PART 1

DC CIRCUITS

CHAPTER 1

BASIC CONCEPTS

It is engineering that changes the world.

—Isaac Asimov

Historical Profiles

Alessandro Antonio Volta (1745–1827), an Italian physicist, invented the electric battery—which provided the first continuous flow of electricity—and the capacitor.

Born into a noble family in Como, Italy, Volta was performing electrical experiments at age 18. His invention of the battery in 1796 revolutionized the use of electricity. The publication of his work in 1800 marked the beginning of electric circuit theory. Volta received many honors during his lifetime. The unit of voltage or potential difference, the volt, was named in his honor.

Andre-Marie Ampere (1775–1836), a French mathematician and physicist, laid the foundation of electrodynamics. He defined the electric current and developed a way to measure it in the 1820s.

Born in Lyons, France, Ampere at age 12 mastered Latin in a few weeks, as he was intensely interested in mathematics and many of the best mathematical works were in Latin. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and the ammeter. The unit of electric current, the ampere, was named after him.

1.1 INTRODUCTION

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

> An electric circuit is an interconnection of electrical elements.

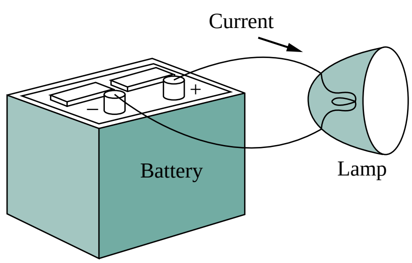

Figure 1.1 A simple electric circuit.

A simple electric circuit is shown in Fig. 1.1. It consists of three basic components: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a torch light, a search light, and so forth.

A complicated real circuit is displayed in Fig. 1.2, representing the schematic diagram for a radio receiver. Although it seems complicated, this circuit can be analyzed using the techniques we cover in this book. Our goal in this text is to learn various analytical techniques and computer software applications for describing the behavior of a circuit like this.

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

1.2 SYSTEMS OF UNITS

As electrical engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system,

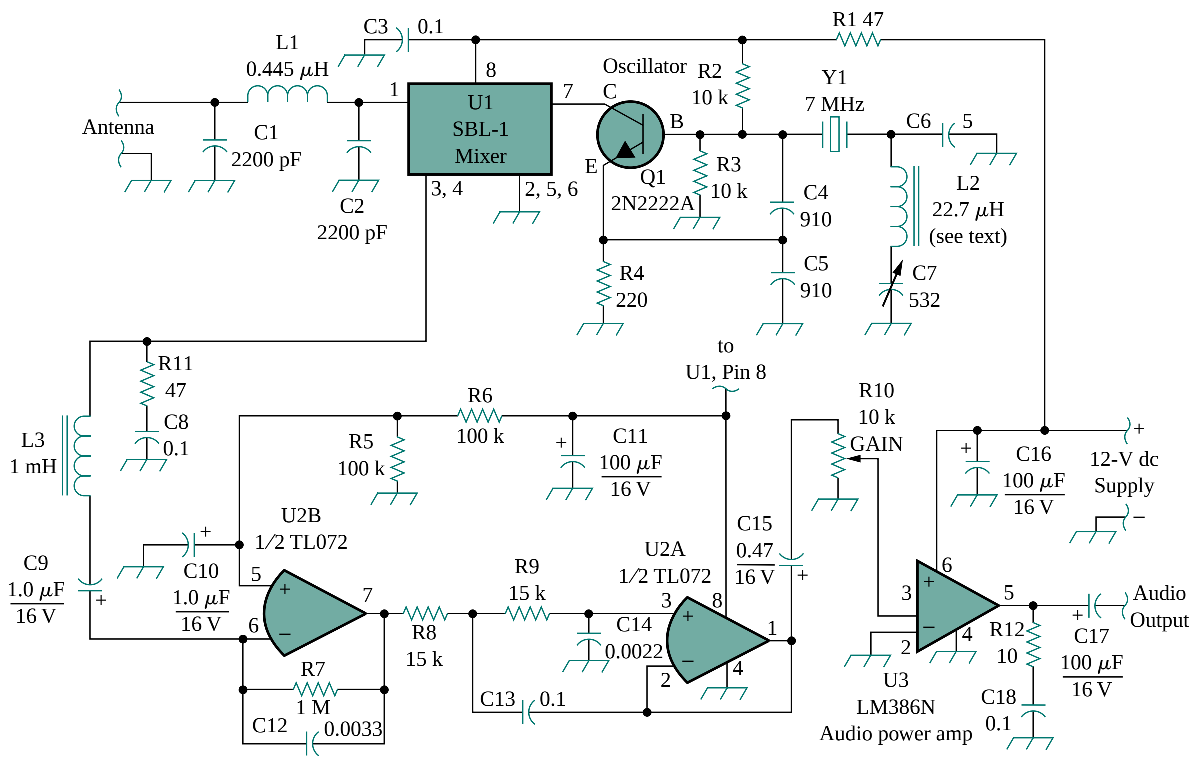

Figure 1.2 Electric circuit of a radio receiver.
(Reproduced with permission from QST, August 1995, p. 23.)

there are six principal units from which the units of all other physical quantities can be derived. Table 1.1 shows the six units, their symbols, and the physical quantities they represent. The SI units are used throughout this text.

