

Half Adder or Binary Adder:

↳ is a combinational circuit that performs the addition of 2 bits. → (2 inputs & 2 outputs)

$$\begin{array}{r}
 a \\
 + b \\
 \hline
 \text{Cout} \quad s
 \end{array}$$



(The Block diagram of half Adder)
(HA)

* The Truth table::

a	b	s	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

* The equations of S, Cout::

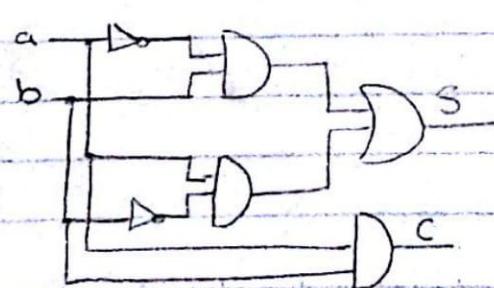
$$\begin{aligned}
 S &= ab + \bar{a}\bar{b} \\
 &= a \oplus b
 \end{aligned}$$

$$\text{Cout} = ab$$

* implementation by using

AND, OR, NOT

In SOP::



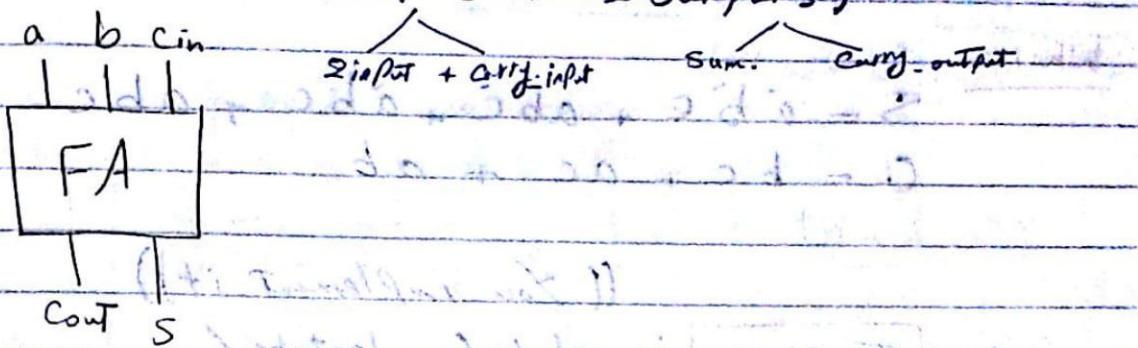
* Implementation by using

XOR::



Full Adder:

(3 inputs & 2 outputs)



Cin	a	b	c	S	Cout
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	1	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	1
1	0	1	0	0	1
1	1	0	0	0	1
1	1	0	1	1	1
1	1	1	1	1	1

$$S = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

1	1	3	2
4	5	7	6
1	1	1	1

$$\begin{aligned} S_{\text{SOP}} &= \bar{a}(b'\bar{c} + b\bar{c}) + a(b'\bar{c} + b\bar{c}) \\ &= \bar{a}(b \oplus c) + a(b \oplus c) \\ &= \boxed{a \oplus b \oplus c} \end{aligned} \quad (\text{No Simplification})$$

$$\begin{aligned} C_{\text{out}} &= \bar{a}bc + ab\bar{c} + abc + abc \\ &\quad \overbrace{\hspace{1cm}}^c \quad \overbrace{\hspace{1cm}}^{ab} \end{aligned}$$

b'	b	c'	b'	b	c'
0	0	0	1	1	0
0	0	1	1	0	1
0	1	0	0	1	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

$$\text{Cout} = bc + ac + ab$$

$$\begin{aligned} &= ab(\bar{c} + c) + c(ab + a\bar{b}) \\ &\quad \overbrace{\hspace{1cm}}^{ab} \quad \overbrace{\hspace{1cm}}^{c(a \oplus b)} \end{aligned}$$

* Implementation the Full Adder (FA) by using:
(AND, OR, NOT, etc.)

