① $2x + 3y = 4$ linear.

② $2x - 4xy = 0$ non-linear.

③ $\frac{3}{y} + \frac{2}{x} = 1$ (xy)

$3x + 2y = xy$ non-linear.

④ $\frac{3}{y} + \frac{2}{x} = 0$ (xy)

⑤ $x^2 + y^2 = 4$ non-linear

e^y | y^2 | $\sqrt{y}$ | $y^{1/2}$

⑥ $2\sin x - y = 14$ non-linear

⑦ $\cos 3x + y = -16$

linear → x, y; x, y, z; x, y, z, w.

$x^2 - x = 0 \Rightarrow x(x-1) = 0$

$x = 0, x = 1.$

① $2x - 4y = 0$ Let $y = t \in \mathbb{R}$.

$2x - 4t = 0 \Rightarrow 2x = 4t \Rightarrow x = 2t$

② $x + y + z = 1$ Let $z = t$, $y = s$

$x + s + t = 1 \Rightarrow x = 1 - s - t$

③ $3x - \frac{1}{2}y = 9$

Let $y = t \Rightarrow 3x - \frac{1}{2}t = 9$

$x = \frac{1}{3}(\quad)$ $3x = 9 + \frac{1}{2}t$

$2x + y = 4$

$\therefore x - y = 2$

$$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & 1 & 4 \end{array}\right]$$

$R_2' = R_2 - 2R_1$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & 0 \end{array}\right]$$

$3y = 0$

$y = 0$

$x - y = 2$

$x = 2$

$(x, y) = (2, 0)$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\{2, 0\}$

$1 - (-2)$

$2x = 4$

$$\left[\begin{array}{cc|c} 2 & 3 & \\ 0 & 5 & 6 \\ 0 & 3 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & 0 \end{array}\right]$$

$3y = 0$

$x = 2$

Consistent

have only one solution

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 0 & 3 \end{array}\right]$$

$0 \cdot x + 0 \cdot y = 3$

$0 \neq 3$

have no solution in curt

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$x - y = 2$

Let $y = t \Rightarrow x = 2 + t$

infinite solutions

$-x+y-z=0$

$2y+z=3$

$\frac{1}{2}z=0$

$$\left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & \frac{1}{2} & 0 \end{array}\right]$$

$z=0$

$2y=3 \Rightarrow y=3/2$

$-x=-y+z-3$

$x=3/2$

homogeneous.

$$5x_1 + 2x_2 + x_3 = 0$$

$$2x_1 + x_2 = 0$$

$$\left[\begin{array}{ccc|c} 5 & 2 & 1 & 0 \\ 2 & 1 & 0 & \end{array}\right]$$

$$5x_1 + 2t + t = 0$$

$$5x_1 = -5t \Rightarrow x_1 = -t$$

$x_1 = -t$

$$\left[\begin{array}{ccc} 5 & 2 & 1 \\ 2 & 1 & 0 \end{array}\right] \begin{matrix} R_1 \\ R_2 \end{matrix}$$

$$\left[\begin{array}{ccc} 5 & 2 & 1 \\ 0 & 1 & -2 \end{array}\right] \quad R_2' = 5R_2 - 2R_1$$

$x_3 = t$

$x_2 - 2x_3 = 0$

$x_2 = 2t$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & \\ 0 & 0 & 3 & \end{array}\right]$$

Let $x_2 = t$

$3x_3 = 0 \Rightarrow x_3 = 0$

$x_1 + 2x_2 + x_3 = 0$ (t, 0)

$x_1 = -2t$

$x_1 + x_2 + x_3 = 0$

$\checkmark x_2$ $= 0$ $\begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $x_2 = 0$

