
STRAIGHT LINES

Distance between two points

Distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

midpoint of line joining two points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope of a line

The slope of a line is the '*tan*' of the angle the line makes with the positive direction of the x-axis. If θ is the angle then, slope = $\tan\theta$.

The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$

- The slope of the x-axis is zero and that of the y-axis is not defined.
- Parallel lines have the same slope.
- The product of the slopes of perpendicular lines is -1.
- The slope is positive if $\theta < 90^\circ$. The slope is negative if $\theta > 90^\circ$.
- If three points A, B, and C are collinear, then AB and BC have the same slope.
- If m_1 and m_2 be slopes of two lines then, θ the angle between is given by $\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$, $1 + m_1 m_2 \neq 0$

Equation of a line

- Equation of x-axis is $y = 0$.
- Equation of y-axis is $x = 0$.
- The equation of a horizontal line is $y = a$. If ' a ' is positive then the line is above the x-axis and if negative it will be below the x-axis.
- The equation of a vertical line is $x = a$. If ' a ' is positive then the line is to the right of the x-axis and if negative it will be to the left of the x-axis.

Point-slope form

$y - y_1 = m(x - x_1)$, where ' m ' is the slope and (x_1, y_1) is a point on the line.

Two-Point form

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ where (x_1, y_1) and (x_2, y_2) are two point on the line.

Slope intercept form

1. $y = mx + c$, where m is the slope and c is the y-intercept. 2. $y = m(x - d)$, where m is the slope and d is the x-intercept.

Intercept form

$\frac{x}{a} + \frac{y}{b} = 1$, where a and b are x and y intercept respectively.

Normal form

$x \cos\theta + y \sin\theta = p$, where p is the length of the normal from the origin to the line and θ is the angle the normal makes with the positive direction of the x-axis.

General Equation of a line

General equation of a Line: $ax + by + c = 0$, where a , b and c are real constants.

- x intercept of the line $ax + by + c = 0$ is $-\frac{c}{a}$
- y intercept of the line $ax + by + c = 0$ is $-\frac{c}{b}$
- Slope of the line $ax + by + c = 0$ is $-\frac{a}{b}$
- Parallel lines differ in constant term, i.e; a line parallel to $ax + by + c = 0$ is $ax + by + k = 0$.
- A line perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$.
- The equation of the family of lines passing through the intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is of the form $a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$.

-
- The perpendicular distance of a point (x_1, y_1) from the line $ax + by + c = 0$ is $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$
 - The distance between the parallel lines $ax + by + c = 0$ and $ax + by + k = 0$ is

$$\left| \frac{c-k}{\sqrt{a^2+b^2}} \right|$$

- Normal form of the equation $ax + by + c = 0$ is $x \cos \theta + y \sin \theta = p$; where $\cos \theta = \pm \frac{a}{\sqrt{a^2+b^2}}$: $\sin \theta = \pm \frac{b}{\sqrt{a^2+b^2}}$ and $p = \pm \frac{c}{\sqrt{a^2+b^2}}$