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Suez Canal University		Course: Electromagnetic fields	
Department of Electrical Engineering		Lecturer: Dr. Ahmed Magdy	
Third Year	Midterm Exam		November 2018	
Total marks [20]	Time allowed : 1 hour	1 page	ELC 214

Answer two Questions only from the following questions :

Question (1):

a) If $\bar{E} = 25\,y^3\,\bar{a}_x + 125\,x\,y^2\,\bar{a}_y$, find **(1)** |E| at P(1, 2, -4); **(2)** a unit vector in the direction of E at P; **(3)** the equation of the direction line passing through P; **(4)** a unit vector (a,b,0) that is perpendicular to part (2) at P and has b>0.

b) A finite filament having length L meter, the charge is distributed uniformly on this line charge with charge density ρ_L. The electric field strength around the central portion of the filament is to be compared with the field of a filament of infinite length. At what distance from the filament will the actual field be within 70% and 99% of the field of infinite filament? (1-b) sheet 2 No $\boxed{20}$

Question (2): Three concentric spherical surfaces having radii a, b and c in free space and carrying uniform charge $Q_a = 2$ nC, $Q_b = -1$ nC and $Q_c = 1$ nC respectively. Assume a = 1 cm, b = 2 cm and c = 3 cm.

a) Find the electric field strength and the electric flux density in all regions, sketch these fields versus r.

b) Find the absolute potential of each sphere.

c) Determine the stored energy in the system.

d) Find the points at which the electric field intensity equals to zero if a negative point charge Q = -1 nC is located at the center of the spheres.

Question (3):

a) What are the properties of the electric flux lines under the static electric field conditions ?

b) Given that $\bar{D} = 30\,e^{-\rho}\,\bar{a}_\rho - 2\,z\,\bar{a}_z$ in cylindrical coordinates.

1) Evaluate both sides of the divergence theorem for the volume enclosed by $\rho = 2$. Z = 0 and z = 5 m.

2) Find volume charge density at $\rho = 1.5$ m.

3) If an infinite line charge with uniform charge density of 20 nC/m is located on the cylindrical axis given in part (1), find the electric field intensity in the point P (3, 60^0, 2) in cylindrical coordinates.

with all my best wishes

Midterm Nov. 2018

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(a) $\bar{E} = 25 y^3 \bar{a}_x + 125 x y^2 \bar{a}_y$

(1) $|E|$ at $P(1,2,-4)$

$$\bar{E}\Big|_{P(1,2,-4)} = 200\,\bar{a}_x + 500\,\bar{a}_y$$

(2) Unit vector

$$\bar{a}_E = \frac{\bar{E}}{|E|} = 0.371\,\bar{a}_x + 0.928\,\bar{a}_y$$

(3) equation of field lines

$$\frac{dy}{dx} = \frac{125 x y^2}{25 y^3} = \frac{5x}{y}$$

$$\therefore \int y\,dy = \int 5x\,dx$$

$$y^2/2 = 5x^2/2 + c \quad \text{at } P(1,2,-4)$$

$$c = 0.5$$

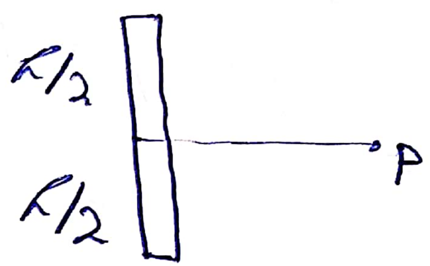
$$\therefore \boxed{0.5 y^2 = 2.5 x^2 + 0.5} \Rightarrow \boxed{y^2 = 5x^2 + 1}$$

(4) $b>0$: $\bar{a}_\perp \cdot \bar{a}_E = 0$

$$\boxed{\bar{a}_\perp = -0.928\,\bar{a}_x + 0.371\,\bar{a}_y}$$

* For finite line of length (L):

$$\bar{E} = \frac{2\rho_L \rho}{4\pi\varepsilon_0} \left. \frac{z}{\rho^2\sqrt{\rho^2+z^2}} \right|_0^{l/2}$$



$$= \frac{2\rho_L \rho}{4\pi\varepsilon_0} \frac{l/2}{\rho^2\sqrt{\rho^2+(l/2)^2}} \bar{a}_\rho$$

$$= \boxed{\frac{\rho_L \rho}{2\pi\varepsilon_0 \rho} \left[\frac{l/2}{\sqrt{\rho^2+(l/2)^2}}\right] \bar{a}_\rho}$$

