

CHAPTER 6 - CAPACITORS AND INDUCTORS

List of topics for this chapter :

Capacitors
Series and Parallel Capacitors
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Series and Parallel Inductors
Applications

CAPACITORS

Problem 6.1 For the circuit shown in Figure 6.1, find $i_c(t)$ given

- (a) $v(t) = (2t + 6) \text{ V}$
(b) $v(t) = 2 \cos(\omega t) \text{ V}$

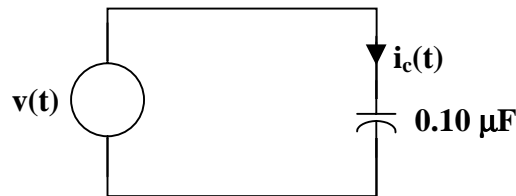


Figure 6.1

- (a) $i_c(t) = C \frac{dv_c}{dt} = 10^{-7} \frac{d(2t + 6)}{dt} = 2 \times 10^{-7} = \underline{\underline{0.2 \mu\text{A}}}$
(b) $i_c(t) = C \frac{dv_c}{dt} = 10^{-7} \frac{d(2 \cos(\omega t))}{dt} = (10^{-7})(-2\omega) \sin(\omega t)$
 $i_c(t) = \underline{\underline{-0.2\omega \sin(\omega t) \mu\text{A}}}$

Problem 6.2 Find $v_c(t)$ as shown in Figure 6.1, given that

- (a) $i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 1 \text{ A} & 0 < t < 5 \\ 0 \text{ A} & 5 < t \end{cases}$ (b) $i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ t^2 \text{ A} & 0 < t < 2 \\ 0 \text{ A} & 2 < t \end{cases}$



Figure 6.1

$$(a) \quad v_c(t) = \frac{1}{C} \int i_c(t) dt$$

$$\text{For } t < 0, \quad v_c(t) = 0\text{ V}$$

$$\text{For } 0 < t < 5, \quad v_c(t) = \frac{1}{10^{-5}} \int_0^t 1 dt = \frac{t}{10^{-5}} = t \times 10^5\text{ V}$$

$$\text{For } 5 < t, \quad v_c(t) = 5 \times 10^5\text{ V}$$

$$v_c(t) = \begin{cases} 0\text{ V} & t < 0 \\ t \times 10^5\text{ V} & 0 < t < 5 \\ 5 \times 10^5\text{ V} & 5 < t \end{cases}$$

These voltages are quite large. This is due to the large currents and small capacitances. Normally, the currents would be quite small, in the μA range.

$$(b) \quad v_c(t) = \frac{1}{C} \int i_c(t) dt$$

$$\text{For } t < 0, \quad v_c(t) = 0\text{ V}$$

$$\text{For } 0 < t < 2, \quad v_c(t) = \frac{1}{10^{-5}} \int_0^t t^2 dt = \frac{t^3}{3} \times 10^5\text{ V}$$

$$\text{For } 2 < t, \quad v_c(t) = \frac{8}{3} \times 10^5\text{ V}$$

$$v_c(t) = \begin{cases} 0\text{ V} & t < 0 \\ \frac{t^3}{3} \times 10^5\text{ V} & 0 < t < 2 \\ 2.667 \times 10^5\text{ V} & 2 < t \end{cases}$$

Problem 6.3 [6.11] A voltage of $60\cos(4\pi t)\text{ V}$ appears across the terminals of a 3-mF capacitor. Calculate the current through the capacitor and the energy stored in it from $t = 0$ to $t = 0.125\text{ s}$.

$$i = C \frac{dv}{dt} = (3 \times 10^{-3}) \frac{d(60 \cos(4\pi t))}{dt}$$

$$i = (3 \times 10^{-3})(60)(4\pi)[- \sin(4\pi t)] = \underline{-0.72\pi \sin(4\pi t) \text{ A}}$$

$$p = vi = [60 \cos(4\pi t)][-0.72\pi \sin(4\pi t)] = -21.6\pi \sin(8\pi t) \text{ W}$$

$$w = \int_0^t p \, dt = -21.6\pi \int_0^{1/8} \sin(8\pi t) \, dt$$

$$w = \frac{21.6\pi}{8\pi} \cos(8\pi t) \Big|_0^{1/8} = \underline{-5.4 \text{ J}}$$