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

$$600{,}000{,}000 \text{ mm} \qquad 600{,}000 \text{ m} \qquad 600 \text{ km}$$

TABLE 1.1 The six basic SI units.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

TABLE 1.2 The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 CHARGE AND CURRENT

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge.* We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

> **Charge** is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.[1]
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

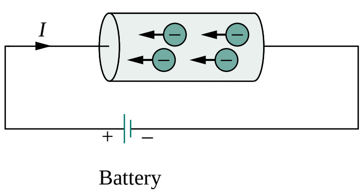

Figure 1.3 Electric current due to flow of electronic charge in a conductor.

A convention is a standard way of describing something so that others in the profession can understand what we mean. We will be using IEEE conventions throughout this book.

We now consider the flow of electric charges. A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges, that is, opposite to the flow of negative charges, as Fig. 1.3 illustrates. This convention was introduced by Benjamin Franklin (1706–1790), the American scientist and inventor. Although we now know that current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges. Thus,

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[1]However, a large power supply capacitor can store up to 0.5 C of charge.

Electric current is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i, charge q, and time t is

$$\boxed{i = \frac{dq}{dt}} \tag{1.1}$$

where current is measured in amperes (A), and

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$\boxed{q = \int_{t_0}^{t} i\,dt} \tag{1.2}$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways that may be represented by different kinds of mathematical functions.

If the current does not change with time, but remains constant, we call it a *direct current* (dc).

A **direct current** (dc) is a current that remains constant with time.

By convention the symbol I is used to represent such a constant current.

A time-varying current is represented by the symbol i. A common form of time-varying current is the sinusoidal current or *alternating current* (ac).

An **alternating current** (ac) is a current that varies sinusoidally with time.

Such current is used in your household, to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4 shows direct current and alternating current; these are the two most common types of current. We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of −5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of +5 A flowing in the opposite direction.

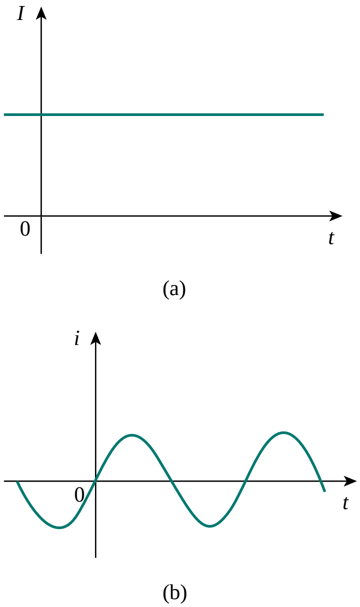


Figure 1.4 Two common types of current: (a) direct current (dc), (b) alternating current (ac).

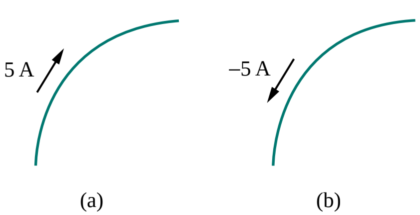


Figure 1.5 Conventional current flow: (a) positive current flow, (b) negative current flow.

EXAMPLE 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

PRACTICE PROBLEM 1.1

Calculate the amount of charge represented by two million protons.

Answer: $+3.204 \times 10^{-13}$ C.

EXAMPLE 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

PRACTICE PROBLEM 1.2

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$ s.

Answer: 7.36 mA.

EXAMPLE 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

PRACTICE PROBLEM 1.3

The current flowing through an element is

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 1 \\ 2t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 6.667 C.

1.4 VOLTAGE

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$v_{ab} = \frac{dw}{dq} \tag{1.3}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton meter/coulomb}$$

Thus,

> Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

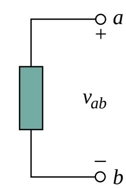


Figure 1.6 Polarity of voltage v_{ab}.

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (−) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) point a is at a potential of v_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is v_{ab}. It follows logically that in general

$$v_{ab} = -v_{ba} \tag{1.4}$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b; in Fig. 1.7(b), point b is −9 V above point a. We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a. In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a.

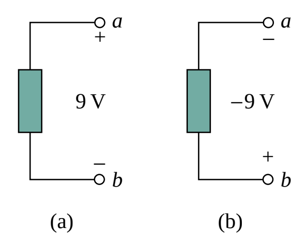


Figure 1.7 Two equivalent representations of the same voltage v_{ab}: (a) point a is 9 V above point b, (b) point b is −9 V above point a.

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V, whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v. A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

1.5 POWER AND ENERGY

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

> **Power** is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$\boxed{p = \frac{dw}{dt}} \tag{1.5}$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \tag{1.6}$$

or

$$\boxed{p = vi} \tag{1.7}$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a $+$ sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a $-$ sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention*. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

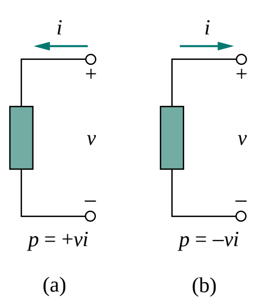


Figure 1.8 Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

> **Passive sign convention** is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of −12 W because a positive current enters the negative terminal. Of course, an absorbing power of +12 W is equivalent to a supplying power of −12 W. In general,

When the voltage and current directions conform to Fig. 1.8(b), we have the *active sign convention* and $p = +vi$.

$$\text{Power absorbed} = -\text{Power supplied}$$

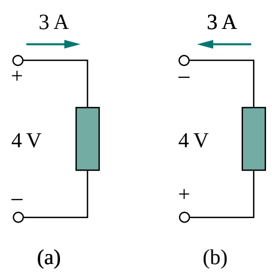


Figure 1.9 Two cases of an element with an absorbing power of 12 W:
(a) $p = 4 \times 3 = 12$ W,
(b) $p = 4 \times 3 = 12$ W.

