

11.22

a. $I_{D1} = I_{D2} = 120 \ \mu\text{A} = 100(V_{GS1} - 1.2)^2$

$\Rightarrow V_{GS1} = V_{GS2} = 2.30 \text{ V}$

For $v_1 = v_2 = -5.4$ V and $V_{DS1} = V_{DS2} = 12$ V

$\Rightarrow V_0 = -5.4 - 2.30 + 12 = 4.3 \text{ V}$

$R_D = \dfrac{10 - 4.3}{0.12} \Rightarrow \underline{R_D = 47.5 \text{ k}\Omega}$

$I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = I_1 = 240 \ \mu\text{A}}$

$I_1 = 240 = 200(V_{GS3} - 1.2)^2 \Rightarrow V_{GS3} = 2.30 \text{ V}$

$R_1 = \dfrac{20 - 2.3}{0.24} \Rightarrow \underline{R_1 = 73.75 \text{ k}\Omega}$

b. $r_{04} = \dfrac{1}{\lambda I_Q} = \dfrac{1}{(0.01)(0.24)} = 416.7 \text{ k}\Omega$

$\Delta I_Q = \dfrac{1}{r_{04}} \cdot \Delta V_{DS} = \dfrac{5.4}{416.7} \Rightarrow \underline{\Delta I_Q \cong 13 \ \mu\text{A}}$

11.23

a. $R_D = \dfrac{10 - 7}{0.5} \Rightarrow \underline{R_D = 6 \text{ k}\Omega}$

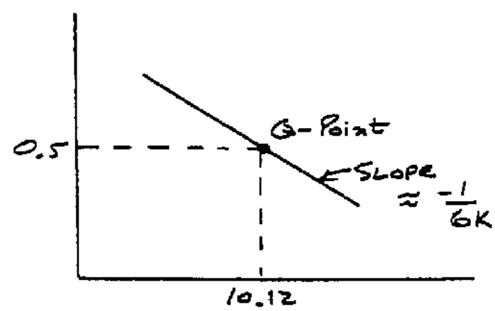
$I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = 1 \text{ mA}}$

b. $10 = I_D(6) + V_{DS} - V_{GS}$

and $V_{GS} = \sqrt{\dfrac{I_D}{K_n}} + V_{TN}$

For $I_D = 0.5$ mA, $V_{GS} = \sqrt{\dfrac{0.5}{0.4}} + 2 = 3.12$ V

and $V_{DS} = 10.12$



Load line is actually nonlinear.

c. Maximum common-mode voltage when M_1 and M_2 reach the transition point, or

$V_{DS}(sat) = V_{GS} - V_{TN} = 3.12 = 2 = 1.12 V$

Then

$v_{cm} = v_{02} - v_{DS}(\text{sat}) + V_{GS} = 7 - 1.12 + 3.12$

Or $\underline{v_{cm}(\max) = 9 \text{ V}}$

Minimum common-mode voltage, voltage across I_Q becomes zero.

So $v_{cm}(\min) = -10 + 3.12$

$\Rightarrow \underline{v_{cm}(\min) = -6.88 \text{ V}}$

11.24

a. $I_{D1} = I_{D2} = 0.5$ mA

$v_{01} - v_{02} = [V^+ - I_{D1}R_{D1}] - [V^+ - I_{D2}R_{D2}]$

$v_{01} - v_{02} = I_{D2}R_{D2} - I_{D1}R_{D1} = I_D(R_{D2} - R_{D1})$

i. $R_{D1} = R_{D2} = 6\ k\Omega$, $\underline{v_{01} - v_{02} = 0}$

ii. $R_{D1} = 6\ k\Omega$, $R_{D2} = 5.9\ k\Omega$

$v_{01} - v_{02} = (0.5)(5.9 - 6)$

$\Rightarrow \underline{v_{01} - v_{02} = -0.05\ V}$

b. $K_{n1} = 0.4\ mA/V^2$, $K_{n2} = 0.44\ mA/V^2$

$V_{GS1} = V_{GS2}$

$I_Q = (K_{n1} + K_{n2})(V_{GS} - V_{TN})^2$

$1 = (0.4 + 0.44)(V_{GS} - V_{TN})^2$

$\Rightarrow (V_{GS} - V_{TN})^2 = 1.19$

$I_{D1} = (0.4)(1.19) = 0.476\ mA$

$I_{D2} = (0.44)(1.19) = 0.524\ mA$

i. $R_{D1} = R_{D2} = 6\ k\Omega$

$v_{01} - v_{02} = (0.524 - 0.476)(6)$

$\Rightarrow \underline{v_{01} - v_{02} = 0.288\ V}$

ii. $R_{D1} = 6\ k\Omega$, $R_{D2} = 5.9\ k\Omega$

$v_{01} - v_{02} = (0.524)(5.9) - (0.476)(6)$

$= 3.0916 - 2.856$

$\Rightarrow \underline{v_{01} - v_{02} = 0.236\ V}$

11.25

(a) From Equation (11.51)

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

$$0.90 = 0.50 - \sqrt{\frac{0.1}{2(0.25)}} \cdot v_d \sqrt{1 - \left[\frac{0.1}{2(0.25)}\right] v_d^2}$$

$$-0.40 = -(0.4472) v_d \sqrt{1 - (0.2) v_d^2}$$

$$0.8945 = v_d \sqrt{1 - (0.2) v_d^2}$$

Square both sides

$$0.80 = v_d^2 (1 - [0.2] v_d^2)$$

$$(0.2)(v_d^2)^2 - v_d^2 + 0.80 = 0$$

$$v_d^2 = \frac{1 \pm \sqrt{1 - 4(0.2)(0.80)}}{2(0.2)} = 4V^2 \text{ or } 1V^2$$

Then $v_d = 2$ V or 1 V

But $|v_d|_{max} = \sqrt{\frac{I_Q}{k_n}} = \sqrt{\frac{0.25}{0.1}} = 1.58$

So $\underline{v_d = 1\ V}$

b. From part (a), $\underline{v_{d,max} = 1.58\ V}$

11.26

$$A_d = \frac{g_m R_D}{2}$$

For $v_{CM} = 2.5V$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25\ mA$$

Let $V_{D1} = V_{D2} = 3V$, then $R_D = \frac{10-3}{0.25} \Rightarrow$

$\underline{R_D = 28\ k\Omega}$

Then $100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14\ mA/V$

And $g_m = 2\sqrt{\frac{k_n'}{2}\left(\frac{W}{L}\right) I_D}$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow$$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 1274$ (Extremely large transistors to meet the gain requirement.)

Need $|A_{CM}| = 0.10$

From Eq.(11.64(b))

$$|A_{CM}| = \frac{g_m R_D}{1 + 2 g_m R_o}$$

So $0.10 = \frac{(7.14)(28)}{1 + 2(7.14)R_o} \Rightarrow R_o = 140\ k\Omega$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.5)} = 200\ k\Omega$$

This current source is adequate to meet common-mode gain requirement.

11.27

a. $I_S = \frac{-V_{GS1} - (-5)}{R_S}$

and $I_S = 2I_D = 2K_n(V_{GS1} - V_{TN})^2$

$$\frac{5 - V_{GS1}}{20} = 2(0.050)(V_{GS1} - 1)^2$$

$$5 - V_{GS1} = 2(V_{GS1}^2 - 2V_{GS1} + 1)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2 + 4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186\ V$$

$I_S = \frac{5 - 2.186}{20} \Rightarrow \underline{I_S = 0.141\ mA}$

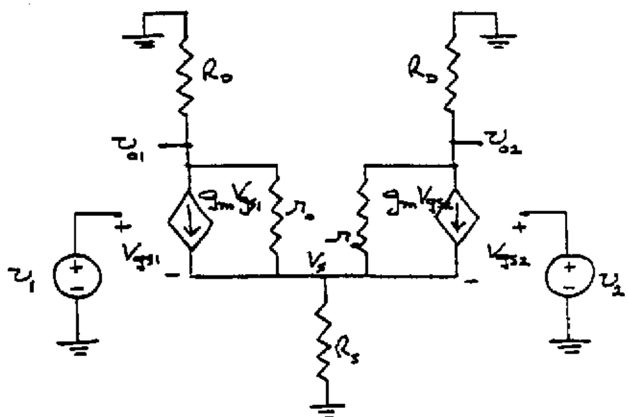
$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.0704\ mA}$

$v_{02} = 5 - (0.0704)(25) \Rightarrow \underline{v_{02} = 3.24\ V}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.05)(2.186 - 1)$

$g_m = 0.119\ \text{mA/V}$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710\ \text{k}\Omega$$



$V_{gs1} = v_1 - V_S,\quad V_{gs2} = v_2 - V_S$

$$\frac{v_{01}}{R_D} + g_m V_{gs1} + \frac{v_{01} - V_S}{r_0} = 0$$

$$v_{01}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m(v_1 - V_S) - \frac{V_S}{r_0} = 0 \qquad (1)$$

$$\frac{v_{02}}{R_D} + g_m V_{gs2} + \frac{v_{02} - V_S}{r_0} = 0$$

$$v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m(v_2 - V_S) - \frac{V_S}{r_0} = 0 \qquad (2)$$

$$g_m V_{gs1} + \frac{v_{01} - V_S}{r_0} + \frac{v_{02} - V_S}{r_0} + g_m V_{gs2} = \frac{V_S}{R_S}$$

$$g_m(v_1 - V_S) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} - \frac{2V_S}{r_0} + g_m(v_2 - V_S) = \frac{V_S}{R_S}$$

$$g_m(v_1 + v_2) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\} \qquad (3)$$

From (1)

$$v_{01} = \frac{V_S\left(g_m + \frac{1}{r_0}\right) - g_m v_1}{\left(\frac{1}{R_D} + \frac{1}{r_0}\right)}$$

Then

$$g_m(v_1 + v_2) + \frac{V_S\left(g_m + \frac{1}{r_0}\right) - g_m v_1}{r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right)} + \frac{v_{02}}{r_0} = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\} \qquad (3)$$

$$g_m(v_1 + v_2)r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + V_S\left(g_m + \frac{1}{r_0}\right) - g_m v_1 + v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\} \cdot r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right)$$

$$g_m(v_1 + v_2)\left(1 + \frac{r_0}{R_D}\right) - g_m v_1 + v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{\left(2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right)\left(1 + \frac{r_0}{R_D}\right) - \left(g_m + \frac{1}{r_0}\right)\right\}$$

$$g_m\left(v_1 \cdot \frac{r_0}{R_D} + v_2 + v_2 \cdot \frac{r_0}{R_D}\right) + v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S} + 2g_m \cdot \frac{r_0}{R_D} + \frac{2}{R_D} + \frac{r_0}{R_S R_D} - g_m - \frac{1}{r_0}\right\}$$

$$g_m\left(v_1 \cdot \frac{r_0}{R_D} + v_2 + v_2 \cdot \frac{r_0}{R_D}\right) + v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{1}{r_0} + \frac{1}{R_S}\left(1 + \frac{r_0}{R_D}\right) + \frac{2}{R_D}(1 + g_m r_0)\right\} \qquad (4)$$

Then substituting into (2),

$$v_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m v_2 = V_S\left(g_m + \frac{1}{r_0}\right)$$

