

\Rightarrow (Consensus term) (For minimization)

$$\Rightarrow A't_1 \not\in A't_2 \Rightarrow t_1t_2$$

$$\Rightarrow \underline{A'}BC + \underline{AC}d \Rightarrow BCd \text{ is a consensus term}$$

$\Rightarrow \underline{A'}Bc + \underline{A}B'cd \Rightarrow$ there isn't any consensus term because more than (Variable & its complement) are found.

Ex:

Simplify:

$$F = \underset{\uparrow}{a'b'c'} + \underset{\downarrow}{a'b'c} + \underset{\uparrow}{a'b'c} + \underset{\uparrow}{ab'c}$$

$$= b'c'(a' + a) + a'b(c' + c)$$

$$= b'c' + a'b \quad (\text{Consensus term } \rightarrow X)$$

Ex:

Simplify: $F = \underset{\leftarrow}{a'b'c'} + a'b'c + abd + acd$

$$= b'c'(a' + a) + abd + acd$$

$$= \underset{\uparrow}{b'c'} + \cancel{abd} + \underset{\uparrow}{acd}$$

(abd) is a consensus term
disjoint

$$= b'c' + acd$$

EX:

$$\text{Simplify: } F = \underbrace{A'BCD}_{\cancel{A'BD}} + \underbrace{A'BC\bar{D}}_{\cancel{A'BD}} + \underbrace{B'EF}_{\cancel{B'EF}} + CDE^G + A'DE\bar{F} + \underbrace{A'B\bar{E}F}_{\cancel{B'EF}}$$

$$= \cancel{A'BD} + \cancel{B'EF} + CDE^G + \cancel{A'DE\bar{F}}$$

A'DEF
 is a consensus term
 $\cancel{A'BD} \cancel{B'EF}$

$$F = A'BD + B'EF + CDE^G$$

EX:

Minimize:

$$F = w\bar{y} + w\bar{x}\bar{z} + \cancel{w\bar{x}y} + w\bar{y}\bar{z}$$

consensus term $w\bar{x}y$
simplification

$$F = w\bar{y} + w\bar{x}\bar{z} + w\bar{x}y + w\bar{y}\bar{z} + \cancel{w\bar{x}y} \leftarrow \text{cancel } w\bar{x}y$$

$$F = w\bar{y} + w\bar{x}\bar{z} + w\bar{y}\bar{z} + \cancel{w\bar{x}y}$$

$$F = w\bar{y} + w\bar{y}\bar{z} + \cancel{w\bar{x}y}$$

\leftarrow Simplify:

No. 9)
Assignment 1

$$(A+B)(\bar{A}+C)$$

$$= A \cdot \cancel{A} + AC + \bar{A}B + BC$$

$$= AC + \bar{A}B + \cancel{BC}$$

\cancel{AC}

$$= AC + \bar{A}B$$

No. 10)

Assignment

$$F = a + \bar{a}b + \bar{a}bc + (\bar{a}bc\bar{d} + \bar{a}bcde)$$

$$= a + \bar{a}b + \bar{a}bc + \bar{a}bc(d + de)$$

$\bar{a}bc$

$(d + d) (d + e)$

$$= a + \bar{a}b + [\bar{a}bc + \bar{a}bc(d + e)]$$

$\bar{a}bc$

$$= a + \bar{a}b + \bar{a}b(c + \bar{c}(d + e))$$

$(c + \bar{c}) (c + d + e)$

$$= a + [\bar{a}b + \bar{a}b(c + d + e)]$$

$$= a + \bar{a}(b + b(c + d + e))$$

$(b + b) (b + c + d + e)$

$$= \bar{a} + \bar{a}(b + c + d + e)$$

$(\bar{a} + \bar{a})(a + b + c + d + e)$

$$= (a + b + c + d + e)$$

De Morgan's Theorems:

$$1) (A \cdot B)^c = A^c + B^c$$

$$2) (A + B)^c = A^c \cdot B^c$$

Ex:

Find the complement of $F = a \cdot b d^c + b^c c + a^c c d + a^c b^c d$

Solution

$$F^c = (a b d^c + b^c c + a^c c d + a^c b^c d)^c$$

$$= [(a b d^c)^c \cdot (b^c c)^c \cdot (a^c c d)^c \cdot (a^c b^c d)^c]$$

$$= (a^c + b^c + d)(b + c)(a + c^c + d^c)(a + b^c + c + d^c)$$

Ex:

If $f = (a+b)(b^c+c) + d^c(a^c b + c)$ Find f^c

Solution

$$f^c = [(a+b)(b^c+c) + d^c(a^c b + c)]^c$$

$$= [(a+b)(b^c+c)]^c \cdot [d^c(a^c b + c)]^c$$

$$= [(a+b)^c + (b^c+c)] [d + (a^c b + c)^c]$$

$$= (a^c b^c + b^c c^c) (d + (a+b) \cdot c^c)$$

$$= (a^c b^c + b^c c^c) (d + a^c c^c + b^c c^c)$$

Date:

Simplify..