Where:

$$S = \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}b\bar{c} + abc$$

$$C = \bar{b}c + ac + ab$$

(From K-map)

((You implement it))

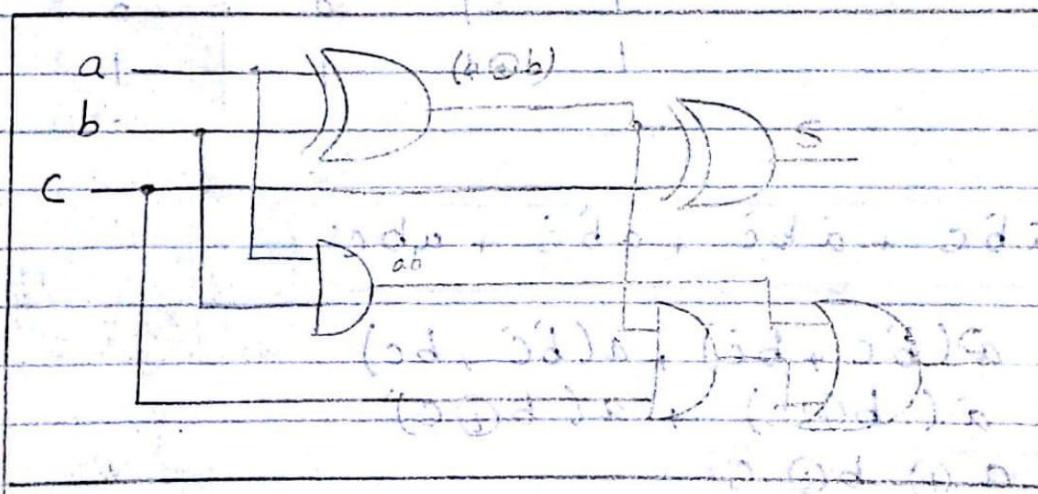
1 2 3 4 5 6 7 8

* How to implement A full Adder by using Two half Adder

From the equations of S, C :

$$S = (a \oplus b) \oplus c$$

$$C = ab + c(a \oplus b)$$

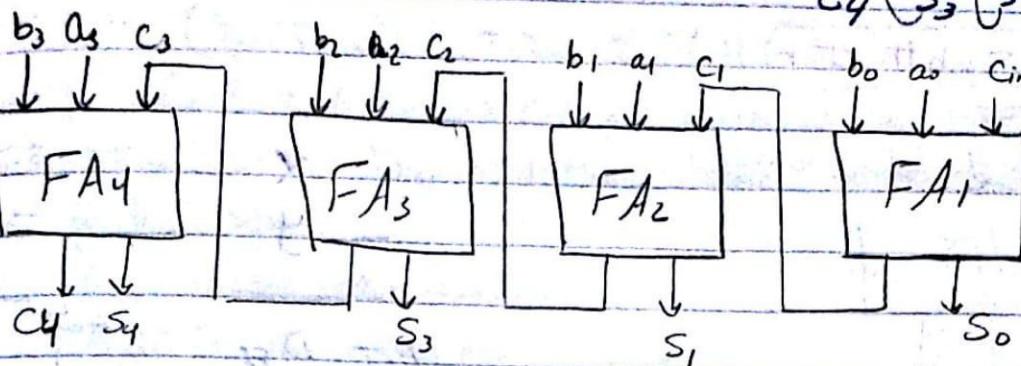
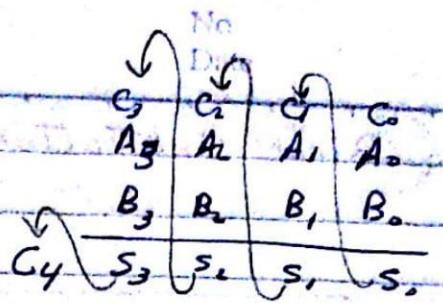


Full Adder, the addition is -

1. $a = 10101010$
 $b = 10101010$

Ans: 100000000

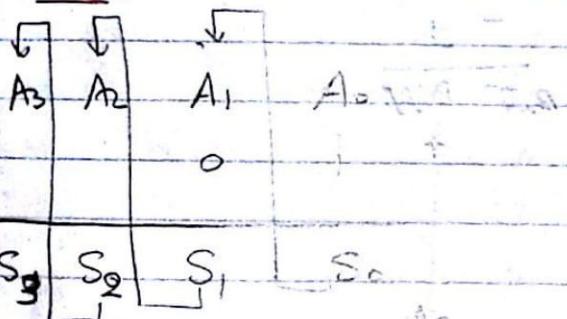
* Cascade 4-bit Full Adder..