Substitute numbers:

$$(0.119)\left[v_1\frac{710}{25} + v_2 + v_2\frac{710}{25}\right] + v_{02}\left[\frac{1}{25} + \frac{1}{710}\right] = V_S\left\{0.119 + \frac{1}{710} + \frac{1}{20}\left(1 + \frac{710}{25}\right) + \frac{2}{25}[1 + (0.119)(710)]\right\} \qquad (4)$$

$$(0.119)[28.4v_1 + 29.4v_2] + (0.0414)v_{02} = V_S\{0.1204 + 1.470 + 6.8392\} = V_S(8.4296)$$

or

$$V_S = 0.4010v_1 + 0.4150v_2 + 0.00491v_{02}$$

Then

$$v_{02}\left(\frac{1}{25} + \frac{1}{710}\right) + (0.119)v_2 = V_S\left(0.119 + \frac{1}{710}\right) \qquad (2)$$

$$v_{02}(0.0414) + v_2(0.119) = (0.1204)[0.401v_1 + 0.4150v_2 + 0.00491v_{02}]$$

$$v_{02}(0.0408) = (0.04828)v_1 - (0.0690)v_2$$

$$v_{02} = (1.183)v_1 - (1.691)v_2$$

Now $\quad \nu_1 = \nu_{cm} + \dfrac{\nu_d}{2}$

$\quad \nu_2 = \nu_{cm} - \dfrac{\nu_d}{2}$

So

$$\nu_{02} = (1.183)\left(\nu_{cm} + \frac{\nu_d}{2}\right) - (1.691)\left(\nu_{cm} - \frac{\nu_d}{2}\right)$$

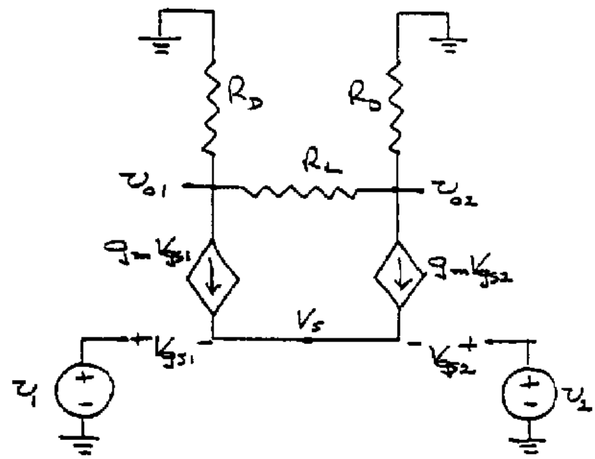
Or $\nu_{02} = 1.437\nu_d - 0.508\nu_{cm}$

$\Rightarrow \underline{A_d = 1.437}, \quad \underline{A_{cm} = -0.508}$

$$CMRR_{dB} = 20\log_{10}\left(\frac{1.437}{0.508}\right)$$

$\Rightarrow \underline{CMRR_{dB} = 9.03 \text{ dB}}$

11.28



KVL:

$\nu_1 = V_{gs1} - V_{gs2} + \nu_2$

So $\nu_1 - \nu_2 = V_{gs1} - V_{gs2}$

KCL:

$g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$

So $V_{gs1} = \dfrac{1}{2}(\nu_1 - \nu_2), \quad V_{gs2} = -\dfrac{1}{2}(\nu_1 - \nu_2)$

Now

$$\frac{\nu_{02}}{R_D} + \frac{\nu_{02} - \nu_{01}}{R_L} = -g_m V_{gs2} = \nu_{02}\left(\frac{1}{R_D} + \frac{1}{R_L}\right) - \frac{\nu_{01}}{R_L} \tag{1}$$

$$\frac{\nu_{01}}{R_D} + \frac{\nu_{01} - \nu_{02}}{R_L} = -g_m V_{gs1} = \nu_{01}\left(\frac{1}{R_D} + \frac{1}{R_L}\right) - \frac{\nu_{02}}{R_L} \tag{2}$$

From (1): $\quad \nu_{01} = \nu_{02}\left(1 + \dfrac{R_L}{R_D}\right) + g_m R_L V_{gs2}$

Substitute into (2):

$$-g_m V_{gs1} = \nu_{02}\left(1 + \frac{R_L}{R_D}\right)\left(\frac{1}{R_D} + \frac{1}{R_L}\right) + g_m R_L\left(\frac{1}{R_D} + \frac{1}{R_L}\right)V_{gs2} - \frac{\nu_{02}}{R_L}$$

$$-g_m \cdot (\nu_1 - \nu_2) + g_m\left(1 + \frac{R_L}{R_D}\right)\left(\frac{1}{2}\right)(\nu_1 - \nu_2) = \nu_{02}\left(\frac{1}{R_D} + \frac{R_L}{R_D^2} + \frac{1}{R_D}\right)$$

$$\frac{1}{2}g_m\left(\frac{R_L}{R_D}\right)(\nu_1 - \nu_2) = \frac{\nu_{02}}{R_D}\left(2 + \frac{R_L}{R_D}\right)$$

$$\Rightarrow \underline{A_{d2} = \frac{\nu_{02}}{\nu_1 - \nu_2} = \frac{\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

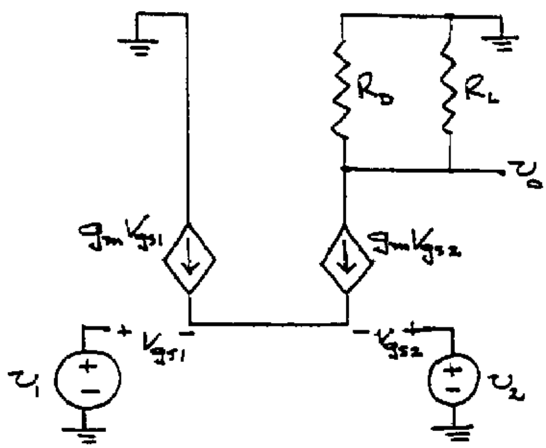
From symmetry

$$\underline{A_{d1} = \frac{\nu_{01}}{\nu_1 - \nu_2} = \frac{-\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

Then

$$\underline{A_\nu = \frac{\nu_{02} - \nu_{01}}{\nu_1 - \nu_2} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

11.29



$\nu_1 - \nu_2 = V_{gs1} - V_{gs2}$

and $g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$

Then $\nu_1 - \nu_2 = -2V_{gs2}$

Or $V_{gs2} = -\dfrac{1}{2}(\nu_1 - \nu_2)$

$v_0 = -g_m V_{gs2}(R_D \| R_L) = \frac{g_m}{2}(R_D \| R_L)(v_1 - v_2)$

Or $\underline{A_d = \frac{g_m}{2}(R_D \| R_L)}$

11.30

From Equation (11.64(a)), $A_d = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D$

We need $A_d = \frac{2}{0.2} = 10$

Then $10 = \sqrt{\frac{K_n(0.5)}{2}} \cdot R_D$ or $\sqrt{K_n} \cdot R_D = 20$

If we set $\underline{R_D = 20\ k\Omega}$, then $\underline{K_n = 1\ mA/V^2}$

For this case $V_D = 10 - (0.25)(20) = 5$ V

$V_{GS} = \sqrt{\frac{0.25}{1}} + 1 = 1.5$ V

$V_{DS}(sat) = V_{GS} - V_{TN} = 1.5 - 1 = 0.5\,V$

Then $v_{cm}(\max) = V_D - V_{DS}(\text{sat}) + V_{GS}$

$= 5 - 0.5 + 1.5$

Or $\underline{v_{cm}(\max) = 6\ V}$

11.31

$V_{d1} = -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s)$

$V_{d2} = -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s)$

Now

$V_o = V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s))$

$V_o = g_m R_D (V_1 - V_2)$

Define $V_1 - V_2 \equiv V_d$

Then

$\underline{A_d = \frac{V_o}{V_d} = g_m R_D}$

and

$\underline{A_{cm} = 0}$

11.32

(a) $K_{n1} = K_{n2} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) = \left(\frac{0.080}{2}\right)(10) = 0.40\ mA/V^2$

$V_{GS1} = V_{GS2} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.1}{0.4}} + 1 = 1.5\,V$

$V_{DS1}(sat) = 1.5 - 1 = 0.5\,V$

For $v_{CM} = +3\,V \Rightarrow V_{D1} = V_{D2} = v_{CM} - V_{GS1} + V_{DS1}(sat)$

$= 3 - 1.5 + 0.5 \Rightarrow V_{D1} = V_{D2} = 2\,V$

$R_D = \frac{10-2}{0.1} \Rightarrow \underline{R_D = 80\ k\Omega}$

(b) $A_d = \frac{1}{2} g_m R_D$ and $g_m = 2\sqrt{(0.4)(0.1)} = 0.4\ mA/V$

Then $A_d = \frac{1}{2}(0.4)(80) = 16$

$CMRR_{dB} = 45 \Rightarrow CMRR = 177.8 = \frac{16}{A_{cm}}$

So $|A_{cm}| = 0.090$

$|A_{cm}| = \frac{g_m R_D}{1 + 2 g_m R_o}$

$0.090 = \frac{(0.4)(80)}{1 + 2(0.4)R_o} \Rightarrow R_o = 443\ k\Omega$

If we assume $\lambda = 0.01\,V^{-1}$ for the current source transistor, then

$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.2)} = 500\ k\Omega$

So the CMRR specification can be met by a 2-transistor current source.

Let $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 1$

Then $K_{n3} = K_{n4} = \left(\frac{0.080}{2}\right)(1) = 0.040\ mA/V^2$

and $V_{GS3} = \sqrt{\frac{I_Q}{K_{n3}}} + V_{TN} = \sqrt{\frac{0.2}{0.04}} + 1 = 3.24\,V$

For $v_{CM} = -3\,V$, $V_{D3} = -3 - V_{GS1} = -3 - 1.5 = -4.5\,V$

$\Rightarrow V_{DS3}(\min) = -4.5 - (-10) = 5.5\,V > V_{DS3}(sat)$

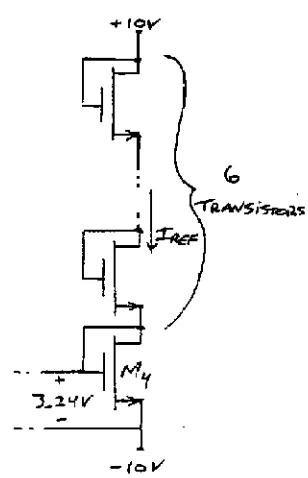
So design is OK.

On reference side: For $\left(\frac{W}{L}\right) \geq 1$, $V_{GS}(\max) = 3.24\,V$

$20 - V_{GS3} = 20 - 3.24 = 16.76\,V$

Then

$\frac{16.67}{3.24} = 5.17 \Rightarrow$ We need six transistors in series.



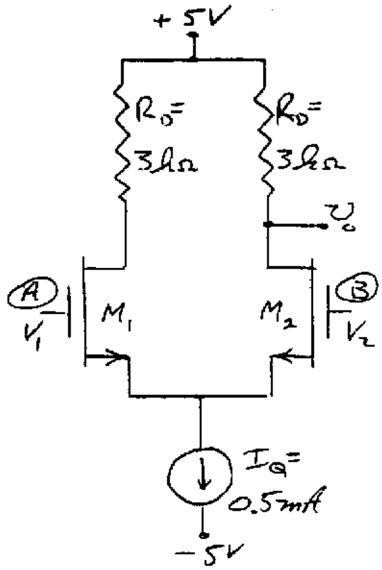
$V_{GS} = \frac{20 - 3.24}{6} = 2.793\,V$

$I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

$0.2 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(2.793 - 1)^2 \Rightarrow$

$\left(\frac{W}{L}\right) = 1.56$ for each of the 6 transistors.

