SEQUENCES AND SERIES

A **sequence** can be regarded as a function whose domain is the set of natural numbers or some subset of it of the type $\{1, 2, 3, \ldots.., k\}$. Generally denoted by $a_1, a_2, \ldots, a_n, \ldots$

Let $a_1, a_2, \ldots, a_n, \ldots$ be a sequence. Then the expression $a_1 + a_2 + \cdots + a_n + \ldots$ is called the **series** associated with the given sequence.

Arithmetic Progression (A.P.)

A sequence $a_1, a_2, \ldots, a_n, \ldots$ is called an **arithmetic sequence** or arithmetic progression if $a_{n+1} = a_2 + d, n \in N$, where a_1 is called the first term and the constant term d is called the common difference of the AP.

- $a, a+d, a+2d, \ldots$ where a is the first term and d is a common difference is the **standard form** of AP
- If a constant is added to each term of an AP, the resulting sequence is also an AP.
- If a constant is subtracted to each term of an AP, the resulting sequence is also an AP.
- If each term of an AP is multiplied by a constant k, the resulting sequence is also an AP. But the resulting AP will have a common difference kd.
- If each term of an AP is divided by a constant k, the resulting sequence is also an AP. But the resulting AP will have a common difference $\frac{d}{k}$.
- n^{th} **term (general term)** of the A.P. is

$$a_n = a + (n-1)d$$

- Sum of first n terms of the AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a_1 + a_n]$$

- Arithmetic mean between a and b is $\frac{a+b}{2}$

Geometric Progression(G.P)

A sequence $a_1, a_2, \ldots, a_n, \ldots$ is called **Geometric sequence** or Geometric progression if $\frac{a_{k+1}}{a_k} = r, k \geq 1$, where a_1 is called the first term and the constant term r is called the common ratio of the GP

- $a, ar, ar^2, ar^3, \ldots$ where a is the first term and r is the common ratio is the **standard form** of GP
- n^{th} term of the GP is $t_n = ar^{n-1}$
- Sum of n terms

$$S_n = \frac{a(1-r^n)}{1-r}, r < 1$$

$$S_n = \frac{a(r^n-1)}{r-1}, r > 1$$

- Geometric mean between a and b is $\sqrt{ab}$
- Arithmetic mean $\geq$ Geometric mean
- Sum of an infinite GP $a_1, a_2, \ldots, a_n, \ldots$ is

$$S_\infty = \frac{a}{1-r}$$

Sum to n terms of Special Series

- $1 + 2 + 3 + 4 + \cdots = \frac{n(n+1)}{2}$
- $1^2 + 2^2 + 3^2 + 4^2 + \cdots = \frac{n(n+1)(2n+1)}{6}$
- $1^3 + 2^3 + 3^3 + 4^3 + \cdots = [\frac{n(n+1)}{2}]^2$