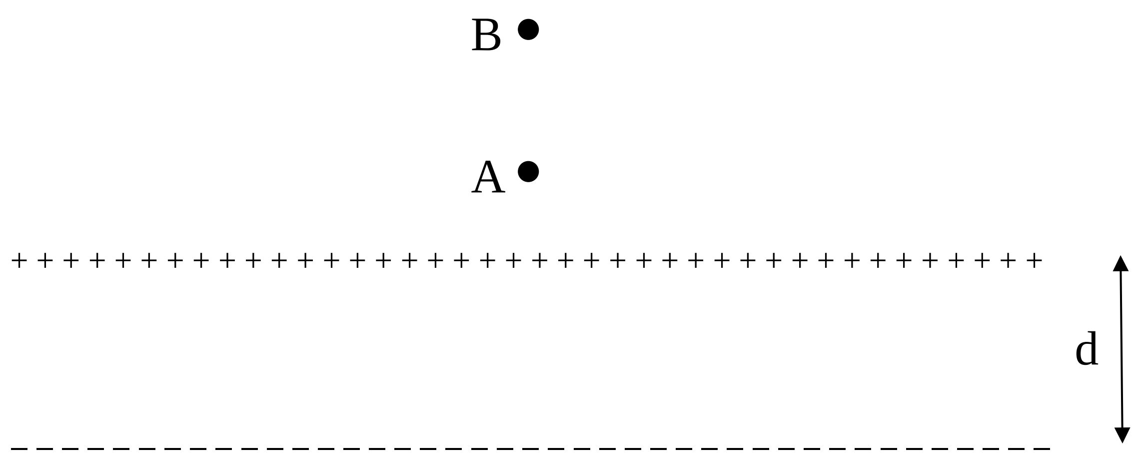
CAPCT-0

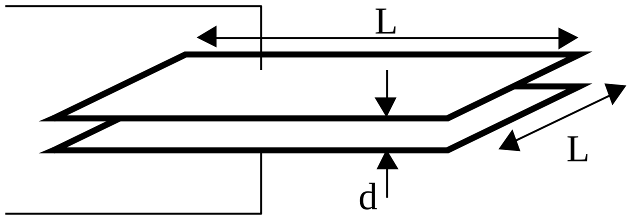
Two parallel infinite plane of charge with equal and opposite charge densities $+\sigma$ and $-\sigma$ are a distance d apart, as shown (side view). How do the magnitudes of the electric fields at points A and B above the planes compare?



A) $E_A > E_B > 0$ and both vectors points up
B) $E_A = E_B$ and both vectors point up
C) $E_A = E_B = 0$

Answer: $E_A = E_B = 0$. The magnitude of the E-field due to an infinite plane of charge is $\sigma/(2\varepsilon_0)$ [sigma/(2*epsilon-naught)], **independent** of distance from the plane. At both points A and B, the E-fields from the two planes exactly cancel. Everywhere between the planes they add up to is σ/ε_0 [sigma/epsilon-naught].

CAPCT-1.
A parallel-plate capacitor has square plates of edge length L, separated by a distance d. A second capacitor is made with L doubled and d decreased by a factor of 2.



$$\mathrm{L} \to 2\mathrm{L}, \quad \mathrm{d} \to \mathrm{d}/2$$

By what factor is the capacitance of the new capacitor increased?

A: 1 B: 2 C: 4 D: 16 E: None of these.

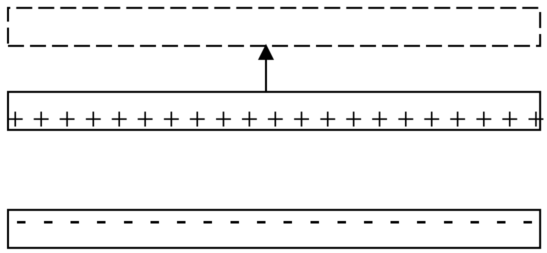
Answer: None of these. The capacitance increased by a factor of 8. The area (L×L) increased by a factor of 4. Another factor of 2 comes from the separation d.