No. 13)

$$F = AB + (AC) + A\bar{B}C (AB + \bar{C})$$

Assignment 1

$$= AB + (\bar{A} + \bar{C}) + \cancel{AB}\cancel{C} \cancel{AB} + \cancel{AB}\cancel{C} \cancel{C}$$

$$= (\cancel{AB} + \bar{A}), (\cancel{C} + \cancel{AB}\bar{C})$$

$$= (\cancel{\bar{A}} + \bar{A}) (\bar{A} + \bar{B}) + (\cancel{\bar{C}} + \bar{C}) (\bar{C} + AB)$$

$$= \cancel{\bar{A}} + \bar{B} + \bar{C} + \cancel{AB}$$

$$= (\cancel{\bar{A}} + \bar{A}) + \bar{B} + \bar{C}$$

$$= (\cancel{\bar{A}} + \bar{A}) (\bar{A} + \bar{B}) + \bar{B} + \bar{C}$$

$$= \bar{A} + (\cancel{\bar{B}} + \bar{B}) + \bar{C} = \bar{A} + 1 + \bar{C} = \boxed{1}$$

lecture 2

$$F = \cancel{\bar{C}} ((a+b) + bcd + abc) (\cancel{bcd})$$

slide 82

$$= b\bar{c}d (a\bar{b} + bcd + abc)$$

$$= (\cancel{b\bar{c}d} \cdot \cancel{ab}) + \cancel{b\bar{c}d} \cdot \cancel{b\bar{c}d} + b\bar{c}d \cdot abc$$

$$= \boxed{abc\bar{d}}$$

From SOP to POS & VS

→ To convert From SOP (Sum of Product) to POS (Product of Sum) and vs., We use These Rules:

1) $a(b+c) = ab + ac$

2) $a + bc = (a+b)(a+c)$

3) $\overline{ab} + \overline{ac} = (a+c)(\bar{a}+b)$

Ex:

Convert From SOPs to POSs:

$$F = WXY + XYZ + W\bar{X}\bar{Z}$$

Solution:

$$F = X(WY + YZ) + W\bar{X}\bar{Z}$$

$$= (\cancel{X})(WY + YZ) + (\cancel{X}) \cdot W\bar{X}\bar{Z}$$

$$= (X + W\bar{Z})(\cancel{X} + (WY + YZ))$$

$$= (X + W)(X + \bar{Z})(\cancel{X} + (W + Y)(Y + Z))$$

$$= (X + W)(X + \bar{Z})(W + X + Y)(X + Y + Z)$$

Ex:

Convert From SopS To Pos:

$$\bar{x} + \underline{y}\bar{z} + \underline{\bar{y}}\bar{z}$$

$$\bar{x} + (\bar{y} + \bar{z})(\bar{y} + z)$$

$$(\bar{x} + \bar{y} + \bar{z})(\bar{x} + \bar{y} + z)$$

Ex:

Convert From SopS To Pos:

$$\bar{a}\bar{c}\bar{d} + \bar{a}\bar{c}d + bc$$

$$\bar{a}\bar{c}\bar{d} + c(\bar{a}\bar{d} + b)$$

$$(c + \bar{a}\bar{d}) \cdot (c + (\bar{a}\bar{d} + b))$$

$$(\bar{a} + c)(c + d)(c + (\bar{a} + b)(b + d))$$

$$(\bar{a} + c)(c + d)(\bar{a} + b + c)(b + c + d)$$

Ex:

$$w\underline{x}\bar{y} + \bar{x}\bar{y}\bar{z} + w\bar{x}\bar{z}$$

$$\bar{x} \cdot (\bar{w}\bar{y} + \bar{y}\bar{z}) + \bar{x} \cdot w\bar{z}$$

$$(\bar{x} + w\bar{z})(\bar{x} + (\bar{w}\bar{y} + \bar{y}\bar{z}))$$

$$(\bar{x} + w)(\bar{x} + \bar{z})(\bar{x} + (\bar{y}(\bar{w} + \bar{z})))$$

$$(\bar{x} + w)(\bar{x} + \bar{z})(\bar{x} + \bar{y})(\bar{x} + w + z)$$

EX:

Given the following expression:-

$$F(a, b, c, d) = \underline{b\bar{c}d} + cd + \underline{b\bar{c}d} + abc + bd + \underline{ab\bar{c}d}$$

a) find the minimum sum of products.

b) find the minimum product of sums.

