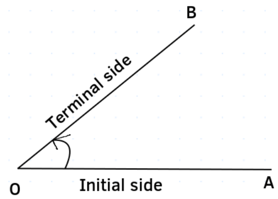
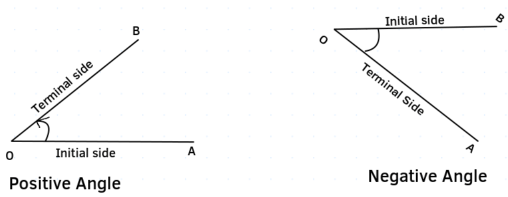
Triganometric Functions

Angle: The rotation of a ray from one position to another along the circumference of a circle is known as an angle. The initial position of the ray is called initial side and the final position of the ray is called terminal side.



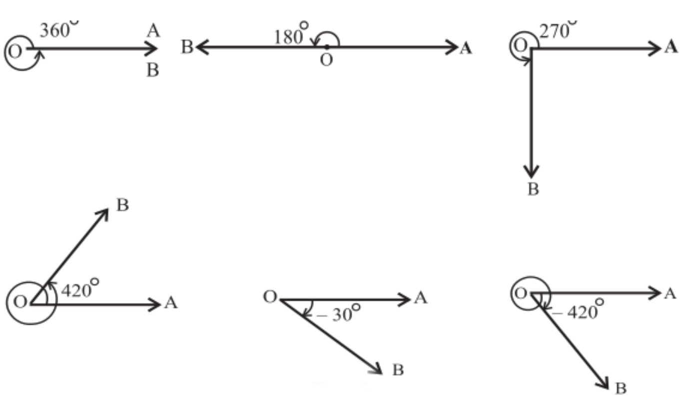
Positive Angle: If the rotation of a ray from one position to another is in anti-clockwise, then the angle is known as positive angle.

Negative Angle: : If the rotation of a ray from one position to another is in anti-clockwise, then the angle is known as positive angle.

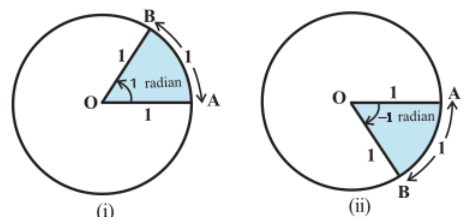


Degree Measure: If a rotation from the initial side to terminal side is $(\frac{1}{360})^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1. A degree is divided into 60 minutes, and a minute is divided into 60 seconds . One sixtieth of a degree is called a minute, written as $1'$, and one sixtieth of a minute is called a second, written as 1". Thus,

$$1 = 60', 1' = 60"$$



Radian Measure: Angle subtended at the centre by an arc of length 1 unit in a unit circle (circle of radius 1 unit) is said to have a measure of 1 radian. In the Fig (i) and (ii), OA is the initial side and OB is the terminal side. The figures show the angles whose measures are 1 radian, -1 radian



Since a circle subtends at the centre an angle whose radian measure is 2π and its degree measure is 360, it follows that

$2\pi\, radian = 360°$ or $\pi\, radian = 180°$

Then we have

$$1\, radian = \frac{180°}{\pi} = 57°16'$$

$$1° = \frac{\pi}{180}\, radian = 0.01746\, radian$$

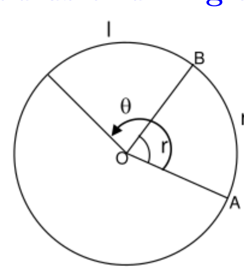
Degree	30°	45°	60°	90°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Note that when an angle is expressed in radians, the word 'radian' is frequently omitted. Thus $\pi = 180°$ and $\frac{\pi}{4} = 45°$ are written with the understanding that π and $\frac{\pi}{4}$ are radian measures. Thus, we can say that

$$\text{Radian Measure} = \frac{\pi}{180} \text{ Degree Measure}$$

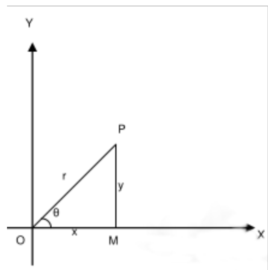
$$\text{Degree Measure} = \frac{180}{\pi} \text{ Radian Measure}$$

Arc, Radius and Angle relation



Length of the arc of a circle having radius $'r'$ and inclination $'\theta'$ radians is $l = r\theta$.

