Relations and functions

A relation from a non-empty set A to a non-empty set B is a subset of A × B.

In this chapter we study different types of relations and functions, composition of functions, and binary operations.

Examples

- $\{(a, b) \in A \times B$: a is brother of b$\}$
- $\{(a, b) \in A \times B$: age of a is greater than age of b$\}$

Types of Relations

- **Empty Relation:** $R : A \to A$ given by $R = \phi \subset A \times A$
- **Universal Relation** $R : A \to A$ given by $R = A \times A$
- **Reflexive Relation** $R : A \to A$ with $(a, a) \in R, \forall a \in A$
- **Symmetric Relation** $R : A \to A$ with $(a, b) \in R \Rightarrow (b, a) \in R, a, b \in A$
- **Transitive Relation** $R : A \to A$ with $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
- **Equivalence Relation** Relation which is Reflexive , Symmetric and Transitive.

Equivalence Class

Let A be an Equivalence Relation in a set A. If $a \in A$, then the subset $\{x \in A, (x, a) \in R\}$ of A is called the Equivalence class corresponding to 'a' and is denoted by $[a]$.

Types of functions

One-One or Injective function.

A function $f : A \to B$ is said to be *One-One* or *Injective*, if the image of distinct elements of A under f are distinct.

i.e;$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Otherwise, f is called many-one.

If lines parallel to x-axis meet the curve at two or more points, then the function is not one-one.

Onto or Surjective function

A function $f : A \to B$ is said to be *Onto* or *Surjective*, if every element of B is some image of some elements of A under f.

ie; If for every element $y \in Y$ then there exists an element x in A such that $f(x) = y$.

Bijective Functions

A function $f : A \to B$ is said to be Bijective if it is both One-One and Onto.

Composition of functions and Invertible function

Composition of Functions.

Let $f : A \to B$ and $g : B \to C$ be two functions. Then the composition of f and g denoted by is gof defined $gof : A \to C$ and $gof(x) = g(f(x))$.

- If $f : A \to B$ and $g : B \to C$ are One-One, then $gof : A \to C$ is One-One.
- If $f : A \to B$ and $g : B \to C$ are Onto, then $gof : A \to C$ is Onto.gof : A → C
- If $f : A \to B$ and $g : B \to C$ are Bijective, $\iff$ $gof : A \to C$ is Bijective.

Inverse Function

If $f : A \to B$ is defined to be invertible, if there exists a function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$. The function g is called the inverse of $'f'$ and is denoted by f^{-1}.

- If function $f : A \to B$ is invertible only if f is bijective.
- $(gof)^{-1} = f^{-1}og^{-1}$.