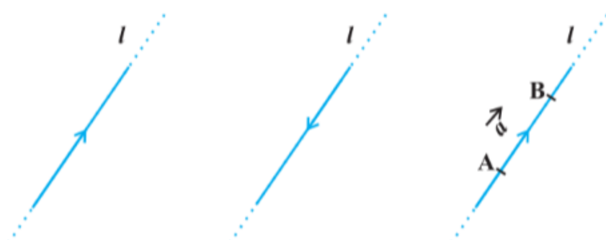
Vector Algebra

Physical quantities we deal are of two types, one that can be specified using a single real number which gives its magnitude and the other which involves the idea of direction as well as magnitude. The first type is called scalar quantity and the second is vector quantity. In this chapter we analyses the basic concepts about vectors, various operations, and their algebraic and geometrical properties.

Basic concepts

Vector

A quantity that has magnitude as well as direction is called a **vector**.

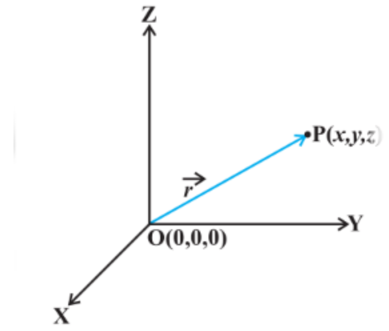


The directed line segments above are vectors .The third vector is denoted as $\vec{AB}$ or simply $\vec{a}$ and read as **vector AB** or **vector a**.

The point A from where the vector $\vec{AB}$ starts is called its **initial point**, and the point B where it ends is called its **terminal point**. The distance between initial and terminal points of a vector is called the **magnitude** (or length) of the vector, denoted as $\vec{AB}$ or $|\vec{a}|$, or a. The arrow indicates the direction of the vector.

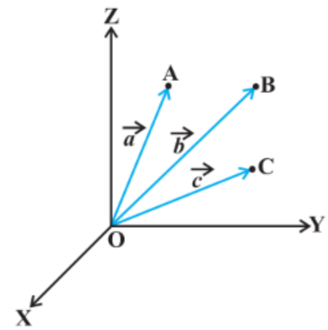
Position Vector

Consider a point P in space, having coordinates (x, y, z) with respect to he origin O (0, 0, 0). Then, the vector $\vec{OP}$ having O and P as its initial and terminal points, respectively , is called the **position vector** of the point P with respect to O.



By distance formula

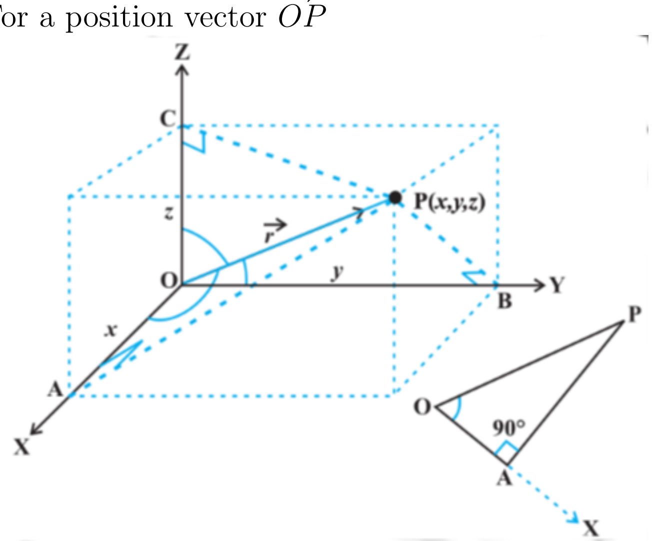
$$|\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$$



the position vectors of points A, B, C, etc., with respect to the origin O are denoted by $\vec{a}, \vec{b}, \vec{c}$ etc.,

Direction Cosines

For a position vector $\vec{OP}$



The angles α, β, γ made by the vector with the positive directions of x, y and z-axes respectively, are called its direction angles. The cosine values of these angles, i.e., $cos\alpha, cos\beta$ and

$cos\gamma$are called direction cosines of the vector , and usually denoted by l, m and n, respectively

Types of Vectors

Zero Vector

A vector whose initial and terminal points coincide, is called a zero vector (or null vector), and denoted as . Zero vector can not be assigned a definite direction as it has zero magnitude.

Unit Vector

A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector. The unit vector in the direction of a given vector is denoted by $\vec{a}$

Coinitial Vector

Two or more vectors having the same initial point are called coinitial vectors.

Collinear Vector

Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

Equal Vector

Two vectors $\vec{a}, \vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as $\vec{a} = \vec{b}$.

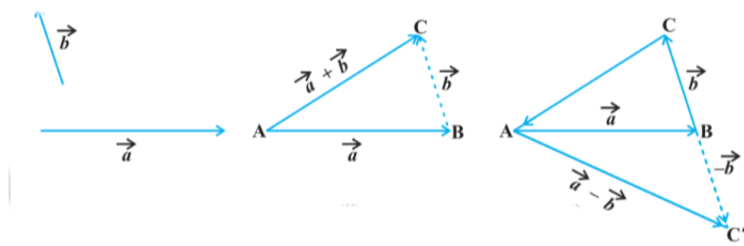
Negative of a vector

A vector whose magnitude is the same as that of a given vector (say, $\vec{AB}$), but direction is opposite to that of it, is called negative of the given vector. For example, vector $\vec{BA}$ is negative of the vector $\vec{AB}$, and written as $\vec{BA} = -\vec{AB}$

