

CHAPTER 12 - THREE-PHASE CIRCUITS

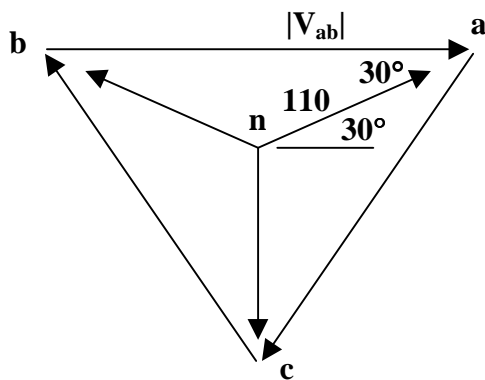
List of topics for this chapter :

- Balanced Three-Phase Voltages
- Balanced Wye-Wye Connection
- Balanced Wye-Delta Connection
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in a Balanced System
- Unbalanced Three-Phase Systems
- PSpice for Three-Phase Circuits
- Applications

BALANCED THREE-PHASE VOLTAGES

Problem 12.1 Given $V_{an} = 110\angle\phi$, $V_{ab} = |V_{ab}|\angle 0^\circ$, and $\phi > 0^\circ$, determine all the phase voltages, line voltages, and the phase sequence.

The easiest way to completely solve for all of the elements of the balanced 3-phase voltages is to do it graphically. All balanced 3-phase voltages generate the same shape. We start off by sketching V_{ab} , which has to be a horizontal line pointing to the right. Next we know that the value of V_{an} has its head connected to “a,” either coming down (indicating its phase angle is negative) or rising up from below (indicating that its phase angle is positive).



It is now easy to determine all of the values and the rotation.

Clearly the rotation is **acb**, or a negative phase sequence.

Because this is a triangle with equal sides, the interior angles are 60° and the line-to-neutral voltages bisect the angle. Thus V_{an} has to be at 30° . This then leads to the following;

$$V_{an} = \underline{110\angle 30^\circ \text{ V}}, V_{bn} = \underline{110\angle 150^\circ \text{ V}}, V_{cn} = \underline{110\angle -90^\circ \text{ V}}$$

and

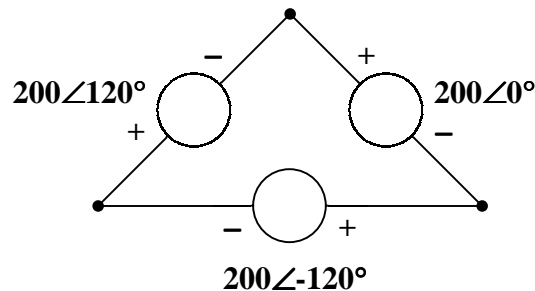
$$|V_{ab}| = 110\sqrt{3} = 190.52$$

therefore

$$V_{ab} = \underline{190.52\angle 0^\circ \text{ V}}, V_{bc} = \underline{190.52\angle 120^\circ \text{ V}}, V_{ca} = \underline{190.52\angle -120^\circ \text{ V}}$$

Problem 12.2 Is it possible to generate the effect of a balanced three-phase delta-connected source with only two voltage sources? If so, how?

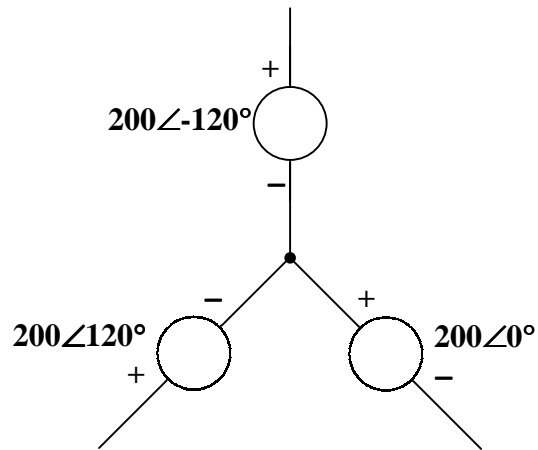
Let us start with three sources arranged in a delta as shown below.



Looking at this figure, we can see that KVL applies and going around the loop leads to a sum of zero volts. Now, if we remove any single source, KVL still states that the voltage across the open part of the circuit still is equal to the same voltage and phase of the source that was removed. Thus, **you can generate a perfect, delta-connected voltage source configuration with just two sources**. This will be expanded on later when we actually look at what happens to power distribution when you have two sources. (Clearly the two remaining sources must make up for the power the third source would have supplied.) Also, you will see later how this leads to being able to measure power delivered in a 3-phase system using only 2 wattmeters.

Problem 12.3 Is it possible to generate the effect of a balanced three-phase wye-connected source with only two voltage sources? Why or why not?

Again, we can look at the sources arranged in a wye and see what would happen. If we were to measure the line-to-line voltages, we would see a balanced 3-phase voltage system. Removing any one of the sources actually takes away two line-to-line voltages. Clearly, unlike the delta configuration, **you cannot deliver 3-phase voltage without 3 sources in a wye configuration**.



BALANCED WYE-WYE CONNECTION

Problem 12.4 [12.7] Obtain the line currents in the three-phase circuit of Figure 12.1.