11.33



$$A_d = \frac{1}{2} g_m R_D$$
$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.25)(0.25)} = 0.50\ mA/V$$
$$A_d = \frac{1}{2}(0.50)(3) = 0.75$$
From Problem 11.17
$$V_1 = V_A = \frac{5(1+\delta)}{2+\delta},\quad V_2 = V_B = 2.5\,V$$
and $V_1 - V_2 = 1.25\delta$

Then
$$V_{o2} = A_d \cdot (V_1 - V_2) = (0.75)(1.25\delta) = 0.9375\delta$$
So for $-0.01 \le \delta \le 0.01$
$$-9.375 \le V_{o2} \le 9.375\ mV$$

11.34

From previous results
$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_{m1} R_1 = \sqrt{2K_{n1} I_{Q1}} \cdot R_1 = 20$$
and
$$A_{d2} = \frac{v_{o3}}{v_{o2} - v_{o1}} = \frac{1}{2} g_{m3} R_2 = \frac{1}{2}\sqrt{2K_{n3} I_{Q2}} \cdot R_2 = 30$$
Set $\frac{I_{Q1} R_1}{2} = 5\,V$ and $\frac{I_{Q2} R_2}{2} = 2.5\,V$

Let $I_{Q1} = I_{Q2} = 0.1\ mA$

Then $R_1 = 100\ k\Omega$, $R_2 = 50\ k\Omega$

Then
$$2\left(\frac{0.06}{2}\right)\left(\frac{W}{L}\right)_1 (0.1) = \left(\frac{20}{100}\right)^2 \Rightarrow$$
$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 6.67$$
and
$$2\left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)_3 (0.1) = \left(\frac{2(30)}{50}\right)^2 \Rightarrow$$
$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 240$$

11.35

a. $$i_{D1} = I_{DSS}\left(1 - \frac{v_{GS1}}{V_P}\right)^2$$
$$i_{D2} = I_{DSS}\left(1 - \frac{v_{GS2}}{V_P}\right)^2$$
$$\sqrt{i_{D1}} - \sqrt{i_{D2}}$$
$$= \sqrt{I_{DSS}}\left(1 - \frac{v_{GS1}}{V_P}\right) - \sqrt{I_{DSS}}\left(1 - \frac{v_{GS2}}{V_P}\right)$$
$$= \frac{\sqrt{I_{DSS}}}{V_P}(v_{GS2} - v_{GS1})$$
$$= -\frac{\sqrt{I_{DSS}}}{V_P} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_P)} \cdot v_d$$
$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$
$$\left(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}}\right)^2 = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$
$$i_{D1} - 2\sqrt{i_{D1}(I_Q - i_{D1})} + (I_Q - i_{D1}) = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$
Then
$$\sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2}\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]$$
Square both sides
$$i_{D1}^2 - i_{D1} I_Q + \frac{1}{4}\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2 = 0$$
$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4\left(\frac{1}{4}\right)\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2}}{2}$$
$$i_{D1} =$$
$$\frac{I_Q}{2} \pm \frac{1}{2}\sqrt{I_Q^2 - \left[I_Q^2 - \frac{2 I_Q I_{DSS} v_d^2}{(-V_P)^2} + \left(\frac{I_{DSS} v_d^2}{(-V_P)^2}\right)^2\right]}$$
Use + sign
$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\sqrt{\frac{2 I_Q I_{DSS}}{(-V_P)^2} \cdot v_d^2 - \left(\frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right)^2}$$
$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\frac{I_Q}{(-V_P)} v_d \sqrt{\frac{2 I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2 \left(\frac{v_d}{V_P}\right)^2}$$
Or
$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left(\frac{1}{-2V_P}\right) \cdot v_d \sqrt{\frac{2 I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2 \left(\frac{v_d}{V_P}\right)^2}$$
We had
$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left(\frac{1}{-2V_P}\right)\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

b. If $i_{D1} = I_Q$, then

$$1 = \frac{1}{2} + \left(\frac{1}{-2V_P}\right)\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

$$|V_P| = v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

Square both sides

$$|V_P|^2 = v_d^2\left[\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2\right]$$

$$\left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2(v_d^2)^2 - \frac{2I_{DSS}}{I_Q}\cdot v_d^2 + |V_P|^2 = 0$$

$$v_d^2 = \frac{\frac{2I_{DSS}}{I_Q} \pm \sqrt{\left(\frac{2I_{DSS}}{I_Q}\right)^2 - 4\left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2(V_P)^2}}{2\left(\frac{2I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2}$$

$$v_d^2 = (V_P)^2\left(\frac{I_Q}{I_{DSS}}\right)$$

Or $\underline{|v_d| = |V_P|\left(\frac{I_Q}{I_{DSS}}\right)^{1/2}}$

c. For v_d small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \left.\frac{di_{D1}}{dv_d}\right|_{v_d=0} = \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot\sqrt{\frac{2I_{DSS}}{I_Q}}$$

Or

$$\Rightarrow \underline{g_f(\max) = \left(\frac{1}{-V_P}\right)\sqrt{\frac{I_Q I_{DSS}}{2}}}$$

11.36

a. $I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = 1\text{ mA}}$

$v_0 = 7 = 10 - (0.5)R_D \Rightarrow \underline{R_D = 6\text{ k}\Omega}$

b. $g_f(\max) = \left(\frac{1}{-V_P}\right)\sqrt{\frac{I_Q\cdot I_{DSS}}{2}}$

$$g_f(\max) = \left(\frac{1}{4}\right)\sqrt{\frac{(1)(2)}{2}}$$

$\Rightarrow \underline{g_f(\max) = 0.25\text{ mA/V}}$

c. $A_d = \frac{g_m R_D}{2} = g_f(\max)\cdot R_D$

$A_d = (0.25)(6) \Rightarrow \underline{A_d = 1.5}$

11.37

a. $I_S = \frac{-V_{GS} - (-5)}{R_S} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$$5 - V_{GS} = (0.8)(20)\left(1 - \frac{V_{GS}}{(-2)}\right)^2$$

$$5 - V_{GS} = 16\left(1 + V_{GS} + \frac{1}{4}V_{GS}^2\right)$$

$$4V_{GS}^2 + 17V_{GS} + 11 = 0$$

$$V_{GS} = \frac{-17 \pm \sqrt{(17)^2 - 4(4)(11)}}{2(4)}$$

$V_{GS} = -0.796$ V

$I_S = \frac{5 - (-0.796)}{20} \Rightarrow \underline{I_S = 0.290\text{ mA}}$

$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.145\text{ mA}}$

$v_{02} = 5 - (0.145)(25) \Rightarrow \underline{v_{02} = 1.375\text{ V}}$

b. Taking into account the r_0 parameters of Q_1 and Q_2, the analysis is identical to that in problem 11.34.

11.38

Equivalent circuit and analysis is identical to that in problem 11.36.

$$A_{d2} = \frac{\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_{d1} = \frac{-\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

11.39

a. Using the results of problem 10.59, the resistance from the base of Q_4 looking toward Q_3:

$$\frac{1}{R_0'} = \frac{1}{r_{01}} + \frac{\left(\frac{1}{r_{\pi3}} + g_{m3} + \frac{1}{r_{03}}\right)}{\left[1 + \left(\frac{1}{r_{\pi3}} + g_{m3} + \frac{1}{r_{03}}\right)R_E\right]}$$

$r_{01} = \frac{120}{0.1} = 1200\text{ k}\Omega,\quad r_{03} = \frac{80}{0.1} = 800\text{ k}\Omega$

Assume $\beta = 100$

$$r_{\pi3} = \frac{(100)(0.026)}{0.1} = 26\text{ k}\Omega$$

$$g_{m3} = \frac{0.1}{0.026} = 3.846\text{ mA/V}$$

$$\frac{1}{R_0'} = \frac{1}{1200} + \frac{\left(\frac{1}{26} + 3.846 + \frac{1}{800}\right)}{\left[1 + \left(\frac{1}{26} + 3.846 + \frac{1}{800}\right)(1)\right]}$$

$$= \frac{1}{1200} + \frac{3.886}{1 + (3.886)(1)} \Rightarrow R_0' = 1.256 \text{ k}\Omega$$

$$R_0 = r_{02}\left[1 + \frac{R_E \| (r_{\pi 2} + R_0')}{r_{02}} + g_{m2}\left(\frac{r_{\pi 2}}{r_{\pi 2} + R_0'}\right)\{R_E \| (r_{\pi 2} + R_0')\}\right]$$

$$R_E \| (r_{\pi 2} + R_0') = (1) \| (26 + 1.256)$$
$$= (1) \| (27.256)$$
$$= 0.965 \text{ k}\Omega$$

$$R_0 = 1200\left[1 + \frac{0.965}{1200} + (3.846)\left(\frac{26}{26 + 1.256}\right)(0.965)\right]$$

$$\Rightarrow R_0 = 5.45 \text{ M}\Omega$$

Then

$$A_v = -g_m(r_{02} \| R_0)$$
$$r_{02} = \frac{120}{0.1} = 1200 \text{ k}\Omega$$
$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_v = -(3.846)[1200 \| 5450]$$
$$\Rightarrow \underline{A_v = -3782}$$

b. For $R = 0$, $r_{04} = \frac{80}{0.1} = 800 \text{ k}\Omega$

$$A_v = -g_m(r_{02} \| r_{04})$$
$$= -(3.846)[1200 \| 800]$$
$$\Rightarrow \underline{A_v = -1846}$$

(c) For part (a), $R_o = (5.45 \| 1.2) = 0.983\ M\Omega$

For part (b), $R_o = (1.2 \| 0.8) = 0.48\ M\Omega$

11.40

$$I_{B5} = \frac{I_{E3}}{1+\beta} = \frac{I_{B3} + I_{B4}}{1+\beta} = \frac{I_{C3} + I_{C4}}{\beta(1+\beta)}$$

Now $I_{C3} + I_{C4} \approx I_Q$

So $I_{B5} \approx \frac{I_Q}{\beta(1+\beta)}$

$$I_{B6} = \frac{I_{E6}}{1+\beta} = \frac{I_{Q1}}{\beta(1+\beta)}$$

For balance, we want $I_{B6} = I_{B5}$

So that $\underline{I_{Q1} = I_Q}$

11.41

a. $A_d = g_m(r_{02} \| r_{04})$

$$r_{02} = \frac{V_{A2}}{I_{C2}} = \frac{150}{0.4} = 375 \text{ k}\Omega$$
$$r_{04} = \frac{V_{A4}}{I_{C4}} = \frac{100}{0.4} = 250 \text{ k}\Omega$$
$$g_m = \frac{I_{C2}}{V_T} = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

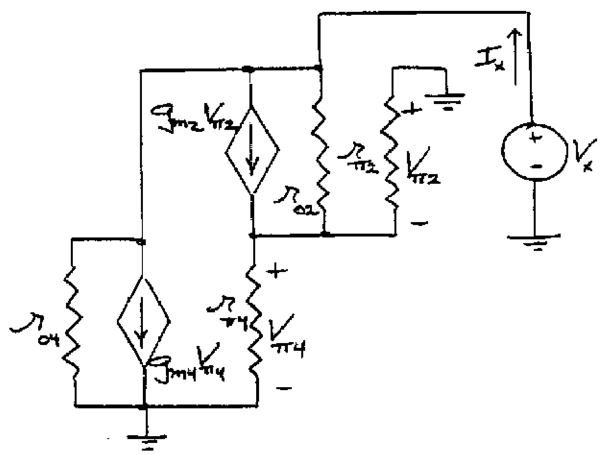
$$A_d = (15.38)(375 \| 250)$$
$$\Rightarrow \underline{A_d = 2307}$$

b. $R_L = r_{02} \| r_{04} = 375 \| 250$

$$\Rightarrow \underline{R_L = 150 \text{ k}\Omega}$$

11.42

(a) For Q_2, Q_4



(1) $I_x = \frac{V_x - V_{\pi 4}}{r_{o2}} + g_{m2}V_{\pi 2} + g_{m4}V_{\pi 4} + \frac{V_x}{r_{o4}}$