Triganometric Functions: Trigonometric functions are periodic and thus many natural phenomena are most readily studies with the help of trigonometric functions. Unlike algebraic functions, these functions are not represented by single letters, instead the abbreviation sin is used for sine function, cos is for cosine function, tan is for tangent function, cosec for cosecant function, sec is for secant function and cot is for cotangent function. For a given angle θ, it is usual to write $sin\theta$ for $sin(\theta)$, etc



$$sin\theta = \frac{opp\ side}{hypotenuse} = \frac{y}{r}$$
$$cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{x}{r}$$
$$tan\theta = \frac{opp\ side}{adjacent\ side} = \frac{y}{x}$$
$$cosec\theta = \frac{1}{sin\theta} = \frac{r}{y}$$
$$sec\theta = \frac{1}{cos\theta} = \frac{r}{x}$$
$$cot\theta = \frac{1}{tan\theta} = \frac{x}{y}$$

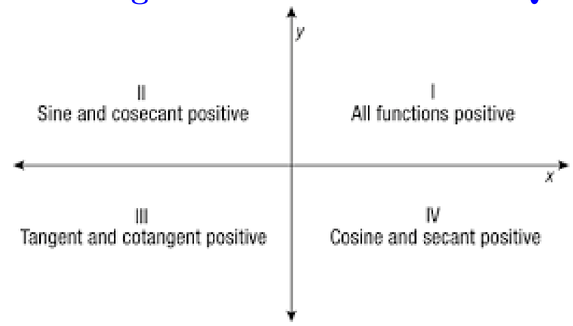
Phythagoras' relations

$$sin^2\theta + cos^2\theta = 1$$
$$sec^2\theta - tan^2\theta = 1$$
$$cosec^2\theta - cot^2\theta = 1$$

Trigonometric Ratios of particular angles

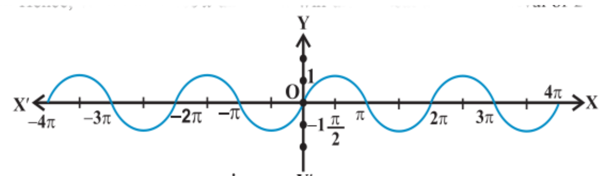
	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0

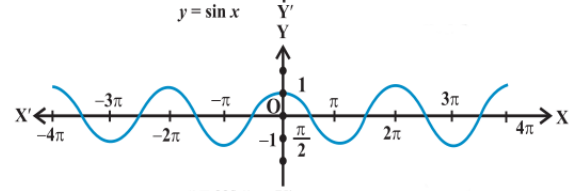
Signs of Trigonometric functions in Quadrants

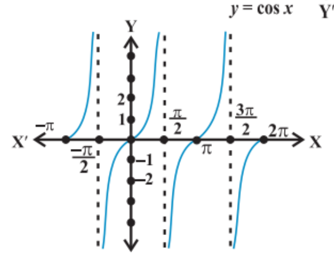


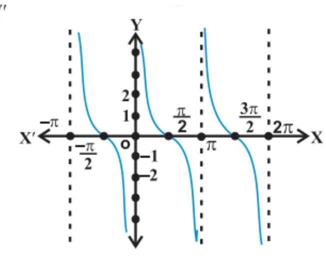
Domain and range of trigonometric functions

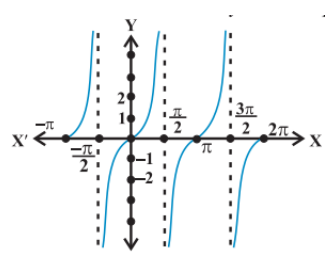
Functions	Domain	Range
(i) sine	R	[–1, 1]
(ii) cosine	R	[–1, 1]
(iii) tangent	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$	R
(iv) cosecant	$R - \{x : x = n\pi, n \in Z\}$	R – [–1, 1]
(v) secant	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$	R – [–1, 1]
(vi) cotangent	$R - \{x : x = n\pi, n \in Z\}$	R