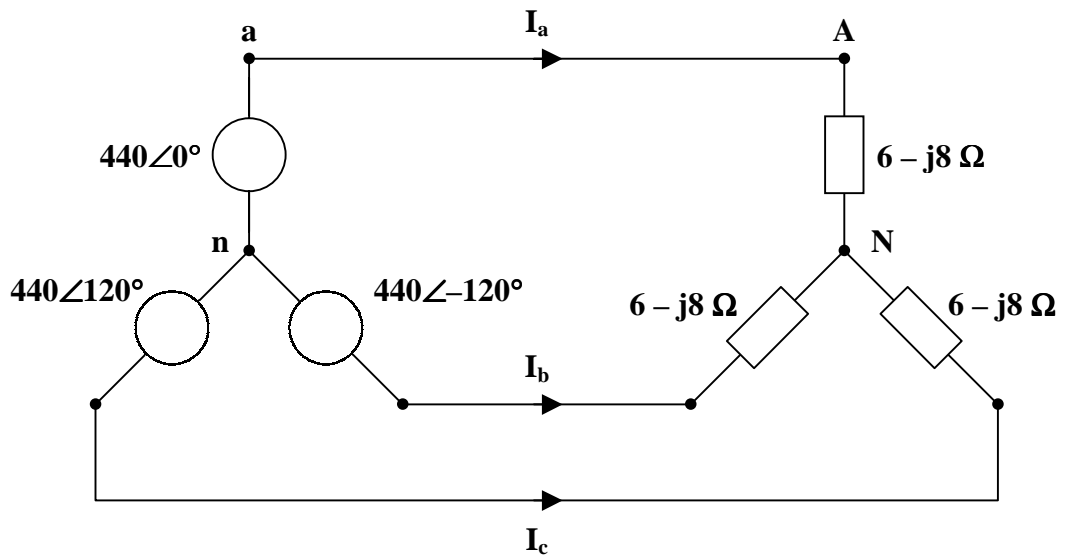
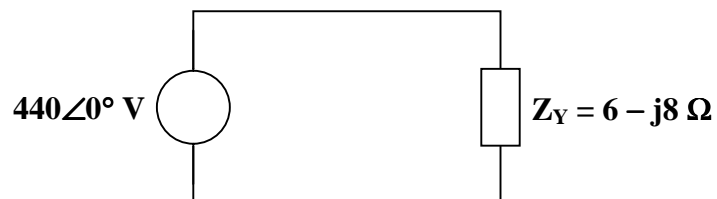


Figure 12.1

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$\mathbf{I}_a = \frac{440\angle 0^\circ}{6 - j8} = \underline{\underline{44\angle 53.13^\circ \text{ A}}}$$

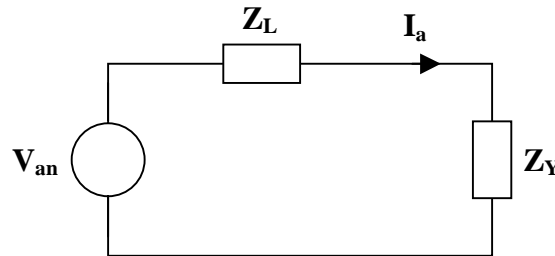
$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \underline{\underline{44\angle -66.87^\circ \text{ A}}}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \underline{\underline{44\angle 173.13^\circ \text{ A}}}$$

BALANCED WYE-DELTA CONNECTION

Problem 12.5 [12.15] In a wye-delta three-phase circuit, the source is a balanced, positive phase sequence with $\mathbf{V}_{an} = 120\angle 0^\circ \text{ V}$. It feeds a balanced load with $\mathbf{Z}_\Delta = 9 + j12 \Omega$ per phase through a balanced line with $\mathbf{Z}_L = 1 + j0.5 \Omega$ per phase. Calculate the phase voltages and currents in the load.

Convert the Δ -connected load to a Y-connected load and use per-phase analysis.



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 3 + j4$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + \mathbf{Z}_L} = \frac{120\angle 0^\circ}{(3 + j4) + (1 + j0.5)} = 19.931\angle -48.37^\circ$$

$$\text{But } \mathbf{I}_a = \mathbf{I}_{AB} \sqrt{3} \angle -30^\circ$$

$$\mathbf{I}_{AB} = \frac{19.931\angle -48.37^\circ}{\sqrt{3} \angle -30^\circ} = \underline{\underline{11.51\angle -18.37^\circ \text{ A}}}$$

$$\mathbf{I}_{BC} = \underline{\underline{11.51\angle -138.4^\circ \text{ A}}}$$

$$\mathbf{I}_{CA} = \underline{\underline{11.51\angle 101.6^\circ \text{ A}}}$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB} \mathbf{Z}_\Delta = (11.51\angle -18.37^\circ)(15\angle 53.13^\circ)$$

$$\underline{V_{AB} = 172.6 \angle 34.76^\circ \text{ V}}$$

$$\underline{V_{BC} = 172.6 \angle -85.24^\circ \text{ V}}$$

$$\underline{V_{CA} = 172.6 \angle 154.8^\circ \text{ V}}$$

BALANCED DELTA-DELTA CONNECTION

Problem 12.6 [12.17] For the Δ - Δ circuit of Figure 12.1, calculate the phase and line currents.

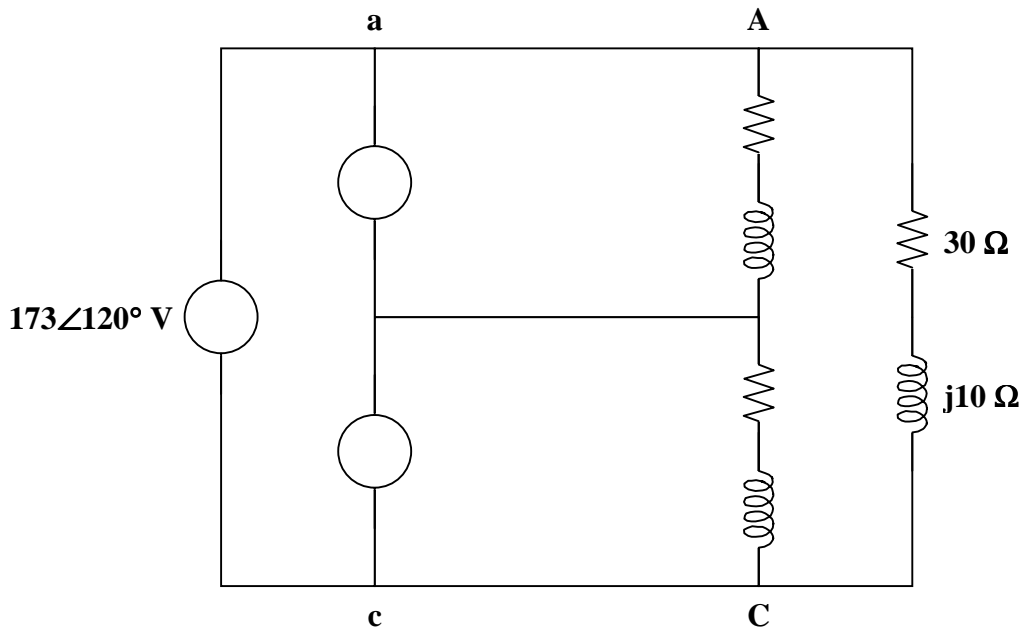


Figure 12.1

$$\underline{Z_{\Delta} = 30 + j10 = 31.62 \angle 18.43^\circ}$$

The phase currents are

$$\underline{I_{AB} = \frac{V_{ab}}{Z_{\Delta}} = \frac{173 \angle 0^\circ}{31.62 \angle 18.43^\circ} = 5.47 \angle -18.43^\circ \text{ A}}$$

$$\underline{I_{BC} = I_{AB} \angle -120^\circ = 5.47 \angle -138.43^\circ \text{ A}}$$

$$\underline{I_{CA} = I_{AB} \angle 120^\circ = 5.47 \angle 101.57^\circ \text{ A}}$$

