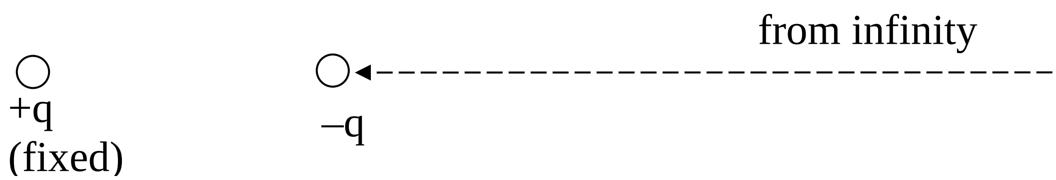
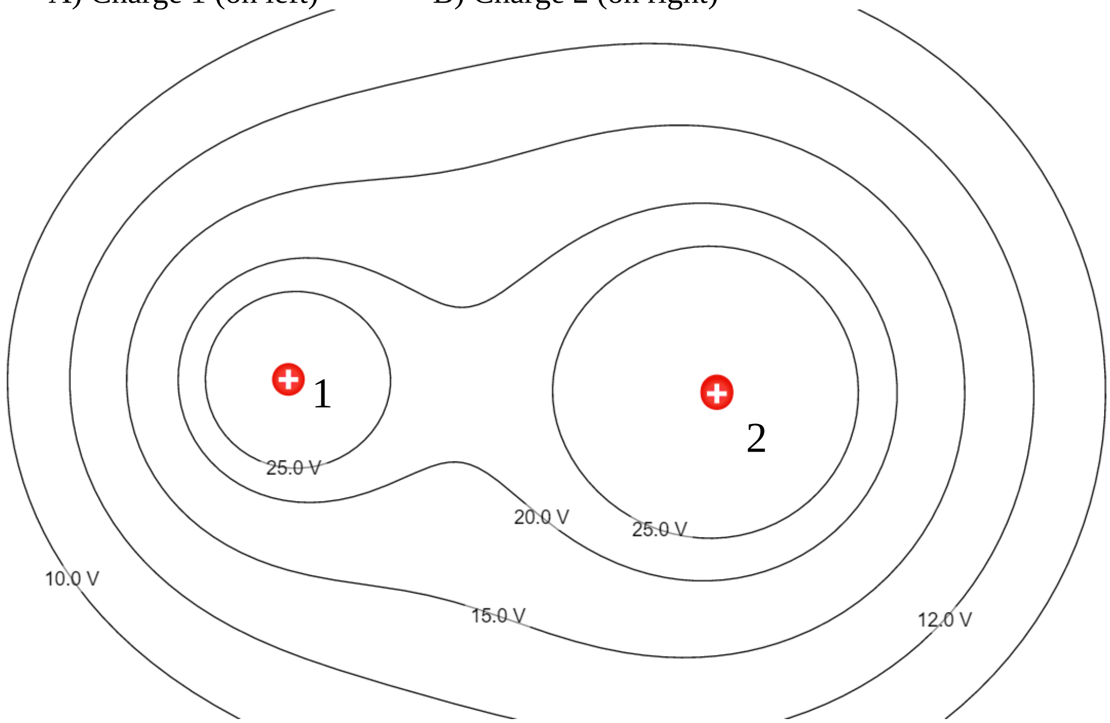
CTVoltageII-1.
A minus charge (–q) is brought in from infinity to be near a plus charge (+q). The work done by the external agent is bringing the minus charge in from infinity is
A) positive B) negative C) zero