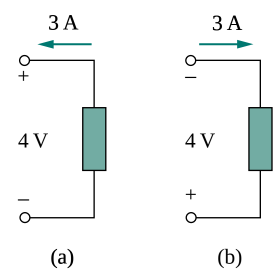


Figure 1.10 Two cases of an element with a supplying power of 12 W:
(a) $p = 4 \times (-3) = -12$ W,
(b) $p = 4 \times (-3) = -12$ W.

In fact, the *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\boxed{\sum p = 0} \tag{1.8}$$

This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.

From Eq. (1.6), the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt \tag{1.9}$$

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

EXAMPLE 1.4

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

PRACTICE PROBLEM 1.4

To move charge q from point a to point b requires -30 J. Find the voltage drop v_{ab} if: (a) $q = 2$ C, (b) $q = -6$ C .

Answer: (a) -15 V, (b) 5 V.

EXAMPLE 1.5

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:

(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75 \cos^2(60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms,

$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \end{aligned}$$

PRACTICE PROBLEM 1.5

Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V, (b) $v = \left(10 + 5\int_0^t i\,dt\right)$ V.

Answer: (a) 17.27 W, (b) 29.7 W.

EXAMPLE 1.6

How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$\begin{aligned} w = pt &= 100\ (\text{W}) \times 2\ (\text{h}) \times 60\ (\text{min/h}) \times 60\ (\text{s/min}) \\ &= 720{,}000\ \text{J} = 720\ \text{kJ} \end{aligned}$$

This is the same as

$$w = pt = 100\ \text{W} \times 2\ \text{h} = 200\ \text{Wh}$$

@ Network Analysis

PRACTICE PROBLEM 1.6

A stove element draws 15 A when connected to a 120-V line. How long does it take to consume 30 kJ?

Answer: 16.67 s.

1.6 CIRCUIT ELEMENTS

As we discussed in Section 1.1, an element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers. Our aim in this section is to gain familiarity with some important active elements.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

> An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1.11 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.11(a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.11(a) can be used for a time-varying voltage source. Similarly, an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to

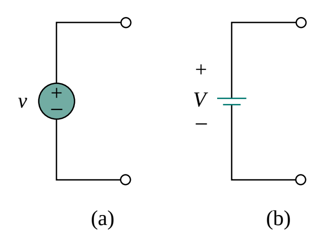


Figure 1.11 Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

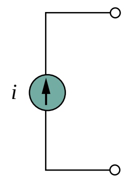


Figure 1.12 Symbol for independent current source.

maintain the designated current. The symbol for an independent current source is displayed in Fig. 1.12, where the arrow indicates the direction of current i.

> An **ideal dependent** (or controlled) **source** is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

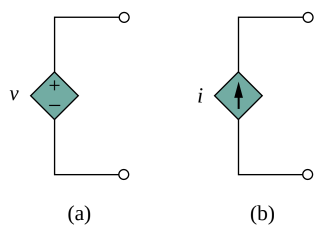


Figure 1.13 Symbols for: (a) dependent voltage source, (b) dependent current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.14, where the voltage $10i$ of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ −) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus an ideal source could in theory supply an infinite amount of energy. It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

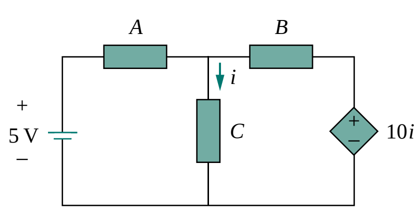


Figure 1.14 The source on the right-hand side is a current-controlled voltage source.

EXAMPLE 1.7

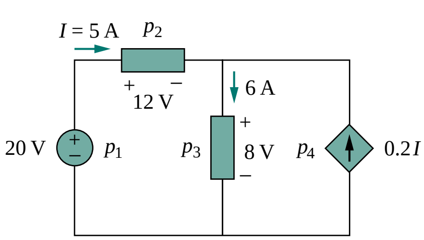


Figure 1.15 For Example 1.7.

Calculate the power supplied or absorbed by each element in Fig. 1.15.

Solution:

We apply the sign convention for power shown in Figs. 1.8 and 1.9. For p_1, the 5-A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100 \text{ W} \qquad \text{Supplied power}$$

For p_2 and p_3, the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60 \text{ W} \qquad \text{Absorbed power}$$
$$p_3 = 8(6) = 48 \text{ W} \qquad \text{Absorbed power}$$

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3, since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \qquad \text{Supplied power}$$

We should observe that the 20-V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the two passive elements are absorbing power. Also,

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

In agreement with Eq. (1.8), the total power supplied equals the total power absorbed.