CapCT-2. A parallel-plate capacitor with capacitance C has a charge Q (meaning +Q on one plate, –Q on the other). The charge is doubled to 2Q (meaning +2Q on one plate, –2Q on the other). The capacitance C of the capacitor...

A) doubled B) decreased by 2X
C) remained constant D) None of these

Answer: The capacitance remained constant. The ratio Q/V is fixed by the geometry (size, shape, separation) of the capacitor. If Q is increased, V also must increase, so the ratio Q/V stays the same.

CAPCT-3. A parallel-plate capacitor is charged up (+Q on one plate, –Q on the other). The plates are isolated so the charge Q cannot change. The plates are then pulled apart so that the plate separation d increases. The total **electrostatic energy** stored in the capacitor



A: increases

B: decreases

C: remains constant.

(Hint: Did the person pulling the plates apart do positive work, negative work or no work?)

As the plates were pulled apart, the energy density (energy per volume) = u = $(1/2)\varepsilon_o E^2$ ....

A) increased B) decreased C) stayed the same.

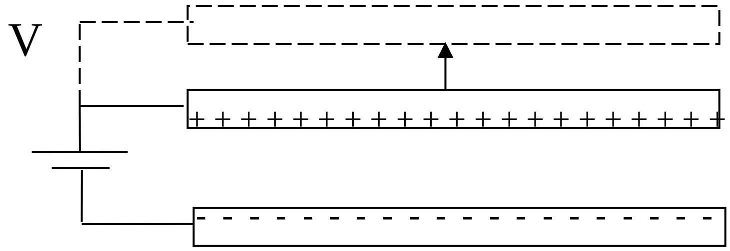
Answer: The energy increased, which we can see in three different ways:

Method I. The Q is fixed and the C decreases (since $C = \varepsilon_0 A/d$), so $U = (1/2)Q^2/C$ increases.

Method II. The total energy stored in the field between the plates is (energy density) x volume = $\frac{1}{2}\varepsilon_o E^2 \times (\text{Volume})$. The field $E = \sigma/\varepsilon_o$ does not change (since the charge and charge density σ do not change) but the volume between the plates increased, so the energy in the field increased.

Method III. The external agent pulling the plates apart did positive work (since the plates attract). That positive work done was stored as increased electrostatic potential energy in the capacitor. The stored energy increased.

CAPCT-4. A parallel plate capacitor is attached to a battery which maintains a constant voltage difference V between the capacitor plates. While the battery is attached, the plates are pulled apart so their separation increases. The electrostatic energy stored in the capacitor



A: increases B: decreases C: stays constant.

Answer: Using $U = (1/2)CV^2$, we can see that the stored energy decreased. The Q on the plates was not constant (since the plates are not isolated) so we cannot use a formula involving the charge Q. The voltage V between the plates is held constant by the battery. (That's what a battery does: it maintains constant voltage difference between its terminals.) The capacitance C decreased as the plates were pulled apart (by $C = \varepsilon_0 A/d$), so $U = (1/2)CV^2$ decreased.

Where did the energy go? Into the battery.

What happened to the charge Q on the plates as the plates were pulled apart (at constant Voltage)? (Hint: What happened to the E-field? E-field is related to charge density.)
Answer: The charge Q decreased. $C = Q/V$, $Q = C V$, since V was constant while C decreased, it must be that Q decreased. Alternatively, we can say that $V = E d$ = constant, so if d increases, E must decrease (since V is constant.). If E decreases, the charge density must decrease (since $E = \sigma/\varepsilon_o$). If charge density decreases, the charge must decrease.

CAPCT-5. A positive charge +Q and a negative charge –Q are held a distance R apart and are then released. The two particles accelerate toward each other as a result of their coulomb attraction. As the particles approach each other, the energy contained in the electric field surrounding the two charges..