The line currents are

$$\underline{I_a = I_{AB} - I_{CA} = I_{AB} \sqrt{3} \angle -30^\circ}$$

$$\mathbf{I}_a = 5.47\sqrt{3} \angle -48.43^\circ = \underline{\underline{9.474 \angle -48.43^\circ \text{ A}}}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \underline{\underline{9.474 \angle -168.43^\circ \text{ A}}}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \underline{\underline{9.474 \angle 71.57^\circ \text{ A}}}$$

BALANCED DELTA-WYE CONNECTION

Problem 12.7 [12.23] In a balanced three-phase Δ -Y circuit, the source is connected in the positive phase sequence, with $\mathbf{V}_{ab} = 220 \angle 20^\circ \text{ V}$ and $\mathbf{Z}_Y = 20 + j15 \Omega$. Find the line currents.

$$\mathbf{I}_a = \frac{\mathbf{V}_{ab} \angle -30^\circ}{\sqrt{3} \mathbf{Z}_Y} = \frac{220 \angle -10^\circ}{\sqrt{3} (20 + j15)}$$

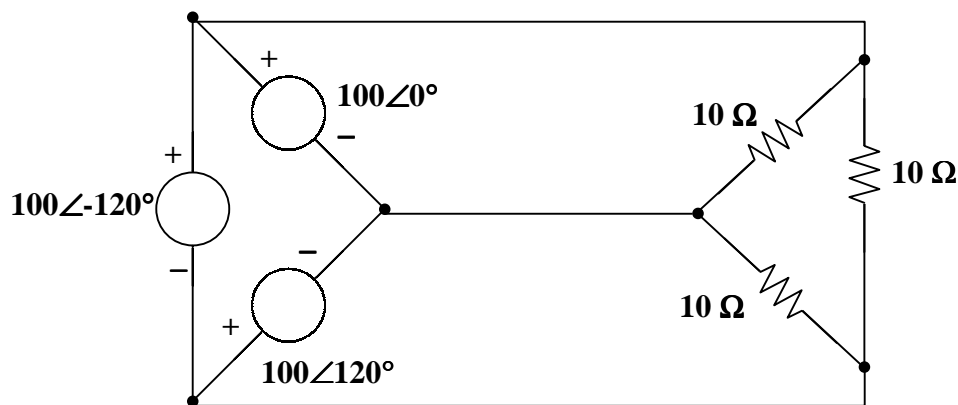
$$\mathbf{I}_a = \underline{\underline{5.081 \angle -46.87^\circ \text{ A}}}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \underline{\underline{5.081 \angle -166.87^\circ \text{ A}}}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \underline{\underline{5.081 \angle 73.13^\circ \text{ A}}}$$

POWER IN A BALANCED SYSTEM

Problem 12.8 Given a delta-connected source with line-to-line voltages of 100 volts-rms and a delta-connected load of 10-ohm resistors, calculate the power absorbed by each resistor and the power delivered by each source.



As we can see from the circuit, there is 100 volts across each 10-ohm resistor.

$$P_{\phi} = (100)^2/10 = \underline{1000 \text{ watts}} \text{ (per phase)}$$

So a total of **3000 watts** is delivered to the load and supplied by the source, with each individual voltage source supplying **1000 watts**.

Problem 12.9 Given $v_{\phi} = 100$ volts for the load in Figure 12.1, determine R_1 and R_2 so that the sources see a balanced load. Find I_1 and I_2 .

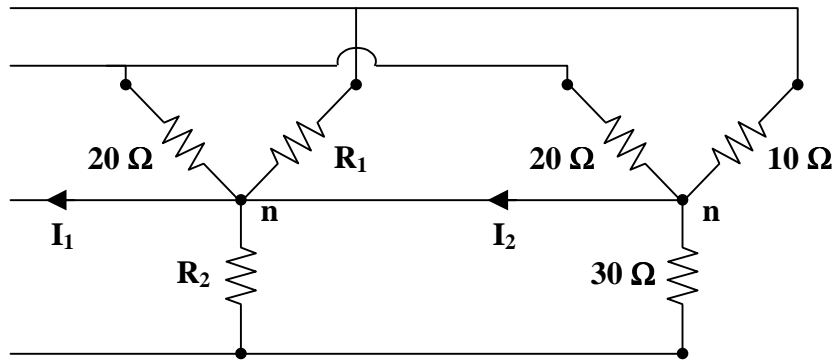
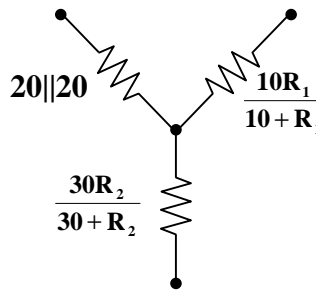


Figure 12.1

We can combine the two circuits as follows since each corresponding resistor is in parallel with each other.



Obviously, the $20||20$ is equal to 10 ohms. This means that the others need to combine into 10 ohms if possible.

$$\frac{10R_1}{10 + R_1} \text{ is } = 10 \text{ if and only if } R_1 = \underline{\infty}$$

$$\frac{30R_2}{30 + R_2} = 10 \longrightarrow 30R_2 = 300 + 10R_2$$

$$\text{or } R_2 = 300/(30 - 10) = \underline{15 \Omega}$$

Clearly, since the load appears balanced to the sources, $I_1 = 0$.

