

### Exercises of Chapter 3

1) Find the duality form of this linear problem

$$\text{Max } z = 7x_1 + 10x_2$$

s. to:

$$3x_1 + 2x_2 \leq 36$$

$$2x_1 + 4x_2 \leq 40$$

$$x_1 \leq 10$$

$$x_1, x_2 \geq 0$$

2) Find the duality form of this linear problem

$$\text{Max } z = 3x_1 + 2x_2 + 4x_3$$

s. to:

$$5x_1 - 4x_2 - x_3 \leq 25$$

$$4x_1 - 4x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

3) Look at this problem carefully:

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

Subject to:

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

and the optimal solution is as follows:

The optimal solution is:

Basic Solution	X1	x2	x3	x4	RHS
Obj.Function line	2	0	0	3	16
x3	3/4	0	1	-1/4	2
x2	1/4	1	0	1/4	2

Find the duality form and the optimal solution from the above tableau.

4) Max  $Z = 5x_1 + 3x_2$

Subject to:

$$4x_1 + 2x_2 \leq 20$$

$$4x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

- Find the duality form
- Find the optimal solution to the primal problem using simplex method
- Find the optimal solution of the duality using simplex method
- Find the optimal solution of the duality using (b)
- Compare (c) and (d).

5) Max  $Z = 3x_1 + 2x_2 + 4x_3$

Subject to:

$$5x_1 + 2x_2 - x_3 \leq 25$$

$$2x_1 - 4x_2 + 6x_3 \leq 50$$

$$x_1, x_2 \geq 0$$

- Find the duality form
- Find the optimal solution to the primal problem using simplex method
- Find the optimal solution of the duality using simplex method
- Find the optimal solution of the duality using (b)
- Compare (c) and (d).

6) Min  $Z = 6x_1 + 4x_2$

Subject to:

$$3x_1 + 4x_2 \geq 60$$

$$x_1 + 2x_2 \geq 20$$

$$3x_1 + 2x_2 \geq 48$$

$$x_1, x_2 \geq 0$$

- Solve the problem by the simplex method
- Find the shadow prices.
- What happen if we change first resource up by 10.

- d) What happen if we change first resource down by 12.
- e) What happen if change the second resource up by 20.
- f) What happen if change the second resource down by 20.
- g) What happen if change the third resource up by 22.
- h) What happen if change the third resource down by 48.
- i) What happen if change the return value of  $X_1$  to 3.
- j) What happen if change the return value  $X_2 = 5$ .
- k) What happen if add a new constraint  $2X_1 + 2X_2 \geq 40$ .
- l) What happen if add a new constraint  $2x_1 + 2x_2 \geq 20$ .