Suez Canal University
Department of Electrical Engineering
Third Year

Course: Electromagnetic fields
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Quiz 1

Answer only one question.

Q1.

a) **Find** the equation of the line passing through the point $(2, 30^0, 0)$ for the field $E = \rho \cos 2\varphi\ \bar{a}_\rho - \rho \sin 2\varphi\ \bar{a}_\varphi$.

b) The spherical surface r = 0.3 m contains a charge density of 100 C/ m^2 , while that at r = 0.7m contains -60 C/ m^2 .

i) **What** charge density must exist on the surface r = 0.5 m so that the total charge is zero.

ii) Using these three values of ρ_S, **calculate** D_r as a function of r and sketch it versus r for $0 < r < 0.9$ m.

iii) **What** point charge must exist on the origin so that the total charge is 2 pC.

Q2.

a) Drive from the first principles the equation of the Electric field intensity at a perpendicular distance z from infinite surface of uniform charge ρ_S.

b) Consider the uniform circular disk of charge on z = 0 plane. The disk has surface charge density ρ_S = 10 pC/ m^2 , radius a = 0.1 m.

i) Find the electric field intensity E at point P (0, 0, 0.5) m.

ii) If a circular disk of radius b (where b < a) is removed out from the circular disk of radius a. Find the radius b which make the new electric field intensity E at the point P (0, 0, 0.5) m produced from the new arrangement (ring disk of charge) equal 25% from its values calculated in part (i).

with all my best wishes

Quiz 1 2018

Q1.

(a) Find equation of line at $P(2, 30^\circ, 0)$

$$\vec{E} = \rho \cos 2\phi \, a_\rho - \rho \sin 2\phi \, \vec{a}_\phi$$

"answer"

$$\frac{d\rho}{\rho \, d\phi} = \frac{E_\rho}{E_\phi} = \frac{-\cancel{\rho} \cos 2\phi}{\cancel{\rho} \sin 2\phi}$$

$$\therefore \int \frac{d\rho}{\rho} = \frac{-1}{2} \int \frac{2 \cos 2\phi}{\sin 2\phi}$$

$$\therefore \ln(\rho) = \frac{-1}{2} \ln(\sin 2\phi) + \underbrace{\ln(C)}_{\text{or } C}$$

$$\therefore \rho = C / \sqrt{\sin 2\phi} \quad \text{at } P(2, 30^\circ, 0)$$

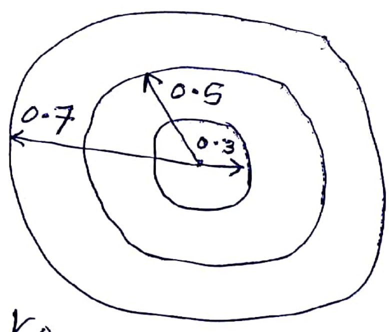
$$\therefore 2 = C / \sqrt{\sin(60)} \Longrightarrow C = 1.86$$

$$\therefore \boxed{\rho = 1.86 / \sqrt{\sin 2\phi}} \qquad \boxed{\rho \sqrt{\sin 2\phi} = 1.86}$$

(b) $a = 0.3\,m$ $\quad \rho_{sa} = 100\ C/m^2$

$b = 0.5\,m$ $\quad \rho_{sb} = ??$

$c = 0.7\,m$ $\quad \rho_{sc} = -60\ C/m^2$



(i) $\rho_{sb} = ??$ $\quad Q_t = \text{Zero}$

$$Q_t = \rho_{sa}(4\pi a^2) + \rho_{sb}(4\pi b^2) + \rho_{sc}(4\pi c^2) = 0$$

$$\therefore \rho_{sb} = -\left(\frac{\rho_{sa} a^2 + \rho_{sc} c^2}{b^2}\right)$$

$$= -\left(\frac{100(0.3)^2 + (-60)\times(0.7)^2}{(0.5)^2}\right) = \boxed{81.6\ C/m^2}$$

(ii) Find $D_r = ??$ and sketch $0 < r < 0.9m$

"answer"

→ $0 < r < 0.3m$:

$$Q_{enc} = 0 \Rightarrow \boxed{D_r = 0}$$

→ $0.3 < r < 0.5$:

$$\oint\!\!\oint \vec{D} \cdot \vec{ds} = Q_{enc}$$

$$D_r(4\pi r^2) = \rho_{sa}(4\pi a^2)$$

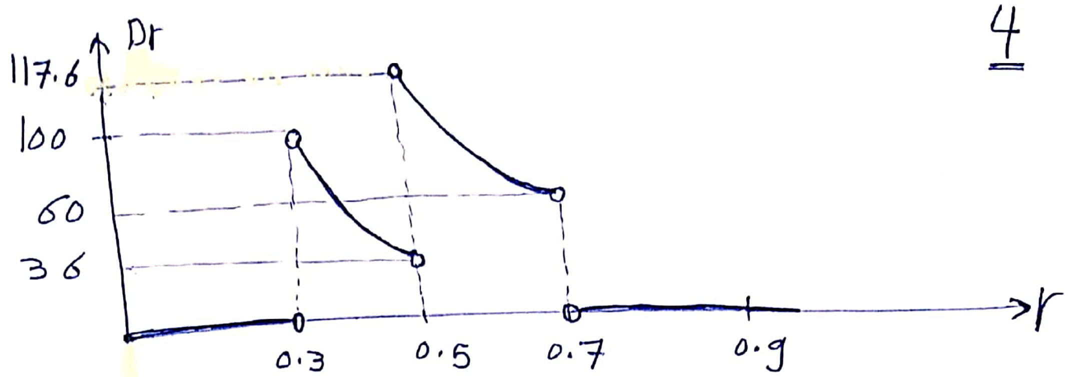
$$D_r = \frac{\rho_{sa} a^2}{r^2} = \boxed{\frac{9}{r^2}\ C/m^2}$$