I_2 can be found by summing all of the currents flowing into it from the second circuit (see Figure 12.3). It is easier if we assume that $V_{20} = V_{AN} = 100\angle 0^\circ$, $V_{10} = V_{BN} = 100\angle 120^\circ$, and $V_{30} = V_{CN} = 100\angle -120^\circ$.

$$\begin{aligned} I_2 &= \frac{V_{AN}}{20} + \frac{V_{BN}}{10} + \frac{V_{CN}}{30} = \frac{100\angle 0^\circ}{20} + \frac{100\angle 120^\circ}{10} + \frac{100\angle -120^\circ}{30} \\ &= 5 + 10\angle 120^\circ + 3.333\angle -120^\circ = 5 + (-5 + j8.66) + (-1.667 - j2.887) \\ &= -1.667 + j5.773 = \underline{\underline{6.009\angle 106.1^\circ \text{ A}}} \end{aligned}$$



UNBALANCED THREE-PHASE SYSTEMS

Problem 12.10 Given a balanced wye-connected source of 200 volts each, calculate the power delivered to each of the resistors in the unbalanced wye-connected load in Figure 12.1.

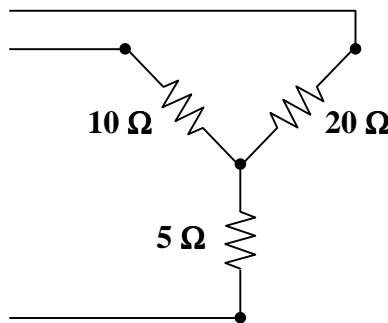
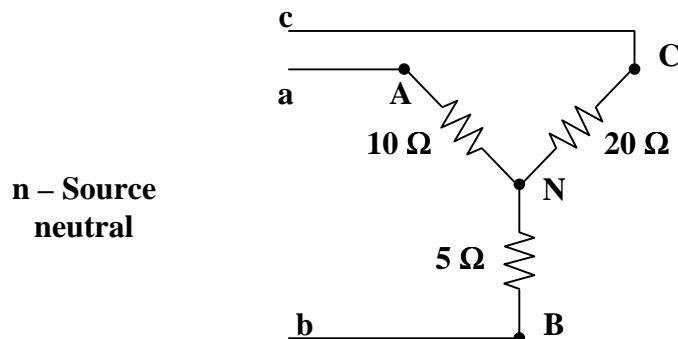


Figure 12.1

We start by labeling the circuit and arbitrarily placing the sources.



Let the voltage from a to the source neutral, n, be $V_{an} = 200\angle 0^\circ$

Likewise, $V_{bn} = 200\angle 120^\circ$ and $V_{cn} = 200\angle -120^\circ$

This can easily be solved if we look at it as a simple circuits problem with only one unknown node voltage, V_N (measured with respect to n).

We now can write a nodal equation at node N.

$$\frac{V_N - V_{an}}{10} + \frac{V_N - V_{bn}}{5} + \frac{V_N - V_{cn}}{20} = 0$$

Solving for V_N we get,

$$\begin{aligned} V_N &= \frac{400}{7} + \frac{800}{7} \angle 120^\circ + \frac{200}{7} \angle -120^\circ \\ &= 57.14 + (-14.285 - j24.74) + (-57.14 + j98.98) \\ &= -14.28 + j74.24 = 75.6 \angle 100.9^\circ \end{aligned}$$

Now that we have V_N , we can either find the currents through the resistors or voltages across them and then calculate the power. Solving for the voltages we get,

$$V_{AN} = V_{an} - V_N = 200 - (-14.28 + j74.24) = 214.28 - j74.24 = 226.8 \angle -19.11^\circ$$

$$\begin{aligned} V_{BN} &= V_{bn} - V_N = -100 + j173.21 - (-14.28 + j74.24) = -85.72 + j98.97 \\ &= 130.93 \angle 130.9^\circ \end{aligned}$$

$$\begin{aligned} V_{CN} &= V_{cn} - V_N = -100 - j173.21 - (-14.28 + j74.24) = -85.72 - j247.45 \\ &= 261.88 \angle -109.11^\circ \end{aligned}$$

Now to calculate the power.

$$P_{10} = \frac{|V_{AN}|^2}{10} = \frac{(226.8)^2}{10} = \underline{\underline{5.144 \text{ kwatts}}}$$

$$P_5 = \frac{|V_{BN}|^2}{5} = \frac{(130.93)^2}{5} = \underline{\underline{3.429 \text{ kwatts}}}$$

$$P_{20} = \frac{|V_{CN}|^2}{20} = \frac{(261.88)^2}{20} = \underline{\underline{3.429 \text{ kwatts}}}$$

Problem 12.11 Given a balanced wye-connected source of 200 volts each, calculate the power delivered to each of the resistors in the unbalanced wye-connected load in Figure 12.1. Explain the difference between this problem and Problem 12.10.

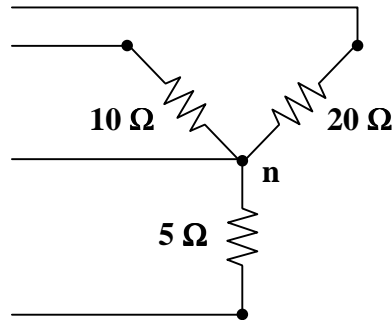


Figure 12.1

First of all, this problem is a lot easier to solve since the neutral connection means that each phase of the wye-connected source is across only its corresponding resistor, unlike the previous problem. In fact, there are no unknown node voltages in this circuit.

Stated mathematically, $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, and $V_{CN} = V_{cn}$

