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Suez Canal University			Course: Electromagnetic fields
Department of Electrical Engineering			Lecturer: Dr. Ahmed Magdy
Third Year	Midterm Exam		December 2019
Total marks [20]		Time allowed : 1 hour	1 page	ELC 214

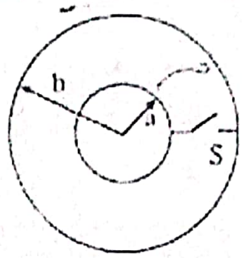
Answer the following questions :

Question (1): Two concentric metallic spherical shells have radii of a and b respectively, as shown in the following Fig. When the switch S is open, the inner sphere has a positive charge q, while the outer has a negative charge -Q.

a) Find the potential of each sphere.

b) What is the potential difference between the spheres after S is closed?

c) What is the value of the charge on each sphere when S is closed and equilibrium is obtained?



Question (2):

A uniform line of charge, ρ_l=20 nC/m, is located at x=1 m, z=4 m and a uniform sheet of charge, ρ_s=20 nC/m^2, is presented at x=3 m in free space.

a) Find the direction of the electric field intensity at P (4,5,6).

b) Give the Cartesian coordinates of one point at which the electric field intensity is negative of the above value.

c) What is the force per meter length on the line charge.

Question (3): Three cylindrical surfaces of radius a = 3 cm, b = 4 cm and b = 5 cm. The inner cylinder has a charge density ρ_{sa} = 8 nC/m^2 and the second charge density ρ_{sb} = -12 nC/m^2 while the outer cylinder has ρ_{sc} . *(6 marks)*

a) **Calculate** ρ_{sc} which make the total charge equal zero.

b) **Write** an expression for $\overline{D}$ and $\overline{E}$ in all regions and **calculate** them if ρ_{sc} = 2 nC/m^2.

c) For the same data of part (b), **what** the value of the line charge density should be placed at the center of the three cylinders to reduce the external field to a zero at the point r = 3.5 cm.

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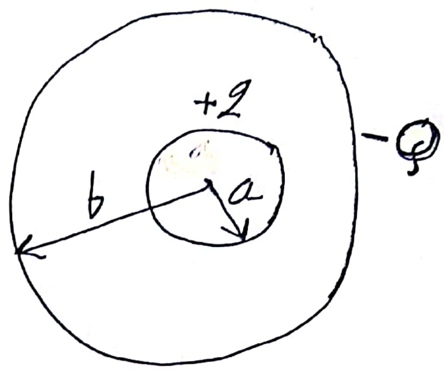
Q = 13

"Mid-term – 12/2019"

$\boxed{1}$ (a) Find V_a and V_b $S \rightarrow$ open

"answer"



Let $V = 0$ at $r \rightarrow \infty$

$$\therefore \boxed{V_b = \frac{q - Q}{4\pi\varepsilon_o b}} \text{ Volt}$$

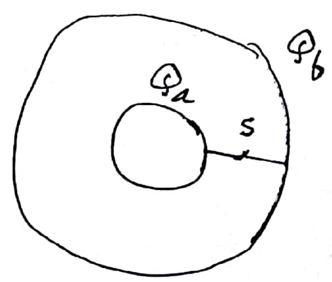
$$V_{ab} = V_a - V_b = \frac{+q}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\therefore V_a = \frac{q}{4\pi\varepsilon_o a} - \frac{q}{4\pi\varepsilon_o b} + \frac{q - Q}{4\pi\varepsilon_o b}$$

$$\therefore \boxed{V_a = \frac{q}{4\pi\varepsilon_o a} - \frac{Q}{4\pi\varepsilon_o b}} \text{ Volt}$$

(b) $S \rightarrow$ closed $\therefore V_a = V_b \Rightarrow \boxed{V_{ab} = \text{Zero}}$

(c) $Q_a + Q_b = q - Q \rightarrow (1)$



$V_a = V_b$

$$\frac{Q_a}{\cancel{4\pi\varepsilon_o} a} = \frac{Q_b}{\cancel{4\pi\varepsilon_o} b}$$

$$\therefore Q_b = \frac{b}{a} Q_a \longrightarrow (2)$$

From (2) into (1)

$$\therefore Q_a + \frac{b}{a} Q_a = 2 - Q$$

$$\therefore Q_a = \frac{2-Q}{1+b/a} = \boxed{\frac{a(2-Q)}{a+b}} \; C$$

$$\therefore Q_b = \frac{b}{\cancel{a}} \frac{\cancel{a}(2-Q)}{a+b} = \boxed{\frac{b(2-Q)}{a+b}} \; C$$

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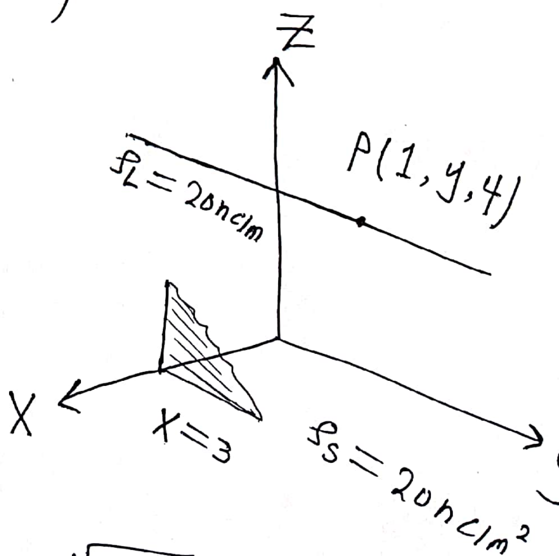
*<u>Note</u>: Potential at $r<a$ equal to Potential at $r=a$. "$E_{in} = zero$"