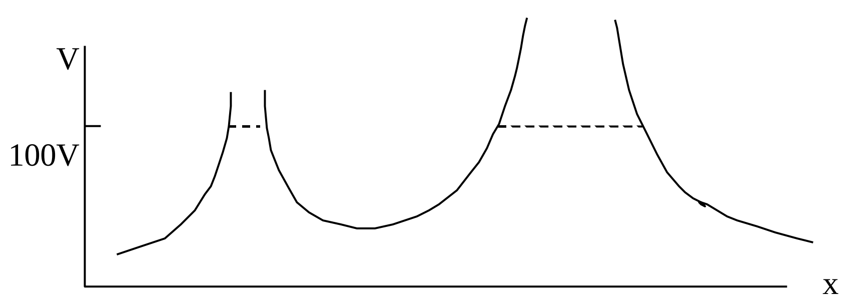
Answer: Negative work was done by the external agent (positive work was done by the field). As the external agent (tweezers) brought the –q charge toward the attractive +q charge, it had to *restrain* (pull back on) the –q charge. So force and displacement are in opposite directions and work done is negative.

CTVoltageII-2.
The equipotential contours around two positive charges (charges 1 and 2) are shown. Which charge is larger?

A) Charge 1 (on left)   B) Charge 2 (on right)

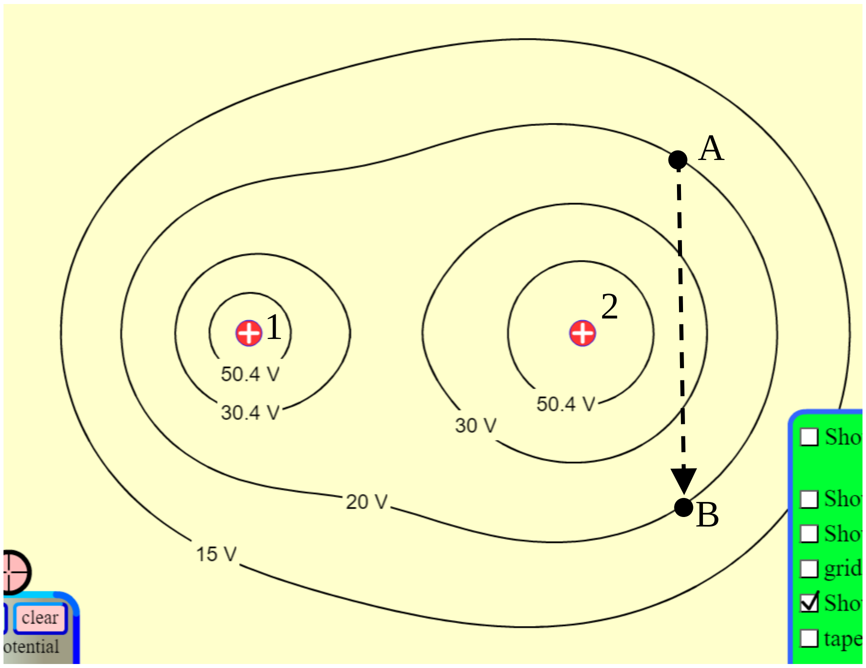


Answers: Charge 2 is the larger charge. Near a point charge, $V = \frac{kq}{r}$. Comparing the two 25V equipotentials, we have $25V = \frac{kq_i}{r_i}$. The charge with the large r, must have the larger q. A better way to see this is to think of voltage as "electrical height" and regard the equipotential lines as contours lines of constant altitude



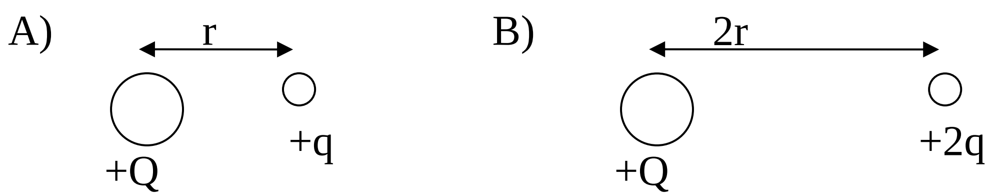
CTVoltageII-3.
A test charge (+q) is carried from point A to point B. The work done by the external agent carrying the test charge is

A) Positive B) Negative C) Zero



Answer: zero. $\Delta U = q\,\Delta V$. Since $\Delta V = 0$, then $\Delta U = 0$ and the total work done is zero. Positive work was done in the first half of the journey; negative work was done in the second half; the net work was zero.