(2) $g_{m2}V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{o2}} = \frac{V_{\pi 4}}{r_{\pi 4} \| r_{\pi 2}}$

(3) $V_{\pi 4} = -V_{\pi 2}$

From (2)

$$\frac{V_x}{r_{o2}} = V_{\pi 4}\left[\frac{1}{r_{\pi 4} \| r_{\pi 2}} + \frac{1}{r_{o2}} + g_{m2}\right]$$

Now

$$I_{C4} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{I_Q}{2}\right) = \left(\frac{120}{121}\right)(0.5) = 0.496\ mA$$

$$I_{C2} = \left(\frac{I_Q}{2}\right)\left(\frac{1}{1+\beta}\right)\left(\frac{\beta}{1+\beta}\right) = (0.5)\left(\frac{120}{(121)^2}\right) \Rightarrow$$

$$I_{C2} = 0.0041\ mA$$

So

$$r_{\pi 2} = \frac{(120)(0.026)}{0.0041} = 761\ k\Omega$$

$$g_{m2} = \frac{0.0041}{0.026} = 0.158\ mA/V$$

$$r_{o2} = \frac{100}{0.0041} \Rightarrow 24.4\ M\Omega$$

$$r_{\pi 4} = \frac{(120)(0.026)}{0.496} = 6.29\ k\Omega$$

$$g_{m4} = \frac{0.496}{0.026} = 19.08\ mA/V$$

$$r_{o4} = \frac{100}{0.496} = 202\ k\Omega$$

Now

$$\frac{V_x}{r_{o2}} = V_{\pi 4}\left[\frac{1}{6.29\|761} + \frac{1}{24400} + 0.158\right] \Rightarrow$$

which yields

$$V_{\pi 4} = \frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o4}} + V_{\pi 4}\left(g_{m4} - g_{m2} - \frac{1}{r_{o2}}\right)$$

$$\frac{I_x}{V_x} = \left[\frac{1}{24400} + \frac{1}{202} + \frac{\left(19.08 - 0.158 - \frac{1}{24400}\right)}{(0.318)(24400)}\right]$$

which yields

$$R_{o2} = \frac{V_x}{I_x} = 135\ k\Omega$$

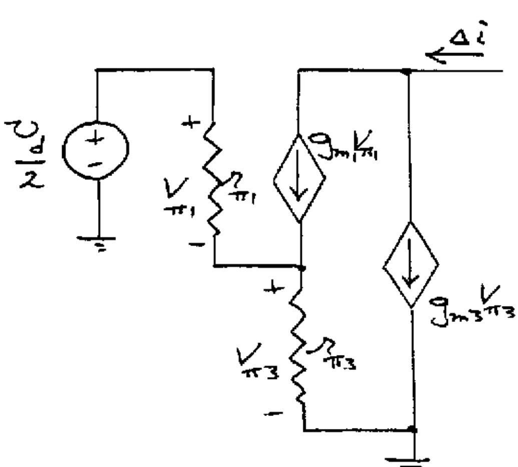
Now

$$r_{o6} = \frac{80}{0.5} = 160\ k\Omega$$

Then

$$R_o = R_{o2}\|r_{o6} = 135\|160 \Rightarrow \underline{R_o = 73.2\ k\Omega}$$

(b) $A_d = g_m^c R_o$ where $g_m^c = \dfrac{\Delta i}{v_d/2}$



$$\Delta i = g_{m1}V_{\pi 1} + g_{m3}V_{\pi 3} \text{ and } V_{\pi 1} + V_{\pi 3} = \frac{v_d}{2}$$

Also $\left(\dfrac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)r_{\pi 3} = V_{\pi 3}$

So $V_{\pi 1}\left(\dfrac{1+\beta}{r_{\pi 1}}\right)r_{\pi 3} = V_{\pi 3}$

Or

$$V_{\pi 1}\left(\frac{121}{761}\right)(6.29) = V_{\pi 3} \cong V_{\pi 1}$$

Then $2V_{\pi 1} = \dfrac{v_d}{2} \Rightarrow V_{\pi 1} = \dfrac{v_d}{4}$

So

$$\Delta i = (g_{m1} + g_{m3})V_{\pi 1} = (0.158 + 19.08)\left(\frac{v_d}{4}\right) = 9.62\left(\frac{v_d}{2}\right)$$

So

$$g_m^c = \frac{\Delta i}{v_d/2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow$$

$\underline{A_d = 704}$

Now

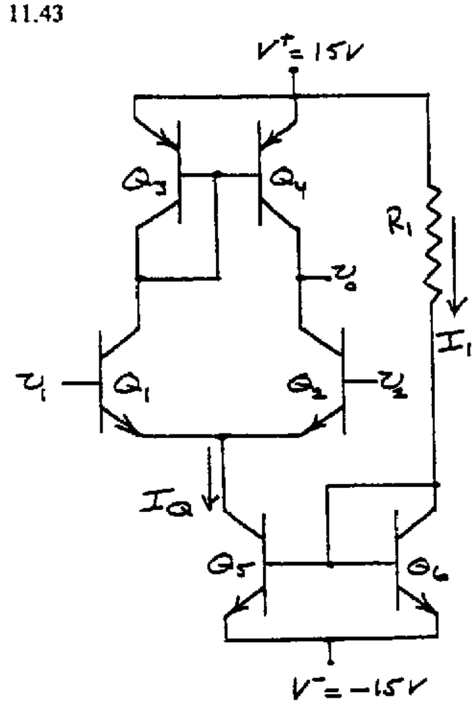
$R_{id} = 2R_i$ where $R_i = r_{\pi 1} + (1+\beta)r_{\pi 3}$

$$R_i = 761 + (121)(6.29) = 1522\ k\Omega$$

Then

$\underline{R_{id} = 3.044\ M\Omega}$

11.43



a. $g_f = \dfrac{I_Q}{4V_T} \Rightarrow I_Q = g_f(4V_T) = (8)(4)(0.026)$

$\Rightarrow \underline{I_Q = 0.832\text{ mA}}$

Neglecting base currents.

$R_1 = \dfrac{30 - 0.7}{0.832} \Rightarrow \underline{R_1 = 35.2\ \text{k}\Omega}$

b. $r_{o4} = r_{o2} = \dfrac{V_A}{I_{CQ}} = \dfrac{100}{0.416} = 240\ \text{k}\Omega$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.416}{0.026} = 16\ \text{mA/V}$

$A_d = g_m(r_{o2}\|r_{o4}) = 16(240\|240)$

$\Rightarrow \underline{A_d = 1920}$

$R_{id} = 2r_\pi,\ \ r_\pi = \dfrac{(180)(0.026)}{0.416} = 11.25\ \text{k}\Omega$

$\Rightarrow \underline{R_{id} = 22.5\ \text{k}\Omega}$

$R_0 = r_{o2}\|r_{o4} \Rightarrow \underline{R_0 = 120\ \text{k}\Omega}$

c. Max. common-mode voltage when

$V_{CB} = 0$ for Q_1 and Q_2.

Therefore

$v_{cm}(\max) = V^+ - V_{EB}(Q_3) = 15 - 0.7$

$v_{cm}(\max) = 14.3\ \text{V}$

Min. common-mode voltage when

$V_{CB} = 0$ for Q_3.

Therefore

$v_{cm}(\min) = 0.7 + 0.7 + (-15) = -13.6\ \text{V}$

So $\underline{-13.6 \le v_{cm} \le 14.3\ \text{V}}$

$R_{icm} \cong \frac{1}{2}(1+\beta)(2R_o)$

$R_0 = \dfrac{V_A}{I_Q} = \dfrac{100}{0.832} = 120\ \text{k}\Omega$

$R_{icm} = (181)(120) \Rightarrow \underline{R_{icm} = 21.7\ \text{M}\Omega}$

11.44

a. $I_0 = I_{B3} + I_{B4} \approx 2\left(\dfrac{I_Q}{2}\right)\left(\dfrac{1}{\beta}\right)$

$I_0 = \dfrac{I_Q}{\beta} = \dfrac{0.2}{100} \Rightarrow \underline{I_0 = 2\ \mu\text{A}}$

b. $r_{o2} = r_{o4} = \dfrac{V_A}{I_{CQ}} = \dfrac{100}{0.1} = 1000\ \text{k}\Omega$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.1}{0.026} = 3.846\ \text{mA/V}$

$A_d = g_m(r_{o2}\|r_{o4}) = (3.846)(1000\|1000)$

$\Rightarrow \underline{A_d = 1923}$

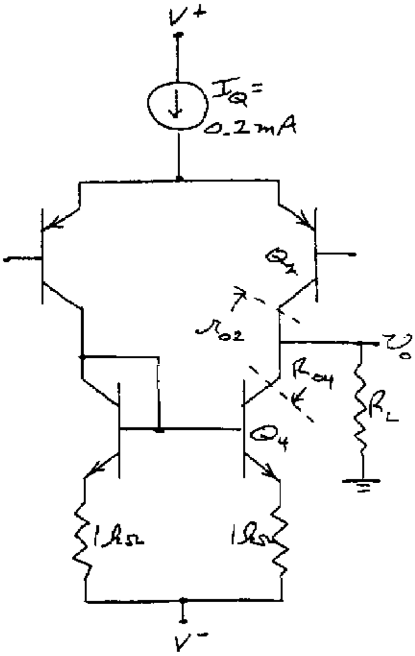
c. $A_d = g_m(r_{o2}\|r_{o4}\|R_L)$

$A_d = (3.846)(1000\|1000\|250)$

$\Rightarrow \underline{A_d = 641}$

11.45

Let $\beta = 100$, $V_A = 100V$



$r_{o2} = \dfrac{V_A}{I_{CQ}} = \dfrac{100}{0.1} = 1000\ k\Omega$

$R_{o4} = r_{o4}[1 + g_m R_E']$ where $R_E' = r_\pi \| R_E$

Now

$r_\pi = \dfrac{(100)(0.026)}{0.1} = 26\ k\Omega$

$g_m = \dfrac{0.1}{0.026} = 3.846\ mA/V$

$R_E' = 26\|1 = 0.963\ k\Omega$

Then

$R_{o4} = 1000[1 + (3.846)(0.963)] = 4704\ k\Omega$

$A_d = g_m(r_{o2}\|R_{o4}) = 3.846(1000\|4704) \Rightarrow$

$\underline{A_d = 3172}$

11.46

a. $A_d = g_m(r_{o2}\|r_{o4}\|R_L)$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{I_Q}{2V_T}$

$r_{o2} = \dfrac{V_{A2}}{I_{CQ}} = \dfrac{125}{I_{CQ}}$

$r_{o4} = \dfrac{V_{A4}}{I_{CQ}} = \dfrac{80}{I_{CQ}}$

If $I_Q = 2$ mA, then $g_m = 38.46$ mA/V

$r_{02} = 125$ kΩ, $r_{04} = 80$ kΩ

So $A_d = 38.46[125\|80\|200]$

Or $A_d = 1508$

For each gain of 1000, lower the current level

For $I_Q = 0.60$ mA, $I_{CQ} = 0.30$ mA

$$g_m = \frac{0.3}{0.026} = 11.54 \text{ mA/V}$$

$$r_{02} = \frac{125}{0.3} = 417 \text{ k}\Omega$$

$$r_{04} = \frac{80}{0.3} = 267 \text{ k}\Omega$$

$A_d = 11.54[417\|267\|200] = 1036$

So $I_Q = 0.60$ mA is adequate

b. For $V^+ = 10$ V, $V_{BE} = V_{EB} = 0.6$ V

For $V_{CB} = 0$,

$v_{cm}(\max) = V^+ - 2V_{EB} = 10 - 2(0.6)$

Or $\underline{v_{cm}(\max) = 8.8 \text{ V}}$

11.48

a. From symmetry,

$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

Or $\underline{V_{DS3} = V_{DS4} = 2 \text{ V}}$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$V_{SD1} = V_{SD2} = V_{SG1} - (V_{DS3} - 10)$$
$$= 2 - (2 - 10)$$