Finite line

$\boxed{20}$ $E|_{finite} = 90\% \; E|_{infinite}$ 9

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$$\therefore \frac{\cancel{\rho_L}}{\cancel{2\pi\varepsilon_0\rho}} \frac{(\ell/2)}{\sqrt{\rho^2+(\ell/2)^2}} = 0.9 \frac{\cancel{\rho_L}}{\cancel{2\pi\varepsilon_0\rho}}$$

$$\therefore \frac{(\ell/2)}{\sqrt{\rho^2+(\ell/2)^2}} = 0.9$$

$$\therefore \frac{\ell}{2} = 0.9\sqrt{\rho^2+(\ell/2)^2}$$

$$\therefore \ell = 1.8\sqrt{\rho^2+(\ell/2)^2}$$

$$\therefore \left(\frac{\ell}{1.8}\right)^2 = \rho^2+(\ell/2)^2$$

$$\therefore \rho = \sqrt{\left(\frac{\ell}{1.8}\right)^2-(\ell/2)^2}$$

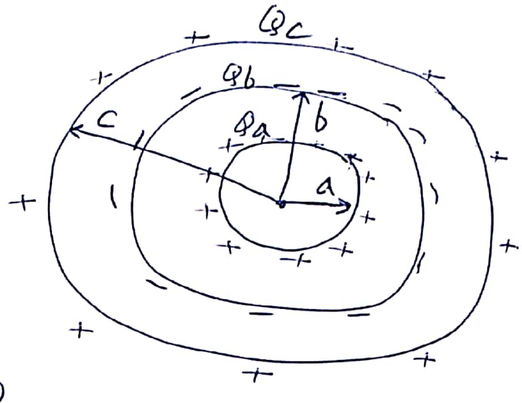
$$\therefore \rho = \ell\sqrt{\left(\frac{1}{1.8}\right)^2-\left(\frac{1}{2}\right)^2}$$

$$\boxed{\rho = 0.242\,L}$$

$\boxed{2}$ $a = 1\ cm$ $\quad Q_a = 2\,nC$

$b = 2\ cm$ $\quad Q_b = -1\,nC$

$c = 3\ cm$ $\quad Q_c = 1\,nC$



(a) Find $\bar{D}$, $\bar{E}$

(i) $r < a$:

$$\oiint \bar{D} \cdot \bar{ds} = Q_{enc} = 0$$

$$\therefore \bar{D}_1 = \bar{E}_1 = \boxed{0}$$

(ii) $a < r < b$:

$$\oiint \bar{D} \cdot \bar{ds} = Q_a$$

$$D_r (4\pi r^2) = 2 \times 10^{-9}$$

$$\therefore \boxed{\bar{D}_2 = \frac{2\times 10^{-9}}{4\pi r^2}\,\bar{a}_r} \quad C/m^2$$

$$\boxed{\bar{E}_2 = \frac{2\times 10^{-9}}{4\pi \varepsilon_0 r^2}\,\bar{a}_r} \quad V/m$$

(iii) $b < r < c$:

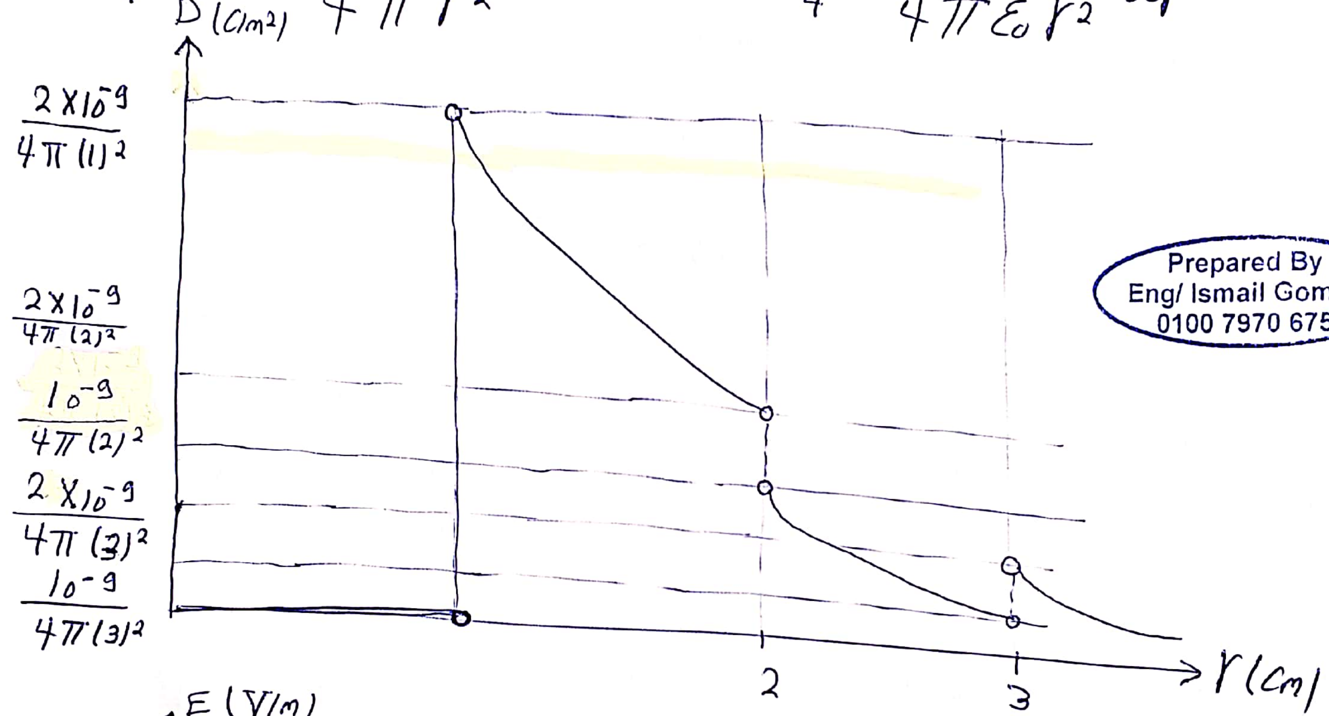
$$\oint\!\!\oint \bar{D} \cdot \bar{ds} = Q_a + Q_b = 1\,nC$$

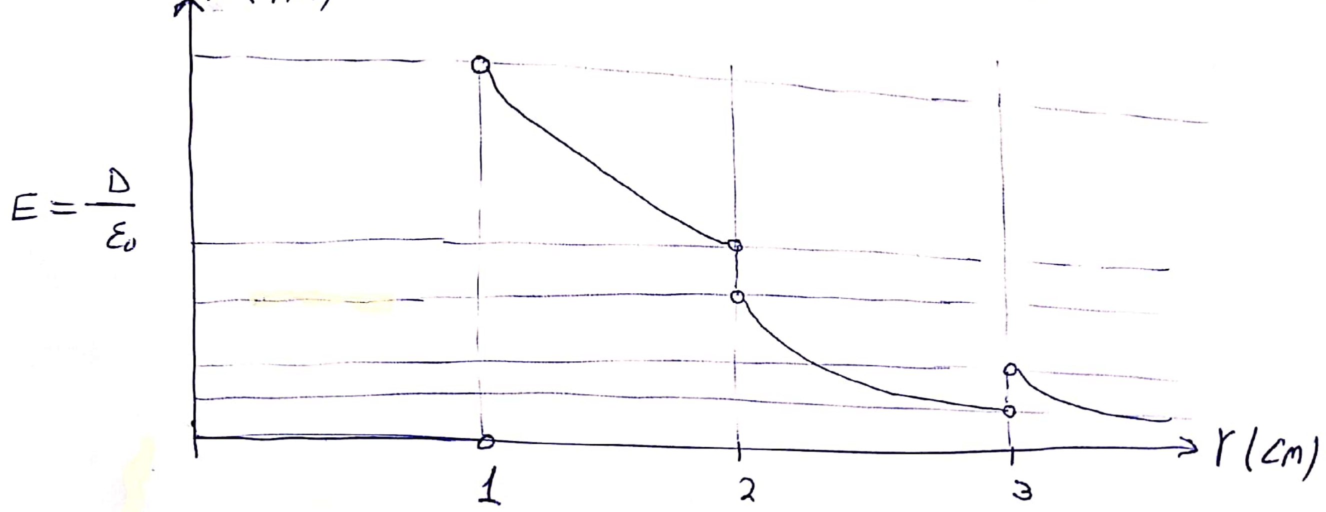
$$\therefore \bar{D}_3 = \frac{10^{-9}}{4\pi r^2}\,\bar{a}_r \quad , \quad \bar{E}_3 = \frac{10^{-9}}{4\pi \varepsilon_0 r^2}\,\bar{a}_r$$