PRACTICE PROBLEM 1.7

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

Answer: $p_1 = -40$ W, $p_2 = 16$ W, $p_3 = 9$ W, $p_4 = 15$ W.

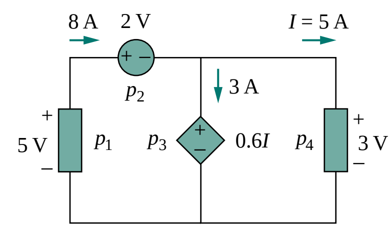


Figure 1.16 For Practice Prob. 1.7.

†1.7 APPLICATIONS[2]

In this section, we will consider two practical applications of the concepts developed in this chapter. The first one deals with the TV picture tube and the other with how electric utilities determine your electric bill.

1.7.1 TV Picture Tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver.[3] The CRT is depicted in

[2]The dagger sign preceding a section heading indicates a section that may be skipped, explained briefly, or assigned as homework.

[3]Modern TV tubes use a different technology.

Fig. 1.17. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun, maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus the beam can be made to "paint" a picture on the TV screen.

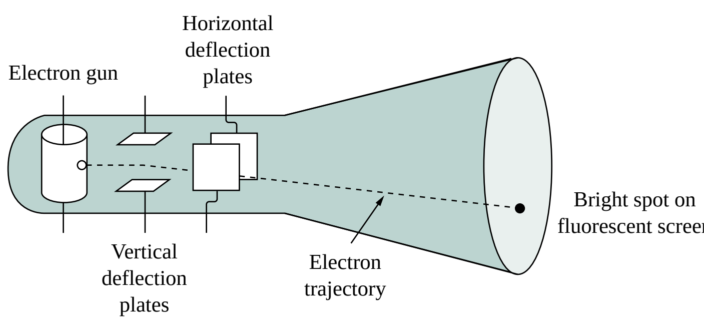


Figure 1.17 Cathode-ray tube.
(Source: D. E. Tilley, Contemporary College Physics [Menlo Park, CA: Benjamin/Cummings, 1979], p. 319.)

EXAMPLE 1.8

The electron beam in a TV picture tube carries 10^{15} electrons per second. As a design engineer, determine the voltage V_o needed to accelerate the electron beam to achieve 4 W.

Solution:

The charge on an electron is

$$e = -1.6 \times 10^{-19}\ \text{C}$$

If the number of electrons is n, then $q = ne$ and

$$i = \frac{dq}{dt} = e\frac{dn}{dt} = (-1.6 \times 10^{-19})(10^{15}) = -1.6 \times 10^{-4}\ \text{A}$$

The negative sign indicates that the electron flows in a direction opposite to electron flow as shown in Fig. 1.18, which is a simplified diagram of the CRT for the case when the vertical deflection plates carry no charge. The beam power is

$$p = V_o i \qquad \text{or} \qquad V_o = \frac{p}{i} = \frac{4}{1.6 \times 10^{-4}} = 25{,}000\ \text{V}$$

Thus the required voltage is 25 kV.

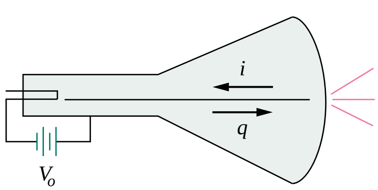


Figure 1.18 A simplified diagram of the cathode-ray tube; for Example 1.8.

PRACTICE PROBLEM 1.8

If an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 30 kV, calculate the power in the beam.

Answer: 48 mW.

1.7.2 Electricity Bills

The second application deals with how an electric utility company charges their customers. The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh). (Other factors that affect the cost include demand and power factors; we will ignore these for now.) However, even if a consumer uses no energy at all, there is a minimum service charge the customer must pay because it costs money to stay connected to the power line. As energy consumption increases, the cost per kWh drops. It is interesting to note the average monthly consumption of household appliances for a family of five, shown in Table 1.3.

TABLE 1.3 Typical average monthly consumption of household appliances.

Appliance	kWh consumed	Appliance	kWh consumed
Water heater	500	Washing machine	120
Freezer	100	Stove	100
Lighting	100	Dryer	80
Dishwasher	35	Microwave oven	25
Electric iron	15	Personal computer	12
TV	10	Radio	8
Toaster	4	Clock	2

EXAMPLE 1.9

A homeowner consumes 3,300 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

Base monthly charge of \$12.00.

First 100 kWh per month at 16 cents/kWh.

Next 200 kWh per month at 10 cents/kWh.

Over 200 kWh per month at 6 cents/kWh.

Solution:

We calculate the electricity bill as follows.

$$\begin{aligned}
\text{Base monthly charge} &= \$12.00\\
\text{First 100 kWh @ \$0.16/kWh} &= \$16.00\\
\text{Next 200 kWh @ \$0.10/kWh} &= \$20.00\\
\text{Remaining 100 kWh @ \$0.06/kWh} &= \$6.00\\
\text{Total Charge} &= \$54.00
\end{aligned}$$

$$\text{Average cost} = \frac{\$54}{100 + 200 + 100} = 13.5 \text{ cents/kWh}$$

PRACTICE PROBLEM 1.9

Referring to the residential rate schedule in Example 1.9, calculate the average cost per kWh if only 400 kWh are consumed in July when the family is on vacation most of the time.