Thus, $P_{10} = (200)^2/10 = \underline{\underline{2 \text{ kwatts}}}$

$P_5 = (200)^2/5 = \underline{\underline{4 \text{ kwatts}}}$

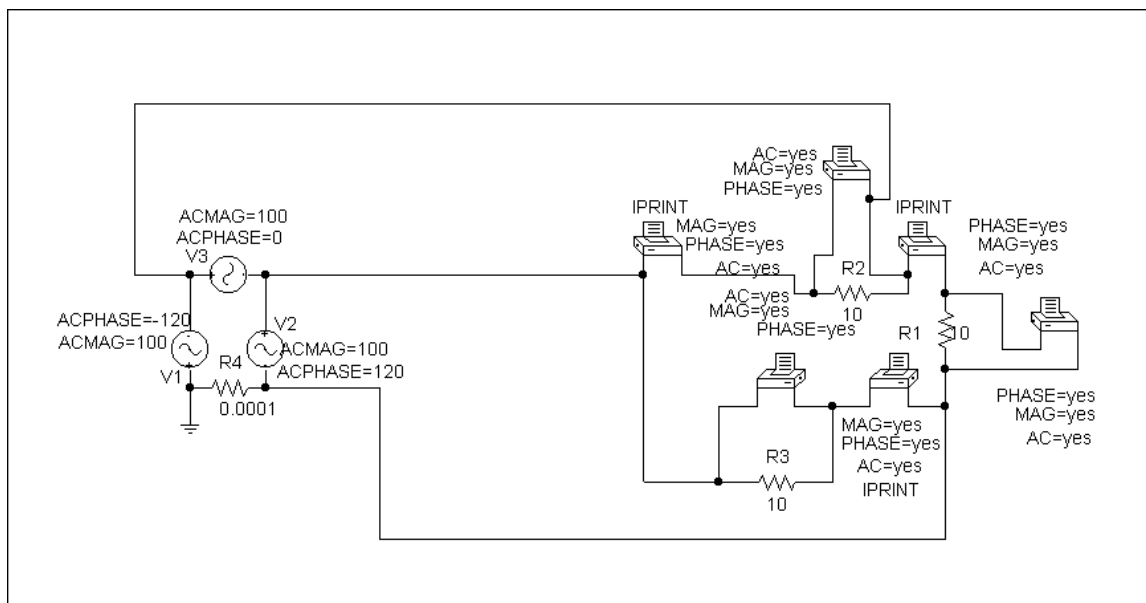
$P_{20} = (200)^2/20 = \underline{\underline{1 \text{ kwatt}}}$

PSPICE FOR THREE-PHASE CIRCUITS

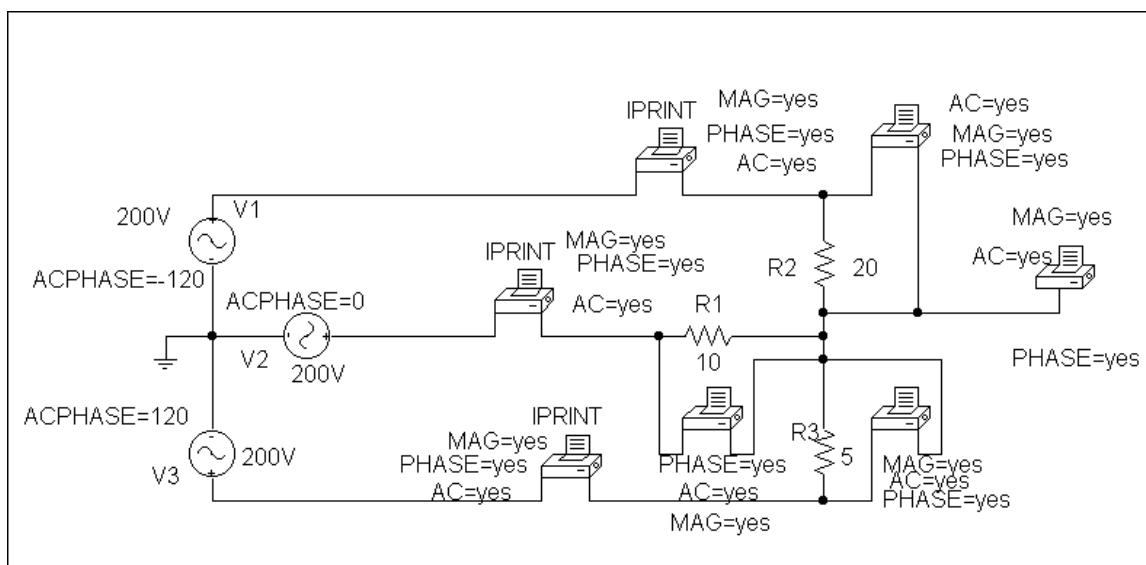
Problem 12.12 Solve Problem 12.8 using PSpice. The circuit was set up as the following schematic, calculating the current through each resistor and the voltage across each resistor. Since this is a resistance circuit, there is no phase angle between the voltage and the current. So all we need to do to calculate power is to multiply the magnitudes of the currents and voltages. Since it is a balanced circuit, each source supplies the same power equal to the power being absorbed by each resistor. After storing the circuit, we run Simulate and then examined the output file. The results yield the following:

$P_{10} = 100 \times 10 = 1 \text{ kwatt}$, for a total of 3 kwatts (both delivered and absorbed)

It is interesting to note that a small resistor needed to be placed in the voltage loop to allow PSpice to run properly.



Problem 12.13 Solve Problem 12.10 using PSpice. We develop the following schematic in PSpice. Note, we are making more measurements than necessary to solve this problem. However, these extra measurements will allow us to check the results of the voltage calculations in 12.10.



After saving the schematic, we run Simulate and open the output file. The results allow us to determine the voltages and currents in the circuit. We then can calculate the power delivered to each resistor.

$$V_{10} = 226.8\angle -19.11^\circ \text{ and } I_{10} = 22.68\angle -19.11^\circ$$

Therefore $P_{10} = (226.8)(22.68) = \mathbf{5.144 \text{ kwatts}}$ (please note that $\cos\theta = 1$ since these are resistors)

$$V_5 = 130.9\angle 130.9^\circ \text{ and } I_5 = 26.19\angle 130.9^\circ$$

$$\text{Therefore } P_{20} = (130.9)(26.19) = \underline{\underline{3.428 \text{ kwatts}}}$$

$$V_{20} = 261.9\angle -109.1^\circ \text{ and } I_{20} = 13.09\angle -109.1^\circ$$

$$\text{Therefore } P_{20} = (261.9)(13.09) = \underline{\underline{3.428 \text{ kwatts}}}$$

Going back to the solution obtained by hand, we see that we have virtually the same values of branch voltages. PSpice also gave us the value of $V_{Nn} = 79.59\angle 100.9^\circ$, which agrees. This shows the value of using PSpice to check results obtained by hand.

Problem 12.14 [12.49] Given the circuit in Figure 12.1, use PSpice to determine currents I_{aA} and voltage V_{BN} .