Solution:

a) minimum SoPs :-

$$F = \underline{\frac{b\bar{c}d}{bc}} + \underline{cd} + \underline{\frac{b\bar{c}d}{ab}} + \underline{\frac{abc}{ab}} + \underline{bd} + \underline{\frac{ab\bar{c}d}{ab}}$$

$$= b\bar{c}(d + \cancel{d}) + cd + ab(c + \cancel{c}d) + bd$$

$$= b\bar{c} + cd + ab(c + \cancel{c})(c + d) + bd$$

$$= b\bar{c} + cd + abc + \underline{\frac{abd}{bd}} + \underline{\frac{bd}{bd}}$$

$$= b\bar{c} + \cancel{cd} + abc + bd(a \neq 1)$$

$$= b\bar{c} + \cancel{cd} + abc$$

$$= b(c + ac) + cd$$

$$= b(\bar{c} + a)(\bar{c} + c) + cd$$

$$= b\bar{c} + ab + cd$$

b) minimum Po s:

$$F = b \underbrace{c}_{\textcircled{c}} + \underbrace{c d}_{\textcircled{d}} + a b$$

$$= (\underbrace{c+d}_{\textcircled{r}})(\underbrace{b+c}_{\textcircled{r}}) + a b$$

$$= (ab + (c+d)) (ab + (b+c))$$

$$= (a+c+d)(b+c+d)(a+b+c)(b+b+c)$$

$$= (a+c+d)(b+c+d)(a+b+c)(b+c)$$

$$= (a+c+d)(b+c+d)[(b+c) + (a+1)]$$

$$= (a+c+d)(b+c+d)(a+b+c)$$

Ex:

Given the following expression :

$$F = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}\bar{c} + a\bar{b}c$$

a) Find the minimum SoP.

b) Find the minimum Pos.

Solution.

a)

$$F = \underbrace{\bar{a}\bar{b}\bar{c}}_{\bar{a}\bar{c}} + \underbrace{\bar{a}\bar{b}\bar{c}}_{\bar{b}\bar{c}} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + \underbrace{a\bar{b}\bar{c}}_{\bar{a}b} + \underbrace{a\bar{b}c}_{ab}$$

$$= \underbrace{\bar{a}\bar{c}}_{\bar{a}} + \underbrace{\bar{a}\bar{b}c}_{\bar{a}} + \underbrace{a\bar{b}\bar{c}}_{\bar{b}} + \underbrace{ab}_{a}$$

$$= \bar{a}(\bar{c} + bc) + a(\bar{b}c + b)$$

$$= \bar{a}(\bar{c} + b)(\bar{c} + c) + a(\bar{b} + b)(c + b)$$

$$= \bar{a}(\bar{c} + b) + a(c + b)$$

$$= \underbrace{\bar{a}\bar{c}}_{\bar{a}} + \underbrace{\bar{a}b}_{\bar{a}} + ac + ab$$

$$= \bar{a}\bar{c} + b(\cancel{\bar{a}} + a) + ac$$

$$= \bar{a}\bar{c} + ac + b$$

\Rightarrow

b) The Minimum Pos:

$$F = \textcircled{a} \cdot \textcircled{c} + \textcircled{a} \cdot c + b$$

$$= (\textcircled{a} + c)(a + \textcircled{c}) + b$$

$$= (\textcircled{a} + b + c)(a + b + \textcircled{c})$$

⇒ Manipulate the following to sum of product expression:

$$F = (b + c)(a + c + d')(c' + d)(a + \textcircled{c} + d')$$

Solution.

$$F = [c + (b \cdot (a + d'))][c' + (d \cdot (a + d'))]$$

$$= [c + (a'b' + b'd')][c' + (ad + d'd)]$$

$$= [\textcircled{c} + (a'b' + b'd')][\textcircled{c}' + ad]$$

$$= (c \cdot ad) + (c \cdot (a'b' + b'd'))$$

$$= acd + a'b'c + b'cd'$$

From Pos to SOP

$$1) (a+b)(a+c) = a+bc$$

Rule → 2) $a(b+c) = ab + ac$

3) $(a+c)(a+b) = ab + a'b$

Ex:

Manipulate the following Function to SOP:

$$F = (x+y)(x+\bar{z})(\bar{x}+y+z)$$

$$= [x + (\bar{y}\bar{z})] (\bar{x} + \bar{y} + z)$$

$$= x \cdot (\bar{y} + z) + \bar{x} \cdot (\bar{y}\bar{z})$$

$$= xy + xz + \bar{x}\bar{y}\bar{z}$$

→ Truth Table (TT)

Ex:

For the function:

$$F = \bar{x}y + z.$$

a) Show the truth table

b) Show The equation F in :

Sum of minterms

or Sum of Product (SOP)

or Disjunctive Form

c) Show F in :

Product of Maxterms

or Product of sum (POS)

or Conjunctive Form.

d) Draw the block diagram of "F"

ANS:

3 Variable (x, y, z)

$2^3 = 8$ prob.