Answer: 13.5 cents/kWh.

†1.8 PROBLEM SOLVING

Although the problems to be solved during one's career will vary in complexity and magnitude, the basic principles to be followed remain the same. The process outlined here is the one developed by the authors over many years of problem solving with students, for the solution of engineering problems in industry, and for problem solving in research.

We will list the steps simply and then elaborate on them.

1. Carefully **Define** the problem.
2. **Present** everything you know about the problem.
3. Establish a set of **Alternative** solutions and determine the one that promises the greatest likelihood of success.
4. **Attempt** a problem solution.
5. **Evaluate** the solution and check for accuracy.
6. Has the problem been solved **Satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.

1. *Carefully **Define** the problem*. This may be the most important part of the process, because it becomes the foundation for all the rest of the steps. In general, the presentation of engineering problems is somewhat incomplete. You must do all you can to make sure you understand the problem as thoroughly as the presenter of the problem understands it. Time spent at this point clearly identifying the problem will save you considerable time and frustration later. As a student, you can clarify a problem statement in a textbook by asking your professor to help you understand it better. A problem presented to you in industry may require that you consult several individuals. At this step, it is important to develop questions that need to be addressed before continuing the solution process. If you have such questions, you need to consult with the appropriate individuals or resources to obtain the answers to those questions. With those answers, you can now refine the problem, and use that refinement as the problem statement for the rest of the solution process.

2. ***Present** everything you know about the problem*. You are now ready to write down everything you know about the problem and its possible solutions. This important step will save you time and frustration later.

3. *Establish a set of* ***Alternative*** *solutions and determine the one that promises the greatest likelihood of success*. Almost every problem will have a number of possible paths that can lead to a solution. It is highly desirable to identify as many of those paths as possible. At this point, you also need to determine what tools are available to you, such as Matlab and other software packages that can greatly reduce effort and increase accuracy. Again, we want to stress that time spent carefully defining the problem and investigating alternative approaches to its solution will pay big dividends later. Evaluating the alternatives and determining which promises the greatest likelihood of success may be difficult but will be well worth the effort. Document this process well since you will want to come back to it if the first approach does not work.

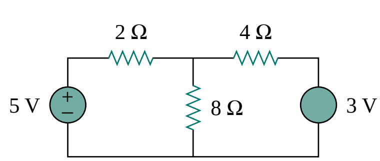
4. ***Attempt*** *a problem solution*. Now is the time to actually begin solving the problem. The process you follow must be well documented in order to present a detailed solution if successful, and to evaluate the process if you are not successful. This detailed evaluation may lead to corrections that can then lead to a successful solution. It can also lead to new alternatives to try. Many times, it is wise to fully set up a solution before putting numbers into equations. This will help in checking your results.

5. ***Evaluate*** *the solution and check for accuracy*. You now thoroughly evaluate what you have accomplished. Decide if you have an acceptable solution, one that you want to present to your team, boss, or professor.

6. *Has the problem been solved* ***Satisfactorily****? If so, present the solution; if not, then return to step 3 and continue through the process again.* Now you need to present your solution or try another alternative. At this point, presenting your solution may bring closure to the process. Often, however, presentation of a solution leads to further refinement of the problem definition, and the process continues. Following this process will eventually lead to a satisfactory conclusion.

Now let us look at this process for a student taking an electrical and computer engineering foundations course. (The basic process also applies to almost every engineering course.) Keep in mind that although the steps have been simplified to apply to academic types of problems, the process as stated always needs to be followed. We consider a simple example.

Assume that we have been given the following circuit. The instructor asks us to solve for the current flowing through the 8-ohm resistor.

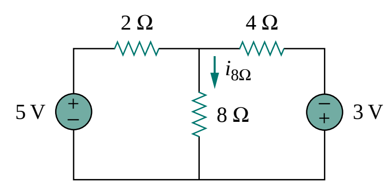


1. *Carefully* ***Define*** *the problem*. This is only a simple example, but we can already see that we do not know the polarity on the 3-V source. We have the following options. We can ask the professor what

the polarity should be. If we cannot ask, then we need to make a decision on what to do next. If we have time to work the problem both ways, we can solve for the current when the 3-V source is plus on top and then plus on the bottom. If we do not have the time to work it both ways, assume a polarity and then carefully document your decision. Let us assume that the professor tells us that the source is plus on the bottom.

2. ***Present*** *everything you know about the problem*. Presenting all that we know about the problem involves labeling the circuit clearly so that we define what we seek.

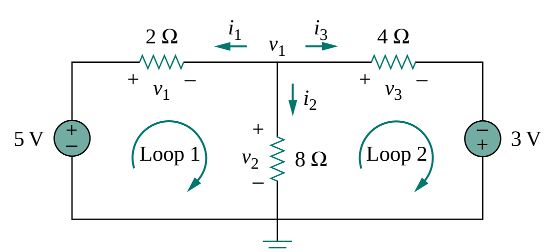
Given the following circuit, solve for $i_{8\Omega}$.



We now check with the professor, if reasonable, to see if the problem is properly defined.