Problem 6.4 Find $v_c(t)$, as shown in Figure 6.1, given that

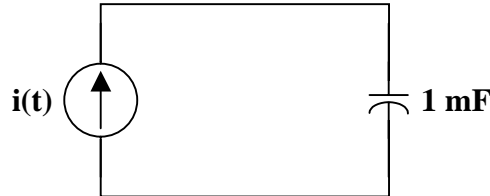
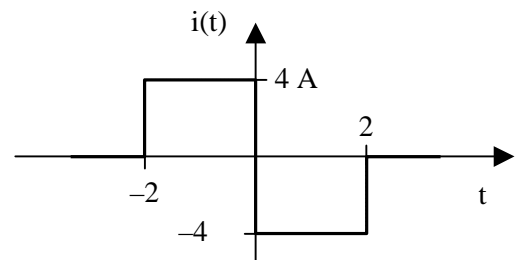
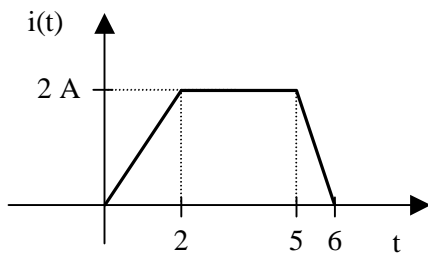


Figure 6.1

$$(a) \quad v_c(t) = \begin{cases} 0 \text{ V} & t < 0 \\ \frac{t^2}{2} \text{ kV} & 0 < t < 2 \\ (2t - 2) \text{ kV} & 2 < t < 5 \\ (-t^2 + 12t - 27) \text{ kV} & 5 < t < 6 \\ 9 \text{ kV} & 6 < t \end{cases}$$

$$(b) \quad v_c(t) = \begin{cases} 0 \text{ V} & t < -2 \\ (4t + 8) \text{ kV} & -2 < t < 0 \\ (-4t + 8) \text{ kV} & 0 < t < 2 \\ 0 \text{ V} & 2 < t \end{cases}$$

SERIES AND PARALLEL CAPACITORS

Problem 6.5 Given the circuit in Figure 6.1, $v_1(0^-) = 0 \text{ V}$, $v_2(0^-) = 100 \text{ V}$, calculate the voltages after the switch closes.

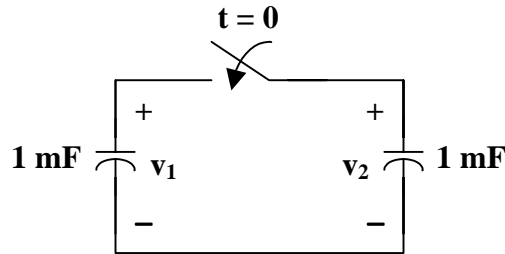


Figure 6.1

➤ **Carefully DEFINE the problem.**

Each component is labeled completely. The problem is clear.

➤ **PRESENT everything you know about the problem.**

Since the capacitors are in parallel, the charge, $q = CV$, must remain the same.

Also, when the switch is closed, the voltages across the capacitors are the same.

➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis can be used to solve this problem.

➤ **ATTEMPT a problem solution.**

$$\text{For } t < 0, \quad w = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} (10^{-3}) (100)^2 = 5 \text{ J} \quad (\text{there is no initial charge on } C_1)$$

$$\text{For } t < 0, \quad q = C_2 V_2 = (10^{-3}) (100) = 0.1 \text{ C}$$

$$\text{For } t > 0, \quad C = C_1 + C_2 = 2 \text{ mF}$$

$$V = \frac{0.1}{2 \times 10^{-3}} = 50 \text{ V}$$

$$\text{Now,} \quad w = \frac{1}{2} CV^2 = \frac{1}{2} (2 \times 10^{-3}) (50)^2 = 2.5 \text{ J}$$

Clearly, the energy has gone from 5 J to 2.5 J. What happened to 2.5 J of energy? Well, the switch cannot close fast enough to keep from having a spark. Thus, 2.5 J of energy must be dissipated in the spark.

