

## ECE 414 Final Project Spring 2019

The problem to be solved in this project is to design a control system that balances and rolls a ball anywhere along a track. The track is controlled by tilting the track about its center point using a permanent magnet brush DC motor coupled to the track through a gearbox having a gear ratio  $N$  between 10 and 50. The position of the ball on the track is measured by a touch sensor that provides  $K_s = 0.1\text{V/cm}$ , quantized every mm. The angle of the track is measured by an optical encoder on the motor shaft that quantizes position into 1024 counts per revolution.

1. The rolling ball is made of either 316 stainless steel or of 1060 aluminum and must be able to move from any fixed position on the 1060 aluminum track to any other fixed position on the track in minimum time while consuming minimum energy. The track is 50cm long and has a cross sectional shape as shown in the attached drawing. The ball is also shown.

2. The parts you have to build the system are:

$J_g$	gearbox inertia	$6.2\text{e-}6\text{ kg-m}^2$ (as seen from the motor)
$J_s$	angular sensor inertia	$1.4\text{e-}7\text{ kg-m}^2$
$G_v$	Voltage amplifier gain	$5\text{ V/V}$ (control signal gain to motor)

Use engineering judgment and choose one of the following motors:

### Motor #1

$J_m$	Motor inertia	$5.0\text{e-}5\text{ kg-m}^2$
$B_m$	Motor viscous friction	$3.0\text{e-}6\text{ Nm-s/rad}$
$K_T$	Motor Torque constant	$0.225 \pm 10\%\text{ Nm/A}$
$R$	Motor Resistance	$8\ \Omega$
$L$	Motor Inductance	$25\text{ mH}$

### Motor #2

$J_m$	Motor inertia	$5.0\text{e-}5\text{ kg-m}^2$
$B_m$	Motor viscous friction	$3.0\text{e-}6\text{ Nm-s/rad}$
$K_T$	Motor Torque constant	$0.175 \pm 10\%\text{ Nm/A}$
$R$	Motor Resistance	$6\ \Omega$
$L$	Motor Inductance	$16\text{ mH}$

### Motor #3

$J_m$	Motor inertia	$5.0\text{e-}5\text{ kg-m}^2$
$B_m$	Motor viscous friction	$3.0\text{e-}6\text{ Nm-s/rad}$
$K_T$	Motor Torque constant	$0.125 \pm 10\%\text{ Nm/A}$
$R$	Motor Resistance	$4\ \Omega$
$L$	Motor Inductance	$7.5\text{ mH}$

### Motor #4

$J_m$	Motor inertia	$5.0\text{e-}5\text{ kg-m}^2$
$B_m$	Motor viscous friction	$3.0\text{e-}6\text{ Nm-s/rad}$
$K_T$	Motor Torque constant	$0.275 \pm 10\%\text{ Nm/A}$
$R$	Motor Resistance	$12\ \Omega$
$L$	Motor Inductance	$32\text{ mH}$

### Grading:

Presentation	30%	(how well you present your work)
Engineering	50%	(your engineering work)

Work Notebook  
Total

20% (engineering notebook contents)  
100%

Documentation requirements:

1. Start an engineering work notebook. This can be a 3 ring binder so both partners can add material as needed. This notebook must have each page dated and numbered per day, *e.g.*, April 11, 2019, page 3 of 4. The notebook must be in chronological order. This notebook is where ALL of your work on the project gets documented, including random thoughts, dead ends, brainstorming, etc.. This is what you WILL do once working. If you do some analysis and later find that it is in error, come back and cross it out, write a brief explanation explaining the error and date it. Never take anything out of this notebook. Engineers learn more from their mistakes and the dead ends they encounter while working on a project, so these errors must be kept. If you make copies of anything, or get valuable material out of a book, copy it and put it in the notebook. If you generate output from MATLAB or SIMULINK, place it in your notebook. When you refer to prior work, identify the date and page, *e.g.*, April 11, 2019, page 2. Write on the figures to annotate them. The notebook is meant to be free flowing and informal, but readable to any engineer. The rule of thumb is that another engineer who is not working on the project should be able to pick up your notebook and duplicate what you did in the order in which you did it. These notebooks often play a key role in getting and defending a patent.
2. Show that the system performs as desired. This is demonstrated by simulation plots of your choosing. Be sure to impress your boss with fewer high quality plots rather than tons of low quality plots.
3. Description of the engineering reasons for the gear ratio you chose.
4. Description of the engineering reasons for choosing the motor you did.
5. Description of how your controller was chosen and how its parameters were determined.
6. Plot of the nominal plant pole locations, controller zero and pole locations, and the overall closed loop transfer function poles and zeros on s-plane plots.
7. A detailed analytic (*e.g.*, no numbers, just variables) signal flow graph of the plant and controller.
8. Analytic (*e.g.*, no numbers) transfer functions for the plant and controller.

All items 2 through 8 above should be documented in your engineering work notebook as they progress from the start of your work until the end. All final results are to be written in a "final report" that is not to exceed the equivalent of 10 single spaced typed pages excluding figures. No more than 10 additional pages of figures are allowed. Plots must have axes labeled and must be described in the body of your report. All figures must have captions at the top or bottom.

Motor electrical dynamics:

$$v = Ri + L \frac{di}{dt} + K_e \dot{\theta}_m$$

where  $v$  is the motor input voltage, which is related to the control input signal  $u$  by  $v = G_v u$ ,  $i$  is the motor current,  $K_e$  is the back EMF constant, and  $\theta_m$  is the motor shaft position.

Motor mechanical dynamics:

$$T_m = K_T i$$

where  $T_m$  is the motor torque,  $K_T$  is the torque constant, and  $i$  is the motor current.

Mechanical system dynamics:

$$J_{eff} \ddot{\theta}_m + B_m \dot{\theta}_m = T_m$$

where  $J_{eff}$  is the effective inertia seen by the motor and  $B_m$  is the motor viscous friction.

Gearbox relationship:

$$\theta = \frac{\theta_m}{N}$$

where  $N$  is the gearbox ratio.

Ball and track dynamics:

$$\left(m + \frac{J_b}{r^2}\right) \ddot{x} + \left(\frac{J_b}{r}\right) \ddot{\theta} - m x \dot{\theta}^2 = m g \sin(\theta)$$

where  $m$  is the mass of the ball,  $J_b = (2/5)mR^2$  is the ball inertia,  $g$  is the acceleration due to gravity, and  $x$ ,  $r$ ,  $R$ , and  $\theta$  are as defined in the drawings.