

أول ملخص

Math $(V(F), \oplus, \odot)$

التعبير

$$① r_1 \oplus r_2 \in V \quad \forall r_1, r_2 \in V$$

$$② (r_1 \oplus r_2) \oplus r_3 = r_1 \oplus (r_2 \oplus r_3) \quad \forall r_1, r_2, r_3 \in V$$

مؤثر

$$③ 0 \in V$$

$$④ r \in V \quad \forall r \in V$$

$$⑤ r_1 \oplus r_2 = r_2 \oplus r_1 \quad \forall r_1, r_2 \in V$$

$$⑥ \text{let } \alpha, \beta \in F$$

$$\alpha \odot r \in V \quad \forall r \in V$$

$$⑦ (\alpha + \beta) \odot r = \alpha \odot r \oplus \beta \odot r \quad \forall r \in V$$

$$⑧ \alpha \odot (r_1 \oplus r_2) = \alpha \odot r_1 \oplus \alpha \odot r_2 \quad r_1, r_2 \in V$$

$$⑨ \alpha \odot (\beta \odot r) = (\alpha \odot \beta) \odot r \quad \forall r \in V$$

$$⑩ 1 \odot r = r \quad \forall r \in V$$

Subject: _____

Date: _____

ex 1) $V = \{(x, y, z) : x + y > 0, \forall x, y, z \in V\}$

Does V be a vector space?

No, why $0 \notin V$

$$0 \in V \quad \text{مفروض}$$

ex 2) $V = \{(x, y, z) : x \geq 0, z, x, y \in V\}$

No, why? $-x \notin V$

Let $r = \{x, y, z\} \pm x \in V$ *مفروض*

$$-r = \{-x, -y, -z\} \Rightarrow -x \leq 0$$

$$-r \notin V$$

EX 3) $V = \{x, y, z\} : x = z + 1$

$y, z \in \mathbb{R}$

V is not a vector space, why?

$0 \in V$ ~~yes~~ ~~no~~

~~$y = z = 0 \Rightarrow x = 1$~~

EX 4) $V = \{x, y, z\} \quad x, y, z \in \mathbb{R}$

where $(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 + x_2, y_1 - y_2, z_1 + z_2)$
 $\alpha \odot (x, y, z) = (\alpha x, \alpha z, \alpha y)$?

$1 \cdot r \neq r$ ~~yes~~ ~~no~~ $1 \cdot r = r$ ~~yes~~ ~~no~~
 $x_1 - y_2 \neq y_2 - y_1$ ~~yes~~ ~~no~~ $y_1 + y_2 = y_2 + y_1$ ~~yes~~ ~~no~~

ثاني ملخص

Subject: Lect 2

maths 2

Vector spaces

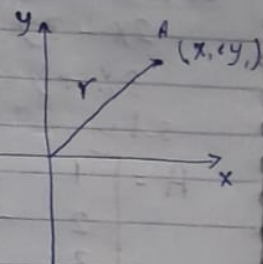
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$$\underline{r}_1 = (x_1, x_2, \dots, x_n)$$

$$\underline{r}_2 = (y_1, y_2, \dots, y_n)$$

$$* \underline{r}_1 + \underline{r}_2 = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

$$\alpha \underline{r}_1 = (\alpha x_1, \alpha x_2, \dots, \alpha x_n)$$



Def. vector space

$$\Rightarrow \underline{0} = (0, 0, \dots, 0)$$

$$(V(F), \oplus, \odot)$$

$$\textcircled{1} \underline{r}_1 \oplus \underline{r}_2 \in V \quad \forall \underline{r}_1, \underline{r}_2 \in V$$

$$\textcircled{2} (\underline{r}_1 \oplus \underline{r}_2) \oplus \underline{r}_3 = \underline{r}_1 \oplus (\underline{r}_2 \oplus \underline{r}_3) \quad \forall \underline{r}_1, \underline{r}_2, \underline{r}_3 \in V$$

$$\textcircled{3} \underline{0} \in V$$

$$\textcircled{4} -\underline{r} \in V \quad \forall \underline{r} \in V$$

$$\textcircled{5} \underline{r}_1 \oplus \underline{r}_2 = \underline{r}_2 \oplus \underline{r}_1 \quad \forall \underline{r}_1, \underline{r}_2 \in V$$

