

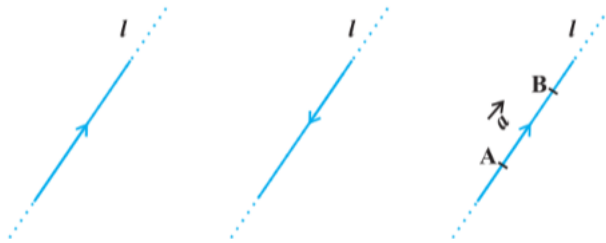
Vector Algebra

Physical quantities we deal are of two types, one that can be specified using a single real number which gives its magnitude and the other which involves the idea of direction as well as magnitude. The first type is called scalar quantity and the second is vector quantity. In this chapter we analyse the basic concepts about vectors, various operations, and their algebraic and geometrical properties.

Basic concepts

Vector

A quantity that has magnitude as well as direction is called a **vector**.

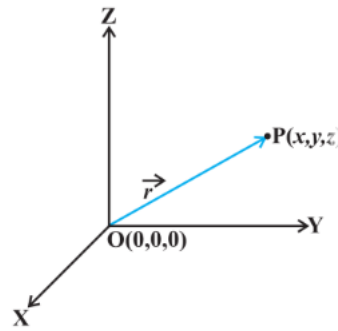


The directed line segments above are vectors. The third vector is denoted as \vec{AB} or simply \vec{a} and read as **vector AB** or **vector a**.

The point A from where the vector \vec{AB} starts is called its **initial point**, and the point B where it ends is called its **terminal point**. The distance between initial and terminal points of a vector is called the **magnitude** (or length) of the vector, denoted as AB or $|\vec{a}|$, or a . The arrow indicates the direction of the vector.

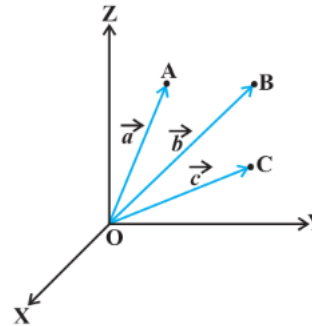
Position Vector

Consider a point P in space, having coordinates (x, y, z) with respect to the origin O $(0, 0, 0)$. Then, the vector \vec{OP} having O and P as its initial and terminal points, respectively, is called the **position vector** of the point P with respect to O.



By distance formula

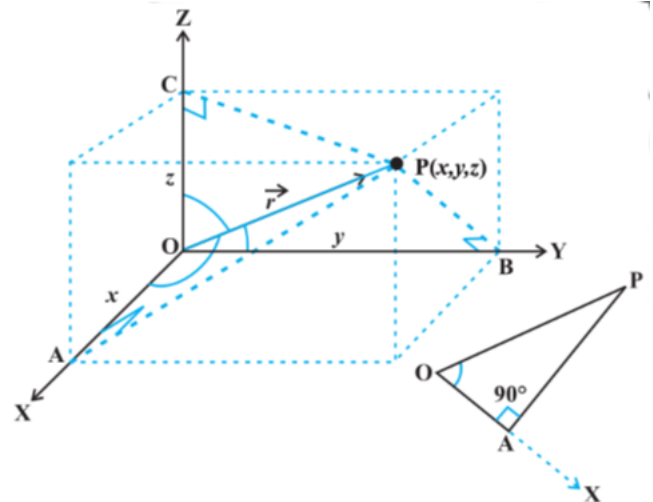
$$|\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$$



the position vectors of points A, B, C, etc., with respect to the origin O are denoted by $\vec{a}, \vec{b}, \vec{c}$ etc.,

Direction Cosines

For a position vector \vec{OP}



The angles α, β, γ made by the vector with the positive directions of x, y and z-axes respectively, are called its direction angles. The cosine values of these angles, i.e., $\cos\alpha, \cos\beta$ and

$\cos\gamma$ are called direction cosines of the vector, and usually denoted by l , m and n , respectively

Types of Vectors

Zero Vector

A vector whose initial and terminal points coincide, is called a zero vector (or null vector), and denoted as $\vec{0}$. Zero vector can not be assigned a definite direction as it has zero magnitude.

Unit Vector

A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector. The unit vector in the direction of a given vector is denoted by \vec{a}

Coinitial Vector

Two or more vectors having the same initial point are called coinital vectors.

Collinear Vector

Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

Equal Vector

Two vectors \vec{a}, \vec{b} are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as $\vec{a} = \vec{b}$.

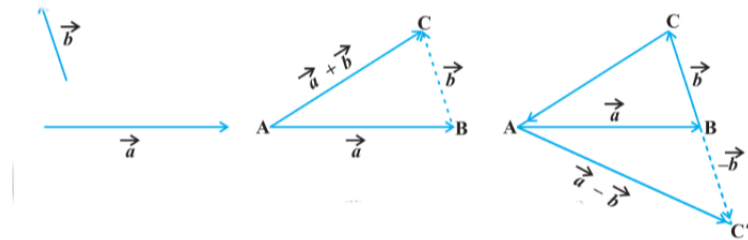
Negative of a vector

A vector whose magnitude is the same as that of a given vector (say, \vec{AB}), but direction is opposite to that of it, is called negative of the given vector. For example, vector \vec{BA} is negative of the vector \vec{AB} , and written as $\vec{BA} = -\vec{AB}$

Addition of Vectors

Triangle Law of Vector Addition

if we have two vectors and then to add them, they are positioned so that the initial point of one coincides with the terminal point of the other



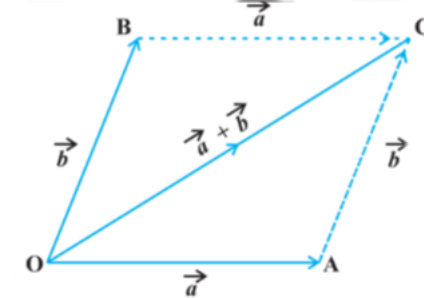
Then, the vector $\vec{a} + \vec{b}$, represented by the third side AC of the triangle ABC, gives us the sum (or resultant) of the vectors \vec{a} and \vec{b} , we have

$$\vec{AB} + \vec{BC} = \vec{AC}$$

similarly the vector $\vec{AC'}$ represents the difference of \vec{a} and \vec{b}

Parallelogram law of Vector Addition

If we have two vectors \vec{a} and \vec{b} represented by the two adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point. This is known as the parallelogram law of vector addition.



Properties of Vector Addition

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (Commutative Property)
- $(\vec{a} + \vec{b}) + \vec{c} =$