CTVoltageII-4.
Two test charges are brought separately into the vicinity of a charge +Q. First test charge +q is brought a distance r from +Q. Then +q is removed and a test charge +2q is brought a distance 2r from +Q. Which charge configuration required more work (done by the external agent moving the test charge) to assemble?

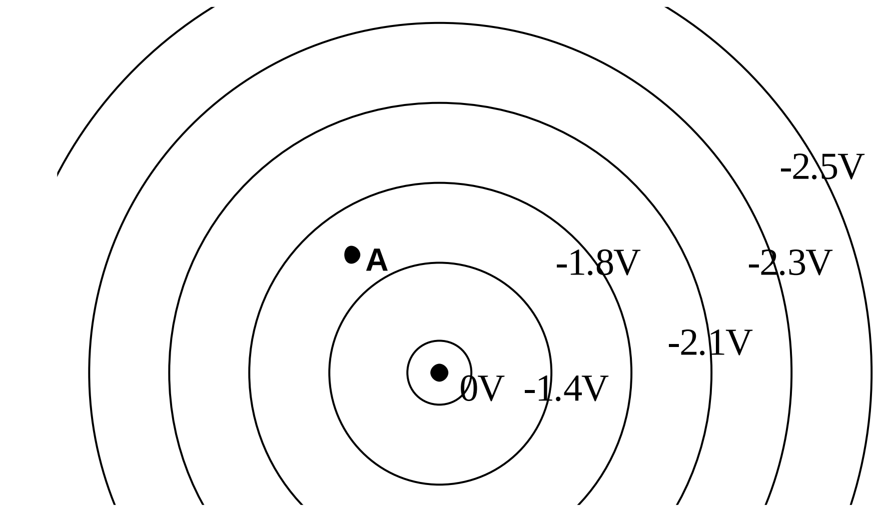


C) Both required the same work.

Answer: Both required the same work. Work done is change in the potential energy = $\Delta U = q\Delta V$.
For case I (on left) this is $\Delta U = q_{test}\Delta V = q\dfrac{kQ}{r}$. For case II (on right), this is

$$\Delta U = q_{test}\Delta V = 2q\frac{kQ}{2r} = q\frac{kQ}{r}$$

CTVoltageII-5.
The equipotential surfaces around a line of charge(into the page) are shown. Each equipotential is 2 m from the nearest-neighbor equipotentials. What is the approximate magnitude of the electric field at point A?



A) 0.1 V/m

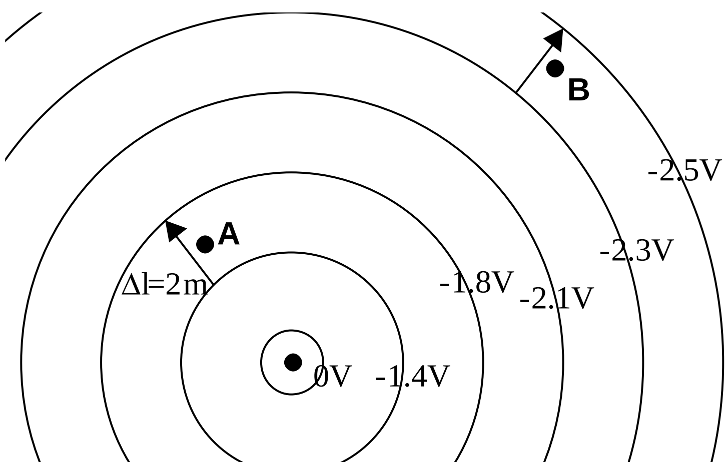
B) 0.2 V/m

C) 0.4 V/m

D) 0.6V/m

E) None of these

The formula $\Delta V = -\vec{E} \cdot \Delta \vec{r}$ can be used if Δr is a short enough distance, so that E is approximately constant over that distance. If $\Delta \vec{r}$ is parallel to $\vec{E}$, then $\Delta V = -\vec{E} \cdot \Delta \vec{r} = -E\,\Delta r$, and we can solve for E, $E = -\frac{\Delta V}{\Delta r} = -\frac{dV}{dr}$. This formula says E is the rate of change of V (volts per meter) if you move along the direction of E. We get the rate of change of V near point A, by taking a relatively short Δr near A (centered on A is best) and reading the ΔV from the graph.