(a)

	x	y	z	\bar{x}	$\bar{x}y$	$F = \bar{x}y + z$
M ₀ - m ₀	0	0	0	1	0	0
M ₁ - m ₁	0	0	1	1	0	1
M ₂ - m ₂	0	1	0	1	1	1
M ₃ - m ₃	0	1	1	1	1	1
M ₄ - m ₄	1	0	0	0	0	0
M ₅ - m ₅	1	0	1	0	0	1
M ₆ - m ₆	1	1	0	0	0	0
M ₇ - m ₇	1	1	1	0	0	1

b)

The sum of minterms \Rightarrow From one's of \bar{F}

\rightarrow in numeric form: $m_1 + m_2 + m_3 + m_5 + m_7$

$$F(x,y,z) = \Sigma m(1,2,3,5,7)$$

\rightarrow in Algebraic form:

$$F(x,y,z) = \bar{x}\bar{y}z + \bar{x}yz + \bar{x}y\bar{z} + \\ xy\bar{z} + xyz$$

c)

The Product of Maxterms \Rightarrow From zero's of \bar{F}

\rightarrow in numeric form: $M_0 + M_4 + M_6$

$$F(x,y,z) = \prod (0,4,6)$$

\rightarrow in Algebraic form:

$$F(x,y,z) = (x+y+z)(\bar{x}+y+\bar{z})(\bar{x}+y+z)$$

d)

$$F = \bar{x}y + \bar{z}$$



Note

The Sum of minterms

Called:

(Canonical Form)

"F" وتصنيف المتغيرات الموجدة في

$$\underline{x, y, z}$$

Ex:-

For the function $F = \sum m(1, 2, 6, 7)$

$$F = \sum m(1, 2, 6, 7)$$

- Show the truth table.
- Show an algebraic expression in sum of Product form.
- Show The minimum Sum of product expression
- Show the minterms of f' (Complement of F) in numeric form & in Algebraic form.

(ANS)

a)

from $0 \rightarrow 7$ is 3 var.

let that 3 variable are a, b, c

m	a	b	c	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$$F = \sum m(1, 2, 6, 7)$$

: if (1) logic F نعم

or m_1 هي المinterm

m_2

or m_8

or m_7

- Algebraic form in SOP:
$$F(a, b, c) = abc' + abc + ab'c + a'b'c$$

c) The minimum of SOP:

$$\begin{aligned} F(a,b,c) &= \overline{a'b'c} + \overline{a'b\bar{c}} + \overline{ab\bar{c}} + \overline{abc} \\ &= \overline{a'b'c} + \overline{a'b\bar{c}} + \overline{ab} \\ &= \overline{a'b'c} + b(\overline{a}\overline{c} + a) \\ &= \overline{a'b'c} + b(\overline{a+a})(\overline{c}+a) \\ &= \overline{a'b'c} + b(\overline{c}+a) \\ &= (\overline{a'b'c} + ab + b\bar{c}) \end{aligned}$$

d) The minterms of f' is:

numeric form \Rightarrow From Truth table "From 0's"
 $f'(a,b,c) = \Sigma m(0,3,4,5)$

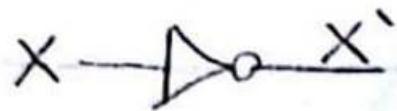
Algebraic form \Rightarrow Every 0's term as SOP.

$$f'(a,b,c) = \overline{a'b\bar{c}} + \overline{a'b\bar{c}} + \overline{ab\bar{c}} + \overline{ab\bar{c}}$$