Or $\underline{V_{SD1} = V_{SD2} = 10 \text{ V}}$

b. $$r_{on} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$r_{op} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$g_m = 2K_p(V_{SG} + V_{TP})$$
$$= 2(0.1)(2-1) = 0.2\, mA/V$$

$$A_d = g_m(r_{on}\|r_{op}) = (0.2)(1000\|667)$$

$\Rightarrow \underline{A_d = 80}$

(c) $I_{D2} = I_{D1} = \dfrac{I_Q}{2} = 0.1\, mA$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.01)(0.1)} = 1000\, k\Omega$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.015)(0.1)} = 667\, k\Omega$$

$$R_o = r_{o2}\|r_{o4} = 667\|1000 = 400\, k\Omega$$

11.49

$$A_d = g_m(r_{02}\|r_{04})$$

$$g_m = 2\sqrt{k_n I_{DQ}} = 2\sqrt{(0.12)(0.075)}$$
$$= 0.1897 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.075)} = 889 \text{ k}\Omega$$

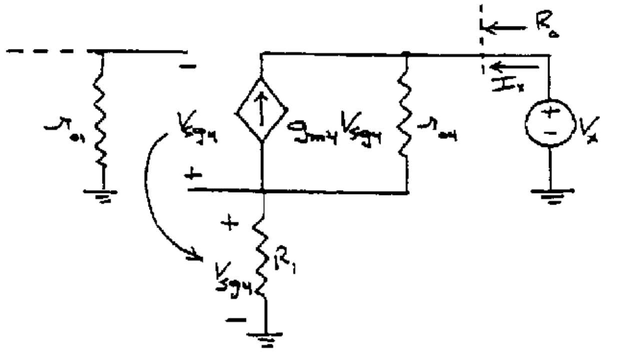
$$r_{04} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.075)} = 667 \text{ k}\Omega$$

$A_d = (0.1897)(889\|667)$

$\Rightarrow \underline{A_d = 72.3}$

11.50

Resistance looking into drain of M_4.



$$V_{sg4} = I_X R_1$$

$$I_X = g_{m4}V_{sg4} = \frac{V_X - V_{sg4}}{r_{04}}$$

$$I_X\left[1 + g_{m4}R_1 + \frac{R_1}{r_{04}}\right] = \frac{V_X}{r_{04}}$$

$$\text{Or } R_0 = r_{04}\left[1 + g_{m4}R_1 + \frac{R_1}{r_{04}}\right]$$

a. $$A_d = g_{m2}(r_{o2}\|R_o)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179\, mA/V$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667\, k\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179\, mA/V$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500\, k\Omega$$

$$R_0 = 500\left[1 + (0.179)(1) + \frac{1}{500}\right] = 590.5 \text{ k}\Omega$$

$A_d = (0.179)[667\|590.5]$

$\Rightarrow \underline{A_d = 56.06}$

b. When $R_1 = 0$, $R_0 = r_{04} = 500\ \text{k}\Omega$

$A_d = (0.179)[667\|500]$

$\Rightarrow \underline{A_d = 51.15}$

(c) For part (a), $R_o = r_{o2}\|R_o = 667\|590.5 \Rightarrow$
$\underline{R_o = 313\ k\Omega}$

For part (b), $R_o = r_{o2}\|r_{o4} = 667\|500 \Rightarrow$
$\underline{R_o = 286\ k\Omega}$

11.51

(a) $A_d = 100 = g_m(r_{o2}\|r_{o4})$

Let $I_Q = 0.5\ mA$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.02)(0.25)} = 200\ k\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.025)(0.25)} = 160\ k\Omega$$

Then

$100 = g_m(200\|160) \Rightarrow g_m = 1.125\ mA/V$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_D}$$

$$1.125 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow \underline{\left(\frac{W}{L}\right)_n = 31.6}$$

Now $\left(\frac{W}{L}\right)_p$ somewhat arbitrary. Let $\left(\frac{W}{L}\right)_p = 31.6$

11.52

$A_d = g_m(r_{o2}\|r_{o4})$

$P = (I_Q + I_{REF})(V^+ - V^-)$

Let $I_Q = I_{REF}$

Then $0.5 = 2I_Q(3-(-3)) \Rightarrow I_Q = I_{REF} = 0.0417\ mA$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(0.0208)} = 3205\ k\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(0.0208)} = 2404\ k\Omega$$

Then

$A_d = 80 = g_m(3205\|2404) \Rightarrow g_m = 0.0582\ mA/V$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n I_D}$$

$$0.0582 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_n (0.0208)} \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_n = 1.02}$$

11.53

$A_d = g_m(r_{o2}\|R_o)$

Want $A_d = 400$

From Example 11.15, $r_{o2} = 1\ M\Omega$

Assuming that $g_m = 0.283\ mA/V$ for the PMOS from Example 11.15, then $R_o = 285\ M\Omega$.

So

$400 = g_m(1000\|285000) \Rightarrow$

$$g_m = 0.4014\ mA/V = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 I_{DQ}}$$

$$0.04028 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_1 (0.1) \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 10.1}$$

11.54

$A_d = g_m(R_{o4}\|R_{o6})$

where

$R_{o4} = r_{o4} + r_{o2}[1 + g_{m4}r_{o4}]$

$R_{o6} = r_{o6} + r_{o8}[1 + g_{m6}r_{o6}]$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667\ k\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.02)(0.040)} = 1250\ k\Omega$$

$$g_{m4} = 2\sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268\ mA/V$$

$$g_{m6} = 2\sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141\ mA/V$$

Then

$R_{o4} = 1667 + 1667[1 + (0.268)(1667)] \Rightarrow 748\ M\Omega$

$R_{o6} = 1250 + 1250[1 + (0.141)(1250)] \Rightarrow 222.8\ M\Omega$

(a)

$R_o = R_{o4}\|R_{o6} = 748\|222.8 \Rightarrow \underline{R_o = 172\ M\Omega}$

(b)

$A_d = g_{m4}(R_{o4}\|R_{o6}) = (0.268)(172000) \Rightarrow$

$\underline{A_d = 46096}$

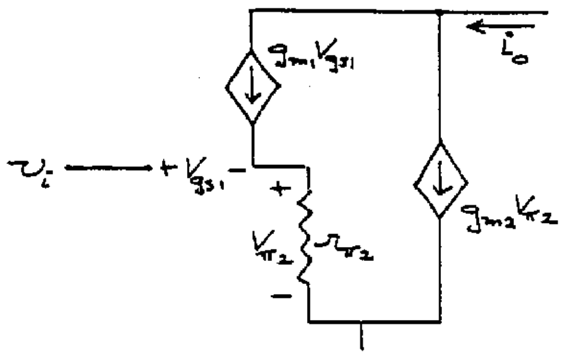
11.55

$$g_{m1} = 2\sqrt{K_n I_{Bias1}} = 2\sqrt{(0.2)(0.25)}$$

$$= 0.447\ \text{mA/V}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{0.75}{0.026} = 28.85\ \text{mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.75} = 4.16\ \text{k}\Omega$$



$$i_0 = g_{m1}V_{gs1} + g_{m2}V_{\pi 2}$$

$V_{\pi 2} = g_{m1}V_{gs1}r_{02}$ and $v_i = V_{gs1} + V_{\pi 2}$

$$i_0 = V_{gs1}(g_{m1} + g_{m2} \cdot g_{m1}r_{\pi 2})$$

$$v_i = V_{gs1} + g_{m1}V_{gs1}r_{\pi 2}$$

and $V_{gs1} = \dfrac{v_i}{1 + g_{m1}r_{\pi 2}}$

$$i_0 = v_i \cdot \frac{g_{m1}(1+\beta)}{1 + g_{m1}r_{\pi 2}}$$

$$g_m^C = \frac{i_0}{v_i} = \frac{g_{m1}(1+\beta)}{1 + g_{m1}r_{\pi 2}}$$

$$= \frac{(0.447)(121)}{1 + (0.447)(4.16)}$$

$\Rightarrow \underline{g_m^C = 18.9 \text{ mA/V}}$

11.56

$$r_0(M_2) = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_0(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$g_m(M_2) = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.2)}$$

$$= 0.4 \text{ mA/V}$$

$$A_d = g_m(M_2)[r_0(M_2)\|r_0(Q_2)]$$

$$= 0.4[500\|400]$$

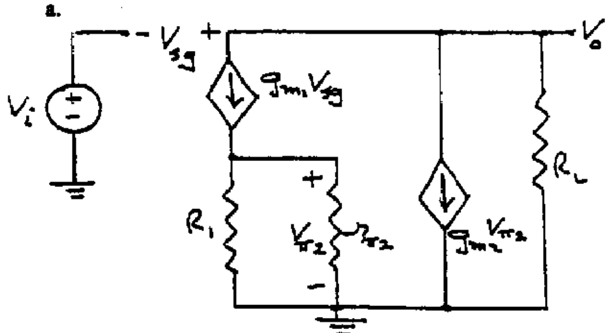
$\Rightarrow \underline{A_d = 88.9}$

If the I_Q current source is ideal,

$\underline{A_{cm} = 0}$ and $\underline{CMRR_{dB} = \infty}$

11.57

a.



b. Assume R_L is capacitively coupled. Then

$$I_{CQ} + I_{DQ} = I_Q$$

$$I_{DQ} = \frac{V_{BE}}{R_1} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_{CQ} = 0.9 - 0.0875 = 0.8125 \text{ mA}$$

$$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$$

$\Rightarrow \underline{g_{m1} = 0.592 \text{ mA/V}}$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{0.8125}{0.026} \Rightarrow \underline{g_{m2} = 31.25 \text{ mA/V}}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.8125} \Rightarrow \underline{r_{\pi 2} = 3.2 \text{ k}\Omega}$$

c. $V_0 = (-g_{m1}V_{sg} - g_{m2}V_{\pi 2})R_L$

$$V_i + V_{sg} = V_0 \Rightarrow V_{sg} = V_0 - V_i$$

$$V_{\pi 2} = (g_{m1}V_{sg})(R_1\|r_{\pi 2})$$

$$V_0 = -[g_{m1}V_{sg} + g_{m2}g_{m1}V_{sg}(R_1\|r_{\pi 2})]R_L$$

$$V_0 = -(V_0 - V_i)[g_{m1} + g_{m2}g_{m1}(R_1\|r_{\pi 2})]R_L$$

$$A_v = \frac{V_0}{V_i} = \frac{[g_{m1} + g_{m2}g_{m1}(R_1\|r_{\pi 2})]R_L}{1 + [g_{m1} + g_{m2}g_{m1}(R_1\|r_{\pi 2})]R_L}$$

We find

$$g_{m1} + g_{m2}g_{m1}(R_1\|r_{\pi 2})$$

$$= 0.592 + (31.25)(0.592)(8\|3.2)$$

$$= 42.88$$

Then $\underline{A_v = \dfrac{(42.88)(R_L)}{1 + (42.88)(R_L)}}$

11.58

a. Assume R_L is capacitively coupled.

$$I_{DQ} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_{CQ} = 1.2 - 0.0875 = 1.11 \text{ mA}$$

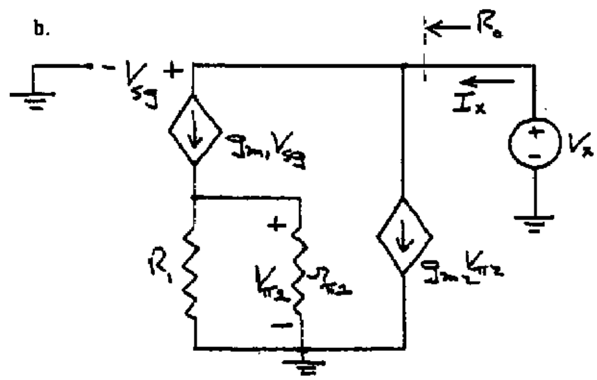
$$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$$

$\Rightarrow \underline{g_{m1} = 0.592 \text{ mA/V}}$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{1.11}{0.026} \Rightarrow \underline{g_{m2} = 42.7 \text{ mA/V}}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.11} \Rightarrow \underline{r_{\pi 2} = 2.34 \text{ k}\Omega}$$

b.