For the Δl =2m chosen on the graph, the voltage change is

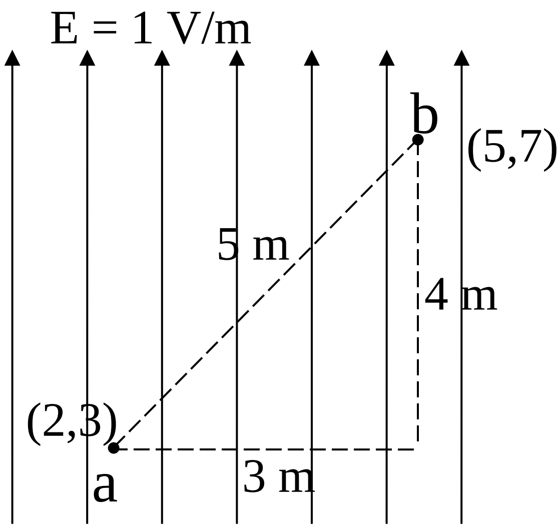
ΔV = (–1.8V – (–1.4V))=–0.4V.
So the E-field at **point A**

$$E = -\frac{\Delta V}{\Delta r} = -\frac{-0.4V}{2m} = 0.2V/m$$

The E-field at **point B** is

$$E = -\frac{\Delta V}{\Delta r} = -\frac{-2.5V - (-2.3V)}{0.2m} - \frac{-0.2V}{2m} = 0.1V/m$$

CTVoltageII-6.
A constant uniform E-field of magnitude E = 1V/m points up. Point a is (2m, 3m) Point b is (5m, 7m).
What is the voltage difference V(b) – V(a) ?