➤ **EVALUATE the solution and check for accuracy.**

After the switch closes, the charge of 0.1 C remains the same and the voltage across both is now the same, 50 V.

Our check for accuracy was successful.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**
This problem has been solved satisfactorily.

$$v_1 = v_2 = \underline{50 \text{ V}}$$

Problem 6.6

Find the equivalent capacitance for the collection of capacitors shown in Figure 6.1.

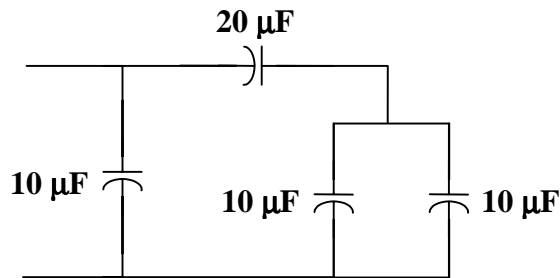


Figure 6.1

$$10 \text{ in parallel with } 10 = 10 + 10 = 20$$

$$20 \text{ in series with } 20 = \frac{(20)(20)}{20 + 20} = 10$$

$$10 \text{ in parallel with } 10 = 10 + 10 = 20$$

Therefore, $C_{\text{eq}} = \underline{20 \mu\text{F}}$

Problem 6.7

Given that the equivalent capacitance of the collection of capacitors shown in Figure 6.1 is $30 \mu\text{F}$, find C_1 and C_2 .

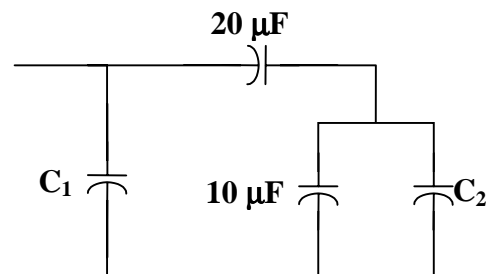


Figure 6.1

There is an infinite number of solutions.

To find one solution, let $C_2 = 10 \mu\text{F}$, the network is similar to the one shown in Problem 6.6.

The first two combinations are the same. Hence, we have

$$10 \text{ in parallel with } C_1 = C_{\text{eq}}$$

or $10 + C_1 = 30 \longrightarrow C_1 = 20$

Therefore, $C_1 = \underline{20 \mu\text{F}}$ and $C_2 = \underline{10 \mu\text{F}}$ produce $C_{\text{eq}} = 30 \mu\text{F}$

Problem 6.8 [6.17] Calculate the equivalent capacitance for the circuit in Figure 6.1. All capacitances are in mF.