$V_{sg} = V_X$

$I_X = g_{m2}V_{\pi 2} + g_{m1}V_{sg}$

$(g_{m1}V_{sg})(R_1 \| r_{\pi 2}) = V_{\pi 2}$

$I_X = V_X[g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2})]$

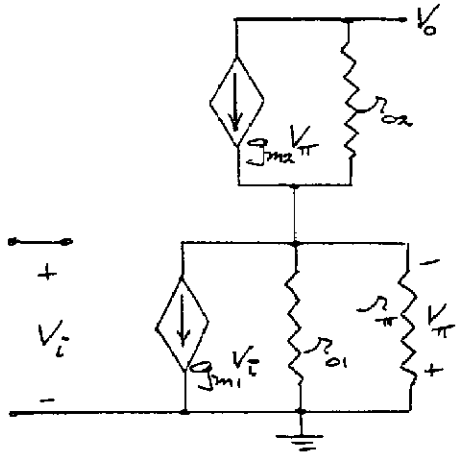
$$R_0 = \frac{V_X}{I_X} = \frac{1}{g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2})}$$

$$= \frac{1}{0.592 + (0.592)(42.7)(8 \| 2.34)}$$

$\Rightarrow \underline{R_0 = 21.6\ \Omega}$

11.59

(a)



(1) $g_{m2}V_\pi + \dfrac{V_o - (-V_\pi)}{r_{o2}} = 0$

(2) $g_{m2}V_\pi + \dfrac{V_o - (-V_\pi)}{r_{o2}} = g_{m1}V_i + \dfrac{-V_\pi}{r_{o1}} + \dfrac{-V_\pi}{r_\pi}$

or

$$0 = g_{m1}V_i - V_\pi\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)$$

Then

$$V_\pi = \frac{g_{m1}V_i}{\left(\dfrac{1}{r_{o1}} + \dfrac{1}{r_\pi}\right)}$$

From (1)

$$\left(g_{m2} + \frac{1}{r_{o2}}\right)V_\pi + \frac{V_o}{r_{o2}} = 0$$

$$V_o = -r_{o2}\left(g_{m2} + \frac{1}{r_{o2}}\right)V_\pi = -r_{o2}g_{m1}V_i\frac{\left(g_{m2} + \dfrac{1}{r_{o2}}\right)}{\left(\dfrac{1}{r_{o1}} + \dfrac{1}{r_\pi}\right)}$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o2}\left(g_{m2} + \dfrac{1}{r_{o2}}\right)}{\left(\dfrac{1}{r_{o1}} + \dfrac{1}{r_\pi}\right)}$$

Now

$g_{m1} = 2\sqrt{K_n I_Q} = 2\sqrt{(0.25)(0.025)} = 0.158\ mA/V$

$g_{m2} = \dfrac{I_Q}{V_T} = \dfrac{0.025}{0.026} = 0.9615\ mA/V$

$r_{o1} = \dfrac{1}{\lambda I_Q} = \dfrac{1}{(0.02)(0.025)} = 2000\ k\Omega$

$r_{o2} = \dfrac{V_A}{I_Q} = \dfrac{50}{0.025} = 2000\ k\Omega$

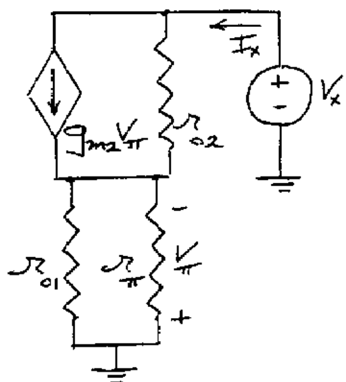
$r_\pi = \dfrac{\beta V_T}{I_Q} = \dfrac{(100)(0.026)}{0.025} = 104\ k\Omega$

Then

$$A_v = \frac{-(0.158)(2000)\left(0.9615 + \dfrac{1}{2000}\right)}{\left(\dfrac{1}{2000} + \dfrac{1}{104}\right)} \Rightarrow$$

$\underline{A_v = -30039}$

To find R_o; set $V_i = 0 \Rightarrow g_{m1}V_i = 0$



$I_x = g_{m2}V_\pi + \dfrac{V_x - (-V_\pi)}{r_{o2}}$

$V_\pi = -I_x(r_{o1} \| r_\pi)$

Then

$$I_x = \left(g_{m2} + \frac{1}{r_{o2}}\right)(-I_x)(r_{o1} \| r_\pi) + \frac{V_x}{r_{o2}}$$

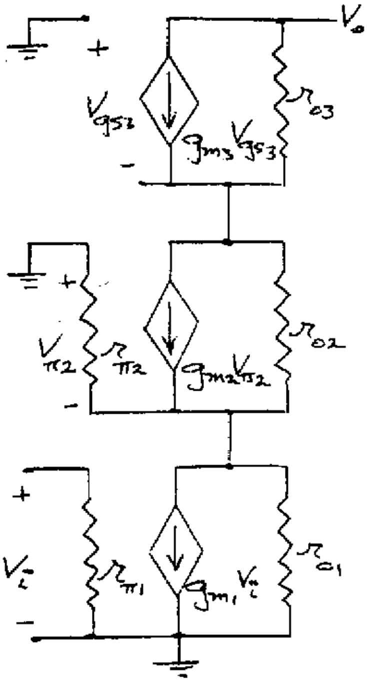
Combining terms,

$$R_o = \frac{V_x}{I_x} = r_{o2}\left[1 + (r_{o1} \| r_\pi)\left(g_{m2} + \frac{1}{r_{o2}}\right)\right]$$

$$= 2000\left[1 + (2000 \| 104)\left(0.9615 + \frac{1}{2000}\right)\right] \Rightarrow$$

$\underline{R_o = 192.2\ M\Omega}$

(b)



(1) $g_{m3}V_{gs3}+\dfrac{V_o-(-V_{gs3})}{r_{o3}}=0$

(2) $g_{m3}V_{gs3}+\dfrac{V_o-(-V_{gs3})}{r_{o3}}=g_{m2}V_{\pi 2}+\dfrac{-V_{gs3}-(-V_{\pi 2})}{r_{o2}}$

or

$$0=V_{\pi 2}\left(g_{m2}+\frac{1}{r_{o2}}\right)-\frac{V_{gs3}}{r_{o2}}$$

(3) $\dfrac{V_{\pi 2}}{r_{\pi 2}}+g_{m2}V_{\pi 2}+\dfrac{-V_{gs3}-(-V_{\pi 2})}{r_{o2}}=g_{m1}V_i+\dfrac{(-V_{\pi 2})}{r_{o1}}$

From (2), $V_{\pi 2}=\dfrac{V_{gs3}}{r_{o2}\left(g_{m2}+\dfrac{1}{r_{o2}}\right)}$

Then

(3) $V_{\pi 2}\left(\dfrac{1}{r_{\pi 2}}+g_{m2}+\dfrac{1}{r_{o2}}+\dfrac{1}{r_{o1}}\right)=g_{m1}V_i+\dfrac{V_{gs3}}{r_{o2}}$

or

$$\frac{V_{gs3}}{r_{o2}\left(g_{m2}+\dfrac{1}{r_{o2}}\right)}\left[\frac{1}{r_{\pi 2}}+g_{m2}+\frac{1}{r_{o2}}+\frac{1}{r_{o1}}\right]=g_{m1}V_i+\frac{V_{gs3}}{r_{o2}}$$

$$\frac{V_{gs3}}{2000\left(0.9615+\dfrac{1}{2000}\right)}\left[\frac{1}{104}+0.9615+\frac{1}{2000}+\frac{1}{2000}\right]$$

$$=0.9615V_i+\frac{V_{gs3}}{2000}$$

Then $V_{gs3}=1.83x10^5V_i$

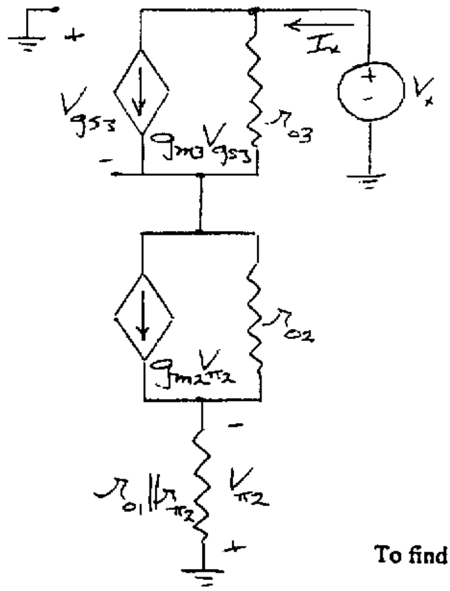
From (1),

$$\left(g_{m3}+\frac{1}{r_{o3}}\right)V_{gs3}=\frac{-V_o}{r_{o3}}$$

or

$$V_o=-2000\left(0.158+\frac{1}{2000}\right)(1.83x10^5)V_i$$

$$A_v=\frac{V_o}{V_i}=-5.80x10^7$$



To find R_o

(1) $I_x=g_{m3}V_{gs3}+\dfrac{V_x-(-V_{gs3})}{r_{o3}}$

(2) $g_{m3}V_{gs3}+\dfrac{V_x-(-V_{gs3})}{r_{o3}}=g_{m2}V_{\pi 2}+\dfrac{-V_{gs3}-(-V_{\pi 2})}{r_{o2}}$

(3) $V_{\pi 2}=-I_x(r_{o1}\|r_{\pi 2})$

From (1) $I_x=V_{gs3}\left(g_{m3}+\dfrac{1}{r_{o3}}\right)+\dfrac{V_x}{r_{o3}}$

$$I_x=V_{gs3}\left(0.158+\frac{1}{2000}\right)+\frac{V_x}{2000}$$

So

$$V_{gs3}=\frac{I_x-\dfrac{V_x}{2000}}{0.1585}$$

From (2),

$$V_{gs3}\left[g_{m3}+\frac{1}{r_{o3}}+\frac{1}{r_{o2}}\right]+\frac{V_x}{r_{o3}}=V_{\pi 2}\left(g_{m2}+\frac{1}{r_{o2}}\right)$$

$$V_{gs3}\left[0.158+\frac{1}{2000}+\frac{1}{2000}\right]+\frac{V_x}{2000}$$

$$=V_{\pi 2}\left(0.9615+\frac{1}{2000}\right)$$

Then

$$\left[\frac{I_x-V_x/2000}{0.1585}\right](0.159)+\frac{V_x}{2000}=-I_x(2000\|104)(0.962)$$