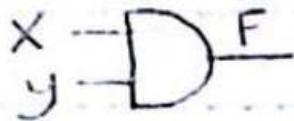
* Gates

⇒ Schematic of AND, OR, Not, NAND, NOR, XOR and XNOR

NOT gate



AND gate



$$F = x \cdot y$$

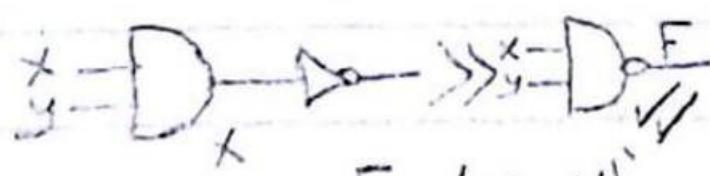
A	B	F
0	0	0
0	1	0

NAND gate

⇒ Not AND

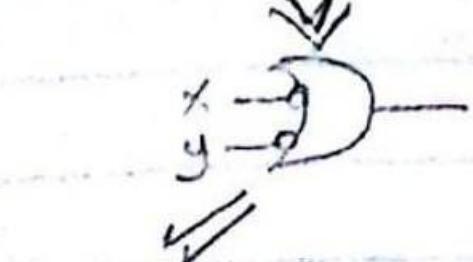
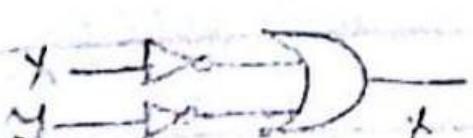
⇒ AND Then AND

⇒ Complement of AND

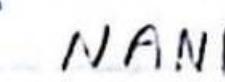


$$F = (\bar{x} \cdot \bar{y})$$

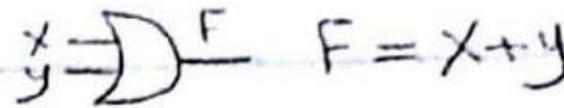
$$= \bar{x} + \bar{y}$$



2 shape for
NAND



OR gate



$$\begin{array}{ccccc} \text{OR} & & F & = & x + y \\ \oplus & & \frac{0}{0} & \frac{1}{1} & \frac{1}{1} \end{array}$$

NOR gate



XOR

$$\begin{array}{ccccc} \text{XOR} & & F & = & x \oplus y \\ \oplus & & \frac{0}{0} & \frac{1}{1} & \frac{0}{1} \end{array}$$

$$\begin{array}{ccccc} \oplus & & \frac{0}{0} & \frac{1}{1} & \frac{0}{1} \\ & & \frac{0}{0} & \frac{1}{1} & \frac{1}{1} \end{array}$$

Invert
output

XNOR



$$F = (x \oplus y)$$

$$= \bar{x}\bar{y} + xy$$

→ Show the block diagram of:-

or
→ Show the schematic for the following function.

a) $F = ab\bar{d} + b\bar{d}\bar{e} + \bar{b}cd + \bar{a}ce$

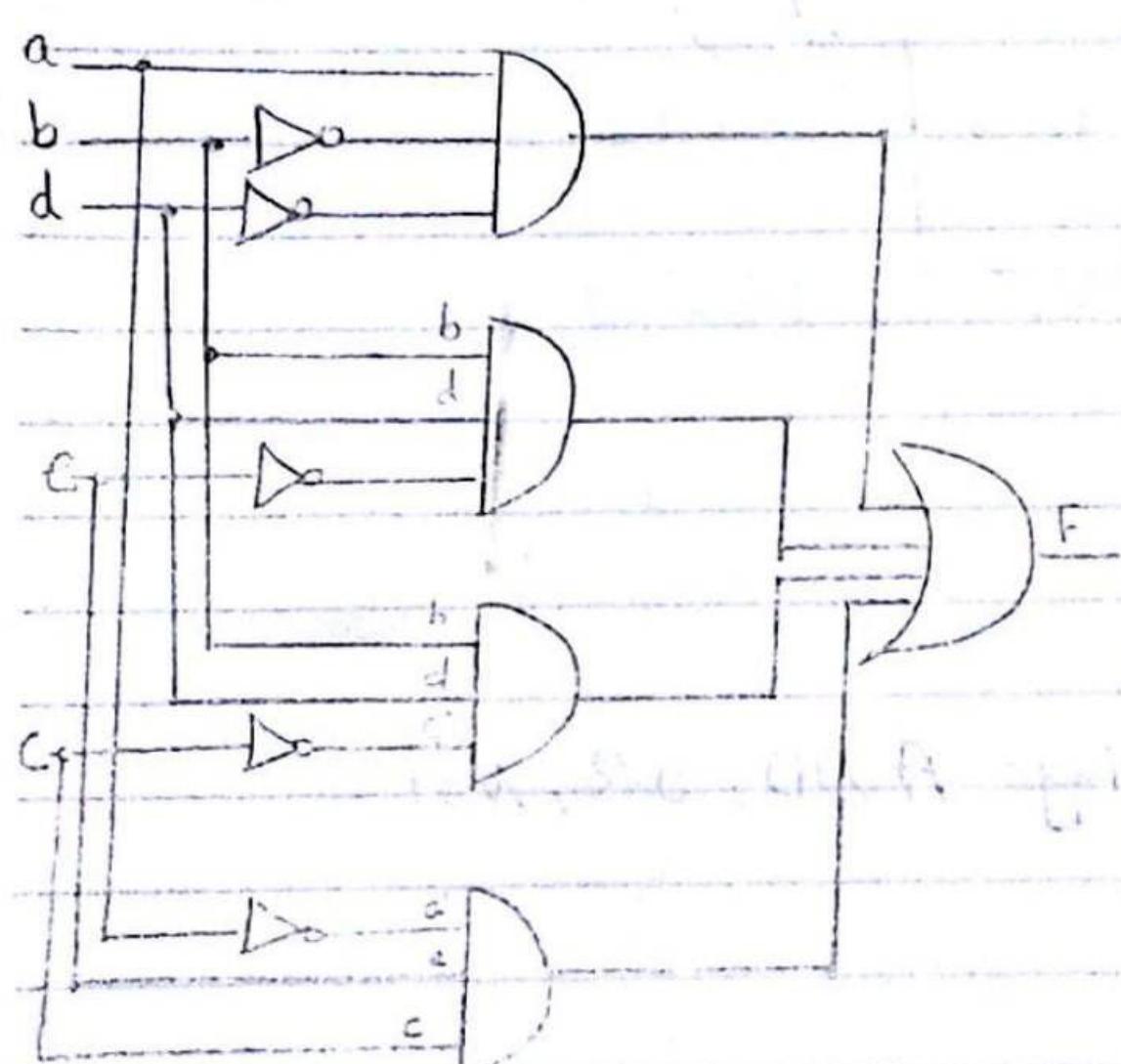
b) $f = b(\bar{c}d + \bar{c}e) + (\bar{a} + ce)(\bar{a} + \bar{b}d)$