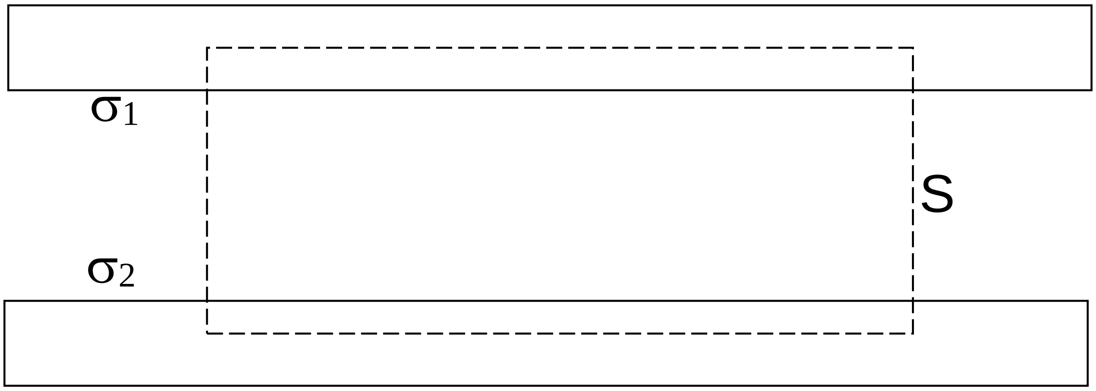
A: increases B: decreases C: stays the same.

Answer: The energy in the E-field decreases. The energy ***density*** of the E-field is $\frac{1}{2}\varepsilon_o E^2$; this is NOT the energy, it is the energy **per volume**. If the E-field is constant, the total energy is $\frac{1}{2}\varepsilon_o E^2 \times \text{volume}$. If the E-field is not constant, the total energy is $\int \frac{1}{2}\varepsilon_o E^2 dV$, where dV is a volume element. In this problem, the volume in which the E-field is large is roughly the space between the particles. This volume is decreasing rapidly as the particles approach each other. Although the E-field between the particles is increasing as they approach, the volume is decreasing more rapidly than the E-field is increasing, so the total energy is decreasing. Actually, I am glossing over some details: the exact calculation is pretty tricky, due to the divergence of the E-field near the point charges.

Another way to see this is : When the two particles are infinitesimally close, so that they "on top of each other", the (+) charge cancels the (–) charge, and the E-field is zero. Then there is no energy in the field, so the field energy must have been decreasing as they approached.

Where did the field energy go? It goes into the increased KE of the particles as they accelerated toward each other, and it goes into **light** that is emitted. Total energy is always constant in an isolated system.

CAPCT-6.
The inner surfaces of the two plates of a capacitor have uniform charge per area σ_1 and σ_2. Consider the gaussian surface S.



From Gauss's Law, you can conclude that
A) $\sigma_1 = \sigma_2$ always. B) $\sigma_1 = -\sigma_2$ always.
C) You cannot conclude anything from Gauss's Law: σ_1 and σ_2 can be anything

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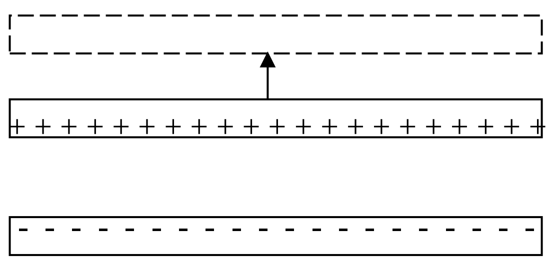
Answer: $\sigma_1 = -\sigma_2$ Gauss's Law is $\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$. We can argue that $\oint_S \vec{E}\cdot d\vec{A} = 0$ because (1) the top and bottom of the surface S is inside the metal where E = 0, and (2) on the sides of S, the E-field is perpendicular to the area vector $\vec{E} \perp d\vec{A} \Rightarrow \vec{E}\cdot d\vec{A} = 0$. From Gauss's Law, we conclude that $Q_{enc} = 0$. Therefore the charge on the top plate must be the opposite of the charge on the bottom plate. Since the areas are equal, and the charges are equal-sized, the charge per areas must also be equal-sized (but opposite in sign).