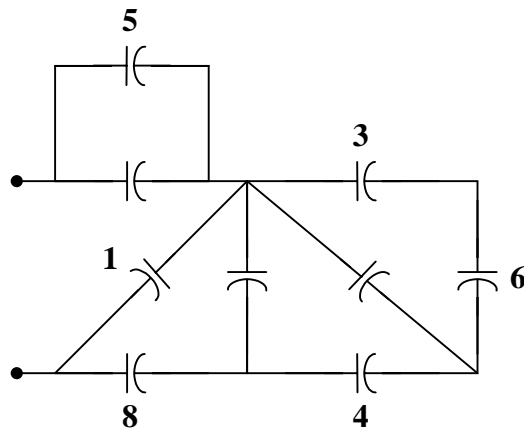


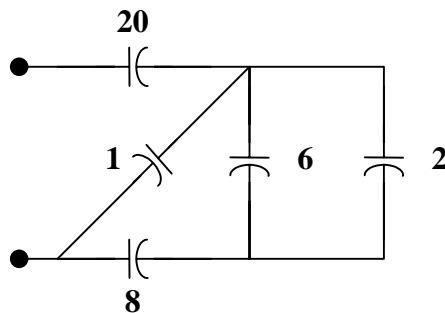
Figure 6.1

$$3 \text{ in series with } 6 = \frac{(6)(3)}{6+3} = 2$$

$$2 \text{ in parallel with } 2 = 2+2=4$$

$$4 \text{ in series with } 4 = \frac{(4)(4)}{4+4} = 2$$

The circuit is reduced to that shown below:



6 in parallel with 2 = $6 + 2 = 8$

8 in series with 8 = $\frac{(8)(8)}{8+8} = 4$

4 in parallel with 1 = $4 + 1 = 5$

5 in series with 20 = $\frac{(5)(20)}{5+20} = 4$

Therefore, $C_{eq} = \underline{4 \text{ mF}}$

INDUCTORS

Problem 6.9 For the circuit shown in Figure 6.1,

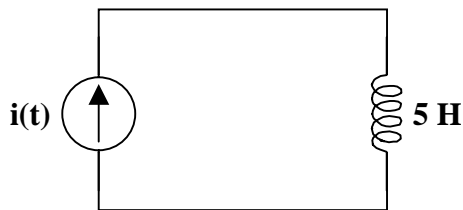


Figure 6.1

calculate $v_L(t)$ given that

(a) $i(t) = (5t + 6) \text{ A}$

(b) $i(t) = 3 \sin(\omega t + 30^\circ) \text{ A}$

(a) $v_L(t) = L \frac{di(t)}{dt} = 5 \frac{d(5t + 6)}{dt} = \underline{25 \text{ V}}$

(b) $v_L(t) = L \frac{di(t)}{dt} = 5 \frac{d(3 \sin(\omega t + 30^\circ))}{dt}$
 $v_L(t) = \underline{15\omega \cos(\omega t + 30^\circ) \text{ V}}$