① using AND, OR, NOT

② using only NAND

(Ans)

a)



① by using NOT, AND, OR

To use NAND

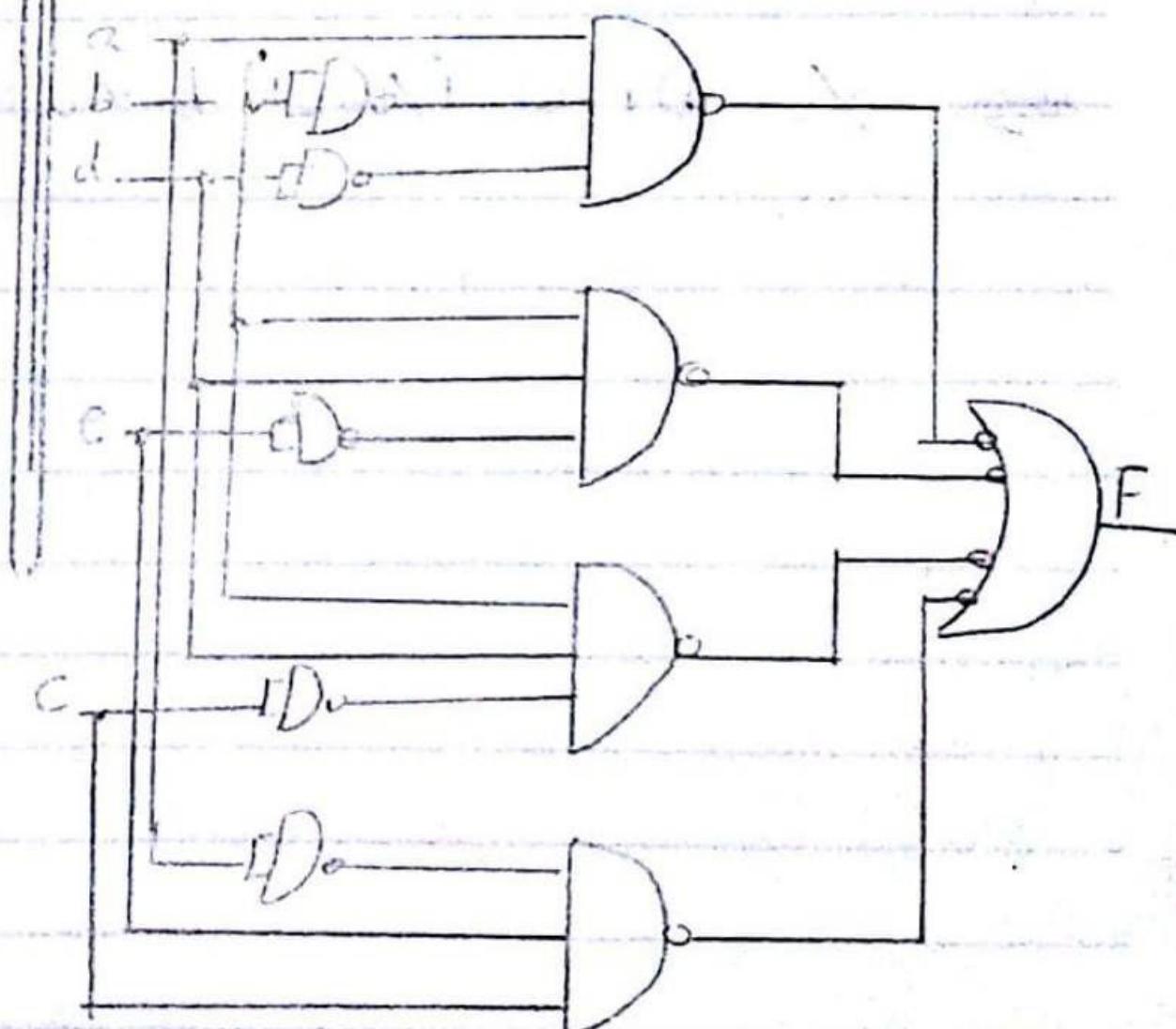
Draw by AND, OR, NOT Then convert each AND to NAND