Q 4.11 (a)

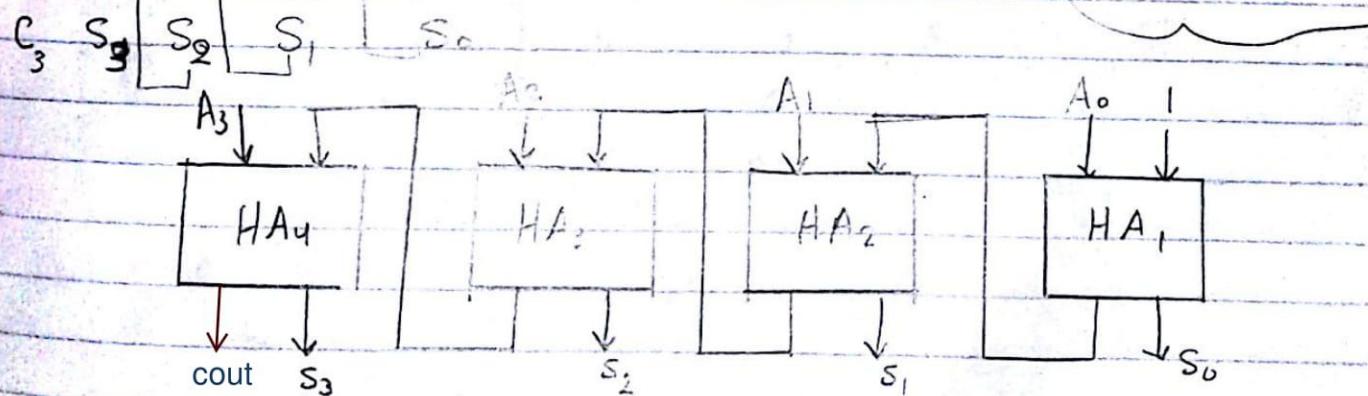
* Using four half Adders, design a 4-bits Combinational Circuit incrementer (a circuit that adds 1 to a 4-bits binary number).

Ans.



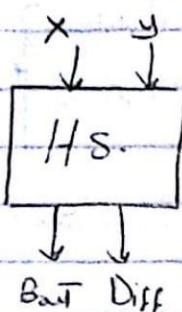
⇒ We will use (Half Adder) (HA)

Where → (2 inputs & 2 outputs)



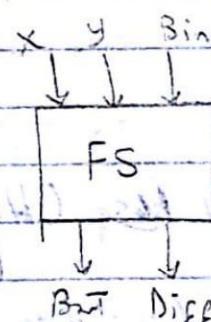
Binary Subtractor ::

* (Half Subtractor) (HS) (2 input, 2 output)



$$\begin{array}{r}
 & X \\
 & - Y \\
 \hline
 \text{BNOT Diff} & 1 \\
 \text{BNOT Borrow} & 1
 \end{array}$$

* (Full Subtractor) (FS)



$$\begin{array}{r}
 & X \\
 & - Y \\
 \hline
 \text{BNOT Diff} &
 \end{array}$$

(Note: The diagram shows a feedback loop where the BNOT Borrow from the FS stage is fed back to the BNOT Borrow input of the next FS stage.)

Binary Subtractor:

Q4.12 (b)

- * Design a full subtractor circuit with three inputs, X, y, Bin and two outputs Diff and Bout . (The circuit subtracts $X - y - \text{Bin}$ Where:
 Bin is the input Borrow.
 Bout is the output Borrow and.
 Diff is the difference result.

Ans:

The Truth Table of the subtractor:

X	y	Bin(Z)	Diff	Bout	Full Subtractor
0	0	0	0	0	
0	0	1	1	1	
0	1	0	1	1	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	0	0	
1	1	0	0	0	
1	1	1	1	1	

Let: $\text{Bin} = Z$

0	1	2	3
4	5	6	7

$$\text{Diff} = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}z + xy\bar{z}$$

$$= \bar{x}(y\bar{z} + \bar{y}z) + x(\bar{y}z + y\bar{z})$$

$$= \bar{x}(y \oplus z) + x(y \oplus z)$$

$$= x \oplus y \oplus z$$

No Simplification.