Problem 6.10 For the circuit shown in Figure 6.1, calculate $v_L(t)$ given that

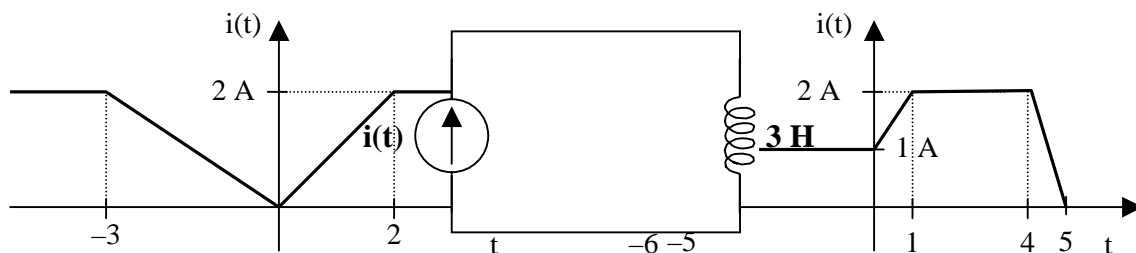


Figure 6.1

$$v_L(t) = L \frac{di_L(t)}{dt}$$

(a) For $t < -3$, $v_L(t) = 3 \frac{d(2)}{dt} = \underline{0 \text{ V}}$

For $-3 < t < 0$, $v_L(t) = 3 \frac{d(-2t/3)}{dt} = \underline{-2 \text{ V}}$

For $0 < t < 2$, $v_L(t) = 3 \frac{d(t)}{dt} = \underline{3 \text{ V}}$

For $2 < t$, $v_L(t) = 3 \frac{d(2)}{dt} = \underline{0 \text{ V}}$

(b) For $t < -6$, $v_L(t) = 3 \frac{d(0)}{dt} = \underline{0 \text{ V}}$

For $-6 < t < -5$, $v_L(t) = 3 \frac{d(t+6)}{dt} = \underline{3 \text{ V}}$

For $-5 < t < 0$, $v_L(t) = 3 \frac{d(1)}{dt} = \underline{0 \text{ V}}$

For $0 < t < 1$, $v_L(t) = 3 \frac{d(t+1)}{dt} = \underline{3 \text{ V}}$

For $1 < t < 4$, $v_L(t) = 3 \frac{d(2)}{dt} = \underline{0 \text{ V}}$

$$\text{For } 4 < t < 5, \quad v_L(t) = 3 \frac{d(-2t + 10)}{dt} = \underline{-6 \text{ V}}$$

$$\text{For } 5 < t, \quad v_L(t) = 3 \frac{d(0)}{dt} = \underline{0 \text{ V}}$$

Problem 6.11 [6.35] The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

$$i = \frac{1}{L} \int_0^t v \, dt + i(0) = \frac{1}{2} \int_0^t (20)(1 - e^{-2t}) \, dt + 0.3$$

$$i = (10) \left(t + \frac{1}{2} e^{-2t} \right)_0^t + 0.3 = (10t + 5e^{-2t} - 4.7) \text{ A}$$

At $t = 1$ s,

$$i = 10 + 5e^{-2} - 4.7 = \underline{5.977 \text{ A}}$$

$$w = \frac{1}{2} Li^2 = \underline{35.72 \text{ J}}$$

Problem 6.12 For the circuit shown in Figure 6.1, calculate $i_L(t)$ given that

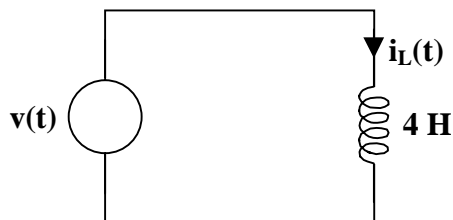
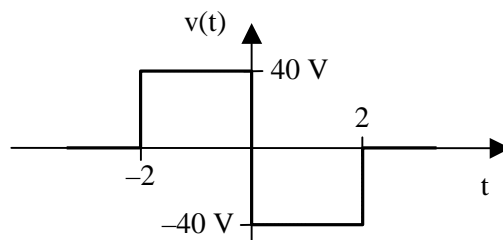
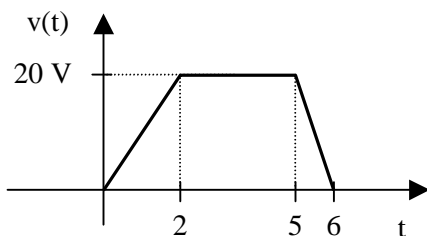


Figure 6.1

$$(a) \quad i_L(t) = \begin{cases} 0 \text{ A} & t < 0 \\ (5/4)t^2 \text{ A} & 0 < t < 2 \\ (5t - 5) \text{ A} & 2 < t < 5 \\ (-2.5t^2 + 30t - 67.5) \text{ A} & 5 < t < 6 \\ 22.5 \text{ A} & 6 < t \end{cases} \quad (b) \quad i_L(t) = \begin{cases} 0 \text{ A} & t < -2 \\ (10t + 20) \text{ A} & -2 < t < 0 \\ (-10t + 20) \text{ A} & 0 < t < 2 \\ 0 \text{ A} & 2 < t \end{cases}$$

SERIES AND PARALLEL INDUCTORS

Problem 6.13 Given the collection of inductors shown in Figure 6.1, find the value of the equivalent inductance.

