Analysis of Dependence of Options Prices on Model Parameters

M Sidik Augi Rahmat

1 A Useful Identity for Black-Scholes Option Prices

Before we begin analysis parameters of option, we derive some useful identity for Black-Scholes equation. Note from this we denote V^{ec} as European option call and V^{ep} as European option put :

$$V^{ec}(S,t) = Se^{-q(T-t)}N(d_1) - Ee^{-r(T-t)}N(d2) \tag{1}$$

$$V^{ep}(S,t) = Ee^{-r(T-t)}N(-d2) - Se^{-q(T-t)}N(-d1) \tag{2}$$

where

$$d_1 = \frac{ln\frac{S}{E} + (r - q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{3}$$

We begin with computing the difference $(d_1^2 - d_2^2)/2$. Since $d_2 = d_1 - \sigma\sqrt{T-t}$ we obtain

$$\frac{d_1^2 - d_2^2}{2} = \frac{(d_1+d_2)(d_1-d_2)}{2} = \frac{2ln\frac{S}{E} + 2(r-q)(T-t)\sigma\sqrt{T-t}}{\sigma\sqrt{T-t}2} \tag{4}$$

$$= ln\frac{S}{E} + (r-q)(T-t) \tag{5}$$

and hence

$$\frac{d_1^2}{2} = \frac{d_2^2}{2} + ln\frac{S}{E} + (r-q)(T-t) \tag{6}$$

For derivative of cumulative distribution function $N'(d)$ of the standardized normal distribution we have

$$N'(d) = \frac{1}{\sqrt{2\pi}}exp(-d^2/2) \tag{7}$$

Using the above identity for the difference $(d_1^2 - d_2^2)$ we finally obtain an important identity:

$$Se^{-q(T-t)}N'(d_1) - Ee^{-r(T-t)}N'(d_2) = 0 \tag{8}$$

2 Delta of an Option

The basic sensitivity factor, which is often evaluated when analyzing the market data, is dependece of a change of derivative price with respect to a change of the price with respect to a change of the price og the underlying asset stock. In the infinitesimal form this factor can be written as partial derivative:

$$\Delta = \frac{\partial V}{\partial S} \tag{9}$$

For European call and put options we are able to derive an explicit formulae for the factor Δ. We differentiate the functions V^{ec} and V^{ep} with respect to S. For a call option, it the relationship $\partial d_1/\partial S = \partial d_2/\partial S$ and using the identity (8) we got:

$$\Delta^{ec} = \frac{\partial V^{ec}}{\partial S} = Se^{-q(T-t)}N'(d_1)\frac{\partial d_1}{\partial S} - Ee^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S} \tag{10}$$

$$+ e^{-q(T-t)}N(d_1) = e^{-q(T-t)N(d_1)} \tag{11}$$

for a European put we obtain:

$$\Delta^{ep} = \frac{\partial V^{ec}}{\partial S} = Se^{-q(T-t)}N'(-d_1)\frac{\partial d_1}{\partial S} - Ee^{-r(T-t)}N'(-d_2)\frac{\partial d_2}{\partial S} \tag{12}$$

$$- e^{-q(T-t)}N(-d_1) = -e^{-q(T-t)}N(-d_1) \tag{13}$$