3. *Establish a set of* ***Alternative*** *solutions and determine the one that promises the greatest likelihood of success*. There are essentially three techniques that can be used to solve this problem. Later in the text you will see that you can use circuit analysis (using Kirchoff's laws and Ohm's law), nodal analysis, and mesh analysis.

To solve for $i_{8\Omega}$ using circuit analysis will eventually lead to a solution, but it will likely take more work than either nodal or mesh analysis. To solve for $i_{8\Omega}$ using mesh analysis will require writing two simultaneous equations to find the two loop currents indicated in the following circuit. Using nodal analysis requires solving for only one unknown. This is the easiest approach.



Therefore, we will solve for $i_{8\Omega}$ using nodal analysis.

4. ***Attempt*** *a problem solution*. We first write down all of the equations we will need in order to find $i_{8\Omega}$.

$$i_{8\Omega} = i_2, \qquad i_2 = \frac{v_1}{8}, \qquad i_{8\Omega} = \frac{v_1}{8}$$

$$\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} = 0$$

Now we can solve for v_1.

$$8\left[\frac{v_1-5}{2}+\frac{v_1-0}{8}+\frac{v_1+3}{4}\right]=0$$

leads to $(4v_1-20)+(v_1)+(2v_1+6)=0$

$$7v_1=+14, \qquad v_1=+2\text{ V}, \qquad i_{8\Omega}=\frac{v_1}{8}=\frac{2}{8}=\underline{\mathbf{0.25\ A}}$$

5. ***Evaluate*** *the solution and check for accuracy*. We can now use Kirchoff's voltage law to check the results.

$$i_1=\frac{v_1-5}{2}=\frac{2-5}{2}=-\frac{3}{2}=-1.5\text{ A}$$

$$i_2=i_{8\Omega}=0.25\text{ A}$$

$$i_3=\frac{v_1+3}{4}=\frac{2+3}{4}=\frac{5}{4}=1.25\text{ A}$$

$$i_1+i_2+i_3=\underline{\mathbf{-1.5+0.25+1.25=0}} \qquad \text{(Checks.)}$$

Applying KVL to loop 1,

$$\begin{aligned}-5+v_1+v_2&=-5+(-i_1\times 2)+(i_2\times 8)\\&=-5+(-(-1.5)2)+(0.25\times 8)\\&=\underline{\mathbf{-5+3+2=0}} \qquad \text{(Checks.)}\end{aligned}$$

Applying KVL to loop 2,

$$\begin{aligned}-v_2+v_3-3&=-(i_2\times 8)+(i_3\times 4)-3\\&=-(0.25\times 8)+(1.25\times 4)-3\\&=\underline{\mathbf{-2+5-3=0}} \qquad \text{(Checks.)}\end{aligned}$$

So we now have a very high degree of confidence in the accuracy of our answer.

6. Has the problem been solved **Satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again. This problem has been solved satisfactorily.

The current through the 8-ohm resistor is 0.25 amp flowing down through the 8-ohm resistor.

1.9 SUMMARY

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the six principal units, the units of other physical quantities can be derived.
3. Current is the rate of charge flow.

$$i=\frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge through an element.

$$v = \frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p = \frac{dw}{dt} = vi$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
9. Two areas of application of the concepts covered in this chapter are the TV picture tube and electricity billing procedure.

REVIEW QUESTIONS

1.1 One millivolt is one millionth of a volt.
(a) True (b) False

1.2 The prefix *micro* stands for:
(a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}

1.3 The voltage 2,000,000 V can be expressed in powers of 10 as:
(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV

1.4 A charge of 2 C flowing past a given point each second is a current of 2 A.
(a) True (b) False

1.5 A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.
(a) True (b) False

1.6 The unit of current is:
(a) Coulomb (b) Ampere
(c) Volt (d) Joule

1.7 Voltage is measured in:
(a) Watts (b) Amperes
(c) Volts (d) Joules per second

1.8 The voltage across a 1.1 kW toaster that produces a current of 10 A is:
(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V

1.9 Which of these is not an electrical quantity?
(a) charge (b) time (c) voltage
(d) current (e) power

1.10 The dependent source in Fig. 1.19 is:
(a) voltage-controlled current source
(b) voltage-controlled voltage source
(c) current-controlled voltage source
(d) current-controlled current source

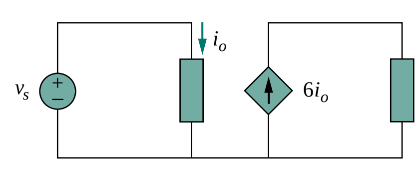


Figure 1.19 For Review Question 1.10.

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5a, 1.6b, 1.7c, 1.8c, 1.9b, 1.10d.

PROBLEMS

Section 1.3 Charge and Current

1.1 How many coulombs are represented by these amounts of electrons:

(a) 6.482×10^{17} (b) 1.24×10^{18}
(c) 2.46×10^{19} (d) 1.628×10^{20}

1.2 Find the current flowing through an element if the charge flow is given by:

(a) $q(t) = (t + 2)$ mC
(b) $q(t) = (5t^2 + 4t - 3)$ C
(c) $q(t) = 10e^{-4t}$ pC
(d) $q(t) = 20 \cos 50\pi t$ nC
(e) $q(t) = 5e^{-2t} \sin 100t$ μC

1.3 Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3$ A, $q(0) = 1$ C
(b) $i(t) = (2t + 5)$ mA, $q(0) = 0$
(c) $i(t) = 20 \cos(10t + \pi/6)$ μA, $q(0) = 2$ μC
(d) $i(t) = 10e^{-30t} \sin 40t$ A, $q(0) = 0$

1.4 The current flowing through a device is $i(t) = 5 \sin 6\pi t$ A. Calculate the total charge flow through the device from $t = 0$ to $t = 10$ ms.