A) +5 V

B) +4 V

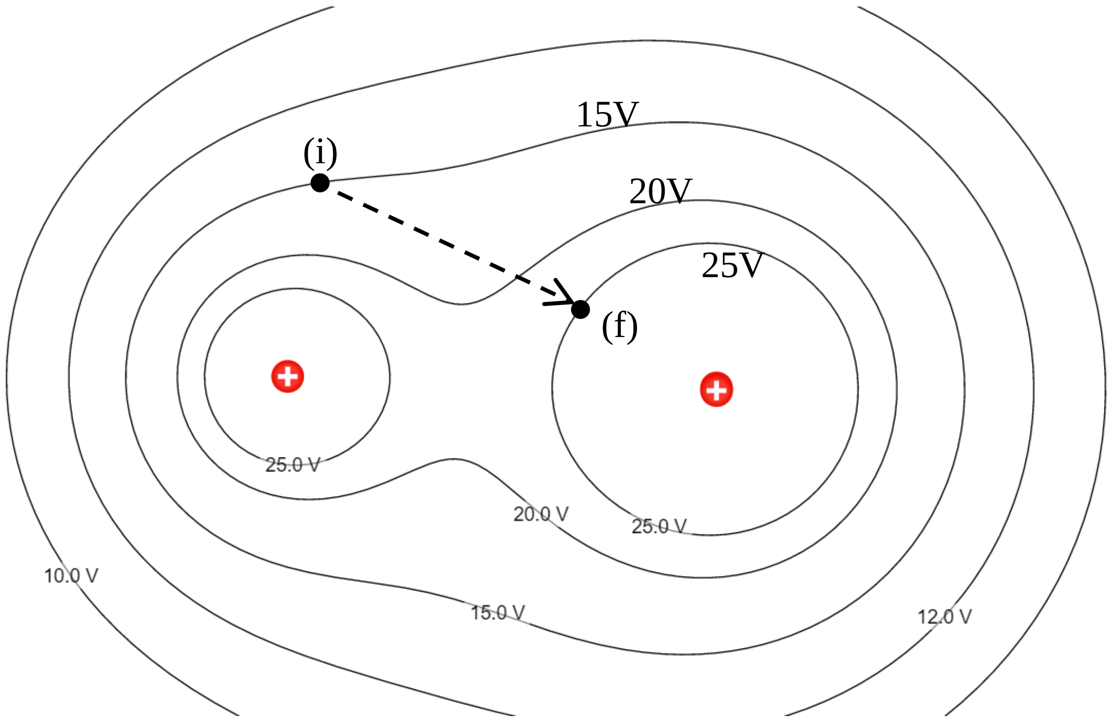
C) +3 V

D) None of these

Answer: None of these. The voltage difference V(b) – V(a) is – 4V. Point b is at a lower voltage than point A. Remember: E-fields point from higher voltage to lower voltage.

$$\Delta V = -\vec{E}\cdot\Delta\vec{r} = -E_y\Delta y = -(1\,V/m)(4\,m) = -4V$$

CTVoltageII-7.
An electron in the vicinity of charges 1 and 2 is moved (by an external agent) at constant speed from position (i) to position (f). What is the work required to move the electron from i to f?

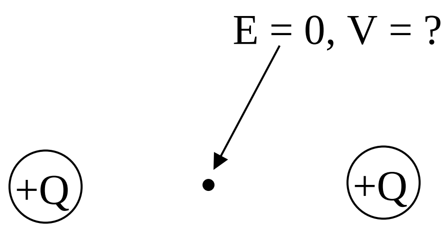


A) +10 eV　　B) More than +10 eV　　C) Between 0 and +10eV
D) –10 eV　　E) None of these.

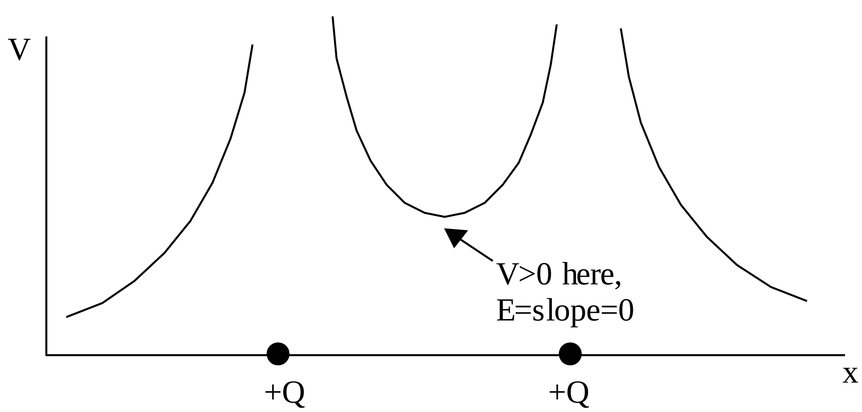
Answer: –10eV. From the voltages on the equipotential lines, you can see that the two charges are both positive. The work done by an external agent in moving the electron is **negative** because in moving a negative electron closer to a positive charge, the external agent actually has to restrain the electron, hold it back. Since the external force and the displacement have opposite directions, the work done by the ext agent is negative.

$$+W_{ext} = \Delta PE = q\Delta V = (-e)(V_f - V_i) = -e(25V - 15V) = (-e)(10V) = -10eV$$

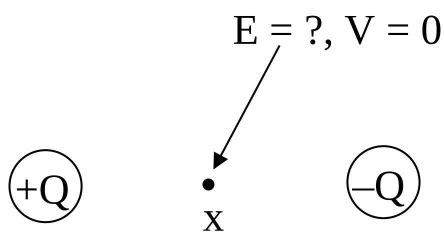
CTVoltageII-8. Two identical charges, +Q and +Q, are fixed in space. The electric field at the point X midway between the charges is zero. The voltage at that point is..
A) Zero B) Non-zero.



