Probability

- An experiment is said to be a **random experiment** if there is more than one possible outcome, and it is impossible to predict the outcome in advance.An experiment is said to be a random experiment if there is more than one possible outcome, and it is impossible to predict the outcome in advance.
- All possible results of an experiment are called its **outcomes**.
- Set of all these outcomes is known as the **sample space** and is denoted by 'S'.
- Any subset E of a sample space S is called an **event**.
- the event E of a sample space S is said to have **occurred** if the outcome of the experiment is such that $\omega \in E$. If the outcome ω is such that $\omega \notin E$, we say that the event E has **not occurred**.

Types of events

1. **Impossible event and sure event**:The empty set ϕ and the sample space S describe the impossible event and sure event respectively.
2. **Simple event** : An event E having only one sample point of a sample space.
3. **Compound event**: An event having more than one sample point of a sample space.

Example:

Consider the experiment of rolling a die. The associated sample space is

$$S = \{1, 2, 3, 4, 5, 6\}$$

- The event " the number appears on the die is a multiple of 7" is impossible (E=ϕ)
- The Event "the number turns up is odd or even" is sure ($E = \{1, 2, 3, 4, 5, 6\}$)
- The Event" the number appears on the die is an odd prime" is simple(E=$\{2\}$)
- The Event" the number appears on the die is even" is compound (E=$\{2,4,6\}$)

Algebra of events

Let A, B, C be events associated with an experiment whose sample space is S.Then

1. Event 'not A' = A'
2. Event 'A or B' = $A \cup B$
3. Event 'A and B' = $A \cap B$
4. Event 'A but not B' = A − B =$A \cap B'$
5. Event 'Neither A nor B' ='not A and not B' =$A' \cap B'$=$(A \cup B)'$

Let $A, B, E_1, E_2, \ldots, E_n$ are events of an experiment .Then

- If $A \cap B = \phi$, then A and B are mutually exclusive events or disjoint events.
- If $E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n = S$, then we say that $E_1, E_2, E_3, \ldots, E_n$ are exhaustive events.
- If $E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n = S$, and $E_i \cap E_j = \phi$, $i \neq j$ then we say that $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive events and exhaustive events.

Axiomatic Approach to Probability

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms

(i) For any event E, $P(E) \geq 0$

(ii) P(S) = 1

(iii) If E and F are mutually exclusive events, then P($E \cup F$) = P(E) + P(F).

Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$, i.e., $S = \{\omega_1, \omega 2, \ldots, \omega_n\}$ It follows from the axiomatic definition of probability that
(i) $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
(ii) $P(\omega_1) + P(\omega_2) + \cdots + P(\omega_n) = 1$
(iii) For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$.

Probability of an event

Let S is a sample space and E be an event, such that n(S) = n and n(E) = m. If each outcome is equally likely, then it follows that $P(E) = \frac{m}{n}$.

1. If A and B are any two events, then $P(A \cup B) = P(A) + P(B) – P(A \cap B)$
2. If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$, $P(A \cap B) = \phi$
3. If A is any events, then P(A′) = 1 − P(A)

Permutations and Combinations

Fundamental Principle of Counting

If an event can occur in ‘m’ different ways, following which another event can occur in ‘n’ different ways, then the total number of occurrences of the events in the given order is m × n.

Example : Mohan has 3 pants and 2 shirts.Then there are $3 \times 2 = 6$ pairs of pant and shirt

Permutation

A permutation is the arrangement of some or all of a number of different objects.

Factorial notation

The notation n! represents the product of first n natural numbers,

$$\begin{aligned} n! &= 1.2.3\dots n \\ &= n(n-1)! \\ &= n(n-1)(n-2)! \\ &= n(n-1)(n-2)(n-3)! \end{aligned}$$

- 1!=1
- 0!=1

Theorem

The number of permutation of ‘n’ different objects taken ‘r’ at a time, where the objects do not repeat is n(n − 1)(n − 2)......(n − r + 1) which is denoted by ${}^nP_r$

- ${}^nP_r = \frac{n!}{(n-r)!}, 0 < r \leq n$
- ${}^nP_n = n!$
- ${}^nP_0 = 1$
- ${}^nP_1 = n$

Theorem

The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

Permutation when all the objects are not distinct.

1. The number of permutations of ‘n’ objects, where ‘p’ objects are of the same kind and rest all different $= \frac{n!}{p!}$
2. The number of permutations of ‘n’ objects, where ‘p_1’ objects are of one kind, ‘p_2’ objects are of the second kind, ..., ‘p_k‘ objects are of a k^{th} kind and rest all different $= \frac{n!}{p_1! \times p_2! \times \dots p_k!}$

Combinations

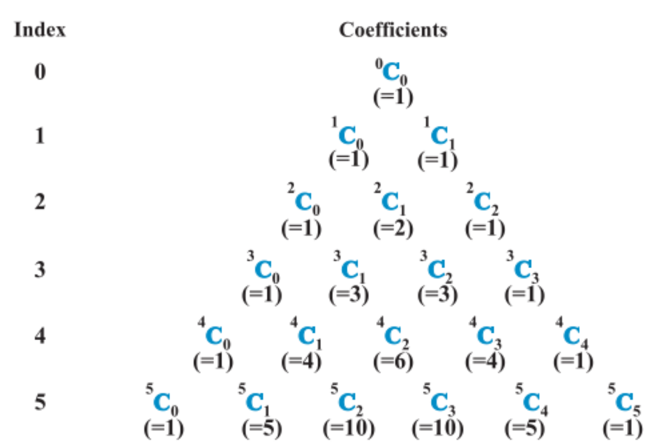
A combination is a selection of some or all of a number of different objects (the order of selection is not important). The number of selection of ‘n’ things taken ‘r’ at a time is ${}^nC_r = \frac{n!}{(n-r)!r!}$

- ${}^nP_r = {}^nC_r.r!$
- ${}^nC_r = \frac{n!}{(n-r)!}, 0 < r‘n$
- ${}^nC_n = {}^nC_0 = 1$
- ${}^nC_1 = 1$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

Binomial Theorem

The expansion of a binomial for any positive integral ‘n’ is given by

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + {}^nC_nb^n$$



Pascal’s triangle

Observations

- The notation $\Sigma_{k=0}^{n}{}^{n}C_k a^k b^{n-k}$ stands for ${}^{n}C_0 a^n +^n C_1 a^{n-1} b +^n C_2 a^{n-2} b^2 + \cdots +^n C_n b^n$
- The general term in the expansion is $t_{r+1} =^n C_r a^{n-r} b^r$
- The coefficients ${}^{n}C_r$ occuring in the binomial theorem are known as binomial
- There are (n+1) terms in the expansion of $(a+b)^n$, i.e., one more than the index coefficients.
- Middle term in the expansion is $\left(\frac{n}{2}+1\right)^{th}$ term if n is even , $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{b_t}$ term if n is odd