التاريخ :

الموضوع :

Liner combination

steps:

$X = C_1V_1 + C_2V_2 + \_\_\_\_ + C_nV_n$ ① نفرض

② نشتق system من هذه المعادلة

③ نقوم بحل system وله حالتان

has Solution
X is linear combination

NO solution
X is not linear combination

EX:

If Possible express $X = (2,1,5) \in R^3$ as linear combination of vectors $V_1 = (1,2,1), V_2 = (1,0,2), V_3 = (1,1,0)$

Sol:

let $X = C_1V_1 + C_2V_2 + C_3V_3$

$(2,1,5) = C_1(1,2,1) + C_2(1,0,2) + C_3(1,1,0)$

$(2,1,5) = C_1, 2C_1, C_1 + C_2, 0, 2C_2, C_3, C_3, 0$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 0 & 5 \end{array}\right| = \begin{array}{l} C_1 + C_2 + C_3 = 2 \\ 2C_1 + \_\_\_ + C_3 = 1 \\ C_1 + 2C_2 + \_\_\_ = 5 \end{array} \longrightarrow \begin{array}{l} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - R_1 \\ R_3 = R_3 - 2R_2 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right| \begin{array}{l} C_1 = 1 \\ C_2 = 2 \\ C_3 = -1 \end{array}$$

X is linear combination of V_1, V_2 and V_3

$X = V_1 + 2V_2 - V_3$ نعوض في المعادلة

EX: let $S = \{(2,1,3), (1,0,1), (2,1,2)\}$

is the vector $(5,5,8)$ a linear comb of S? or express $(5,5,8)$?

Sol) $(5,5,8) = C_1(2,1,3) + C_2(1,0,1) + C_3(2,1,2)$

$$\left|\begin{array}{ccc|c} 2 & 1 & 2 & 5 \\ 1 & 0 & 1 & 5 \\ 3 & 1 & 2 & 8 \end{array}\right| \xrightarrow{R_1 \leftrightarrow R_2} \left|\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 2 & 1 & 2 & 5 \\ 3 & 1 & 2 & 8 \end{array}\right| \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left|\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -5 \\ 0 & 1 & -1 & -7 \end{array}\right| \xrightarrow{R_3 - R_2} \left|\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & -1 & -2 \end{array}\right|$$

The system has exactly 1 sol + is a lin comb.

$$\xrightarrow{R_1 + R_3} \left|\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & -1 & -2 \end{array}\right| \xrightarrow{-R_3} \left|\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right| \quad \begin{array}{l} \bullet\ C_1 = 3 \\ \bullet\ C_2 = -5 \\ \bullet\ C_3 = 2 \end{array} \longrightarrow (5,5,8) = 3(2,1,3) - 5(1,0,1) + 2(2,1,2)$$

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EX:- let $S=\left\{\begin{bmatrix}1&0\\-2&-2\end{bmatrix},\begin{bmatrix}1&1\\-2&-1\end{bmatrix},\begin{bmatrix}0&0\\1&1\end{bmatrix}\right\}$ في الشكل Z

express $\begin{bmatrix}-11&8\\-2&6\end{bmatrix}$ as a linear comb. of S!

Sol:-

$$\begin{bmatrix}-11&8\\-2&6\end{bmatrix}=C_1\begin{bmatrix}1&0\\-2&-2\end{bmatrix}+C_2\begin{bmatrix}1&1\\-2&-1\end{bmatrix}+C_3\begin{bmatrix}0&0\\1&1\end{bmatrix}$$

$$\left|\begin{array}{ccc|c}1&1&0&-11\\0&1&0&8\\-2&-2&1&-2\\-2&-1&1&6\end{array}\right| \xrightarrow[R_4+2R_1]{R_3+2R_1} \left|\begin{array}{ccc|c}1&1&0&-11\\0&1&0&8\\0&0&1&-24\\0&1&1&-16\end{array}\right| \xrightarrow[R_4-R_2]{R_1-R_2} \left|\begin{array}{ccc|c}1&0&0&-19\\0&1&0&8\\0&0&1&-24\\0&0&1&-24\end{array}\right|$$

$$\xrightarrow{R_4-R_3} \left|\begin{array}{ccc|c}1&0&0&-19\\0&1&0&8\\0&0&1&-24\\0&0&0&0\end{array}\right|$$

The system has exactly one sol.

$C_1=19 \quad C_2=8 \quad C_3=-24$

EX:- Let $W=\{1+2X, X^3, X^2-1, -X^2+X^3-X\}$

write $3-2X^3$ as a linear comb. of W.

Sol:-

$3-2X^3=K_1(1+2X)+K_2(X^3)+K_3(X^2-1)+K_4(-X^2+X^3-X)$

$$\begin{array}{c}a\\X\\X^2\\X^3\end{array}\left|\begin{array}{cccc|c}1&0&1&0&3\\2&0&0&-1&0\\0&0&1&-1&0\\0&1&0&1&-2\end{array}\right| \longrightarrow \left|\begin{array}{cccc|c}1&0&0&0&-3\\0&1&0&0&4\\0&0&1&0&-6\\0&0&0&1&6\end{array}\right| \rightarrow \begin{array}{l}\bullet K_1=-3\\\bullet K_2=4\\\bullet K_3=-6\\\bullet K_4=-6\end{array}$$

الحل $= 3-2X^3=-3(1+2X)+4(X^3)+6(X^2-1)-6(-X^2+X^3-X)$

EX:- Given the vectors: $U=(2,0,1)$, $V=(11,1,5)$ and $W=(7,3,2)$

Is the vector $(5,3,2)$ a linear comb?

Sol:- $(5,3,2)=K_1(2,0,1)+K_2(11,1,5)+K_3(7,3,2)$

$$\left|\begin{array}{ccc|c}2&11&7&5\\0&1&3&3\\1&5&2&2\end{array}\right| \xrightarrow{R_3\leftarrow R_1} \left|\begin{array}{ccc|c}1&5&2&2\\0&1&3&3\\2&11&7&5\end{array}\right| \xrightarrow{R_3-2R_1} \left|\begin{array}{ccc|c}1&5&2&2\\0&1&3&3\\0&1&3&1\end{array}\right| \xrightarrow{R_3-R_2} \left|\begin{array}{ccc|c}1&5&2&2\\0&1&3&3\\0&0&0&-2\end{array}\right|$$

$0\neq 2$ = The system doesn't have a sol

Not a lin. comb of U, V, W

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Rank

التاريخ: ..... الموضوع: .....

K_2: is Free variable
The system has infinitely many solutions.
linear combination

K_1	1	2	0	0	-1
K_2	0	0	1	0	2
K_3	0	0	0	1	1

: اذا كان

ملحوظة:

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ∴ Zero vector is a linear comb of any set of vectors ←

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rank A = عدد المتجهات
independent

rank A < عدد المتجهات
depent

- $\{(1,2),(0,0)\}$ depent
- $\{(1,0,1),(0,1,0),(1,1,1),\}$ depent
- $\{(1,1,1),(2,2,2),(3,3,3)\}$ depent
- $\{(1,1),(0,1),(1,0)\}$ depent