CAPCT-7. The charged plates of a capacitor are electrically isolated (not connected to a battery or anything) and are slowly pulled apart. How many of the following 6 quantities **remain constant** as the plates are pulled apart?
Capacitance C, charge Q, electric field E between the plates, voltage difference V between the plates, energy U,
energy density u between the plates

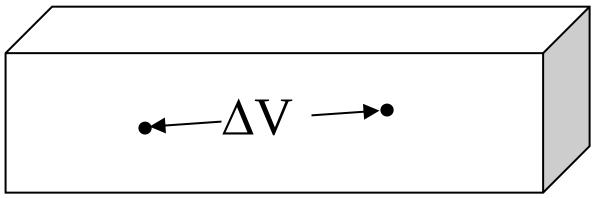


A) 1 B) 2 C) 3 D) 4 E) more than 4

Answer: 3. The capacitance decreases. The charge remains constant. The E-field remains constant. The voltage increases. The energy U increases. The energy density u remains constant.

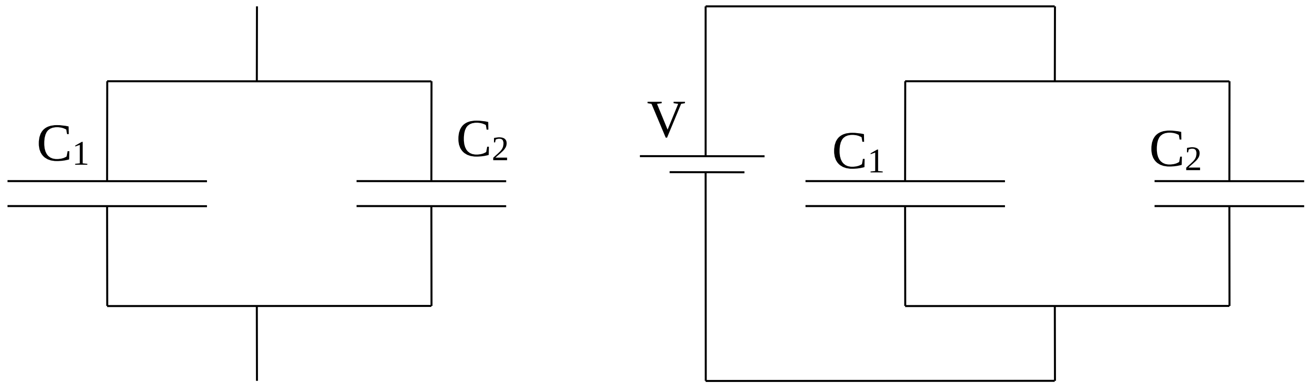
CAPCT-8. Consider a metal in electrostatic equilibrium. The voltage difference between two different points in the metal ..

A) is zero always.
B) could be positive, negative, or zero, depending on the charges on the surface.



Answer: ΔV is zero always. Since E = 0 inside a metal in electrostatic equilibrium, ΔV must be zero between any two points in the metal since $\Delta V = -\int_i^f \vec{E} \cdot d\vec{r}$

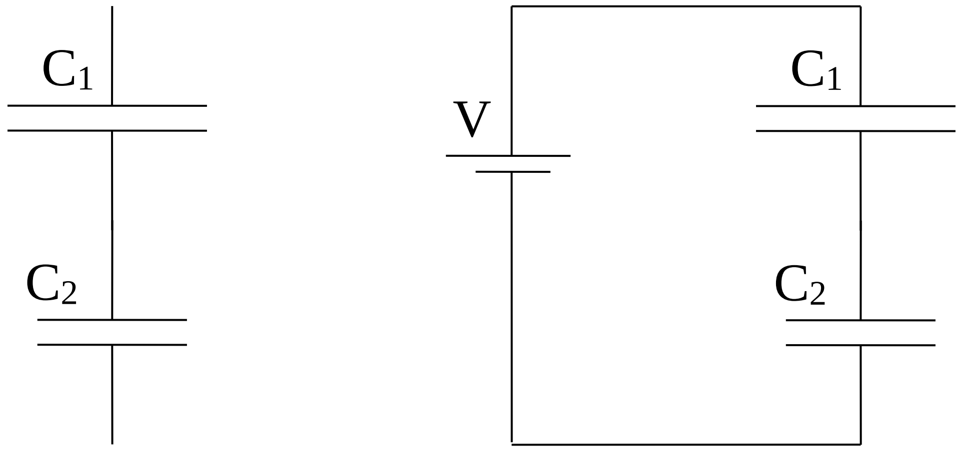
CAPCT-9. Consider any two capacitors C_1 and C_2 hooked together in parallel. They may or may not be hooked to a battery.



