CONIC SECTIONS

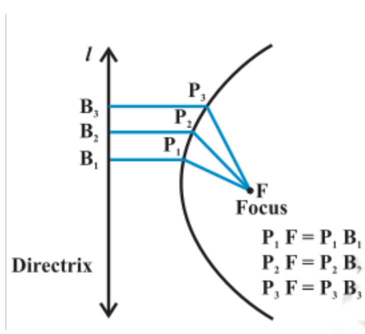
Circle

A circle is the set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is the centre and the fixed distance is the radius

- Equation of a circle with centre origin and radius r is $x^2 + y^2 = r^2$.
- Equation of a circle with centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$
- General form of the equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

Parabola

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane. The fixed line is called the **directrix** of the parabola and the fixed point F is called the **focus**.

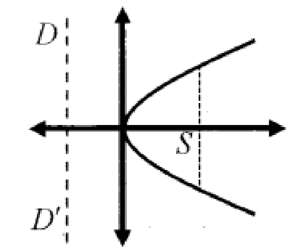


A line through the focus and perpendicular to the directrix is called the **axis** of the parabola. The point of intersection of parabola with the axis is called the **vertex** of the parabola.

Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola

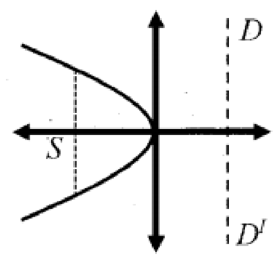
Standard equations of parabola

1. $y^2 = 4ax$



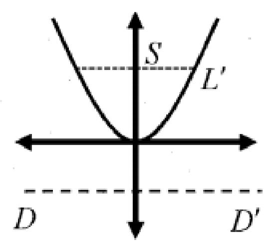
Vertex: (0, 0)
Focus(S): (a, 0)
Length of Latus rectum: (LL') = 4a
Equation of directrix (DD') is x = -a

2. $y^2 = -4ax$



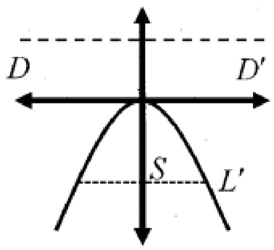
Vertex: (0, 0)
Focus(S): (-a, 0)
Length of Latusrectum (LL') = 4a
Equation of directrix (DD') is x = a

3. $x^2 = 4ay$



Vertex: (0, 0)
Focus(S): (0, a)
Length of Latusrectum (LL') = 4a
Equation of directrix (DD') is y = -a

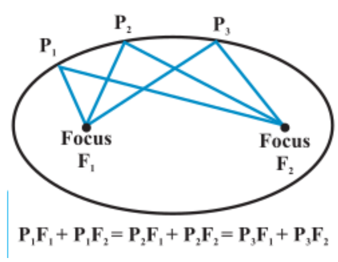
4. $x^2 = -4ay$



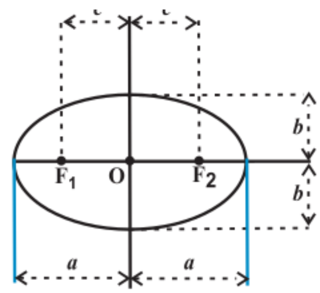
Vertex: (0, 0)
Focus(S): (0, -a)
Length of Latusrectum (LL') = 4a
Equation of directrix (DD') is y = a

Ellipse

An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

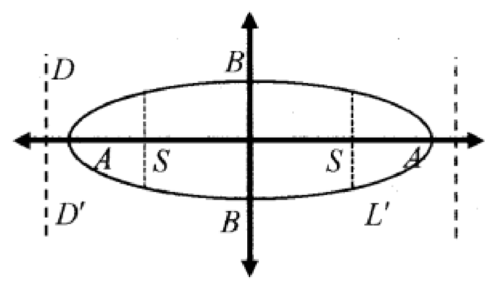


- The two fixed points are called the **foci** (plural of 'focus') of the ellipse(F_1, F_2)
- The mid point of the line segment joining the foci is called the **centre** of the ellipse.(O)
- The line segment through the foci of the ellipse is called the **major axis** and the line segment through the centre and perpendicular to the major axis is called the **minor axis**.
- The end points of the major axis are called the **vertices** of the ellipse
- The **eccentricity** of an ellipse is the ratio of the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse (eccentricity is denoted by e) i.e., $e = \frac{c}{a}$
- if 2a is the length of major axis and 2b is the length of minor axis and c is the focal lenth then $a^2 = b^2 + c^2$