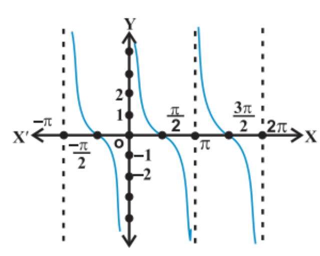

y = sin x

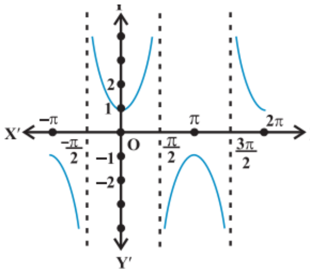

y = cos x


y = tan x

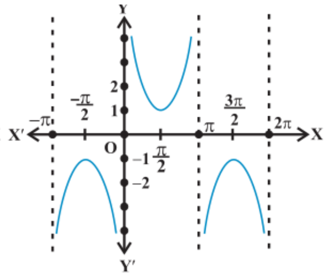

y = cot x


y = tan x


y = cot x



$y = \sec x$



$y = \operatorname{cosec} x$

Triganometric Functions of larger angles

For an angle θ and integer n

$$\begin{aligned} sin(n\pi \pm \theta) &= sin\theta \\ cos(n\pi \pm \theta) &= cos\theta \\ tan(n\pi \pm \theta) &= tan\theta \\ cosec(n\pi \pm \theta) &= cosec\theta \\ sec(n\pi \pm \theta) &= sec\theta \\ cot(n\pi \pm \theta) &= cot\theta \end{aligned}$$

(even multiple of $90^0 + \theta$)

$$\begin{aligned} sin((2n+1)\frac{\pi}{2} \pm \theta) &= cos\theta \\ cos((2n+1)\frac{\pi}{2} \pm \theta) &= sin\theta \\ tan((2n+1)\frac{\pi}{2} \pm \theta) &= cot\theta \\ cot((2n+1)\frac{\pi}{2} \pm \theta) &= tan\theta \\ cosec((2n+1)\frac{\pi}{2} \pm \theta) &= sec\theta \\ sec((2n+1)\frac{\pi}{2} \pm \theta) &= cosec\theta \end{aligned}$$

(even multiple of $90^0 + \theta$)

Compound Angle Formulas

- $sin(x+y) = sin\, xcosy + cosx\, siny$
- $sin(x–y) = sinx\, cosy–cosx\, siny$
- $cos(x+y) = cosx\, cosy–sinx\, siny$
- $cos(x–y) = cosx\, cosy + sinx\, siny$
- $tan(x+y) = \frac{tanx+tany}{1-tanx\, tany}$
- $tan(x–y) = \frac{tanx-tany}{1+tanx\, tany}$
- $cot(x+y) = \frac{cotx\, coty-1}{cotx+coty}$
- $cot(x-y) = \frac{cotx\, coty+1}{cotx-coty}$

Multiple Angle Formulas

- $sin2x = 2sinx\, cosx = \frac{2tanx}{1+tan^2x}$
- $sin3x = 3sinx–4sin^3x$
- $cos2x = cos^2x - sin^2x = 1-2sin^2x = 2cos^2x - 1 = \frac{1-tan^2x}{1+tan^2x}$
- $cos3x = 4cos^3x - 3cosx$
- $tan2x = \frac{2tan/,x}{1-tan^2x}$

Sum Formulas

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Product Formula

$2\cos x\cos y = \cos(x+y) + \cos(x-y)$
$-2\sin x\sin y = \cos(x+y) - \cos(x-y)$
$2\sin x\cos y = \sin(x+y) + \sin(x-y)$
$2\cos x\sin y = \sin(x+y) - \sin(x-y)$.

Solution of Trigonometric Equations

sin x = 0 gives $x = n\pi$, where $n \in Z$
cos x = 0 gives $x = (2n+1)\pi$, where $n \in Z$
tanx = 0 gives $x = n\pi$, where $n \in Z$
sin x = sin y gives $x = n\pi + (-1)^n y$, where $n \in Z$
cos x = cos y gives $x = 2n\pi y$, where $n \in Z$
tan x = tan y gives $x = n\pi + y$, where $n \in Z$
Principal solution is the solution which lies in the interval $0 \le x \le 2\pi$.