We find

$$R_o=\frac{V_x}{I_x}=6.09x10^{10}\ \Omega$$

11.60

Assume emitter of Q_1 is capacitively coupled to signal ground.

$$I_{CQ} = 0.2\left(\frac{80}{81}\right) = 0.1975\, mA$$

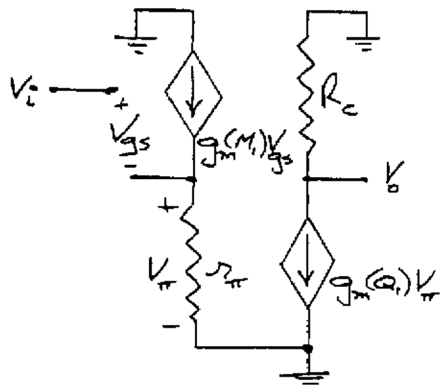
$$I_{DQ} = \frac{0.2}{81} = 0.00247\, mA$$

$$r_\pi = \frac{(80)(0.026)}{0.1975} = 10.5\, k\Omega$$

$$g_m(Q_1) = \frac{0.1975}{0.026} = 7.60\, mA/V$$

$$g_m(M_1) = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1) = 0.0445\, mA/V$$



$V_i = V_{gs} + V_\pi$ and $V_\pi = g_m(M_1)V_{gs}r_\pi$

or

$$V_{gs} = \frac{V_\pi}{g_m(M_1)r_\pi}$$

Then

$$V_i = V_\pi\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)$$

or

$$V_\pi = \frac{V_i}{\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)}$$

$$V_o = -g_m(Q_1)V_\pi R_C \Rightarrow$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m(Q_1)R_C}{\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)}$$

Then

$$A_v = \frac{-(7.60)(20)}{\left(1 + \frac{1}{(0.0445)(10.5)}\right)} \Rightarrow$$

$$\underline{A_v = -48.4}$$

11.61

Using the results from Chapter 4 for the emitter-follower:

$$R_0 = R_4 \,\Big\|\, \left[\frac{r_{\pi 8} + \frac{r_{\pi 9} + r_{07} + R_{011}}{1+\beta}}{1+\beta}\right]$$

$$r_{\pi 8} = \frac{\beta V_T}{I_{C8}} = \frac{(100)(0.026)}{1} = 2.6\text{ k}\Omega$$

$$I_{C9} \approx \frac{I_{C8}}{\beta} = \frac{1}{100} = 0.01\text{ mA}$$

$$r_{\pi 9} = \frac{(100)(0.026)}{0.01} = 260\text{ k}\Omega$$

$$r_{07} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500\text{ k}\Omega$$

$$r_{011} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500\text{ k}\Omega$$

$$R_{011} = r_{011}\left[1 + g_m R'_E\right],\quad g_m = \frac{0.2}{0.026} = 7.69$$

$$r_{\pi 11} = \frac{(100)(0.026)}{0.2} = 13\text{ k}\Omega$$

$$R'_E = 0.2\|13 = 0.197\text{ k}\Omega$$

$$R_{011} = 500[1 + (7.69)(0.197)] = 1257\text{ k}\Omega$$

Then

$$R_0 = 5 \,\Big\|\, \left[\frac{2.6 + \frac{260 + 500 + 1257}{101}}{101}\right]$$

$$= 5\|0.223$$

$$\Rightarrow \underline{R_0 = 0.213\text{ k}\Omega}$$

11.62

$$R_i = r_{\pi 1} + (1+\beta)r_{\pi 2}$$

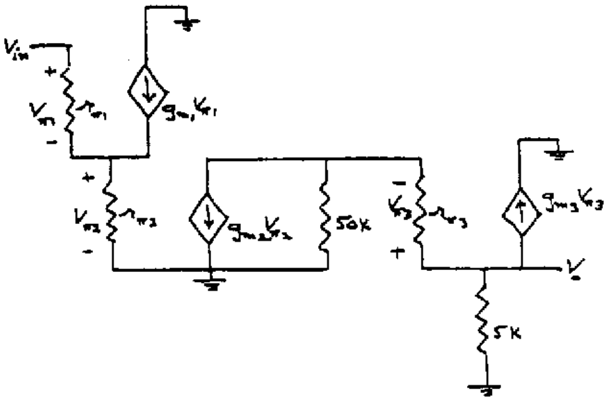
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2\text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2(0.026)}{0.5} = 520\text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow \underline{R_i \cong 1.05\text{ M}\Omega}$$

$$R_0 = 5 \,\Big\|\, \frac{r_{\pi 3} + 50}{101},\quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6\text{ k}\Omega$$

$$R_0 = 5 \,\Big\|\, \frac{2.6 + 50}{101} = 5\|0.521 \Rightarrow \underline{R_0 = 0.472\text{ k}\Omega}$$



$$V_0 = -\left(\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3}\right)(5)$$

$$V_0 = -V_{\pi 3}\left(\frac{1+\beta}{r_{\pi 3}}\right)(5) \quad (1)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2}V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2}V_{\pi 2} = V_{\pi 3}\left(\frac{1}{r_{\pi 3}} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)r_{\pi 2} = V_{\pi 1}\left(\frac{1+\beta}{r_{\pi 1}}\right)r_{\pi 2} \quad (3)$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3}\left(\frac{101}{2.6}\right)(5) \quad (1)$$

$$\Rightarrow V_{\pi 3} = -V_0(0.005149)$$

And

$$19.23V_{\pi 2} = -V_0(0.005149)\left(\frac{1}{2.6} + \frac{1}{50}\right) - \frac{V_0}{50} = -V_0(0.02208) \quad (2)$$

Or $V_{\pi 2} = -V_0(0.001148)$

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0(0.001148) \quad (4)$$

So

$$-V_0(0.001148) = [V_{in} + V_0(0.001148)]\left(\frac{101}{520}\right)(5.2) \quad (3)$$

$$-V_0(0.001148) - V_0(0.001159) = V_{in}(1.01)$$

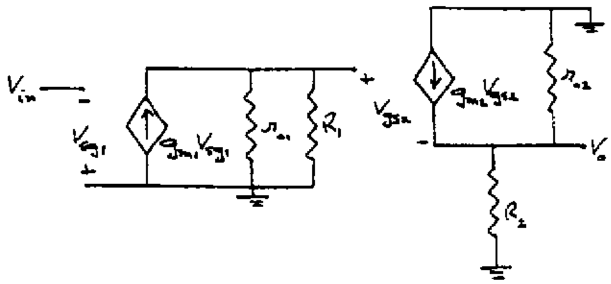
$$\Rightarrow A_v = \frac{V_0}{V_{in}} = -438$$

11.63

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2}V_{gs2})(R_2 \| r_{02})$$

$$V_{gs2} = (g_{m1}V_{sg1})(r_{01} \| R_1) - V_0$$

and $V_{sg1} = -V_{in}$

So

$$V_{gs2} = -(g_{m1}V_{in})(r_{01} \| R_1) - V_0$$

Then

$$V_0 = g_{m2}(R_2 \| r_{02})[-(g_{m1}V_{in})(r_{01} \| R_1) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2}(R_2 \| r_{02})g_{m1}(r_{01} \| R_1)}{1 + g_{m2}(R_2 \| r_{02})}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414 \ mA/V$$

$$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406 \ mA/V$$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2 \| r_{02} = 5 \| 100 = 4.76 \text{ k}\Omega$$

$$R_1 \| r_{01} = 35 \| 485 = 32.6 \text{ k}\Omega$$

Then

$$A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

So

$\Rightarrow \underline{A_v = -11.5}$

Output Resistance—From the results for a source follower in Chapter 6.

$$R_0 = \frac{1}{g_{m2}} \,\Big\|\, R_2 \,\Big\|\, r_{02} = \frac{1}{1.414} \,\Big\|\, 5 \,\Big\|\, 100$$

$$= 0.707 \| 4.76$$

So $\underline{R_0 = 0.616 \text{ k}\Omega}$

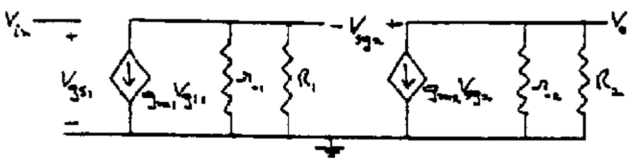
11.64

a. $R_2 = \frac{5}{0.5} \Rightarrow \underline{R_2 = 10 \text{ k}\Omega}$

$$V_{SG2} = \sqrt{\frac{I_{D2}}{K_{p2}}} - V_{TP2} = \sqrt{\frac{0.5}{0.25}} + 1 = 2.41V$$

$$R_1 = \frac{5 - (-2.41)}{0.1} \Rightarrow \underline{R_1 = 74.1 \text{ k}\Omega}$$

b.



$$V_0 = -(g_{m2}V_{sg2})(r_{02}\|R_2)$$

$$V_{sg2} = V_0 - [-(g_{m1}V_{gs1})(r_{01}\|R_1)]$$

and $V_{gs1} = V_{in}$

$$A_v = \frac{V_0}{V_{in}} = \frac{-(g_{m2})(r_{02}\|R_2)(g_{m1})(r_{01}\|R_1)}{1 + (g_{m2})(r_{02}\|R_2)}$$

$$g_{m1} = 2\sqrt{K_{n1}I_{D1}} = 2\sqrt{(0.1)(0.1)} = 0.2 \; mA/V$$

$$g_{m2} = 2\sqrt{K_{p2}I_{D2}} = 2\sqrt{(0.25)(0.5)} = 0.707 \; mA/V$$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

$$r_{02}\|R_2 = 200\|10 = 9.52 \text{ k}\Omega$$

$$r_{01}\|R_1 = 1000\|74.1 = 69.0 \text{ k}\Omega$$

Then

$$A_v = \frac{-(0.707)(9.52)(0.2)(69)}{1 + (0.707)(9.52)}$$

So

$\Rightarrow \underline{A_v = -12.0}$

$$R_0 = \frac{1}{g_{m2}} \,\Big\|\, R_2 \,\Big\|\, r_{02} = \frac{1}{0.707} \,\Big\|\, 10 \,\Big\|\, 200$$

$$= 1.414 \| 9.52$$

Or $\underline{R_0 = 1.23 \text{ k}\Omega}$

11.65

a. $I_{C2} = 0.25$ mA

$$R = \frac{5-2}{0.25} \Rightarrow \underline{R = 12 \text{ k}\Omega}$$

$$I_{C3} = \frac{v_{02} - V_{BE}(\text{on})}{R_{E1}} \Rightarrow R_{E1} = \frac{2 - 0.7}{0.5}$$

$\Rightarrow \underline{R_{E1} = 2.6 \text{ k}\Omega}$

$$R_C = \frac{5 - v_{03}}{I_{C3}} = \frac{5-3}{0.5} \Rightarrow \underline{R_C = 4 \text{ k}\Omega}$$

$$I_{C4} = \frac{[v_{03} - V_{BE}(\text{on})] - (-5)}{R_{E2}}$$

$$R_{E2} = \frac{3 - 0.7 + 5}{3} \Rightarrow \underline{R_{E2} = 2.43 \text{ k}\Omega}$$

b. Input resistance to base of Q_3.

$$R_{i3} = r_{\pi 3} + (1+\beta)R_{E1}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$R_{i3} = 5.2 + (101)(2.6) = 267.8 \text{ k}\Omega$$

$$A_{d1} = \frac{v_{02}}{v_d} = \frac{1}{2} g_{m2}(R\|R_{i3})$$

$$g_{m2} = \frac{0.25}{0.026} = 9.62 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}(9.62)(12\|267.8) \Rightarrow \underline{A_{d1} = 55.2}$$