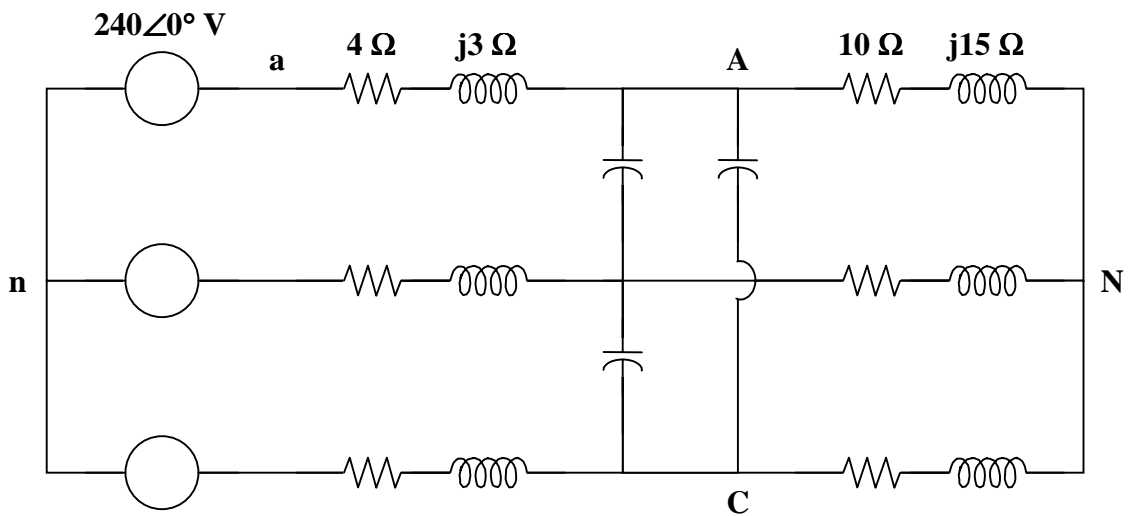
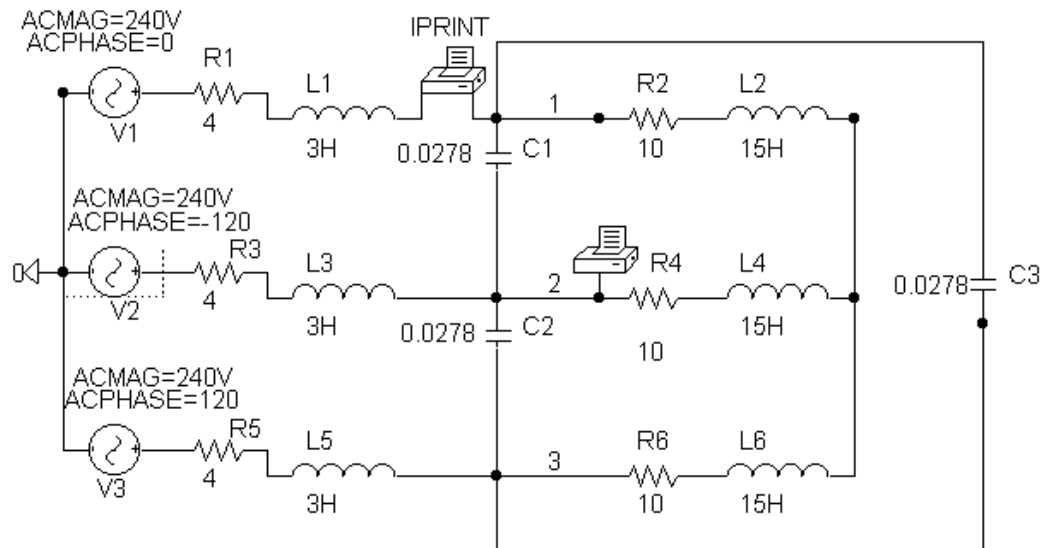


Figure 12.1

The schematic is shown below.



Pseudocomponents IPRINT and PRINT are inserted to measure I_{aA} and V_{BN} . In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E-01	2.308 E+02	-1.334 E+02

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \underline{11.15 \angle 37^\circ \text{ A}}, \quad V_{BN} = \underline{230.8 \angle -133.4^\circ \text{ V}}$$

APPLICATIONS

Problem 12.15 [12.59] For the circuit displayed in Figure 12.1, find the wattmeter readings.

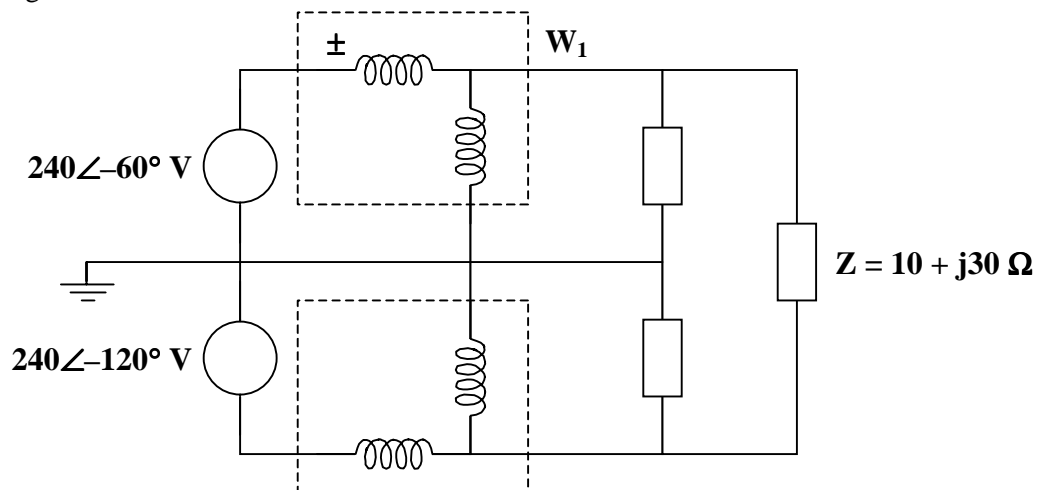
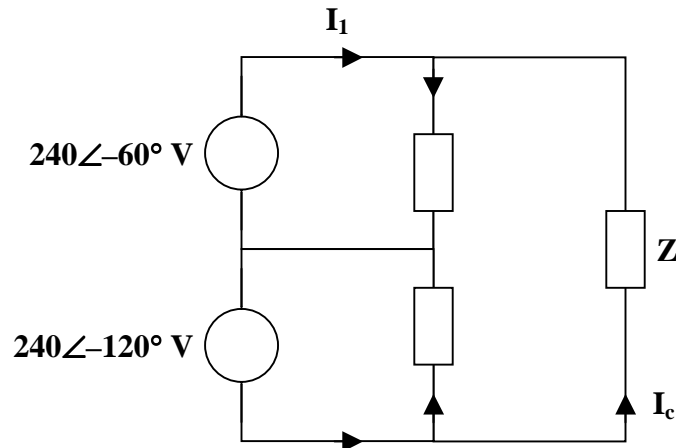