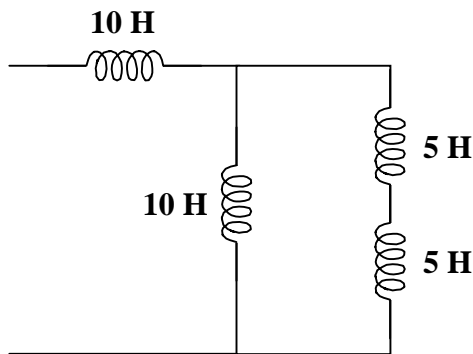


Figure 6.1

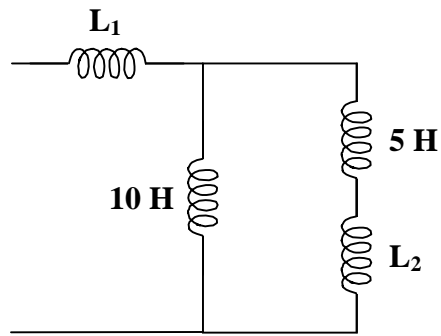
5 in series with 5 = 5 + 5 = 10

10 in parallel with 10 = $\frac{(10)(10)}{10 + 10} = 5$

5 in series with 10 = 5 + 10 = 15

Thus, $L_{eq} = \underline{15 \text{ H}}$

Problem 6.14 Given the collection of inductors shown in Figure 6.1, find the values of L_1 and L_2 , when the equivalent inductance is 20 H.



Problem 6.15

Figure 6.1

There is an infinite number of solutions.

However, if $L_2 = 5 \text{ H}$, the network is similar to the one shown in Problem 6.13. The first two combinations are the same. Hence, we have

$$5 \text{ in series with } L_1 = L_{eq}$$

or $5 + L_1 = 20 \longrightarrow L_1 = 15$

Therefore, $L_1 = \underline{15 \text{ H}}$ and $L_2 = \underline{5 \text{ H}}$ produce $L_{eq} = 20 \text{ H}$

APPLICATIONS

Problem 6.16 Calculate the voltage across the current source in Figure 6.1 given that $i(t) = (t + 5) \text{ A}$.

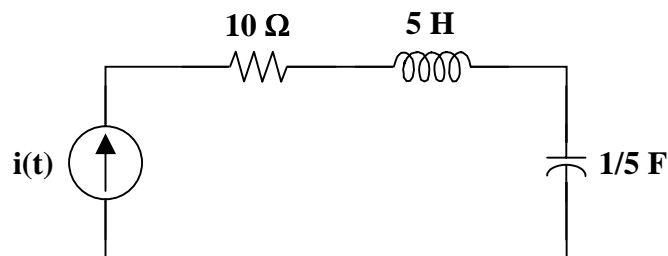


Figure 6.1

➤ **Carefully DEFINE the problem.**

Each component is labeled completely. The problem is clear, except for the value of the capacitor voltage at some point in time.

➤ **PRESENT everything you know about the problem.**

We know the current as well as the values of the elements. However, we do not know the initial condition on the voltage across the capacitor. We will solve for the voltage across the current source assuming that the capacitor voltage at $t = 0$ is equal to $v_C(0)$.

➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis will be used to solve this problem.

➤ **ATTEMPT a problem solution.**

$$v(t) = v_R(t) + v_L(t) + v_C(t)$$

$$v(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(\tau) d\tau$$

$$v(t) = (10)(t + 5) + (5)(1) + 5 \int_0^t (\tau + 5) d\tau + v_C(0)$$

$$v(t) = 10t + 50 + 5 + (5) \left(\frac{t^2}{2} + 5t \right) + v_C(0)$$

$$v(t) = 10t + 50 + 5 + 2.5t^2 + 25t + v_C(0)$$

$$v(t) = [2.5t^2 + 35t + 55] V + v_C(0)$$

➤ **EVALUATE the solution and check for accuracy.**

The current through each element is the same. The voltage across each element was determined while attempting a problem solution.

$$\text{For the resistor, } i_R(t) = \frac{v_R(t)}{10} = \frac{10t + 50}{10} = t + 5$$

$$\text{For the inductor, } i_L(t) = \frac{1}{L} \int v_L(\tau) d\tau = \frac{1}{5} \int_0^t 5 d\tau = \frac{1}{5} \cdot 5t + i_L(0) = t + i_L(0)$$

$$\text{For the capacitor, } i_C(t) = C \frac{dv_C(t)}{dt} = \frac{1}{5} \cdot \frac{d}{dt} (2.5t^2 + 25t) = \frac{1}{5} (5t + 25) = t + 5$$