$\boxed{2}$

(a) Direction $\overline{E}$ at $P(4,5,6)$



$$\overline{E}_t = \overline{E}_L + \overline{E}_s$$

$$\overline{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \overline{a}_\rho$$

$$\overline{\rho} = (4,5,6) - (1,y,4)$$
$$= 3\,\overline{a}_x + 2\,\overline{a}_z$$

$$|\rho| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\therefore \overline{E}_L = \frac{20 \times 10^{-9}}{2\pi \times 8.85 \times 10^{-12} \times \sqrt{13}} \frac{3\,\overline{a}_x + 2\,\overline{a}_z}{\sqrt{13}}$$

$$\therefore \overline{E}_L = 83\,\overline{a}_x + 55.33\,\overline{a}_z$$

$$* \; \overline{E}_s = \frac{\rho_s}{2\varepsilon_0}\overline{a}_N = \frac{20 \times 10^{-9}}{2 \times 8.85 \times 10^{-12}}\,\overline{a}_x$$

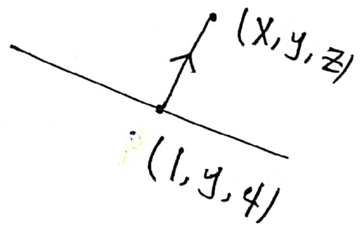
$$= 1129.9\,\overline{a}_x$$

$$\therefore \overline{E}_t = \overline{E}_L + \overline{E}_s = \boxed{1213\,\overline{a}_x + 55.33\,\overline{a}_z} \; V/m$$

(b) $\overline{E}_T = -1213\,\overline{a}_x - 55.33\,\overline{a}_y$

let $P(x, y, z)$

$$\overline{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\,\overline{a}_\rho$$



$$\overline{\rho} = (x, y, z) - (1, y, 4) = (x-1)\,\overline{a}_x + (z-4)\,\overline{a}_z$$

$$\therefore \overline{E}_L = \frac{20 \times 10^{-9}}{2\pi\varepsilon_0} \frac{(x-1)\,\overline{a}_x + (z-4)\,\overline{a}_z}{(x-1)^2 + (z-4)^2}$$

$$\overline{E}_s = \frac{\rho_s}{2\varepsilon_0} a_N = \frac{20 \times 10^{-9}}{2\varepsilon_0}(-a_x) \quad \text{let } x < 3$$

$$= -1129.9\,\overline{a}_x$$

$$\therefore \bar{E}_t = -83\,\bar{a}_x - 55.33\,\bar{a}_z$$

$$= \frac{20\times 10^{-9}}{2\pi\varepsilon_0}\left[\frac{(x-1)\bar{a}_x}{(x-1)^2+(z-4)^2} + \frac{(z-4)\bar{a}_z}{(x-1)^2+(z-4)^2}\right]$$

$$\therefore -83 = \frac{10^{-8}}{\pi\times 8.85\times 10^{-12}} \frac{x-1}{(x-1)^2+(z-4)^2} \rightarrow (1)$$

$$-55.33 = \frac{10^{-8}}{\pi\times 8.85\times 10^{-12}} \frac{z-4}{(x-1)^2+(z-4)^2} \rightarrow (2)$$

divide (1) $\div$ 2

$$\frac{83}{55.33} = \frac{x-1}{z-4} \Rightarrow (x-1) = 1.5(z-4)$$

sub. into (1)

$$\therefore -83 = \frac{10^{-8}}{\pi\times 8.85\times 10^{-12}} \frac{1.5(z-4)}{[1.5(z-4)]^2+(z-4)^2}$$

$\therefore z = 2\,m$, $(x-1) = -3$

$\therefore x = -2$

$\therefore$ $\boxed{\text{Point}(-2, 0, 2)}$ Generally $(-2, y, 2)$

(c) $F = Q E_s$

$$Q = \int \rho_L \, dL = \rho_L \times L = 20 \times 10^{-9} \times 1m$$

$$\therefore Q = 20 \times 10^{-9} C$$

$$\bar{E}_s = \frac{\rho_s}{2\varepsilon_0} \bar{a}_N = \frac{20 \times 10^{-9}}{2 \times 8.85 \times 10^{-12}} (-\bar{a}_x)$$

$$= -1129.9 \, \bar{a}_x$$

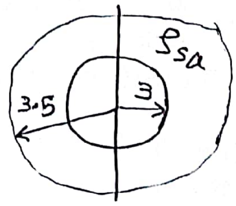
$$\therefore F = Q\bar{E} = \boxed{-2.26 \times 10^{-5} \bar{a}_x}$$

Q-3 : sheet 3 Number $\boxed{2}$

(a) ✓

(b) ✓

(c) $E|_{\rho = 3.5cm} = \text{zero} \quad \therefore Q_t = 0$



$$\therefore \rho_L (L) + 2\pi a L \rho_{sa} = 0$$

$$\therefore \rho_L = -2\pi a \rho_{sa}$$

$$= -2\pi \times 3 \times 10^{-2} \times 8n = \boxed{-1.51 \, nC/m}$$