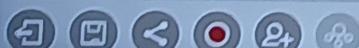


$$\begin{array}{l} \text{Ex: } \begin{array}{c|cc} 3 & & \\ \hline x_1 - 2x_2 + x_3 & = 3 \\ 2x_1 + x_2 - x_3 & = 5 \\ 3x_1 - x_2 + 2x_3 & = 12 \end{array} \\ \text{Solution} \end{array}$$

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 1(2-1) + 2(4+3) + !(-2-3) = 1+14-5 = 10$$

$$\begin{aligned} D_{x_1} &= \begin{vmatrix} + & -2 & +1 \\ 1 & 1 & -1 \\ 12 & -1 & ? \end{vmatrix} = 3(2-1) + 2(10+2) + 1(-5-12) \\ &= 3 + 44 - 17 = 30 \\ D_{x_2} &= \begin{vmatrix} + & - & + \\ 1 & 3 & 1 \\ 2 & 5 & -1 \\ 3 & 12 & 2 \end{vmatrix} = 1(16+12) - 3(4+3) + 1(24-15) \\ &= 22 - 21 + 9 = 10 \\ D_{x_3} &= \begin{vmatrix} + & -2 & +3 \\ 1 & 1 & 5 \\ 2 & -1 & 12 \end{vmatrix} = 1(12+5) + 2(24-15) + 3(-2-3) \\ &= 17 + 18 - 15 = 20 \\ x_1 &= \frac{D_{x_1}}{D} = \frac{30}{10} = 3, \quad x_2 = \frac{D_{x_2}}{D} = \frac{10}{10} = 1, \quad x_3 = \frac{D_{x_3}}{D} = \frac{20}{10} = 2 \end{aligned}$$



Ex: Solve the following linear system using Cramer's rule:

$$2x + y = 7 \quad \checkmark$$

$$3x - 4y = 5 \quad \checkmark$$

Solution

$$\Delta = \begin{vmatrix} 2 & 1 \\ 3 & -4 \end{vmatrix} = 8 - 3 = -11$$

$$\Delta_x = \begin{vmatrix} 7 & 1 \\ 5 & -4 \end{vmatrix} = -28 - 5 = -33$$

$$\Delta_y = \begin{vmatrix} 2 & 7 \\ 3 & 5 \end{vmatrix} = |0 - 2| = -11$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-33}{-11} = 3$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-11}{-11} = 1$$

Solution of linear equations System

* Cramer's Rule

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

⋮

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

* System of two equations:

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

Solution

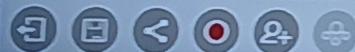
$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta x_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = ?$$

$$x_1 = \frac{\Delta x_1}{\Delta} =$$

$$\Delta x_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = ?$$

$$x_2 = \frac{\Delta x_2}{\Delta}$$



Ex: Express the matrix

$$B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

as a sum of symmetric
and skew symmetric
matrices.

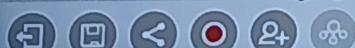
Solution

$$B = P_{\text{symm}} + Q_{\text{skew}}$$

$$P = \frac{1}{2}(B + B') = \frac{1}{2} \left(\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix}$$

$$Q = \frac{1}{2}(B - B') = \frac{1}{2} \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$



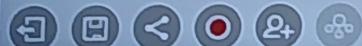
$$A = P + Q$$

↙ Symmetric ↘ Skew

$$= \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\begin{aligned} A &= \frac{1}{2}A + \frac{1}{2}A \\ &= \frac{1}{2}A + \frac{1}{2}A + \underbrace{\frac{1}{2}A' - \frac{1}{2}A'}_{\text{Skew}} \\ &= \frac{1}{2}(A+A') + \frac{1}{2}(A-A') \end{aligned}$$

↙ Symm ↘ Skew



$$B = \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 7 \\ 4 & -7 & 0 \end{bmatrix}$$

Solution

$$B' = \begin{bmatrix} 0 & -5 & 4 \\ 5 & 0 & -7 \\ -4 & 7 & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 7 \\ 4 & -7 & 0 \end{bmatrix}$$

$$= -B \quad \text{Skew symmetric}$$

* For any square matrix A

$A + A'$ is symmetric

$A - A'$ is skew symmetric
: $\sim i$

$$(A + A')' = A' + A$$

$$(A - A')' = A' - A = -(-A' + A)$$



* Symmetric and
Skew Symmetric Matrix

$$A' = A \quad \text{Symmetric}$$

$$A' = -A \quad \text{Skew symmetric}$$

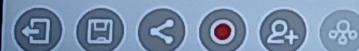
Ex: Determine if A is symmetric or skew symmetric

$$A = \begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -15 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

Solution

$$A' = \begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -15 & -1 \\ 3 & -1 & 1 \end{bmatrix} = A$$

Symmetric



* Transpose of a matrix:

Let $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \\ -1 & 7 \end{bmatrix}$

Find the transpose of

A

Solution

$$A^t \text{ or } A' = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 5 & 7 \end{bmatrix}$$

* $(A')' = A$

* $(A \pm B)^t = A^t \pm B^t$

* $(A \pm B)' = A' \pm B'$

* $(AB)' = B' A'$