Hence,

$$i(t) = i_R(t) = i_L(t) = i_C(t), \quad \text{when } i(0) = 5 = i_L(0)$$

Our check for accuracy was successful.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**
This problem has been solved satisfactorily.

$$v(t) = \underline{[2.5t^2 + 35t + 55] V + v_C(0)}$$

Problem 6.17 Given the circuit in Figure 6.1, find $v(t)$ for $v_C(t) = (10 + 5t) \text{ V}$.

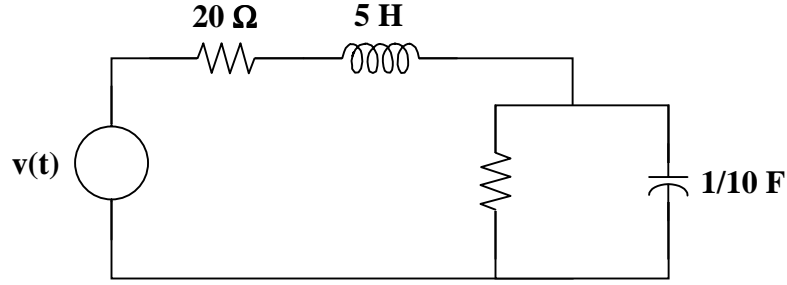


Figure 6.1

$$i_C(t) = C \frac{dv_C(t)}{dt} = \frac{1}{10} \frac{d}{dt}(10 + 5t) = \left(\frac{1}{10}\right)(5) = \frac{1}{2} \text{ A}$$

$$i_{10\Omega}(t) = \frac{v_C(t)}{R} = \frac{10 + 5t}{10} = \left(1 + \frac{1}{2}t\right) \text{ A}$$

$$i_L(t) = \frac{1}{2} + \left(1 + \frac{1}{2}t\right) = \left(\frac{3}{2} + \frac{1}{2}t\right) \text{ A}$$

$$v_L(t) = L \frac{di(t)}{dt} = (5) \frac{d}{dt}\left(\frac{3}{2} + \frac{1}{2}t\right) = (5)\left(\frac{1}{2}\right) = 2.5 \text{ V}$$

$$v_{20\Omega}(t) = i_{20\Omega}(t)R = i_L(t)R = \left(\frac{3}{2} + \frac{1}{2}t\right)(20) = (30 + 10t) \text{ V}$$