&

each OR to NAND

&

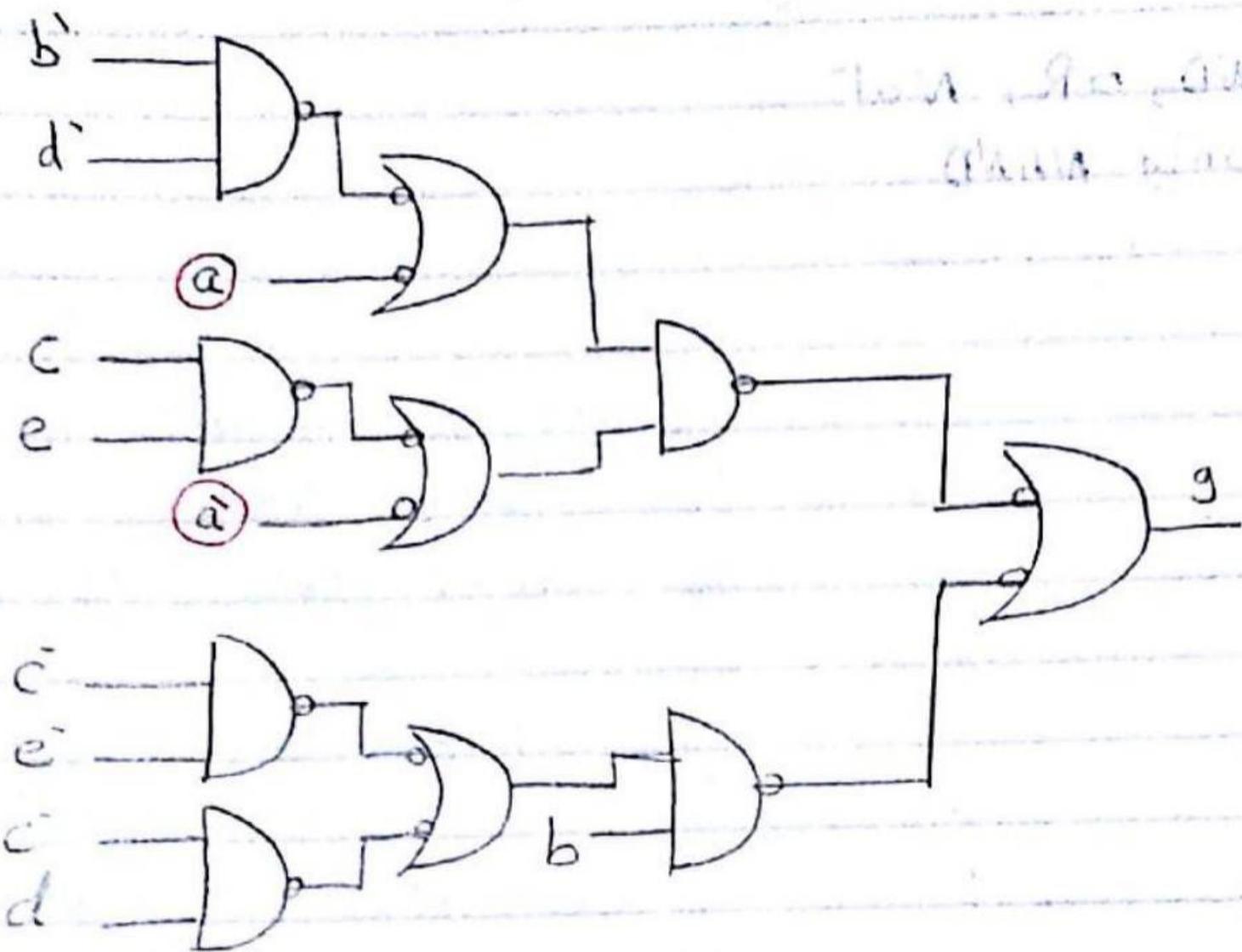
each NOT to NAND



b)