→ $0.5 < r < 0.7$:

$$D_r = \frac{\rho_{sa} a^2 + \rho_{sb} b^2}{r^2} = \boxed{\frac{29.4}{r^2}\ C/m^2}$$

→ $r > 0.7m$:

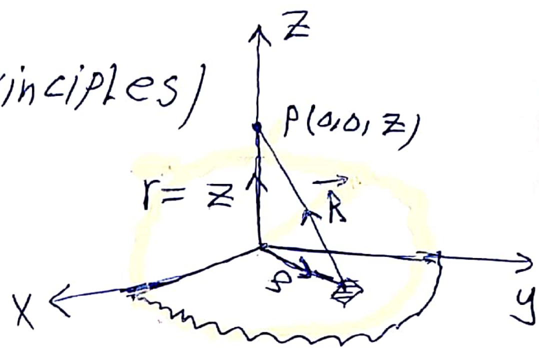
$$D_r = \frac{\rho_{sa} a^2 + \rho_{sb} b^2 + \rho_{sc} c^2}{r^2} = \boxed{\text{zero}}$$



(iii) $Q = \boxed{2\,PC}$

Q2:

(a) Infinite sheet (First principles)



$$d\bar{E} = \frac{dQ}{4\pi \varepsilon_0 R^2} a_R$$

$$dQ = \rho_s\, ds = \rho_s\, \rho\, d\rho\, d\phi$$

$$\vec{R} = \vec{r} - \vec{r}' = z a_z - \rho a_\rho \qquad |R| = \sqrt{z^2 + \rho^2}$$

$$\therefore d\bar{E} = \frac{\rho_s\, \rho\, d\rho\, d\phi}{4\pi \varepsilon_0} \frac{z a_z - \rho a_\rho}{(z^2+\rho^2)^{3/2}}$$

From symmetry D_ρ is cancelled

$$\therefore \vec{E} = \frac{\rho_s z}{4\pi \varepsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{\rho}{(z^2+\rho^2)^{3/2}} d\rho\, d\phi\; a_z$$

$$= \frac{\rho_s z}{4\pi \varepsilon_0} (2\pi)\, \cancel{\tfrac{1}{2}} \frac{(z^2+\rho^2)^{-\frac{1}{2}}}{-\cancel{\tfrac{1}{2}}} \Bigg|_0^{\infty} a_z$$

$$= \frac{\rho_s z}{2\varepsilon_0} \frac{-1}{\sqrt{z^2+\rho^2}} \Bigg|_0^{\infty} a_z$$

$$= \frac{-\rho_s z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2+\infty}} - \frac{1}{\sqrt{z^2+0}}\right]\bar{a}_z$$

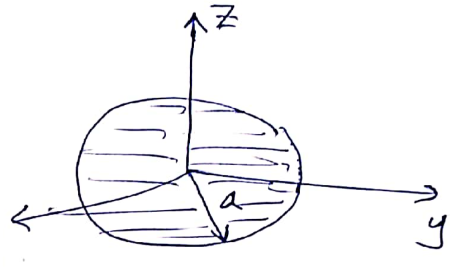
$$= \boxed{\frac{\rho_s}{2\varepsilon_0}\,\bar{a}_z}$$

(b) circular disk $\rho_s = 10\ pC/m^2$ $a = 0.1m$

(i) $\vec{E}$ at $P(0,0,0.5)$

"answer"

From the previous proof



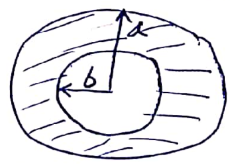
$$\vec{E} = \frac{\rho_s z}{\varepsilon_0}\,\frac{-1}{\sqrt{z^2+\rho^2}}\Bigg|_0^a\, a_z$$

$$\therefore \vec{E} = \frac{-\rho_s z}{\varepsilon_0}\left[\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+0}}\right] a_z$$

$$\therefore \vec{E} = \frac{-(10\times10^{-12})\times0.5}{8.85\times10^{-12}}\left[\frac{1}{\sqrt{(0.5)^2+(0.1)^2}} - \frac{1}{0.5}\right] a_z$$

$$\therefore \boxed{\vec{E} = 0.022\, a_z}\ V/m$$

(ii) $b = ??$ $\vec{E}\Big|_{new} = 0.25\,\vec{E}\Big|_{old}$



$$\frac{-\rho_s z}{\varepsilon_0}\left[\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+b^2}}\right] = 0.25\,\frac{-\rho_s z}{\varepsilon_0}\left[\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+0}}\right]$$

$\because a = 0.1 \quad \& \quad Z = 0.5$

$$\therefore \frac{1}{\sqrt{(0.5)^2 + (0.1)^2}} - \frac{1}{\sqrt{(0.5)^2 + b^2}} = 0.25 \left[\frac{1}{\sqrt{(0.5)^2 + (0.1)^2}} - \frac{1}{0.5} \right]$$

$$\therefore \boxed{b = 0.086 \, m}$$

<u>Note:</u>

$p \longrightarrow 10^{-12}$

$n \longrightarrow 10^{-9}$

$\mu \longrightarrow 10^{-6}$

$m \longrightarrow 10^{-3}$

$K \longrightarrow 10^{3}$

$M \longrightarrow 10^{6}$

$G \longrightarrow 10^{9}$

$T \longrightarrow 10^{12}$