Answer: The voltage is non-zero. At points near positive charges, there is a positive voltage. The only way to get zero voltage with point charges is to be infinitely distant from all charges or to have the negative voltages from negative charges exactly cancel the positive voltages from positive charges.



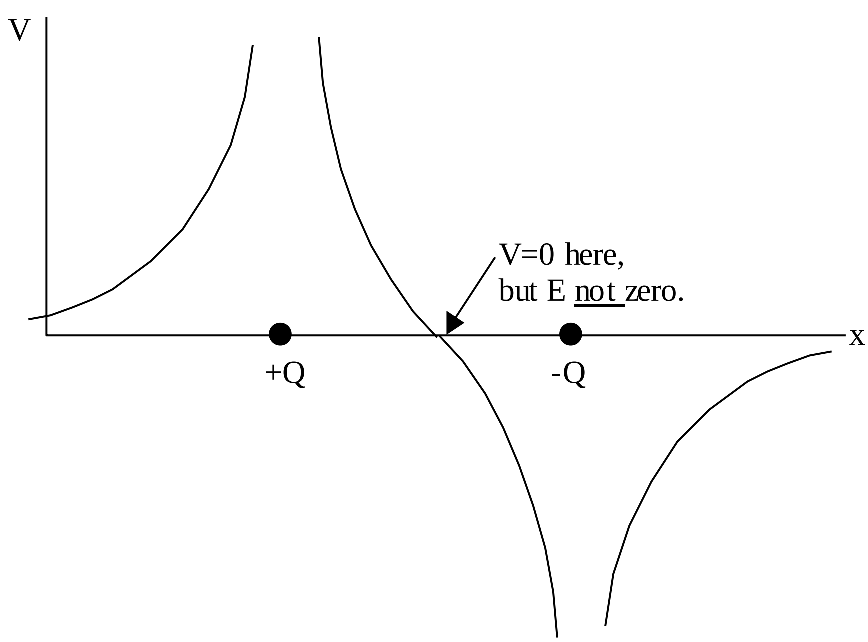
CTVoltageII-9. Two equal and opposite charges +Q and –Q are fixed in space. The voltage at the point X midway between the charges is zero. The electric field at that point is..
A) Zero B) Non-zero.



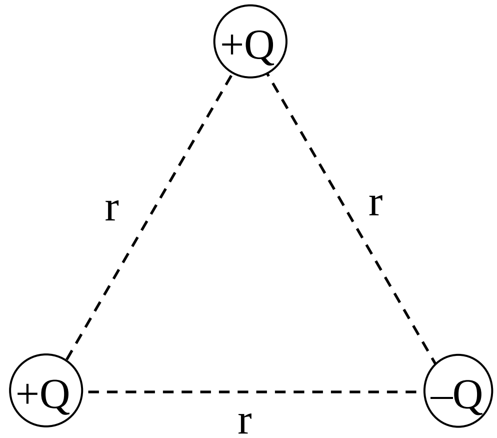
What sign work is done (by an external agent) to bring a negative charge –q from infinity to point X?
A) positive work B) negative work C) zero work

Answers: The E-field is non-zero. The E-fields from the two charges do not cancel there. On a plot of voltage vs. position, the E-field magnitude is proportional to the slope.

The work done by an external agent to bring any charge to that point is zero, since
$W_{ext} = \Delta U = q\Delta V = q \cdot 0 = 0$



CTVoltageII-10. Three charges +Q, +Q, and –Q form an equilateral triangle. The total electrostatic energy of this charge configuration (relative to all charges at infinity) is



A) positive

B) negative

C) zero

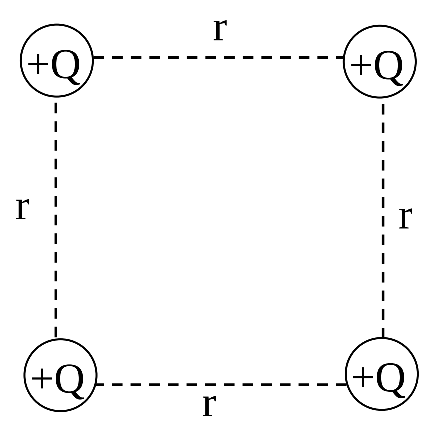
Answer: the total electrostatic potential energy is negative. The ++ pair produces a positive energy; each +- pair produces a negative energy.