$$v(t) = v_{20\Omega}(t) + v_L(t) + v_C(t) = (30 + 10t) + (2.5) + (10 + 5t)$$

$$v(t) = \underline{\underline{(15t + 42.5) \text{ V}}}$$

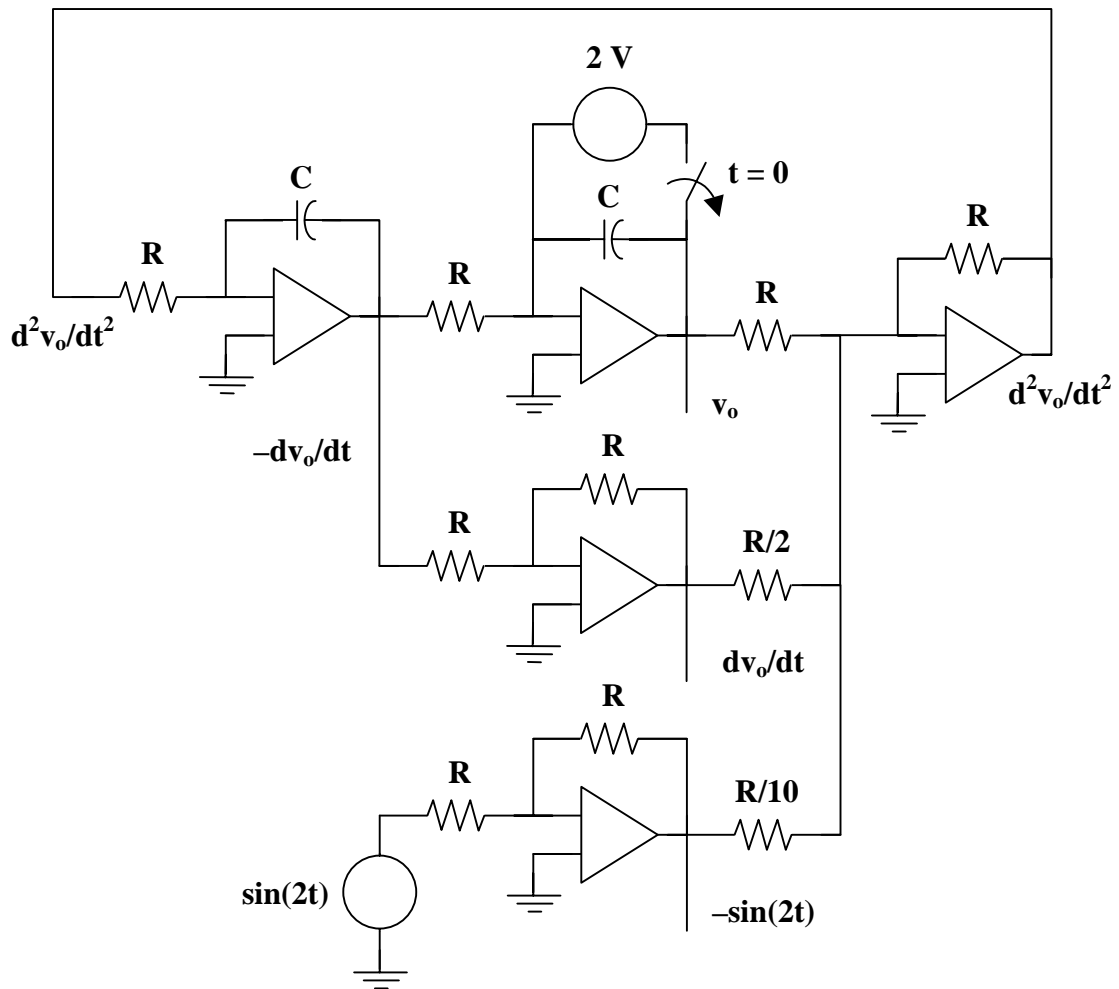
Problem 6.18 [6.67] Design an analog computer to simulate

$$\frac{d^2 v_o}{dt^2} + 2 \frac{dv_o}{dt} + v_o = 10 \sin(2t)$$

where $v_o(0) = 2$ and $v'_o(0) = 0$.

$$\frac{d^2 v_o}{dt^2} = 10 \sin(2t) - 2 \frac{dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below.



Problem 6.19 Calculate $v(t)$ and $v_L(t)$ for the circuit shown in Figure 6.1 and $v_C(0) = -10$ Volts (with the plus side of v_C at the top of the capacitor).

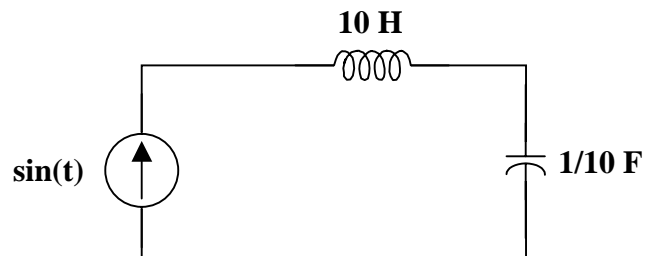


Figure 6.1

$$v(t) = \underline{\underline{0 \text{ V}}}$$

$$v_L(t) = \underline{\underline{10 \cos(t) \text{ V}}}$$