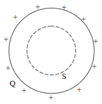
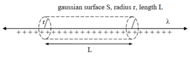
Charges

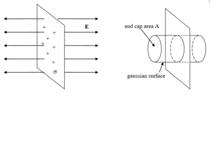
- **Facts & Laws**
  - **Experimental Facts**
    - Electric Charge comes in two types; which we call (+) and (-)
    - Unlike charges attract; and like charges repel according to Coulomb's Law; which says: $\vec{F} = k\frac{|q_1||q_2|}{r^2}$
    - Electric Charge is <u>conserved</u>: The net charge of an isloated system canno change:
      - Conserved in the universe
        - Energy
        - Linear Momentum($\vec{p} = m\vec{v}$)
        - Angular momentum (spin $= \vec{L} = I\vec{\omega}$)
        - Charge - main concept in Electric Filed
    - The charge e is the fundamental unit of charge: you never find a free particle in nature with charge = fraction of e:
  - **Coulomb's Law**
    - Definition
      - Coulomb's Law state that the magnitude F of the force between two charges separated by distance r is given by $\vec{F} = k\frac{|q_1||q_2|}{r^2}$
        - $k = \text{constant} = 9.0 \cdot 10^9 \frac{N \cdot m^2}{C^2}$
        - $e = +1.602 \cdot 10^{-19}$ C
    - Directions
      - We need to use the unit vectors since Coulomb's Law only gives us how strong the force is:
- **Charge Distributions**
  - Point Charge (0D)
    - $Q = Q$
  - Line Charge (1D)
    - $\lambda = \frac{Q}{L}\ [\frac{C}{m}]$ so; $Q = \lambda L$
  - Surface Charge(2D)
    - $\sigma = \frac{Q}{A}\ [\frac{C}{m^2}]$ so; $Q = \sigma A$
  - Volume Charge(3D)
    - $\rho = \frac{Q}{V}\ [\frac{C}{m^3}]$ so; $Q = \rho V$
- **Electric Filed**
  - Definition
    - Surrounding every charge (or group of charges) is a thing; called electric field $\vec{E}$ (it is a vector thing) $\vec{E} = \frac{\vec{F}_{onq}}{q} = \frac{kQ}{r^2}\hat{r}\ [\frac{N}{C}]$
  - Directions
    - Positive Charge → Filed Point away
    - Negative Charge → Filed Point toward
    - Superpostion Pronciple: $E_{total} = E_1 + E_2 + \dots$
  - **Gauss's Law**
    - Core Concept
      - Gauss's Law is one of the four funamental laws of electricity and magnetism called Maxwell's Equation
      - Before diving to Gauss's Law we need to understand a new concept called the electric flux Φ through a surface:
      - we consider an imaginary surface which cuts across some E-field lines: We say that there is some electric flux though this surface:
      - but we must understand something called Surface vector:
      - A surface vector it's an area with direction; $\vec{A} = A\hat{n}$: Magnitude of vector $\vec{A}$ = area a of a surface. Direction of vector $\vec{A}$ = directon prpendicular(normal) to a surface whihc is the direction of the unit normal $\hat{n}$
        - We have an ambiguity in the direction $\hat{n}$: Every flat surface has two perpendicular directions:
      - So the electric flux Φ through a surface A defined as $\Phi = \vec{E} \cdot \vec{A} = EA\cos(\theta)$:
        - The flux Φ has the following geometrical interpretation $|\text{flux}| \propto$ the numbers of electric field lines crossing the surface:
        - Think of the E-filed as rain flowing through an open window of area A.
        - The flux is a measure of the amount of rain flowing through the window
          - To get a big flux; you need a large E; a large A; and you need the area perpendicular to the E-field vector
          - Which means the area vector $\vec{A}$ is <u>parallel</u> to $\vec{E}$:
        - But this formula only works if the surface is flat and the E-filed is constant.
        - if the E-field varies with position and/or the surface is not flat; we need more general definition of flux; we need to integrate how fun is that
          - $\Phi = \int \vec{E} \cdot d\vec{a}$ = "surface integral of E"
    - Definition
      - $\oint \vec{E} \cdot d\vec{a} = \frac{q_{enclosed}}{\epsilon_0}$
        - In words; the electric flux through any <u>closed</u> surface S is a constant ($\frac{1}{\epsilon_0}$) times the total charge inside S:
        - A surface is closed if it has no edges; like a sphere: For a closed surface; the direction of da is alwyas the <u>outward</u> normal:
        - The constant epsilon nought is related to k by $k = \frac{1}{4\pi\epsilon_0}$
          - $\epsilon_0 = 8.85 \cdot 10^{-12}$
        - When the Filed lines exit a closed surface; the flux there is positive: When the Field lines enter a closed surface; the flux is negative:
          - Note: If a closed surface S encloses no charges; then the number of lines entering must equal the number of lines existing:
          - So only charges inside the surface can contribute to the flux through the surface: Positive charges inside produce positive flux; negative charges produce negative flux:
  - Gaussian surfaces & Symmetry
    - Gauss's Law is always true (it's a LAW): but it is not always useful:
    - Only in situations with very high symmetry it is easy to compute the flux integral $\int \vec{E} \cdot d\vec{a}$:
    - In these few cases of high symmetry; we can use Gauss's Law to compute the E-field
    - Using Gauss's Law to solve for the E-Filed
      - Spherical Symmetry → Sphere(Point Charge) we use → $\vec{E} = \frac{kQ}{r^2}$

        - By symmetry; E must be radial (along a radius); and it can only depend on the distance r from the center; so $E = E(r)$:
        - $E = 0$ everywhere inside a hollow uniform sphere of charge:
        - Conclusion: The E-field outside a uniform shell of charge; or outside any spherically charge distribution; is exactly the same as if all teh charge was concentrated at the center:
        - $\vec{E} = \frac{kQ}{r^2}$
      - Cylindrical Symmetry → Cylinder(Line Charge) → $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}$

        - By symmetry; E is in the cylindrically radial direction and $E = E(r)$:
        - Through the end caps we can see that the flux is zero since the E-field ⊥ to the line
        - So the flux simplified to only the side; which gives us
        - The charge inside the Gaussian surface is (charge/length)X length $= \lambda L$; so the equation becomes
        - $\oint = \vec{E} \cdot d\vec{a} = E(2\pi rL) = \frac{q_{enclosed}}{\epsilon_0} \rightarrow E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$ $\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$
        - $\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}$
      - Planar Symmetry → Pillbox (Sheet Charge) → $\vec{E} = \frac{\sigma}{2\epsilon_0}$

        - By symmetry; the E-field must be perpendicular to the plane (either away or towards)
        - On end caps; $\vec{E} \cdot d\vec{a} = Eda$
        - On curved side; $\vec{E} \cdot da = 0$; since the curved side ⊥ to the plane
        - $\oint \vec{E} \cdot d\vec{a} = \frac{q_{enclosed}}{\epsilon_0}$; where $q_{enclosed} = \sigma A$
        - Since we have two ends our integral becomes: $2E\cancel{A} = \frac{\sigma\cancel{A}}{\epsilon_0}$
        - So we get that $E = \frac{\sigma}{2\epsilon_0}$ = constant; regardless of Position!
        - $\vec{E} = \frac{\sigma}{2\epsilon_0}$
  - **Conductors & Electric Fields**
    - Inside a Conductor → $E = 0$
    - Charge moves to the surface