Let $x_3 = t \Rightarrow x_1 = -t$

$$5x_1 - 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 - x_3 = 7$$

$$x_1 - 11x_2 + 4x_3 = 3$$

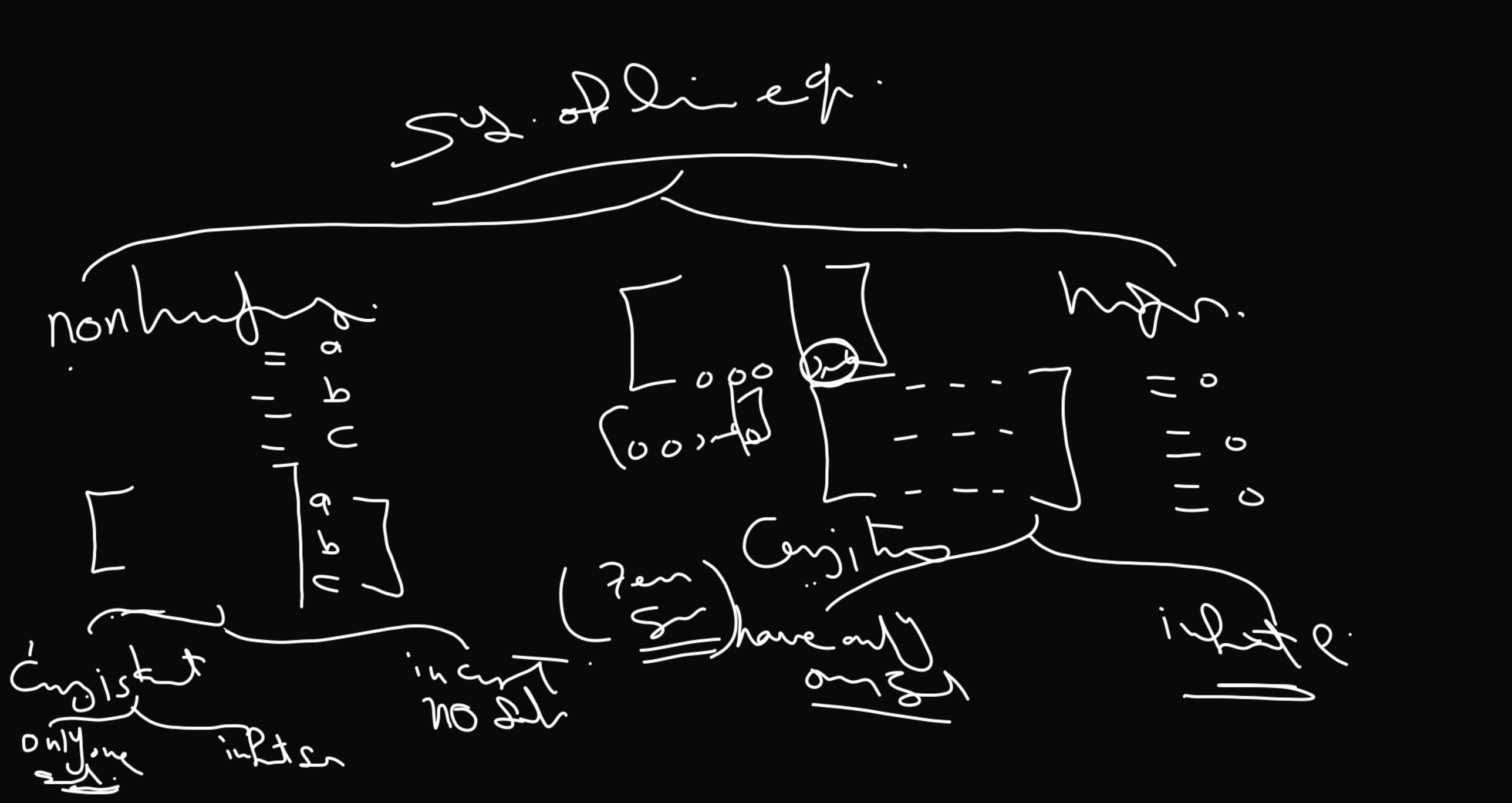
$$\left[\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 5 & -3 & 2 & 3 \\ 2 & 4 & -1 & 7 \end{array}\right] \xrightarrow{\substack{R_2' = R_2 - 5R_1 \\ R_3' = R_3 - 2R_1}} \left[\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 0 & 52 & -18 & -12 \\ 0 & 26 & -9 & 1 \end{array}\right] \xrightarrow{R_2' = \frac{1}{2}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 0 & 26 & -9 & -6 \\ 0 & 26 & -9 & 1 \end{array}\right] \xrightarrow{R_3' = R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 0 & 26 & -9 & -6 \\ 0 & 0 & 0 & 7 \end{array}\right]$$

$0x_1 + 0x_2 + 0x_3 = 7$

$0 \neq 7$

have no soln.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(2)

$xy + 3y = 9$

$\frac{1}{2} \to$

$2 \to x$

(1)

$x + 2y = 3$

$\cos 3x + 2y = 7$

$e^x + \ln y = 3$

$\sin(2x) + 3y = 7$