Figure 12.1

Consider the circuit as shown below.



$$Z = 10 + j30 = 31.62 \angle 71.57^\circ$$

$$I_a = \frac{240 \angle -60^\circ}{31.62 \angle 71.57^\circ} = 7.59 \angle -131.57^\circ$$

$$I_b = \frac{240 \angle -120^\circ}{31.62 \angle 71.57^\circ} = 7.59 \angle -191.57^\circ$$

$$I_c Z + 240 \angle -60^\circ - 240 \angle -120^\circ = 0$$

$$I_c = \frac{-240}{31.62 \angle 71.57^\circ} = 7.59 \angle 108.43^\circ$$

$$I_1 = I_a - I_c = 13.146 \angle -101.57^\circ$$

$$I_2 = I_b + I_c = 13.146 \angle 138.43^\circ$$

$$P_1 = \text{Re} [V_1 I_1^*] = \text{Re} [(240 \angle -60^\circ)(13.146 \angle 101.57^\circ)] = 2360 \text{ W}$$

$$P_2 = \text{Re} [V_2 I_2^*] = \text{Re} [(240 \angle -120^\circ)(13.146 \angle -138.43^\circ)] = -632.8 \text{ W}$$

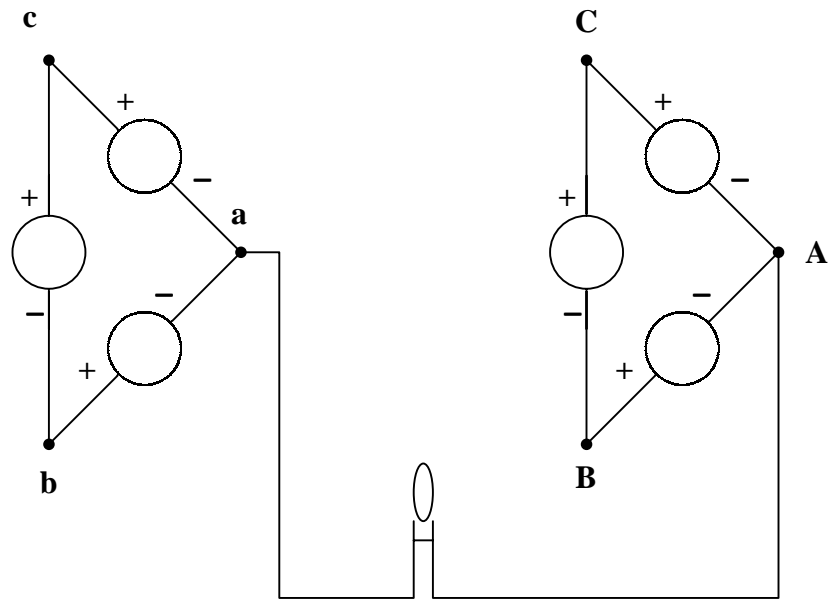
$$P_1 = \underline{\underline{2360 \text{ W}}}$$

$$P_2 = \underline{\underline{-632.8 \text{ W}}}$$

Problem 12.16 Given two three-phase sources with line-to-line voltages of 110 volts rms, is it possible to use standard 100 watt light bulb(s) to see if the sources are in phase with each other? Is it possible to see if they have the same rotation? More than one light bulb can be used.

Actually, only one light bulb is necessary. We need to make two assumptions before we start. The first is that both sources have exactly the same frequency. This is possible if both sources are driven by the same mechanical system. The second is that they share the same ground.

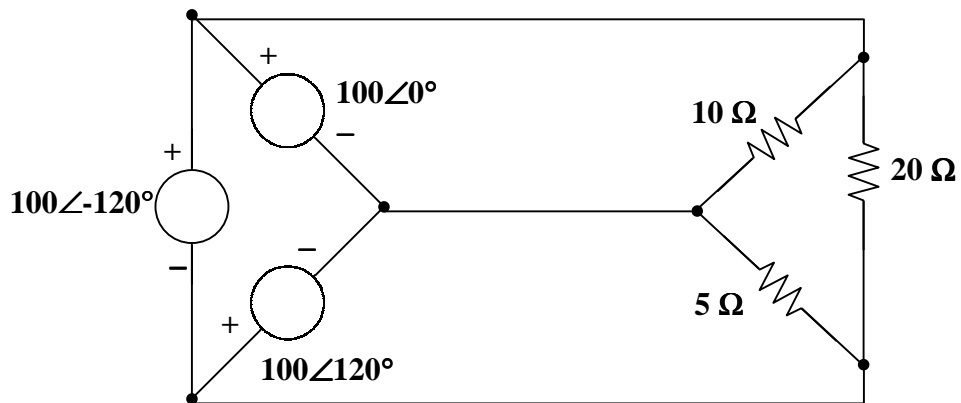
Consider the following representation of the two sources.



Now we connect the light to terminal a. Then we connect it to A, B, and C of the other source. If the sources are out of phase with each other, the light will light at least dimly when connected to each of the terminals. If the sources are reasonably close to being in phase with each other, there will be one terminal where the light will not light. Let us assume that it is the connection between a and A.

If we now connect the light to b and B and the light does not light, then the two sources have the same rotation. If the light lights, then they have opposite rotations. Once the rotations have been determined, the two sources can be connected in parallel.

Problem 12.17 Given an unbalanced-delta connected load, show how you can use two wattmeters to determine the power delivered to the load. Calculate the voltages and currents each meter sees and determine the power delivered to the load. The circuit is shown below.



This can be a difficult problem to solve unless we remember what we found as an answer to Problem 12.2. It is possible to represent a balanced delta source with only two sources. That tells us how to hook up the meters.