Let $\alpha, \beta \in F$

$$\textcircled{6} \alpha \odot \underline{r} \in V, \quad \forall \underline{r} \in V$$

$$\textcircled{7} (\alpha + \beta) \odot \underline{r} = \alpha \odot \underline{r} \oplus \beta \odot \underline{r}, \quad \forall \underline{r} \in V$$

$$\textcircled{8} \alpha \odot (\underline{r}_1 \oplus \underline{r}_2) = \alpha \odot \underline{r}_1 \oplus \alpha \odot \underline{r}_2, \quad \forall \underline{r}_1, \underline{r}_2 \in V$$

$$\textcircled{9} (\alpha \beta) \odot \underline{r} = \alpha \odot (\beta \odot \underline{r}), \quad \forall \underline{r} \in V$$

$$\textcircled{10} 1 \odot \underline{r} = \underline{r}, \quad \forall \underline{r} \in V$$

Date:

No:

EX: 1) $V = \{(x, y, z) : x+y > 0, \forall x, y, z \in \mathbb{R}\}$

Does V be a vector space?

Ans: No, why.

$0 \notin V$ because $x+y > 0 \Rightarrow x=y \neq 0$

2) $V = \{(x, y, z) : x \geq 0, x, y, z \in \mathbb{R}\}$

V is not a vector space, why?

Ans: $-r \in V$ because: let $r = (x, y, z) \in V$
 $\Rightarrow x \geq 0$.

$-r = (-x, -y, -z) \Rightarrow -x \leq 0$.

$\therefore -r \notin V$.

3) $V = \{(x, y, z) : x = z+1, y, z \in \mathbb{R}\}$

V is not a vector space, why?

Ans: $0 \notin V$.

$x = z+1 \Rightarrow z=0, x=1$.

4) $V = \{(x, y, z) : x, y, z \in \mathbb{R}\}$ where

$$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 + x_2, y_1 - y_2, z_1 + z_2)$$

في الحالات في نستبعد الشرط رقم 6، 1

$$\alpha \odot (x, y, z) = (\alpha x, \alpha z, \alpha y)$$

Ans: take $\alpha = 1 \Rightarrow 1 \odot r \neq r$.

$$1 \odot (x, y, z) = (x, y, z) \neq (x, z, y)$$

Date: lect 2 algebra

No: maths 2

Ans 2 let $(x_1, y_1, z_1), (x_2, y_2, z_2) \in V$
 $(x_2, y_2, z_2) \oplus (x_1, y_1, z_1) =$

$(x_2 + x_1, y_2 - y_1, z_2 + z_1) \neq (x_1, y_1, z_1) \oplus (x_2, y_2, z_2)$.
Commutative Property is not satisfied.

5 $V = \{(x, y, z) : x = z^2, y, z \in \mathbb{R}\}$.

Ans:- $-x \notin V$ because $x = z^2 \geq 0$

6 $V = \{(x, y, z) : x = y + z + 1, y, z \in \mathbb{R}\}$.

Ans:- $\underline{r}_1 = (x, y, z) = (y + z + 1, y, z)$.

$\underline{r}_2 = (a, b, c) = (b + c + 1, b, c)$.

$\underline{r}_1 + \underline{r}_2 = (x + z + b + c + 2, y + b, z + c) \notin V$.

ثالث ملخص

(الأخير)

Page: _____

Date: / /

Lesson 2: Vector Spaces.

Def: Any thing that could be defined (numbers, functions...) if it achieves the following conditions.

1- $x + y \in V$

3- $x + y = y + x$

5- for all x in V , there is an object $(-x)$ in V called negative of x such that $x + (-x) = (-x) + x = 0$

6- $C \odot x$ should be in V

7- $C \cdot (x + y) = Cx + Cy$

8- $(C_1 + C_2) \cdot x = C_1x + C_2x$

9- $C_1(C_2x) = C_1C_2x$

10- $x \cdot 1 = x$

* نفرض أن x, y, z في V

2- $x + (y + z) = (x + y) + z$

4- يجب أن يوجد المتجه الصفري " 0_V "

حيث أن C أي عدد صحيح

* $\mathbb{R} \rightarrow$ the real numbers

* $\mathbb{R}^2 \rightarrow$ the vectors in 2 dimensions

* $\mathbb{R}^3 \rightarrow$ " " " 3 "