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Suez Canal University			Course: Electromagnetic fields	
Department of Electrical Engineering			Lecturer: Dr. Ahmed Magdy	
Third Year	Midterm Exam		August 2019	
Total marks [20]		Time allowed : 1 hour	1 page	ELC 214

Answer all the following questions :

Summer 2019

Question (1):

a) A coaxial cable has two concentric cylinders of radii 4 cm (a) and 6 cm (b). It is located along the z - axis. If $\rho_v = 0$, $\varepsilon = 4\varepsilon_0$ between the cylinders, V = 0 at r = 4 cm and V = 400 V at r = 6 cm. Solve Laplace's equation to find :

1) The potential function.
2) The electric field intensity.
3) The surface charge density on both conductors.
4) The capacitance of the system.

b) A uniform line of charge, ρ_l=20 nC/m, is located at x=1 m, z=4 m and a uniform sheet of charge, ρ_s=20 nC/m^2, is presented at x=3 m in free space.

1) Find the magnitude of the electric field intensity at the origin.
2) Find the direction of the electric field intensity at P (4,5,6).
3) What is the force per meter length on the line charge.

Sheet 2 $\boxed{3}$

Question (2):

a) Two concentric cylindrical conductors of radius a = 0.02 m and b = 0.05 m. The inner cylinder has a charge density 40 nc/m^2 while the outer cylinder has ρ_2 such that the electric field exists between the two cylinders but they are zero elsewhere.

1) Find the surface charge density ρ_2,
2) Derive expressions for the field strength in all regions,
3) Derive expressions for the electric flux density in all regions,
4) Find the potential difference between the two cylinders,
5) Find the energy stored in the system per unit length,
6) Draw the distribution of the electric field intensity versus r in all regions if uniform line charge of density ρ_L nc/m is located on the axis of the two cylindrical conductors.

b) A 5 cm radius copper has a capacitance C_1. A uniform dielectric layer of thickness d is placed on the sphere if $\varepsilon_r = 3$, determine d such that the new capacitance is 2 C_1.

with all my best wishes

a $\quad \ln(b/a)$

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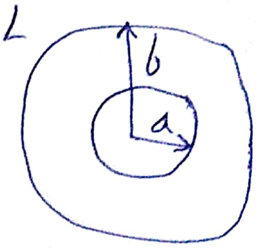
$\boxed{6}$ $a = 4\,cm \;,\; b = 6\,cm$

$\varepsilon = 4\varepsilon_0 \;,\; \rho_v = 0$

Coaxial



$V = 0$ at $r = 4\,cm \;,\; \dfrac{\partial V}{\partial r} = -400\,V/m$ at $r = 6\,cm$

(a) $V(r) = ??$

answer:

$$\nabla^2 V = 0 \Rightarrow \cancel{\frac{1}{r}}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

$$\therefore\; r\frac{\partial V}{\partial r} = A \Rightarrow \frac{\partial V}{\partial r} = \frac{A}{r}$$

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$$\therefore \left.\frac{\partial V}{\partial r}\right|_{r=6cm} = -400$$

$$\therefore \frac{A}{6\times10^{-2}} = -400 \Rightarrow \boxed{A = -24}$$

$$\therefore \frac{\partial V}{\partial r} = \frac{-24}{r}$$

$$\therefore V(r) = -24\ln(r) + B$$

$$\because V(0.04) = 0$$

$$\therefore 0 = -24\ln(0.04) + B \Rightarrow \boxed{B = -77.25}$$

$$\therefore \boxed{V(r) = -24\ln(r) - 77.25}\ V$$

(b) $$\bar{E}(r) = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r = \boxed{\frac{24}{r}\bar{a}_r}\ V/m$$

(c) $$\bar{D}(r) = \varepsilon\bar{E} = 4\varepsilon_0\bar{E} = \boxed{\frac{96\varepsilon_0}{r}\bar{a}_r}\ C/m^2$$

(D) $$\rho_{sa} = |D|_{r=a} = \boxed{\frac{96\varepsilon_0}{0.04}}\ C/m^2$$

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$$\rho_{sb} = |D|_{r=b} = \boxed{\frac{-96\,\varepsilon_0}{0.08}}\ C/m^2$$

(e) $\frac{C}{l} = ??$ $\quad\because C = \frac{2\pi\varepsilon l}{\ln(b/a)}$

$$\therefore \frac{C}{l} = \frac{8\pi\varepsilon_0}{\ln(b/a)} = \boxed{\frac{8\pi\varepsilon_0}{\ln\left(\frac{6}{4}\right)}}\ F/m$$

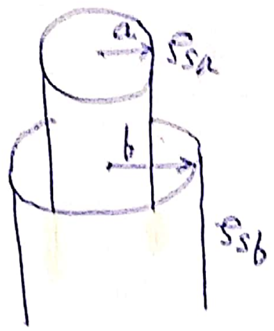
$\boxed{2-a}$ $a = 0.02\,m$ $b = 0.05\,m$

$\rho_{sa} = 40\,nc/m^2$ $\rho_{sb} = ??$

The field exist between the two cylinders, zero elsewhere.