What thing is definitely the same for the two capacitors?
A) the capacitance (C1 = C2)
B) the voltage (V1 = V2) ("V" means ΔV, of course)
C) the charge (Q1 = Q2)
(charge "Q" means +Q on 1 plate , –Q on the other)
D) All of these
E) None of these

Answer: Only the voltage is the same on both. Since the top plates are connected by a wire they must be at the same voltage (see previous concept test). Likewise, the bottom plates must be at the same voltage. So the voltage differences between top and bottom must be the same for the two capacitors.

CAPCT-10. Consider any two capacitors C_1 and C_2 hooked together in series. They may or may not be hooked to a battery.



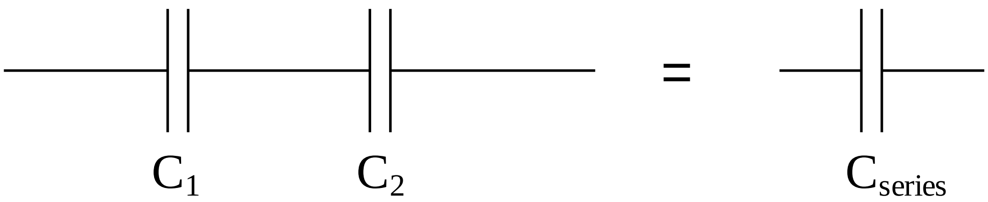
What thing is definitely the same for the two capacitors?
A) the capacitance (C1 = C2)
B) the voltage (V1 = V2) ("V" means ΔV, of course)
C) the charge (Q1 = Q2)
(charge "Q" means +Q on 1 plate , –Q on the other)
D) All of these
E) None of these

Answer: Q1 = Q2. The charges on the two plates of a capacitor are equal and opposite.

How do the voltage differences across the two capacitors (V1 and V2) relate to the voltage V on the battery?
A) V = V1 = V2
B) V = V1+V2
C) V = V1–V2
D) V = V2 – V1
E) V is not related in any definite way to V1 and V2

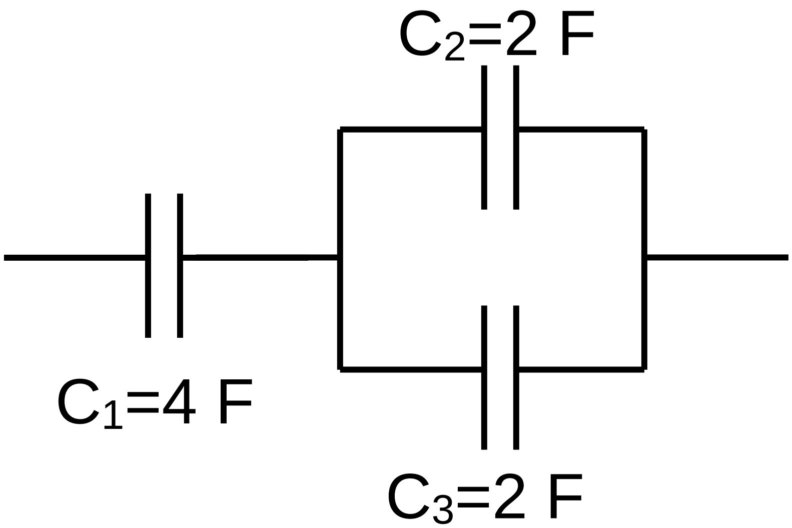
Answer: V = V1+V2

CAPCT-11.
Two capacitors C_1 and C_2 are hooked in series. The equivalent series capacitance of the two capacitors is …
A: Always less than C_1 or C_2
B: Always greater than C_1+C_2
C: Always greater than C_1 or C_2 but always less than C_1+C_2
D: None of these.