$$\text{Bout} = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + xyz$$

\bar{x}	0	1	1	0	1
\bar{y}	1	0	1	0	1
\bar{z}	0	1	0	1	0
x	1	0	1	0	1
y	0	1	0	1	0
z	1	1	0	1	0

$$= \boxed{\bar{x}\bar{y} + \bar{x}z + yz}$$

Adder / Subtractor

$$A - B$$

$$A + (-B)$$

$$\xrightarrow{2^{\text{'}} \text{s Comp.} = (1^{\text{'}} \text{s Comp.} + 1)}$$

$$A + \underline{(B \oplus 1) + 1}$$

$\xrightarrow{1^{\text{'}} \text{s comp.} + 1} \rightarrow 2^{\text{'}} \text{s Comp. of } B.$

$$\begin{array}{c} [0 \oplus 1] \rightarrow 1 \\ [1 \oplus 0] \rightarrow 0 \end{array} \Rightarrow \boxed{\text{XOR}} \text{ for } 1^{\text{'}} \text{s Comp.}$$

\Rightarrow Mode input M

* if $\boxed{M = 0}$

$$A + [(B \oplus M) + M]$$

$$A + [(B \oplus 0) + 0]$$

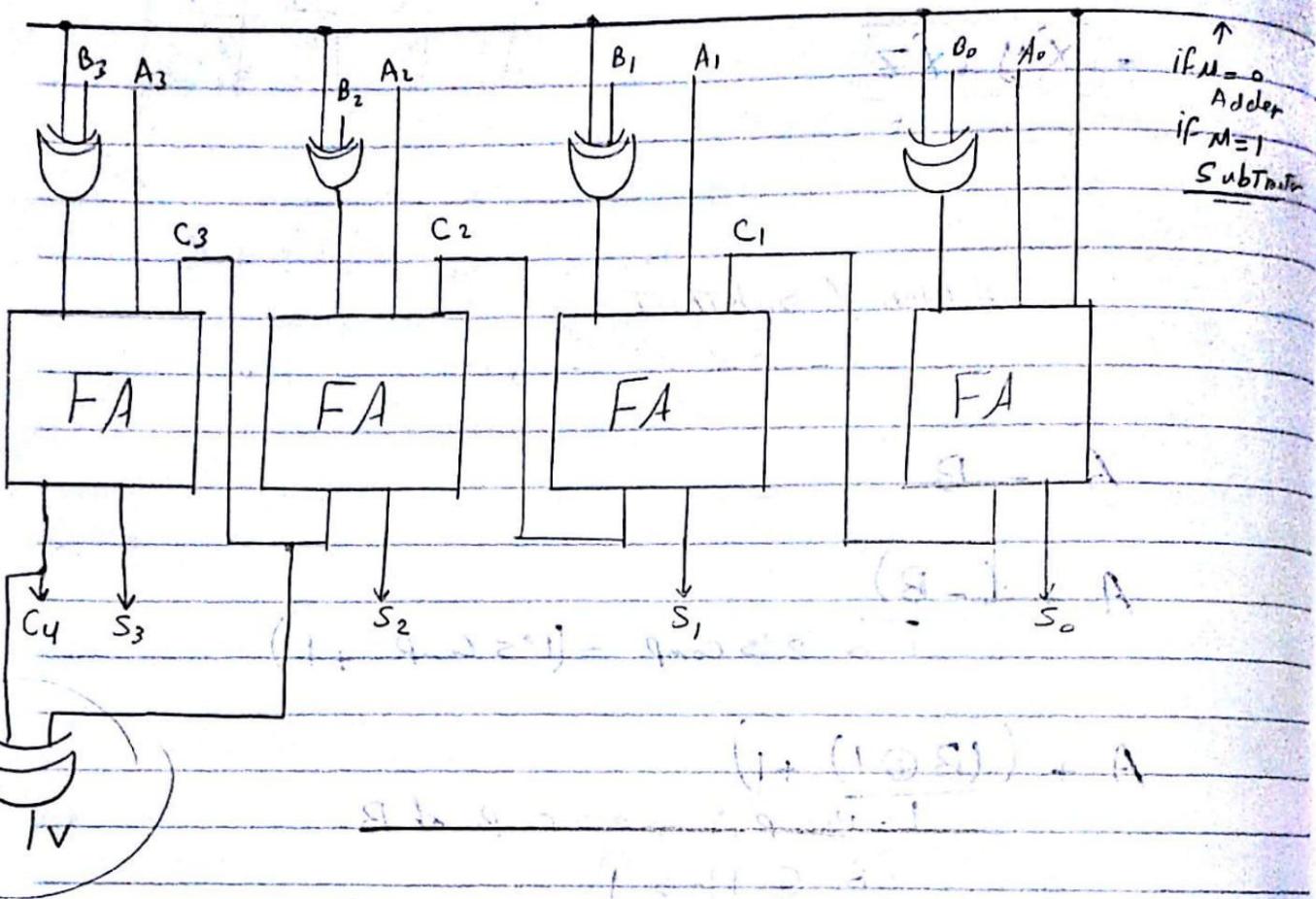
$\xrightarrow{\text{for Adder when }} \underline{M=0}$