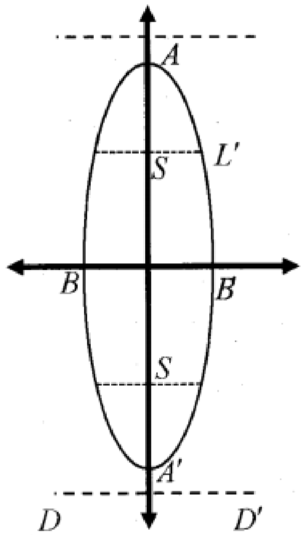
Standard equations of an ellipse

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$



- Eccentricity, $e = \frac{\sqrt{a^2-b^2}}{a}$
- Length of Latusrectum (LL') = $\frac{2b^2}{a}$
- Focii, S(ae, 0) and S'(-ae, 0) or S(c, 0), S'(-c, 0)
- Centre (0, 0)
- Vertices A(a, 0) and A'(-a, 0)
- Equation of directrix (DD') is $x = \frac{a}{e}$ and $x = \frac{-a}{e}$
- Length of major axis (AA') = 2a
- Length of minor axis'(BB') = 2b

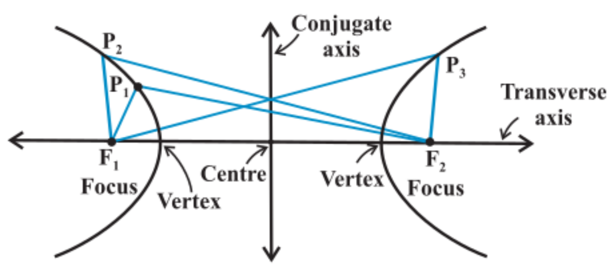
2. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a > b$



- Eccentricity, $e = \frac{\sqrt{a^2 - b^2}}{a}$
- Length of Latus rectum (LL') $= \frac{2b^2}{a}$
- Focii, S(0, ae) and S'(0, -ae) or S(0, c), S'(0, -c)
- Centre (0, 0)
- Vertices A(0, a) and A'(0, -a)
- Equation of directrix (DD') is $y = \frac{a}{e}$ and $y = \frac{-a}{e}$
- Length of major axis (AA') = 2a
- Length of minor axis (BB') = 2b

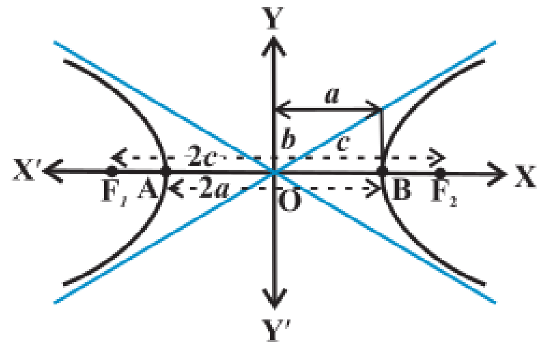
Hyperbola

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.



$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$

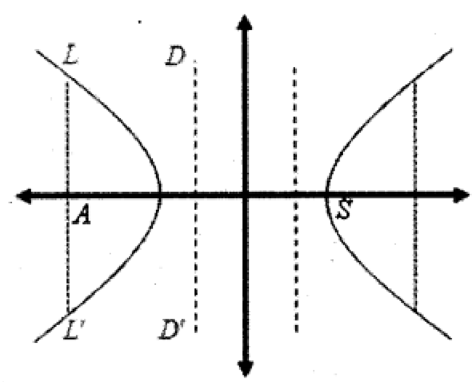
- The two fixed points are called the **foci** of the hyperbola
- The mid-point of the line segment joining the foci is called the **centre** of the hyperbola
- The line through the foci is called the **transverse axis** and the line through the centre and perpendicular to the transverse axis is called the **conjugate axis.**
- The points at which the hyperbola intersects the transverse axis are called the **vertices** of the hyperbola



- We denote the distance between the two foci by 2c, the distance between two vertices (the length of the transverse axis) by 2a and we define the quantity b as $b = \sqrt{c^2 - a^2}$

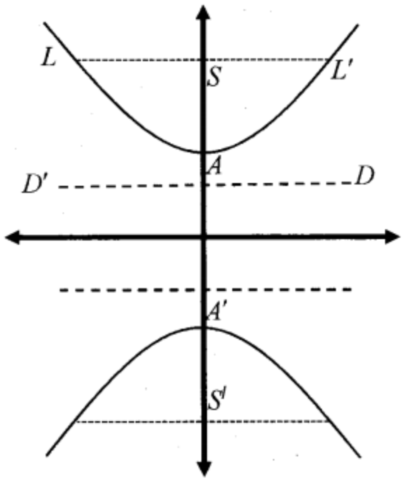
Standard equation of Hyperbola

1. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



- Focii, S(ae, 0) and S'(-ae, 0) or S(c, 0), S'(-c, 0)
- Centre (0, 0)
- Vertices A(a, 0) and A'(-a, 0)
- Equation of directrix (DD') is $x = \frac{a}{e}$ and $x = \frac{-a}{e}$

2. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



- Eccentricity, $e = \frac{\sqrt{a^2+b^2}}{a}$
- Length of Latus rectum (LL') = $\frac{2b^2}{a}$
- Focii, S(0, ae) and S'(0, -ae) or S(0, c), S'(0, -c)
- Centre (0, 0)
- Vertices A(0, a) anti A'(0, -a)
- Equation of directrix (DD') is $y = \frac{a}{e}$ and $y = \frac{-a}{e}$