Answer: C_{series} is less than either C_1 or C_2. Since $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$, that means $\frac{1}{C_{ser}} > \frac{1}{C_1}$ and $\frac{1}{C_{ser}} > \frac{1}{C_2}$. But that means $C_{ser} < C_1$ and means $C_{ser} < C_2$. When you place capacitors in series, you always get a smaller capacitance. What you gain is the maximum voltage that can be placed across the capacitors. Suppose you have two 100 μF capacitors, each rated at 20V (meaning that if you ever put more than 20V across that capacitor, it will be destroyed). If you put these in series, you will have a single equivalent series capacitor with capacitance C_{ser} = 50 μF and which can take 40V maximum voltage across its terminals.

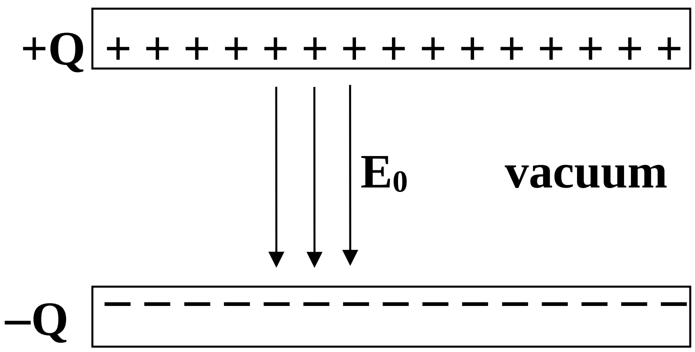
CAPCT-12. What is the effective total capacitance of this arrangement of capacitors?
A) 0.5 F B) 2 F C) 4 F D) 8 F E) Other

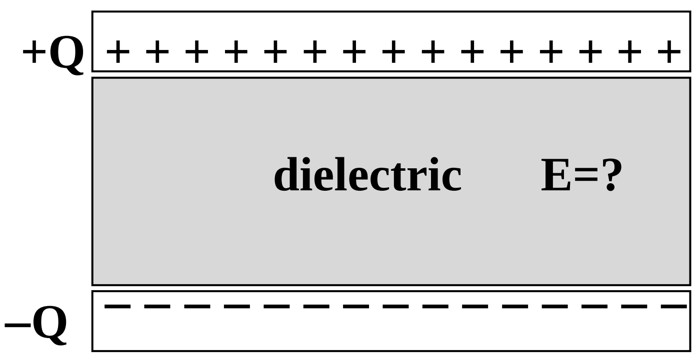


Answer: The effective total capacitance is 2F. (C2 and C3 are in parallel, so their effective capacitance C23 = C2+C3=4 F. C1 is in series with C23. 4F in series with 4F gives 2F.

CAPCT-13.
A charged capacitor has fixed charge Q (+Q on the top plate and –Q on the bottom plate). When there is a vacuum between the plates, the electric field has magnitude E_o. A dielectric material (insulator, like plastic) is placed between the plates. The dielectric becomes polarized when placed in the E-field. In the dielectric, how does the magnitude of the E-field compare to the original E-field?

A) $E = E_0$ B) $E > E_0$ C) $E < E_0$





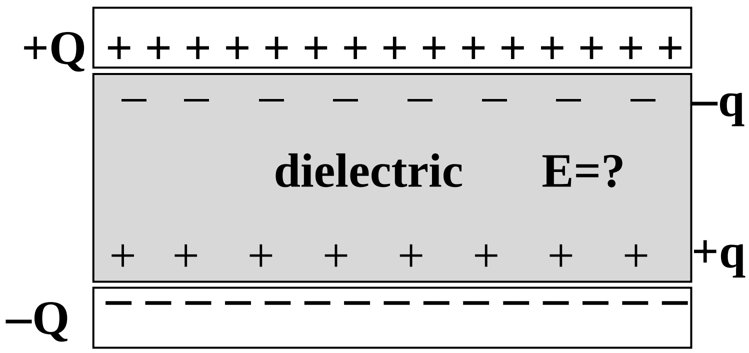
Did the voltage difference V between the plates change?
A) No change. B) V increased. C) V decreased.

Did the capacitance C of the capacitor change?
A) No change. B) C increased. C) C decreased.

