

Linear Algebra

System of Linear equations

$$\begin{array}{l} 2x + y = 1 \\ x + 2y = 3 \end{array}$$

$$X = A^{-1}D$$

~~$AX = D$~~
 ~~$|A| \neq 0$~~

Gauss

Gauss-Jordan

$$\begin{aligned}
 a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 &= d_1 \\
 b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 &= d_2 \\
 &\vdots
 \end{aligned}$$

$$\begin{aligned}
 &\text{or } c_1 x_1 + c_2 x_2 + \dots + c_4 x_4 = d_n \\
 &\text{or } \begin{bmatrix} c_1 & c_2 & \dots & c_4 & d_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_4 \end{bmatrix} = d_n
 \end{aligned}$$

$$\begin{aligned} \text{Ex) } x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

SOL

$$A = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 3 & 1 & -1 & -2 \end{bmatrix}$$

$$\begin{aligned} R_1' &= R_2 - 2R_1 \\ R_3' &= R_3 - 3R_1 \end{aligned} \quad \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ 0 & \boxed{-5} & -10 & -20 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

$$R_3' = 7R_3 + R_2 \quad \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ 0 & 0 & 10 & 30 \end{bmatrix}$$

$$\begin{aligned} 10z = 30 &\Rightarrow z = 3 \\ -7y - 4z = 2 &\Rightarrow y = -2 \\ x + 2y + 3z &= 6 \end{aligned}$$

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E.x)

$$x_1 + 2x_2 + x_3 + 3x_4 = 0$$

$$2x_1 + x_2 - x_3 = 1$$

$$-x_1 + 3x_2 + 3x_3 + 4x_4 = 2$$

$$4x_1 + 5x_2 - x_4 = 1$$

S.o.L

$$A = \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 2 & 1 & -1 & 0 & 1 \\ -1 & 3 & 3 & 4 & 2 \\ 4 & 5 & 0 & -1 & 1 \end{array} \right] \quad \begin{array}{l} R'_2 = R_2 - 2R_1 \\ R'_3 = R_3 + R_1 \\ R'_4 = R_4 - 4R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & -3 & -3 & -6 & 1 \\ 0 & \boxed{5} & 4 & 7 & 2 \\ 0 & \boxed{-3} & -4 & -13 & 1 \end{bmatrix}$$

$$R'_3 = \frac{3R_3 + 5R_2}{}$$

$$R_4 = R_4 - R_2 \quad \sim \begin{bmatrix} \boxed{1} & \boxed{2} & 1 & 3 & 0 \\ 0 & \boxed{-3} & -3 & -6 & 1 \\ \textcircled{6} & 0 & -3 & -9 & 11 \\ 0 & 0 & \boxed{-1} & -7 & 0 \end{bmatrix}$$

↙

$$R_4 = 3R_4 - R_3$$

$$\sim \begin{bmatrix} 1 & \boxed{2} & \boxed{1} & \boxed{3} & 0 \\ 0 & -3 & \boxed{-3} & \boxed{-6} & 1 \\ 0 & 0 & \boxed{-3} & \boxed{-9} & 11 \\ 0 & 0 & 0 & -12 & -11 \end{bmatrix}$$

↙ $-11 \times 4 = -44$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & -3 & -3 & -6 & 1 \\ 0 & 0 & -3 & -9 & 11 \\ 0 & 0 & 0 & 0 & \textcircled{12} \end{bmatrix}$$