Addition of Vectors

Triangle Law of Vector Addition

if we have two vectors and then to add them, they are positioned so that the initial point of one coincides with the terminal point of the other



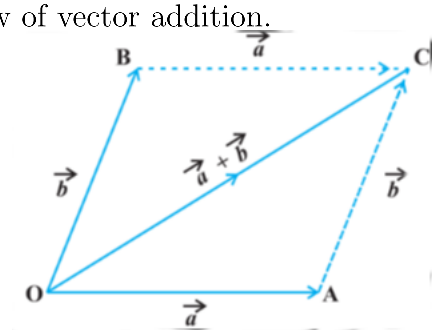
Then, the vector $\vec{a} + \vec{b}$, represented by the third side AC of the triangle ABC, gives us the sum (or resultant) of the vectors $\vec{a}$ and $\vec{b}$, we have

$$\vec{AB} + \vec{BC} = \vec{AC}$$

similarly the vector $\vec{AC'}$ represents the difference of $\vec{a}$ and $\vec{b}$

Parallelogram law of Vector Addition

If we have two vectors $\vec{a}$ and $\vec{b}$ represented by the two adjacent sides of a parallelogram in magnitude and direction , then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point. This is known as the parallelogram law of vector addition.



Properties of Vector Addition

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (Commutative Property)
- $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (Associative Property)

Multiplication of Vector by a Scalar

Let be $\vec{a}$ given vector and λ a scalar. Then the product of the vector by the scalar λ, denoted as $\lambda\vec{a}$, is called the multiplication of vector by the scalar λ.The vector λ has the direction same (or opposite) to that of vector according as the value of λ is positive (or negative). Also, the magnitude of vector $\lambda\vec{a}$ is $|\lambda|$ times the magnitude of the vector $\vec{a}$

$$|\lambda\vec{a}| = |\lambda||\vec{a}|$$

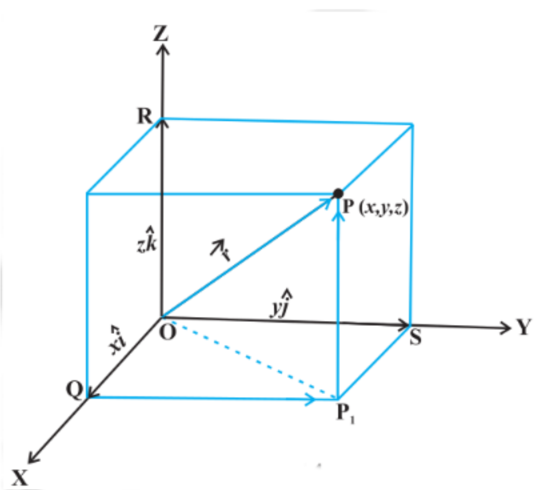
Components of a Vector

Let $\hat{i}, \hat{j}, \hat{k}$ be the unit vectors along the x-axis, y-axis, z-axis respectively. The point P(x, y, z) be a point in space. Then the position vector of the point P can be expressed in component form as

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

and

$$|\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$$



This form of any vector is called its **component form**. Here, x, y and z are called as the **scalar components** of r , and $x\hat{i}, y\hat{j}$ and $z\hat{k}$ are called the **vector components** of r along the respective axes. Sometimes x, y and z are also termed as **rectangular components**.

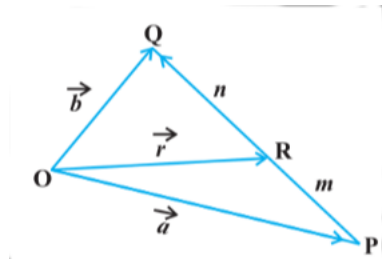
Vector Joining two points

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are two points, then the vector joining P_1 and P_2 is the vector $\vec{P_1P_2}$

$$\vec{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_1 - z_2)\hat{k}$$

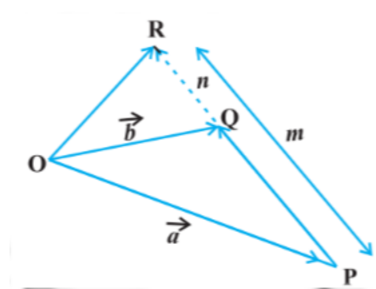
Section formula

Let P and Q be two points represented by the position vectors $\vec{OP}$ and $\vec{OQ}$ with respect to the origin O ,then the position vector of the point R which divides PQ in the ratio $m : n$ **internally** is



$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Position vector of the point R which divides PQ in the ratio $m : n$ **externally** is



$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

If R is the midpoint of PQ then $\vec{r} = \frac{\vec{a}+\vec{b}}{2}$

Product of Two Vectors

Scalar Product(Dot)

The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}.\vec{b}$, is defined as

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos(\theta)$$

where θ is the angle between $\vec{a}$ and $\vec{b}$

Observations

1. $\vec{a}.\vec{b}$ is a real number
2. Let $\vec{a}$ and $\vec{b}$ are two nonzero vectors , then $\vec{a}.\vec{b} = 0$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

3. If $\theta = 0$ then $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$, In purticular $\vec{a}.\vec{a} = |\vec{a}|^2$

4. If $\theta = \pi$ then $\vec{a}.\vec{b} = -|\vec{a}||\vec{b}|$, In purticular $\vec{a}.\vec{a} = -|\vec{a}|^2$

5. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1, \hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

6. The angle between two nonzero vectors is given by
$$cos(\theta) = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

7. Scalar Product is Commutative , ie $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

8. $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

9. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then
$$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$