for the function "g" \Rightarrow if All Variable are Available.

$$g = b((\bar{c}\bar{d} + \bar{c}\bar{e}) + (\bar{a} + \bar{c}e)(\bar{a}\bar{c} + \bar{b}\bar{d}))$$



② by using NAND.

\Rightarrow You Draw (no. 1) by using AND, OR, NOT.

→ Using "only" OR and NOT gates.: draw a schematic for this function.

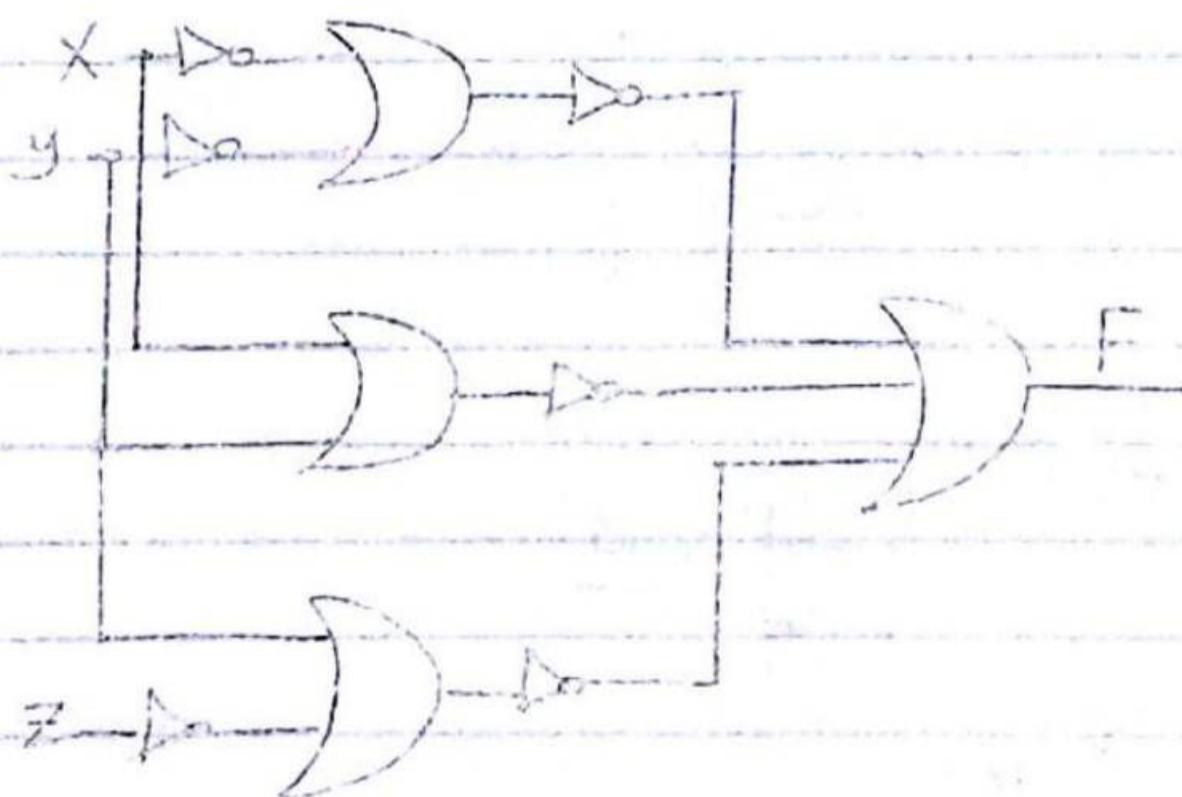
$$F = xy + x'y + yz$$

ANS:

You must manipulate the function to can draw it by using OR, NOT only.

By complement the function twice

$$\begin{aligned}(F')' &= [(xy + x'y + yz)']' \\ &= [(x'+y')(x+y)(y+z')]' \\ &= (x'+y')' + (x+y)' + (y+z')'\end{aligned}$$



See Some Other for this function
(OR, AND, NOT)