Answers: The E-field decreases. The voltage decreases. The capacitance increases.
When the dielectric is placed between the plates, the E-field from the +Q/-Q charges polarizes the dielectric, and polarization charges appear: –q on the top of the dielectric and +q on the bottom of the dielectric. (See diagram below). The total E-field in the dielectric is due to **all** the charges present: both the q's and the Q's (the Q's didn't change). The E-field from the q's partially cancels the E-field from the Q's and the total E-field is smaller than it was in the vacuum.

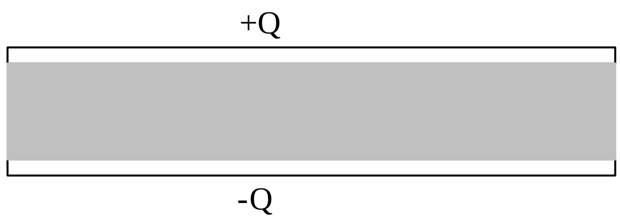
Since $V = E\,d$, when E decreases, V must decrease.

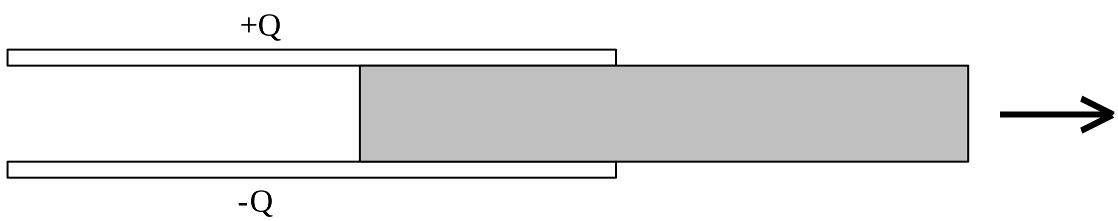
$C = Q/V$, and Q (the charge on the plates) did not change, but V decreased. So C increased.


+Q
−q
dielectric
E=?
+q
−Q

CAPCT-14. A parallel-plate capacitor with a dielectric between the plates is charged so that +Q resides on one plate, –Q on the other. With the plates isolated and the charge Q constant, the dielectric is pulled out from between the plates. The energy stored in the capacitor ...

A: increased B: decreased C: stayed the same.





Hint: Was work done when the dielectric was removed?

Answer: The energy increased.

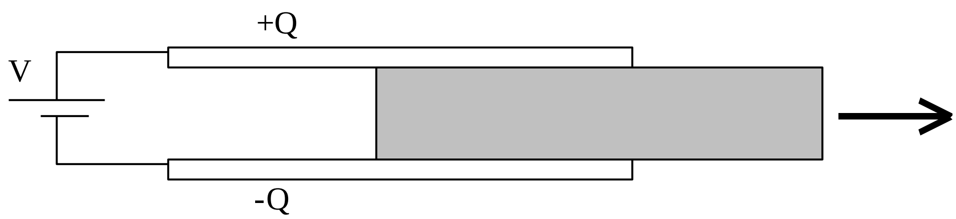
Method I: $U = (1/2)Q^2/C$. Q remains constant, and C decreases, so U increases.

Method II: The E-field increased, so the energy density $(1/2)\varepsilon_0 E^2$ increased.

Method III: The external agent did positive work. The polarization charges on the dielectric result in an electrostatic force that pulls the dielectric back in between the plates. The external agent has to pull hard to overcome this electrostatic force. Energy flowed out of the external agent into the increased energy of the capacitor.

CAPCT-15. Same question, except now the capacitor is hooked to a battery which <u>maintains a constant voltage</u> between the plates as the dielectric is removed.

With the voltage fixed, as the slab was removed, the charge Q on the capacitor ..
A: increased B: decreased C: stayed the same.



The energy stored in the capacitor ...
A: increased B: decreased C: stayed the same.

Answers: The charge Q decreased. The energy U

Note that the capacitance C decreases when the dielectric is removed.
$C = Q/V$. So $Q = CV$. Since V remains constant and C decreases, the Q must decrease.
$U = (1/2)CV^2$. Since C decreased, and V = constant, the energy decreased. The energy went into the battery.

