VECTOR SPACE

V is called a "Vector space" if all the following are satisfied

1: $u+v \in V$, $u+v$ must be in V
2: $u+v = v+u$
3: $u+(v+w) = (u+v)+w$
4: $u+\vec{0} = \vec{0}+u = u$
5: $u+(-u) = (-u)+u = \vec{0}$
6: αu must be in V ; $\alpha u \in V$
7: $\alpha(u+v) = \alpha u + \alpha v$
8: $(\alpha_1+\alpha_2)u = \alpha_1 u + \alpha_2 u$
9: $\alpha_1(\alpha_2 u) = (\alpha_1 \alpha_2)u$
10: $1.u = u$

IMPORTANT

- اذا كان معرف (الدمج بطريقه غير المالوفه) ، أما الضرب ثابت Standard :
  ← فحص الشروط (1,2,3,7,8)
- اذا كان معرف الدمج Standard ، اما الضرب (بطريقه غير المالوفة) :
  ← فحص الشروط (6,7,8,9,10)
- اذا كان هناك متباينة (اكبر او اصغر "<" ">") :
  ← افحص الشرط السادس

Q1: let $V = \{(a,b,0) : a,b \in R\}$ Define the addition by: $(a,b,0)+(c,d,0) = (a+c, b-d, 0)$, with Standard Scalar Multiplication. Is V a vector space:

NO, why?

2: $u+v = v+u$

let $u = (u_1,u_2,0)$, $v = (v_1,v_2,0)$

$u+v = (u_1+v_1, u_2-v_2, 0)$

$v+u = (v_1+u_1, v_2-u_2, 0)$ $\neq$

V is not vector space

Q2: let $V = R^2$, define the addition by: $(u_1+u_2)+(v_1+v_2) = (u_1+v_1, u_2+v_2+1)$ with Standard scalar multiplication. Is V a vector? No why

7: $\alpha(u+v) = \alpha u + \alpha v$

$\alpha(u+v) = \alpha(u_1+v_1, u_2+v_2+1)$ - ونوزع

$\alpha u + \alpha v = (\alpha u_1, \alpha u_2)+(\alpha v_1, \alpha v_2)$ $\neq$

$(\alpha u_1, \alpha u_2)+(\alpha v_1, \alpha v_2) = \alpha u_1+\alpha v_1, \alpha u_2+\alpha v_2+1$

$\alpha u_1+\alpha v_1, \alpha u_2+\alpha v_2+\alpha \neq$ اذا

V is not vector space.

Q3: Let $V = R^2$ with standard addition, but with Scalar multiplication defined by:

$\alpha(x,y) = (\alpha^2 x, \alpha y)$, V is a vector? No, why?

8: $(\alpha_1+\alpha_2)u = \alpha_1 u + \alpha_2 u$

$(\alpha_1+\alpha_2)u = (\alpha_1+\alpha_2)(u_1,u_2) = (\alpha_1+\alpha_2)^2 u_1, (\alpha_1+\alpha_2)u_2$

$\alpha_1 u + \alpha_2 u = \alpha_1(u_1,u_2) + \alpha_2(u_1,u_2) =$ $\neq$

$\alpha_1^2 u_1, \alpha_1 u_2 + \alpha_2^2 u_1, \alpha_2 u_2 = (\alpha_1^2+\alpha_2^2)u_1, (\alpha_1+\alpha_2)u_2$

V is not vector space

Q4: let $V = R^2$ define the addition by: $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$; define the scalar multiplication by: $k(u_1, u_2) = (ku_1, 0)$. V a vector? No, why?

10- $1.u = u$

let $u = (u_1, u_2)$

$1.(u_1, u_2) = u_1, 0$

$(u_1, u_2) \neq u_1, 0$

V is not vector space.

Q5: $V\{(x, y) : x \geq 0\}$ with standard operations on R^2. V is a vector? No, why?

6- let $u = (x, y) = x \geq 0$

$\alpha(x, y) = (\alpha x, \alpha y) : \alpha x \overset{?}{\geq} 0$?

let $\alpha = -1$

$-1x \leq 0$

$\alpha x \leq 0$ not closed under scalar multiplication - V is not vector space.

Q6: let $V = P_1$ with standard addition and define the scalar multiplication by: $\alpha(a_0 + a_1 x) = a_0 + \alpha a_1 x$, V is a vector? No, why

8- $(\alpha_1 + \alpha_2) = \alpha_1 u + \alpha_2 u$

let $u = a_0 + a_1 x$

- $(\alpha_1 + \alpha_2)(a_0 + a_1 x) = a_0 + (\alpha_1 + \alpha_2) a_1 x$
- $\alpha_1 u + \alpha_2 u = \alpha_1(a_0 + \alpha a_1 x) + \alpha_2(a_0 + \alpha a_1 x)$

$= (a_0 + \alpha_1 a_1 x) + (a_0 + \alpha_2 a_1 x)$

$= 2a_0 + (\alpha_1 a_1 + \alpha_2 a_1)x$ $\neq$

V is not vector space

Q7: $V = \{(x, y, z) ; x + y > 0 \ \forall x, y, z \in IR\}$

Does V be a vector space?

No, why:

$0 \in V$ because $x + y > 0$, $x = \cancel{x} = 0$

Q8: $V = \{(x, y, z) : x \geq 0, x, y, z \in IR\}$

V not a vector space why:

$\because u \in V$ because $u (x, y, z) \in V$, $x \geq 0$

$-u = (-x, -y, -z)$, $-x \leq 0$

$-u \notin V$

Q9: $V = \{(x, y, z) : x = z + 1, y, z \in IR\}$

V is not a vector space why?

$\underline{0} \notin V$, $x = z + 1$, $y = z = 0$, $x = 1$

Q10: $V = \{(x, y, z) : x, y, z \in IR\}$ where

$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 + x_2, y_1 - y_2, z_1 + z_2)$

$\alpha \odot (x, y, z) = (\alpha x, \alpha z, \alpha y)$

$\because$ take $\alpha = 1$, $(x, y, z) \neq (x, z, y)$

$1.u \neq u$ because $(x, y, z) \neq (x, z, y)$

Q11: $V = \{(x, y, z) ; x = y + z + 1, y, z \in IR$:

$u(x, y, z) = (y + z + 1, y, z)$

$u_2 = (a, b, c) = (b + c + 1, b, c)$

$(u_1 + u_2) = (y + z + b + c + 2, y + b, z + c) \in V$