We can solve this using either hand calculations or PSpice. Let us use PSpice since we will need to make calculations twice in order to check our results, although the hand calculations are not that difficult.

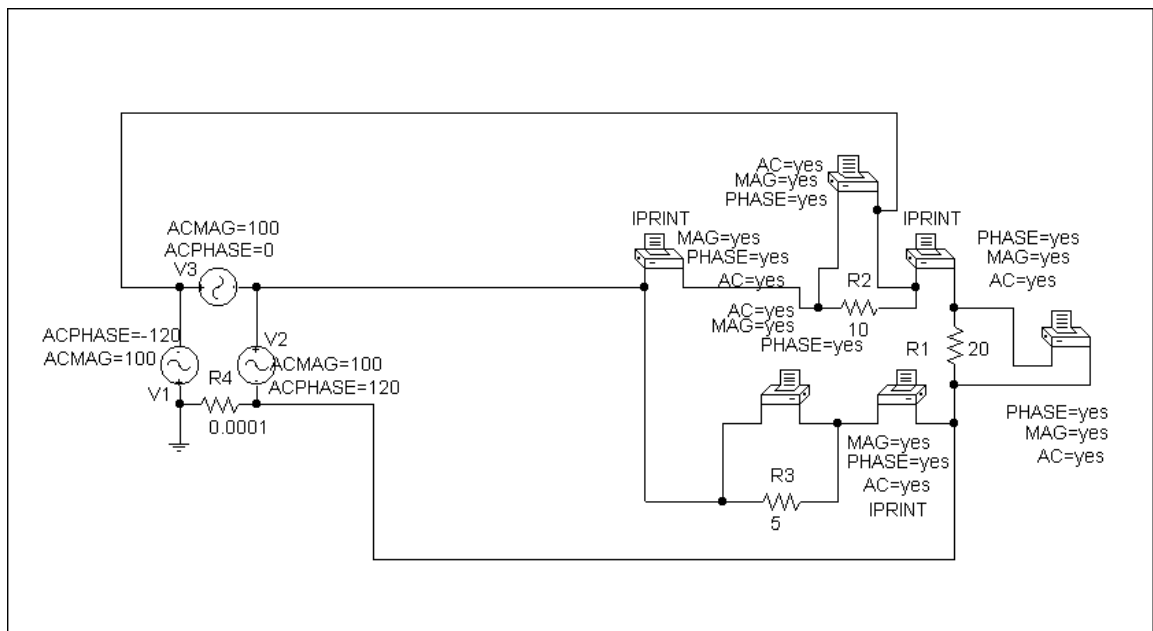
First, we design the schematic and save it. We then simulate and get the results shown here.

$$P_{10} = 100 \times 10 = 1 \text{ kwatt}$$

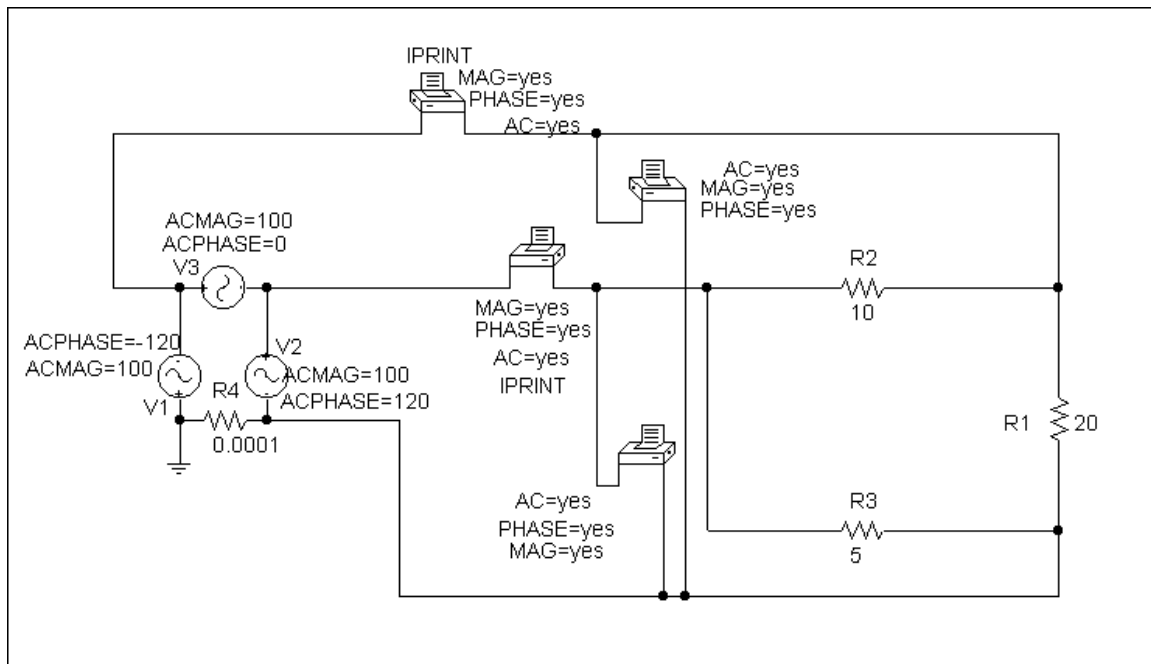
$$P_5 = 100 \times 20 = 2 \text{ kwatts}$$

$$P_{20} = 100 \times 5 = 500 \text{ watts}$$

$$P_{\text{Total}} = 1000 + 2000 + 500 = 3.5 \text{ kwatts}$$



Now look at the following circuit. If we assume that V_3 is not in the circuit for the moment, we see that V_1 and V_2 would supply all the power. So we place an IPRINT in series with V_1 and an IPRINT in series with V_2 . Then all we need to do is measure the Voltages V_1 and V_2 . (Actually in practice there are line losses and these voltages would be measured at the load.)



After we get the results from PSpice, we can make the following calculations.

$$P_1 = 100 \times 13.23 \cos(-120^\circ + 160.9^\circ) = 1 \text{ kwatt}$$

$$P_2 = 100 \times 26.46 \cos(120^\circ - 139.1^\circ) = 2.5 \text{ kwatts}$$

$$P_{\text{Total}} = P_1 + P_2 = \underline{\underline{3.5 \text{ kwatts}}} \text{ (the answer checks!!)}$$