CAPCT-16.
A circuit consists of three identical capacitors $C_1 = C_2 = C_3 = C$, which are connected to a battery of voltage V_0. Compare the charges Q_1, Q_2, and Q_3 on the three capacitors C_1, C_2, and C_3 .

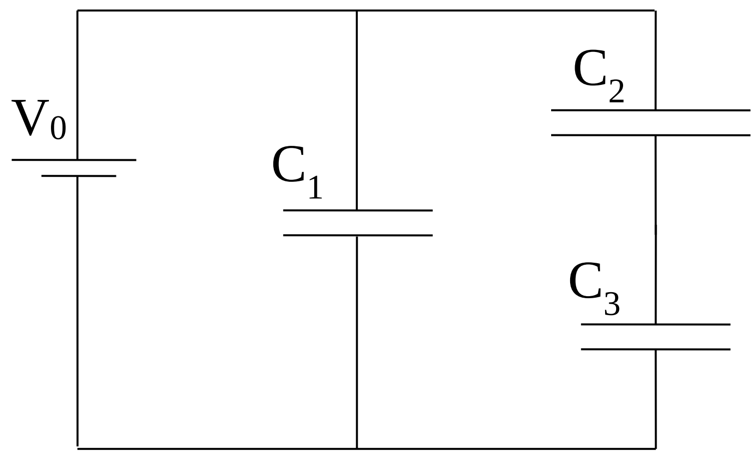
A) $Q_1 > Q_2 > Q_3$

B) $Q_1 > Q_3 > Q_2$

C) $Q_1 > Q_2 = Q_3$

D) $Q_1 = Q_2 = Q_3$

E) $Q_1 < Q_2 = Q_3$



Answer: $Q_1 > Q_2 = Q_3$

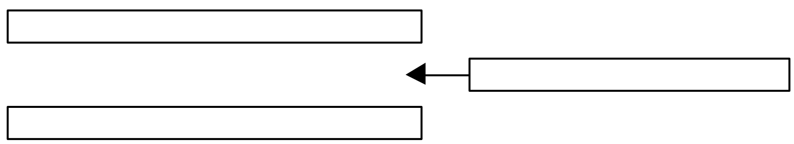
Look at the voltages and use $Q = CV$.

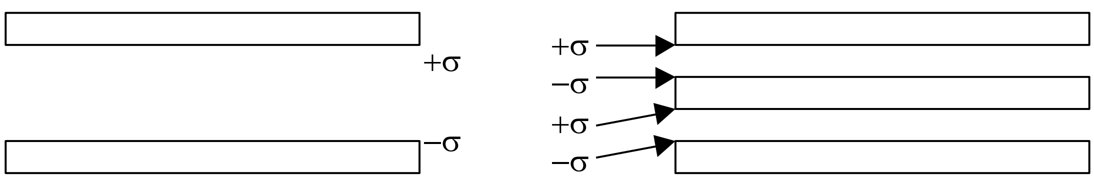
CAPCT-17.

Consider a capacitor made of two isolated parallel metal plates separated by a distance d. The top plate is positively charged with total charge +Q; the bottom plate is negatively charged with total charge –Q. A slab of metal of thickness $t < d$ is inserted between the plates, not connected to either one. Upon insertion of the metal slab, the voltage difference ΔV between the top and bottom plates does what? [Hint: $\Delta V = E \cdot d$]

A: increases B: decreases C: remains constant.

Another hint: The charge density, before and after, looks like:





After insertion of the third plate, the ratio Q/V where Q is the charge on the top plate and V is the voltage difference between top and bottom plates..

A: increases B: decreases C: remains constant.

Answer: ΔV between the top and bottom plates decreases. The E-field between any pair of plates remains the same: $E = \sigma/\varepsilon_0$. But the distance over which this E-field exists decreases: it was d before insertion of the 3rd plate; it is (d-t) after insertion. The voltage difference ΔV was originally $E \cdot d$; after insertion, it is $E \cdot (d-t)$.

The ratio Q/V is the effective capacitance of all 3 plates. Q remained the same, but V decreased, so the effective capacitance increased. That is, the 3-plate combination, with its small gaps, has a larger effective capacitance than the original 2 plates with its big gap.