Now $\dfrac{v_{03}}{v_{02}} = \dfrac{-\beta(R_C\|R_{i4})}{r_{\pi 3} + (1+\beta)R_{E1}}$

where $R_{i4} = r_{\pi 4} + (1+\beta)R_{E2}$

and $\dfrac{v_0}{v_{03}} = \dfrac{(1+\beta)R_{E2}}{r_{\pi 4} + (1+\beta)R_{E2}}$

$$r_{\pi 4} = \frac{(100)(0.026)}{3} = 0.867 \text{ k}\Omega$$

$$\frac{v_0}{v_{03}} = \frac{(101)(2.43)}{0.867 + (101)(2.43)} = 0.9965$$

$$R_{i4} = 0.867 + (101)(2.43) = 246.3 \text{ k}\Omega$$

$$r_{\pi 3} = 5.2 \text{ k}\Omega$$

So

$$\frac{v_{03}}{v_{02}} = \frac{-(100)(4\|246.3)}{5.2 + (101)(2.6)} = -1.47$$

So

$$A_d = \frac{v_0}{v_d} = (55.2)(0.9965)(-1.47)$$

$\Rightarrow \underline{A_d = -80.9}$

c. Using Equation (11.32b)

$$A_{cm1} = \frac{-g_{m2}(R\|R_{i3})}{1 + \dfrac{2(1+\beta)R_o}{r_{\pi 2}}}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.62)(12\|267.8)}{1 + \dfrac{2(101)(100)}{10.4}} = \underline{-0.0569 = A_{cm1}}$$

Then

$$A_{cm} = \left(\frac{v_0}{v_{03}}\right)\left(\frac{v_{03}}{v_{02}}\right)\cdot A_{cm1}$$

$$= (0.9965)(-1.47)(-0.0569)$$

$$\Rightarrow \underline{A_{cm} = 0.08335}$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{80.9}{0.08335}\right)$$

$$\Rightarrow \underline{CMRR_{dB} = 59.7 \text{ dB}}$$

11.66

a. $R_{C1} = \dfrac{10 - v_{01}}{I_{C1}} = \dfrac{10-2}{0.1} \Rightarrow \underline{R_{C1} = 80 \text{ k}\Omega}$

$$R_{C2} = \frac{10 - v_{04}}{I_{C4}} = \frac{10-6}{0.2} \Rightarrow \underline{R_{C2} = 20 \text{ k}\Omega}$$

b. $A_{d1} = \dfrac{v_{01} - v_{02}}{v_d} = -g_{m1}(R_{C1}\|r_{\pi 3})$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80\|23.4) \Rightarrow \underline{A_{d1} = -69.6}$$

$$A_{d2} = \frac{v_{04}}{v_{01} - v_{02}} = \frac{1}{2}g_{m4}R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2}(7.692)(20) = 76.9$$

Then $A_d = (76.9)(-69.6)$

$$\Rightarrow \underline{A_d = -5352}$$

11.67

a. Neglect the effect of r_0 in determining the differential-mode gain.

$$A_{d1} = \frac{v_{02}}{v_d} = \frac{1}{2}g_{m2}(R_C\|R_{i3})$$

where $R_{i3} = r_{\pi 3} + (1+\beta)R_E$

$$A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1+\beta)R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2}\cdot(1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{I_{C3}}$$

$$v_{02} = 12 - \frac{1}{2}(1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2}(37.3)[8\|668] = 147.4$$

$$A_2 = \frac{-(200)(4)}{4.86 + (201)(3.3)} = -1.197$$

Then

$$A_d = A_{d1}\cdot A_2 = (147.4)(-1.197) \Rightarrow \underline{A_d = -176}$$

$$R_0 = r_{05} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}(R_C\|R_{i3})}{1 + \dfrac{2(1+\beta)R_o}{r_{\pi 2}}}$$

$$r_{\pi 2} = \frac{(200)(0.026)}{\frac{1}{2}\cdot(1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8\|668)}{1 + \dfrac{2(201)(41.2)}{5.36}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow \underline{A_{cm} = 0.114}$$

b. $v_d = v_1 - v_2 = 2.015\sin\omega t - 1.985\sin\omega t$

$$v_d = 0.03\sin\omega t \text{ (V)}$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2.0\sin\omega t$$

$$v_{03} = A_d v_d + A_{cm}v_{cm}$$

$$= (-176)(0.03) + (0.114)(2)$$

Or

$\underline{v_{03} = -5.052\sin\omega t}$

Ideal, $A_{cm} = 0$

So

$$v_{03} = A_d v_d = (-176)(0.03)$$

$\underline{v_{03} = -5.28\sin\omega t}$

c. $R_{id} = 2r_{\pi 2} = 2(5.36) \Rightarrow \underline{R_{id} = 10.72 \text{ k}\Omega}$

$$2R_{icm} \cong 2(1+\beta)R_0\|(1+\beta)r_0$$

$$r_0 = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2}\cdot(1.94)} = 82.5 \text{ k}\Omega$$

$$2R_{icm} = [2(201)(41.2)]\|[(201)(82.5)]$$

$$= 16.6 \text{ M}\Omega\|16.6 \text{ M}\Omega$$

So

$$\Rightarrow \underline{R_{icm} = 4.15 \text{ M}\Omega}$$

11.68

a. $I_1 = \dfrac{24 - V_{GS4}}{R_1} = k_n(V_{GS4} - V_{TN})^2$

$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$

$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$

$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$v_{O2} = 12 - \left(\frac{0.375}{2}\right)(40) = 4.5 \text{ V}$$

$$\frac{v_{O2} - V_{GS3}}{R_5} = I_{D3} = k_n(V_{GS3} - V_{TN})^2$$

$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$

$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)} = 0.387 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}g_{m2}R_D = \frac{1}{2}(0.387)(40)$$

$\Rightarrow A_{d1} = 7.74$

$$A_2 = \frac{-g_{m3}R_{D2}}{1 + g_{m3}R_5}$$

$$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)} = 0.434 \text{ mA/V}$$

$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$

So $A_d = A_{d1} \cdot A_2 = (7.74)(-0.482)$

$\Rightarrow \underline{A_d = -3.73}$

$$R_o = r_{o4} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}R_D}{1 + 2g_{m2}R_o} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)} = -0.149$$

$A_{cm} = (-0.149)(-0.482) \Rightarrow \underline{A_{cm} = 0.0718}$

b. $v_d = v_1 - v_2 = 0.3\sin\omega t$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2\sin\omega t$$

$v_{O3} = A_d v_d + A_{cm} v_{cm}$

$= (-3.73)(0.3) + (0.0718)(2)$

$\Rightarrow \underline{v_{O3} = -0.975\sin\omega t \text{ (V)}}$

Ideal, $A_{cm} = 0$

$v_{O3} = A_d v_d = (-3.73)(0.3)$

Or

$\Rightarrow \underline{v_{O3} = -1.12\sin\omega t \text{ (V)}}$

11.69

The low-frequency, one-sided differential gain is

$$A_{v2} = \frac{v_{O2}}{v_d} = \frac{1}{2}g_m R_C\left(\frac{r_\pi}{r_\pi + R_B}\right) = \frac{\frac{1}{2}\cdot\beta R_C}{r_\pi + R_B}$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$A_{v2} = \frac{\frac{1}{2}\cdot(100)(10)}{5.2 + 0.5} \Rightarrow \underline{A_{v2} = 87.7}$$

$C_M = C_\mu(1 + g_m R_C)$

$$g_m = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$C_M = 2[1 + (19.23)(10)] \Rightarrow \underline{C_M = 387 \text{ pF}}$

$$f_H = \frac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)} = \frac{1}{2\pi[5.2\|0.5] \times 10^3 \times (8 + 387) \times 10^{-12}}$$

So

$\Rightarrow \underline{f_H = 883 \text{ kHz}}$

11.70

a. From Equation (11.117),

$$f_Z = \frac{1}{2\pi R_o C_o} = \frac{1}{2\pi(5 \times 10^6)(0.8 \times 10^{-12})}$$

Or $\underline{f_Z = 39.8 \text{ kHz}}$

b. From Problem 11.69, $f_H = 883\ kHz$. From Equation (11.116(b)), the low-frequency common-mode gain is

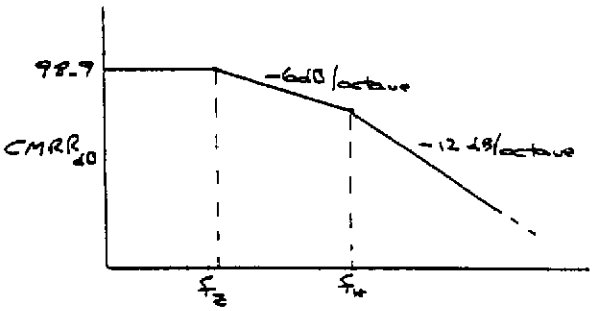
$$A_{cm} = \frac{-g_m R_C}{\left[\left(1 + \frac{R_B}{r_\pi}\right) + \frac{2(1+\beta)R_o}{r_\pi}\right]}$$

$r_\pi = 5.2 \text{ k}\Omega,\ g_m = 19.23 \text{ mA/V}$

So

$$A_{cm} = \frac{-(19.23)(10)}{\left[\left(1 + \frac{0.5}{5.2}\right) + \frac{2(101)(5 \times 10^6)}{5.2 \times 10^3}\right]} = -9.9 \times 10^{-4}$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{87.7}{9.9\times10^{-4}}\right) = 98.9\text{ dB}$$



11.71

a. From Equation (7.72),

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{1}{0.026} = 38.46\text{ mA/V}$$

Then

$$800\times10^6 = \frac{38.46\times10^{-3}}{2\pi(C_\pi + C_\mu)}$$

Or

$$C_\pi + C_\mu = 7.65\times10^{-12}\text{ F} = 7.65\text{ pF}$$

And $C_\pi = 6.65$ pF

$$C_M = C_\mu(1 + g_m R_C) = 1[1 + (38.46)(10)]$$

$$= 386\text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12\text{ k}\Omega$$

$$f_H = \frac{1}{2\pi[3.12\|1]\times10^3\times(6.65+386)\times10^{-12}}$$

Or

$\underline{f_H = 535\text{ kHz}}$

b. From Equation (11.117),

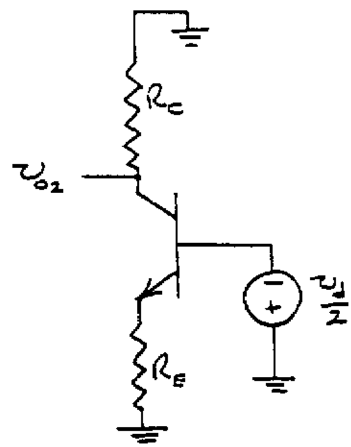
$$f_Z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi(10\times10^6)(10^{-12})}$$

Or

$\underline{f_Z = 15.9\text{ kHz}}$

11.72

The differential-mode half circuit is:



$$v_{02} = \frac{g_m\left(\frac{v_d}{2}\right)R_C}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E} \text{ or } A_v = \frac{\left(\frac{1}{2}\right)\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\text{ k}\Omega$$

$$A_v = \frac{\left(\frac{1}{2}\right)(100)(10)}{5.2+(101)R_E} = \frac{500}{5.2+(101)R_E}$$

a. For $R_E = 0.1$ kΩ : $\underline{A_v = 32.7}$

b. For $R_E = 0.25$ kΩ : $\underline{A_v = 16.4}$