(1) $\rho_2 = ??$

$\because Q_t = \text{zero}$



$\therefore \rho_{sa}(2\pi a L) + \rho_2(2\pi b L) = 0$

$\therefore \rho_{sa}(a) + \rho_2 b = 0$

$\therefore \rho_2 = \dfrac{-\rho_{sa}(a)}{b} = \dfrac{-40 * 0.02}{0.05}$

$= \boxed{-16\,nc/m^2}$

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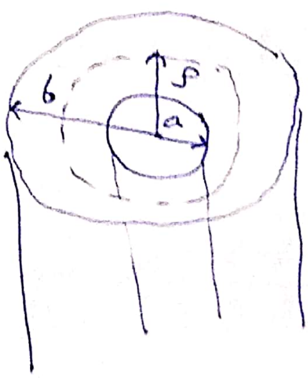
(2) $\overline{E}$ in all regions :

(i) $\rho < a$: $\boxed{\overline{E} = 0}$

(ii) $a < \rho < b$:



$\oint\!\!\oint \overline{D} \cdot \overline{ds} = Q_{enc}$

$D_\rho(2\pi \rho L) = \rho_{sa}(2\pi a L)$

$D_\rho = \dfrac{\rho_{sa}\, a}{\rho}$

$$\therefore \bar{D} = \frac{\rho_{sa}\, a}{\rho}\, \bar{a}_\rho \qquad \therefore \bar{E} = \frac{\rho_{sa}\, a}{\varepsilon_0\, \rho}\, \bar{a}_\rho$$

(iii) $\rho > b$: $\boxed{\bar{E} = 0}$ $\qquad \boxed{\bar{E} = \frac{90.4}{\rho}\, \bar{a}_\rho}$ V/m

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[3]

$$\bar{D} = \begin{cases} 0 & \rho < a \\ \dfrac{\rho_{sa}\, a}{\rho}\, \bar{a}_\rho \ \text{C/m}^2 & a < \rho < b \\ 0 & \rho > b \end{cases}$$

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(4)

$$V_{ab} = -\int_b^a \bar{E} \cdot d\bar{L}$$

$$= -\int_{0.05}^{0.02} \frac{90.4}{\rho}\, d\rho = -90.4 \ln(\rho) \Big|_{0.05}^{0.02}$$

$$= -90.4 \ln\left(\frac{0.02}{0.05}\right) = \boxed{82.83\,\text{V}}$$

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(5)

$$W_E = \frac{\varepsilon_0}{2} \iiint |E|^2\, dV$$

$$= \frac{\varepsilon_0}{2} \int_0^1 \int_0^{2\pi} \int_{0.02}^{0.05} \left(\frac{90.4}{\rho}\right)^2 \rho\, d\rho\, d\phi\, dz$$

$$= \frac{8.85 \times 10^{-12}}{2} * (90.4)^2 \times 2\pi \ln\left(\frac{0.05}{0.02}\right)$$

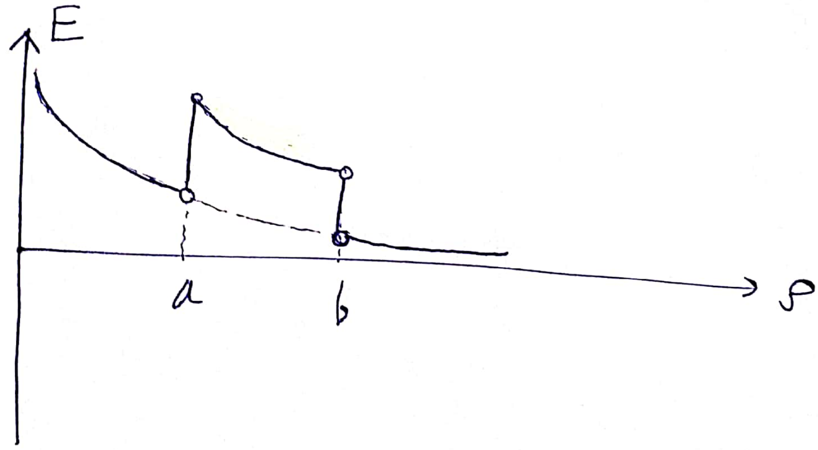
$$\boxed{W_E = 2.06 \times 10^{-7}\ J}$$

$\underline{\underline{10}}$

(b) $\bar{E}_T = \bar{E}_L + \bar{E}|_{\text{cylinders}}$

$$\because \bar{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\bar{a}_\rho$$

$$\therefore \bar{E}_T = \begin{cases} \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\bar{a}_\rho & \rho < a \\ \left(\dfrac{\rho_{sa}\, a}{\varepsilon_0 \rho} + \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\right)\bar{a}_\rho & a < \rho < b \\ \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\bar{a}_\rho & \rho > b \end{cases}$$



3 $a = 5\,cm$ $C = C_1$

$\varepsilon_r = 3$ $d = ??$ $C = 2C_1$

answer

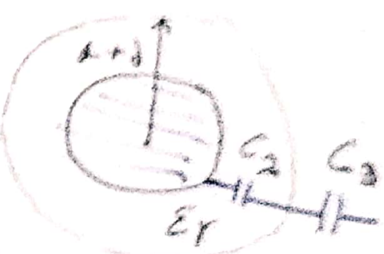
* For isolated sphere:

$$C_1 = 4\pi\varepsilon_0 a = 4\pi\varepsilon_0 \times 5\times10^{-2} = 0.2\pi\varepsilon_0$$

* with dielectric:

$$C = 2C_1 = 2\times0.2\pi\varepsilon_0 = 0.4\pi\varepsilon_0$$

$$\frac{1}{C} = \frac{1}{C_2} + \frac{1}{C_3}$$



$$\frac{1}{0.4\pi\varepsilon_0} = \frac{1}{\dfrac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{a}-\frac{1}{a+d}}} + \frac{1}{\dfrac{4\pi\varepsilon_0}{\frac{1}{a+d}-\frac{1}{\infty}}}$$

$$\frac{1}{0.4} = \frac{\frac{1}{0.05}-\frac{1}{0.05+d}}{4\times3} + \frac{1}{4(0.05+d)}$$

$$10 = \frac{1}{3}\left[\frac{1}{0.05} - \frac{1}{0.05+d}\right] + \frac{1}{0.05+d}$$

$$\therefore d = 0.15\,m = \boxed{15\,cm}$$

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