(iv) $r > c$:

$$\oint\!\!\oint \bar{D} \cdot \bar{ds} = Q_a + Q_b + Q_c = 2\,nC$$

$$\bar{D}_4 = \frac{2 \times 10^{-9}}{4\pi r^2}\,\bar{a}_r \quad , \quad \bar{E}_4 = \frac{2 \times 10^{-9}}{4\pi \varepsilon_0 r^2}\,\bar{a}_r$$





(b) $V_c - \cancel{V_\infty}_{\to 0} = -\int_\infty^c \frac{2\times10^{-9}}{4\pi\varepsilon_0 r^2}\,dr$

$$= \frac{-2\times10^{-9}}{4\pi\varepsilon_0}\left.\frac{-1}{r}\right|_\infty^c$$

$$= \frac{2\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{c} - \cancel{\frac{1}{\infty}}_{\to 0}\right] = \boxed{\frac{2\times10^{-9}}{4\pi\varepsilon_0 c}}$$

Volt

$$\Rightarrow V_b - V_c = -\int_c^b \frac{1\times10^{-9}}{4\pi\varepsilon_0 r^2}\,dr$$

$$\therefore V_b = V_c + \frac{1\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{c}\right]$$

$$\Rightarrow V_a - V_b = -\int_b^a \frac{2\times10^{-9}}{4\pi\varepsilon_0 r^2}\,dr$$

$$\therefore V_a = V_b + \frac{2\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

(c) $W_T = W_1 + W_2 + W_3 + W_4$ $\because W = \frac{\varepsilon}{2}\iiint |E|^2\,dv$

$$W_2 = \int_0^{2\pi}\int_0^{\pi}\int_a^b \left(\frac{2\times10^{-9}}{4\pi\varepsilon_0 r^2}\right)^2 r^2 \sin\theta\,dr\,d\theta\,d\phi$$

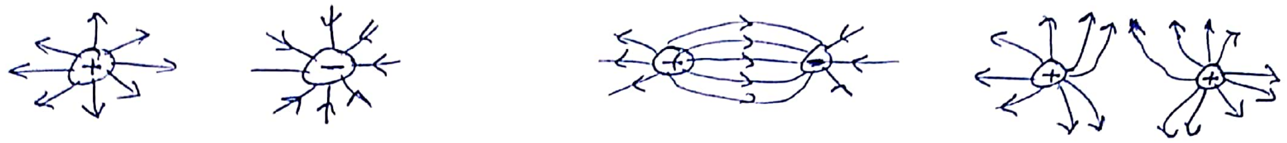
$\Rightarrow \boxed{W_1 = 0}$ $E_1 = 0$

$$\Rightarrow W_3 = \int_0^{2\pi}\int_0^{\pi}\int_b^c \left(\frac{1\times10^{-9}}{4\pi\varepsilon_0 r^2}\right)^2 r^2 \sin\theta\,dr\,d\theta\,d\phi = \boxed{\quad J}$$

$$\Rightarrow W_4 = \int_0^{2\pi}\int_0^{\pi}\int_c^{\infty} \left(\frac{2\times10^{-9}}{4\pi\varepsilon_0 r^2}\right)^2 r^2 \sin\theta\,dr\,d\theta\,d\phi = \boxed{\quad J}$$

(d) $Q_{enc} = 0 \Rightarrow E = 0$ For $b < r < c \Rightarrow \boxed{2cm < r < 3cm}$

$\boxed{3-a}$ Flux lines



(1) moving from $+Q$ to $-Q$.

(2) have the same direction of $\bar{E}$.

(3) The electric flux density $\bar{D} \propto \bar{E}$

(4) The SI unit of $\psi \equiv Q$ is Coloumb.