1.5 Determine the total charge flowing into an element for $0 < t < 2$ s when the current entering its positive terminal is $i(t) = e^{-2t}$ mA.

1.6 The charge entering a certain element is shown in Fig. 1.20. Find the current at:

(a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

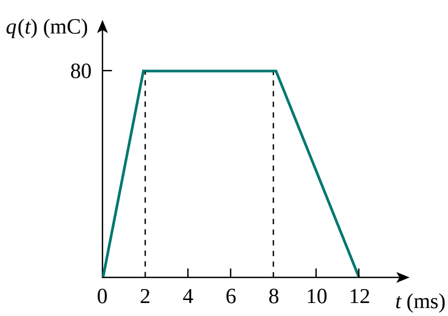


Figure 1.20 For Prob. 1.6.

1.7 The charge flowing in a wire is plotted in Fig. 1.21. Sketch the corresponding current.

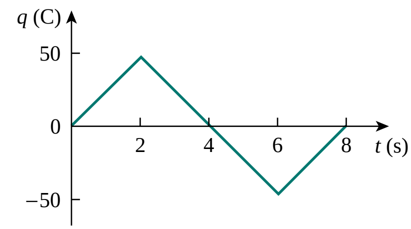


Figure 1.21 For Prob. 1.7.

1.8 The current flowing past a point in a device is shown in Fig. 1.22. Calculate the total charge through the point.

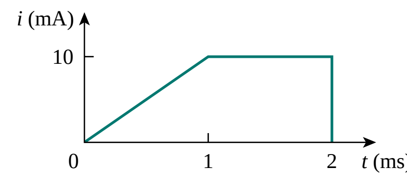


Figure 1.22 For Prob. 1.8.

1.9 The current through an element is shown in Fig. 1.23. Determine the total charge that passed through the element at:

(a) $t = 1$ s (b) $t = 3$ s (c) $t = 5$ s

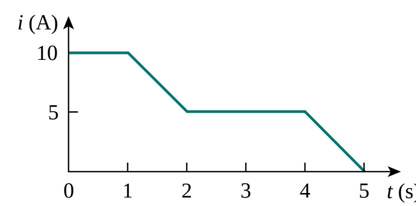


Figure 1.23 For Prob. 1.9.

Sections 1.4 and 1.5 Voltage, Power, and Energy

1.10 A certain electrical element draws the current $i(t) = 10 \cos 4t$ A at a voltage $v(t) = 120 \cos 4t$ V. Find the energy absorbed by the element in 2 s.

1.11 The voltage v across a device and the current i through it are

$$v(t) = 5 \cos 2t \text{ V}, \qquad i(t) = 10(1 - e^{-0.5t}) \text{ A}$$

Calculate:

(a) the total charge in the device at $t = 1$ s
(b) the power consumed by the device at $t = 1$ s.

1.12 The current entering the positive terminal of a device is $i(t) = 3e^{-2t}$ A and the voltage across the device is $v(t) = 5\,di/dt$ V.

(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

1.13 Figure 1.24 shows the current through and the voltage across a device. Find the total energy absorbed by the device for the period of $0 < t < 4$ s.

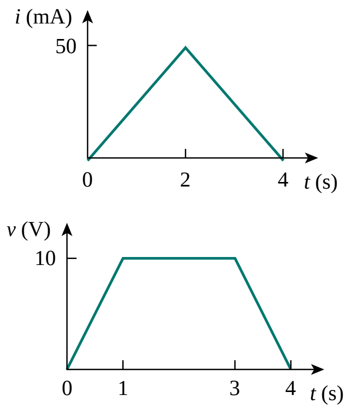


Figure 1.24 For Prob. 1.13.

Section 1.6 Circuit Elements

1.14 Figure 1.25 shows a circuit with five elements. If $p_1 = -205$ W, $p_2 = 60$ W, $p_4 = 45$ W, $p_5 = 30$ W, calculate the power p_3 received or delivered by element 3.

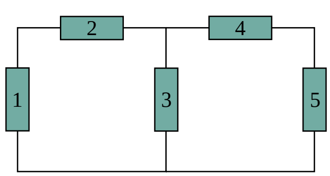


Figure 1.25 For Prob. 1.14.

1.15 Find the power absorbed by each of the elements in Fig. 1.26.

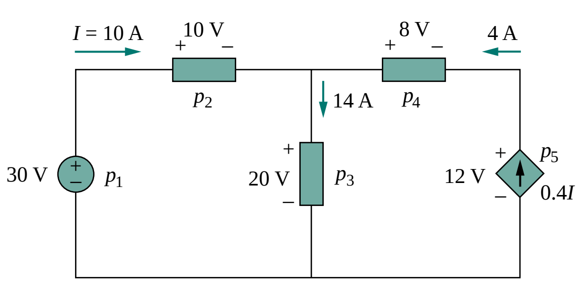


Figure 1.26 For Prob. 1.15.

