SDE Simulation and Statistic

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Summarised: DATE

1 Brownian Motion

A stochastic process is a t-parametric system of random variables $\{X(t), t \in I\}$ where I is and interval of reals or a discrete set of indices. Or as its easy stochastic process is when random variable depends on some parameter, it could be time or another. One of example of stochastic process is Brownian motion. A Brownian motio $\{X(t), t \geq 0\}$ is a t-parametric system of random variables, for which satisfies the following properties:

- $X_0 = 0$
- The Increment are stationary and independent.
- It is a martingale. (Martingale is pretty hard because need to know probability, measusere space and stuff. But basically martingale is a concept where we double the bet if we lose in gambling. It because the random variable is independent each other.)
- It has continuos path, but nowhere differentiable
- $X_t - X_s \approx \mathcal{N}(0, t-s)$ for $t \geq s \geq 0$. It means at the time $t > s$ the variable X has a normal probability distribution.

A Brownian motion with parameters $\mu = 0$, $\sigma^2 = 1$ is called Wiener process. In our simulation, each increment is such that:

$$X_{t_i+\Delta t} - X_{t_i} = \Delta X_i \approx \mathcal{N}(\mu \Delta t, \sigma^2 \Delta t) \tag{1}$$

The process at time T is given by $X_T = \Sigma_i \Delta X_i$ and follows the distribution:

$$X_T \approx \mathcal{N}(\mu T, \sigma^2 T) \tag{2}$$

A Brownian motion $\{X(t), t \geq 0\}$ can be characterized by its deterministic and fluctuatuing components. Its Increments $dX(t)$ can be expressed in the following form of total differential

$$dX(t) = \mu dt + \sigma dW(t) \tag{3}$$

where $dW(t)$ is Wiener process. Equation **??** is called stochastic differential equation.

2 Geometric Brownian Motion

If $\{X(t), t \geq 0\}$ is a Brownian motion with parameters μ, σ and $y_o \in R^+$, then the system of random variables $\{Y(t), t \geq 0\}$

$$Y(t) = y_0 e^{X(t)}, t \geq 0 \tag{4}$$

is called geometric Brownian motion. Expectef value and variance for geometric Brownian motion is defined basically

$$E(Y(t)) = y_0 e^{ut+\frac{\sigma^2 t}{2}}, \quad Var(Y(t)) = y_0^2 e^{2\mu t+\sigma^2 t}(e^{\sigma^2 t} - 1) \tag{5}$$