CTVoltageII-11. Four equal positive charges +Q form a square of edge length r.
The total electrostatic energy of this charge configuration is ..

A) $4\frac{kQ^2}{r}$

B) greater than $4\frac{kQ^2}{r}$

C) less than $4\frac{kQ^2}{r}$



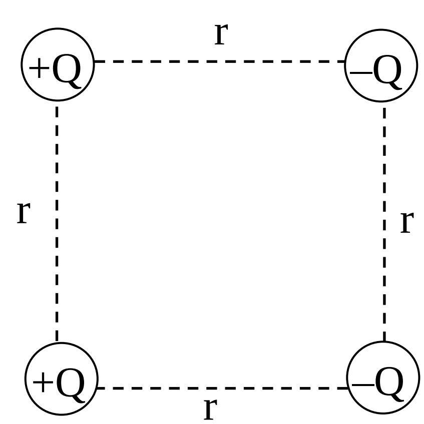
Answer: greater than $4\frac{kQ^2}{r}$. The diagonal pairs each contribute a positive energy.

CTVoltageII-12. Four charges +Q, +Q, –Q, and –Q, form a square as shown.
The total electrostatic energy of this charge configuration is ..

A) zero

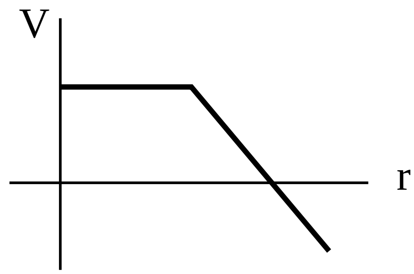
B) positive

C) negative

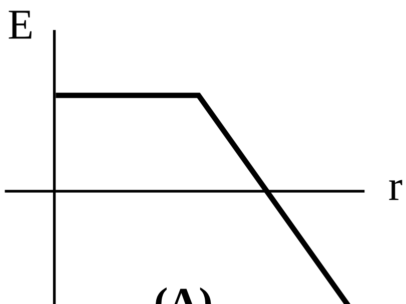


Answer: the total energy is negative. Add up the energies of each pair of charges.

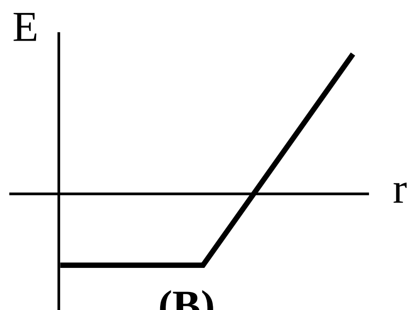
CTVoltageII-13. The voltage (potential) V versus distance r from a collection of charges is as shown.



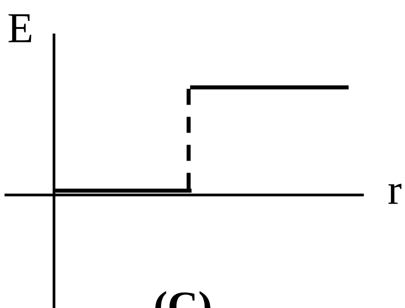
Which graph shows the electric field E versus distance r? E is positive if it points in the direction of increasing r.