1.16 Determine I_o in the circuit of Fig. 1.27.

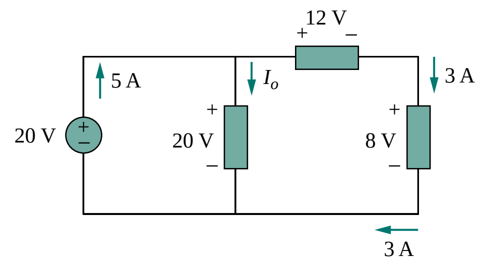


Figure 1.27 For Prob. 1.16.

1.17 Find V_o in the circuit of Fig. 1.28.

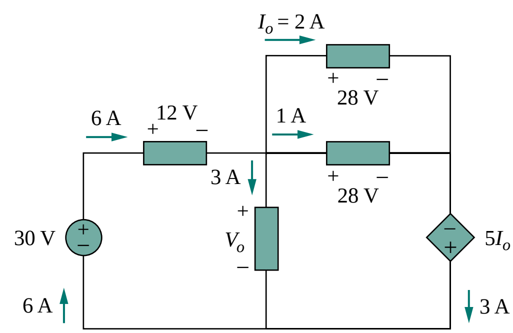


Figure 1.28 For Prob. 1.17.

Section 1.7 Applications

1.18 It takes eight photons to strike the surface of a photodetector in order to emit one electron. If 4×10^{11} photons/second strike the surface of the photodetector, calculate the amount of current flow.

1.19 Find the power rating of the following electrical appliances in your household:

(a) Lightbulb (b) Radio set
(c) TV set (d) Refrigerator
(e) Personal computer (f) PC printer
(g) Microwave oven (h) Blender

1.20 A 1.5-kW electric heater is connected to a 120-V source.

(a) How much current does the heater draw?

(b) If the heater is on for 45 minutes, how much energy is consumed in kilowatt-hours (kWh)?

(c) Calculate the cost of operating the heater for 45 minutes if energy costs 10 cents/kWh.

1.21 A 1.2-kW toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 9 cents/kWh.

1.22 A flashlight battery has a rating of 0.8 ampere-hours (Ah) and a lifetime of 10 hours.
(a) How much current can it deliver?
(b) How much power can it give if its terminal voltage is 6 V?
(c) How much energy is stored in the battery in kWh?

1.23 A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$ V, where t is in hours,
(a) how much charge is transported as a result of the charging?
(b) how much energy is expended?
(c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

1.24 A 30-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:
(a) the current through the lamp,
(b) the cost of operating the light for one non-leap year if electricity costs 12 cents per kWh.

1.25 An electric stove with four burners and an oven is used in preparing a meal as follows.
Burner 1: 20 minutes Burner 2: 40 minutes
Burner 3: 15 minutes Burner 4: 45 minutes
Oven: 30 minutes
If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

1.26 PECO (the electric power company in Philadelphia) charged a consumer $34.24 one month for using 215 kWh. If the basic service charge is $5.10, how much did PECO charge per kWh?

COMPREHENSIVE PROBLEMS

1.27 A telephone wire has a current of 20 μA flowing through it. How long does it take for a charge of 15 C to pass through the wire?

1.28 A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

1.29 The power consumption for a certain household for a day is shown in Fig. 1.29. Determine:
(a) the total energy consumed in kWh
(b) the average power per hour.

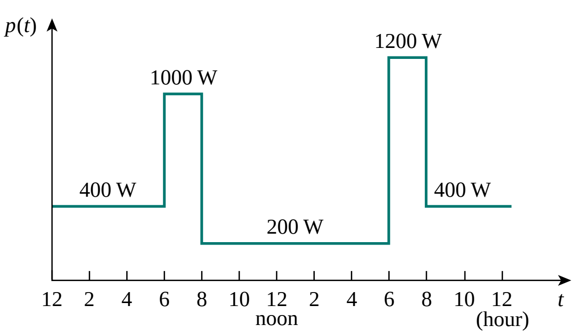


Figure 1.29 For Prob. 1.29.

1.30 The graph in Fig. 1.30 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

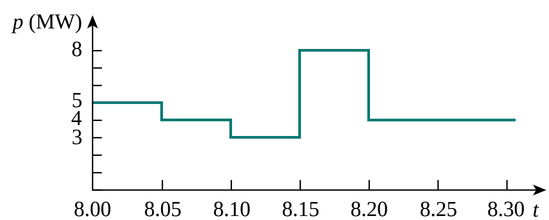


Figure 1.30 For Prob. 1.30.

1.31 A battery may be rated in ampere-hours (Ah). An lead-acid battery is rated at 160 Ah.
(a) What is the maximum current it can supply for 40 h?
(b) How many days will it last if it is discharged at 1 mA?

1.32 How much work is done by a 12-V automobile battery in moving 5×10^{20} electrons from the positive terminal to the negative terminal?

1.33 How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

1.34 A 2-kW electric iron is connected to a 120-V line. Calculate the current drawn by the iron.