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# قاعدة لوبيتال "L'hopital Rule"

$\Rightarrow$ If $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\dfrac{\infty}{\infty}$

then $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{\bar{f}(x)}{\bar{g}(x)}$

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**EX.1:** $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x}$

"الحل"

$$\lim_{x\to 0}\frac{1-\cos x}{x} = \frac{0}{0}$$

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$$= \lim_{x\to 0}\frac{\sin x}{1} = \sin(0) = \boxed{0}$$

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**EX.2:** $\displaystyle\lim_{x\to 0}\frac{e^{x}-1}{x} = \frac{1-1}{0} = \frac{0}{0}$

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$$\therefore \lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \frac{e^x}{1} = e^0 = \boxed{1}$$

3 $$\lim_{x\to -2} \frac{x+2}{x^2+3x+2} = \frac{0}{0}$$

$$= \lim_{x\to -2} \frac{1}{2x+3} = \frac{1}{2(-2)+3} = \boxed{-1}$$

4 $$\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{1-\sin x} = \frac{0}{1-1} = \frac{0}{0}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{\cancel{-}\sin x}{\cancel{-}\cos x} = \frac{+1}{0} = \boxed{\infty}$$

5 $$\lim_{x\to 0} \frac{x+\tan x}{\sin x} = \frac{0+0}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{1+\sec^2 x}{\cos x} = \frac{1+1}{1} = \boxed{2}$$



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* **Case A :** ( الحالة الخاصة الأولى حالة الضرب )

$$\lim_{x \to a} f(x)\, g(x) = 0 \cdot \infty \quad \text{or} \quad 0 \cdot -\infty$$

$$\Rightarrow \lim_{x \to a} \frac{f(x)}{1/g(x)} = \frac{0}{\frac{1}{\infty}} = \frac{0}{0}$$

$$\Rightarrow \lim_{x \to a} \frac{g(x)}{\frac{1}{f(x)}} = \frac{\infty}{\frac{1}{0}} = \frac{\infty}{\infty}$$

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**EX.1 :** $\lim_{x \to 0} \sqrt{x}\, \ln x = \sqrt{0}\, \ln(0) = 0 \cdot -\infty$

$$= \lim_{x \to 0} \frac{\ln x}{\frac{1}{\sqrt{x}}} = \frac{\ln(0)}{\frac{1}{\sqrt{0}}} = \frac{-\infty}{\infty}$$

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$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{2} x^{-3/2}}$$

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$$= \lim_{x \to 0} \frac{1}{-\frac{1}{2} x^{-\frac{1}{2}}} = -2 \lim_{x \to 0} \sqrt{x} = \boxed{0}$$

[2] $\lim_{x\to-\infty} x^2 e^{x} = (-\infty)^2 e^{-\infty} = \infty \cdot 0$

$= \lim_{x\to-\infty} \dfrac{x^2}{e^{-x}} = \dfrac{\infty}{\infty}$

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$= \lim_{x\to-\infty} \dfrac{2x}{-e^{-x}} = \dfrac{-\infty}{-\infty} = \dfrac{\infty}{\infty}$

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$= \lim_{x\to-\infty} \dfrac{2}{e^{-x}} = \dfrac{2}{e^{\infty}} = \dfrac{2}{\infty} = \boxed{0}$

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[3] $\lim_{x\to\infty} x \tan\left(\frac{1}{x}\right) = \infty \tan\left(\frac{1}{\infty}\right) = \infty \cdot 0$

$= \lim_{x\to\infty} \dfrac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} = \dfrac{0}{0}$

$= \lim_{x\to\infty} \dfrac{\sec^2\left(\frac{1}{x}\right) \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$

$= \sec^2\left(\frac{1}{\infty}\right) = \sec^2(0) = \boxed{1}$

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|---|---|---|---|---|---|
| يفضلوا فوق ← | $\sin(1/x)$ | $\sin^{-1}x$ | $\log_3(x)$ | $\ln(x)$ | البسط |
| ينزلوا تحت ← | | $\sin x$ | $3^x$ | $e^x$ | المقام |

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$$= \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x\cos x} = \frac{1-1}{0+0} = \frac{0}{0}$$

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$$= \lim_{x\to 0} \frac{-\sin x}{\cos x + [\cos x - x\sin x]} =$$

$$= \frac{0}{1+1-0} = \frac{0}{2} = 0$$

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[ الحالة الخاصة الثالثة $[f(x)]^{g(x)}$ الأسية ]

* **Case C:**

$$y = \lim_{x\to 0} [f(x)]^{g(x)} = 1^{\infty} \text{ or } 0^{0} \text{ or } \infty^{0}$$

$$\ln y = \lim_{x\to 0} g(x) \ln(f(x))$$

$$= \lim_{x\to 0} \frac{\ln[f(x)]}{\frac{1}{g(x)}} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

باستخدام لوبيتال

$\boxed{1}$ $\lim_{x \to 0} x^{x^2} = 0^0$ 

$$y = \lim_{x \to 0} x^{x^2}$$

$$\ln y = \lim_{x \to 0} \ln (x)^{x^2}$$

$$\ln y = \lim_{x \to 0} x^2 \ln(x) = 0 \cdot \ln(0)$$
$$= 0 \cdot -\infty$$

$$\ln y = \lim_{x \to 0} \frac{\ln(x)}{1/x^2} = \lim_{x \to 0} \frac{\ln(x)}{x^{-2}}$$
$$= \frac{\ln(0)}{1/0} = \frac{-\infty}{\infty}$$

باستخدام قاعدة لوبيتال

$$\ln(y) = \lim_{x \to 0} \frac{1/x}{-2x^{-3}} = \lim_{x \to 0} \frac{x^3}{-2x}$$

$$\ln(y) = \lim_{x \to 0} \frac{x^2}{-2} = \frac{0}{-2} = 0$$

$$\therefore y = e^0 = \boxed{1}$$

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