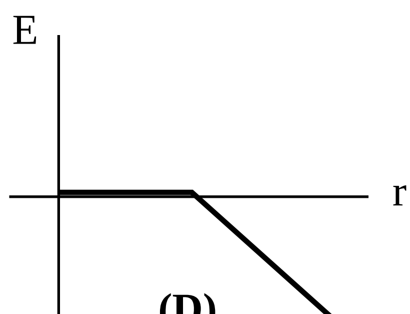
(A)



(B)



(C)

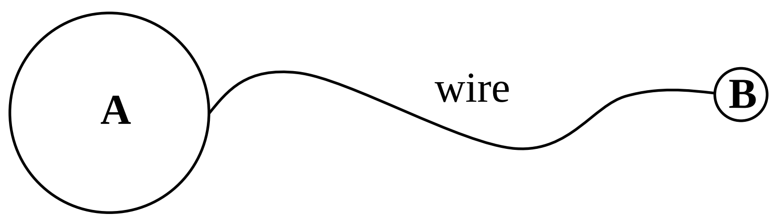


(D)

(E) None of these

Answer: C. $E = -dV/dr$

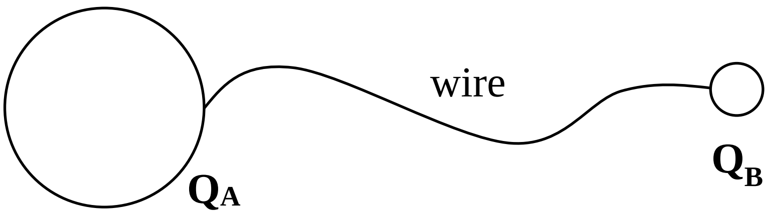
CTVoltageII-14. A large metal sphere A has initial charge +Q. A long metal wire now connects this charged sphere to a (initially uncharged) small metal sphere B. When in equilibrium, how do the voltages (potentials) on the surfaces of the two spheres compare?



A) $V_A = V_B$ B) $V_A > V_B$ C) $V_A < V_B$

Answer: $V_A = V_B$ In equilibrium, a metal is an equipotential (an equal voltage volume). You can think of the two spheres and the wire as a single piece of metal.

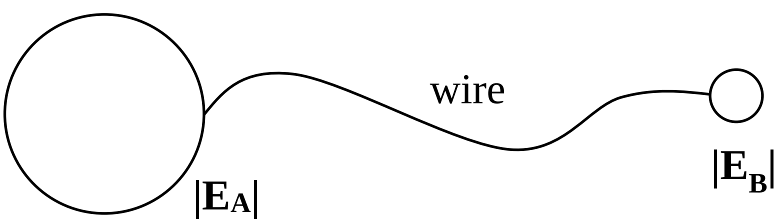
CTVoltageII-15. In equilibrium, how does the charge Q_A on the large sphere compare to the charge Q_A on the small sphere? Hint: The wire is very long, so the spheres are far apart and don't have much effect on each other. The E-field outside a uniform sphere of charge is exactly the same as a point charge (and likewise with the voltage: $V_{\text{outside sphere}} = V_{\text{pt charge}} = kQ/r$).



A) $Q_A = Q_B$ B) $Q_A > Q_B$ C) $Q_A < Q_B$

Answer: $Q_A > Q_B$ Since the voltage near each sphere is the same as that of a point charge (V = kQ/r), and the voltages are the same, we have $V = \dfrac{kQ_A}{r_A} = \dfrac{kQ_B}{r_B}$. So, $\dfrac{Q_A}{Q_B} = \dfrac{r_A}{r_B} > 1$

CTVoltageII-16. In equilibrium, how do the magnitudes of the E-fields at the surfaces of the spheres compare?



A) $E_A > E_B$ B) $E_A < E_B$ C) $E_A = E_B = 0$ D) $E_A = E_B \neq 0$

Answer: $E_A < E_B$

The E-field near each sphere is the same as that of a point charge: $E = kQ/r^2$. So we can set up the ratio $\frac{E_B}{E_A} = \frac{\left(\frac{kQ_B}{r_B^{\ 2}}\right)}{\left(\frac{kQ_A}{r_A^{\ 2}}\right)} = \frac{Q_B}{Q_A}\frac{r_A^{\ 2}}{r_B^{\ 2}} = \frac{r_B}{r_A}\frac{r_A^{\ 2}}{r_B^{\ 2}} = \frac{r_A}{r_B} > 1$ Here, we have used the result from the previous question: $\frac{Q_A}{Q_B} = \frac{r_A}{r_B}$