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(6) $\bar{D} = 30 e^{-\rho} \bar{a}_\rho - 2z\bar{a}_z$

(1) sheet 3 No. $\boxed{23}$ Page 7 نوتة

(2) $\rho_v \big|_{\rho = 1.5\,m} = ??$

$$\rho_v = \nabla \cdot \bar{D} = \frac{30}{\rho}\left[e^{-\rho} - \rho e^{-\rho}\right] - 2$$

$$\therefore \rho_v = \frac{30}{1.5}\left[e^{-1.5} - 1.5 e^{-1.5}\right] - 2 = \boxed{}$$

C/m3

(3) $\bar{E}_T = \bar{E}_{line} + \bar{E}_{cylinder}$ $\quad P(\overset{\rho}{3}, \overset{\phi}{60^\circ}, \overset{z}{2})$

$$\bar{E}_T = \frac{\rho_L}{2\pi\varepsilon_0\rho}\bar{a}_\rho + \frac{\bar{D}}{\varepsilon_0}$$

$$\bar{E}_T = \frac{20\times10^{-9}}{2\pi\times8.85\times10^{-12}\times3}\bar{a}_\rho + \frac{30 e^{-3}\bar{a}_\rho - 2(2)\bar{a}_z}{8.85\times10^{-12}}$$

$$= \boxed{}$$ V/m

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$\boxed{3-23}$ Evaluate both sides of divergence

$$\vec{D} = 30 e^{-\rho} \bar{a}_\rho - 2 z \bar{a}_z$$

cylindrical $r = 2$, $z = 0$ and $z = 5$

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$$\oiint \overline{D}\cdot\overline{\partial s} = \iiint (\nabla\cdot\vec{D})\, dv$$

* R.H.S:

$$\overline{\nabla}\cdot\overline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \cancel{\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi}}^{\to 0} + \frac{\partial D_z}{\partial z}$$

$$\therefore \overline{\nabla}\cdot\overline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(30\rho e^{-\rho}) + \frac{\partial}{\partial z}(-2z)$$

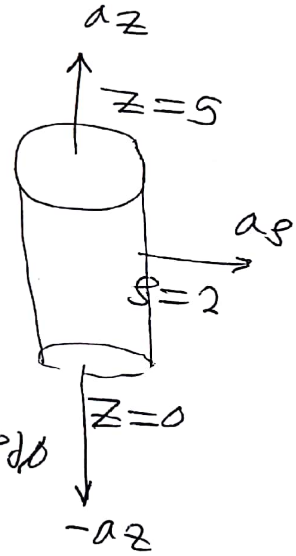
$$= \frac{30}{\rho}[e^{-\rho} - \rho e^{-\rho}] - 2$$

$$\therefore R.H.S = \int_0^5\int_0^{2\pi}\int_0^2 \left[\frac{30}{\rho}[e^{-\rho}-\rho e^{-\rho}]-2\right]\rho\, d\rho\, d\phi\, dz$$

$$= 30\times 2\pi\times 5\int_0^2\left(e^{-\rho}-\rho e^{-\rho}-\frac{2}{30}\right)d\rho$$

$$= \boxed{129.43\, C}$$

* For L.H.S:



$$L.H.S = \oiint \overline{D}\cdot\overline{\partial s}$$

$$\therefore L.H.S = \iint D_z\Big|_{z=5}\rho\, d\rho\, d\phi + \iint -D_z\Big|_{z=0}\rho\, d\rho\, d\phi$$

$$+ \iint D_\rho\Big|_{\rho=2}\rho\, d\phi\, dz$$

$$\therefore L.H.S = \int_0^{2\pi}\int_0^{2} -2z\Big|_{z=5} \rho\, d\rho\, d\phi$$

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$$+ \int_0^{2\pi}\int_0^{2} +2z\Big|_{z=0} \rho\, d\rho\, d\phi + \int_0^{5}\int_0^{2\pi} 30 e^{-\rho}\Big|_{\rho=2} \rho\, d\phi\, dz$$

$$= -10 \left.\frac{\rho^2}{2}\right|_0^2 \times 2\pi + 0 + 30 e^{-2} \times 2 \times 2\pi \times 5$$

$$= -40\pi + 600\pi e^{-2} = \boxed{129.43\, C}$$