* if $\boxed{M = 1}$

$$A + [(B \oplus 1) + 1] \rightarrow \text{for subtractor when }$$

$\underline{M=1}$

~~if X + Y = S + b~~ (Modifying)
M



To detect if there is overflow or no: ($V = C_3 \oplus C_4$)

if $V=1 \rightarrow$ Where (C_3 & C_4 are different)

Then there is an overflow

if $V=0 \rightarrow$ Where (C_3 & C_4 are the same)

Then there isn't overflow.

Ex

if $M=1$, 1101 , 0101

A

B

~~Find sum of A & B~~

~~Step 1: Add 1101 + 0101, then M will be 1 if there is no overflow.~~

~~1101 + 0101 → Result of 4 bits =~~

$$\begin{array}{r} 1101 \\ + 0101 \\ \hline 1000 \end{array}$$

~~Result of 4 bits = $[-2^{4-1}, +2^{4-1}]$
 $= [-8, +7]$~~

$$V = C_3 \oplus C_4 = 1 \oplus 1 = 0 \rightarrow \text{No overflow.}$$

~~also 1101 + 0101~~

~~last result is 1000~~

~~so 1000 + 0101~~

~~→ last result is 1001~~

~~last result is 1001~~

~~which is溢出，所以需要向左移位。~~

~~将低三位移出，再将高三位移入。~~

~~shift left by 1 bit~~

~~1101~~

~~1101~~

~~1101~~

~~1101~~

~~1101~~

~~1101~~

~~1101~~

~~off~~

Q4.13

- * The following Adder/Subtractor circuit has the following values for the mode input M and data inputs A & B.

The Block diagram of
Adder/Subtractor

M A B

- a) 0 0111 0110
- b) 0 1000 1001
- c) 1 1100 1000
- d) 1 0101 1010
- e) 1 0000 0001

- * In each of the previous cases, determine the values of the four sum outputs, the carry C, overflow O/V

ANS

a)

$$\begin{array}{r}
 M = 0 \rightarrow \text{Adder} \\
 \begin{array}{r}
 0111 \\
 + 0110 \\
 \hline
 \underbrace{1101}_{\text{Sum}}
 \end{array}
 \end{array}$$

$$\begin{aligned}
 Y &= C_3 \oplus C_4 \\
 &= 1 \oplus 0 = 1
 \end{aligned}$$

SUM	C	Y
1101	0	1

→ 04

b)

$M = 0 \rightarrow \text{Adder}$

$$\begin{array}{r} & C_3 \\ & 1000 \\ & 1001 \\ \hline 10001 \\ \hline C_4 \quad \text{Sum} \end{array}$$

$$\Rightarrow V = C_3 \oplus C_4 \\ = 0 \oplus 1 \\ = 1$$

Sum	C	V
0001	1	1

c)

$M = 1 \rightarrow \text{Subtractor}$

$$\begin{array}{r} 1100 \\ 1000 \leftarrow 2^{\text{'}}\text{s Comp.} \\ \hline 10100 \\ \hline C_4 \quad \text{Sum} \end{array}$$

$$V = C_3 \oplus C_4 \\ = 0 \oplus 1 = 1$$

Sum	C	V
0100	1	1

d)

$M = 1 \rightarrow \text{Subtractor}$

$$\begin{array}{r} 0101 \\ + 0110 \leftarrow 2^{\text{'}}\text{s Comp.} \\ \hline 01011 \\ \hline C_4 \quad \text{Sum} \end{array} \Rightarrow V = C_3 \oplus C_4 \\ = 1 \oplus 0 \\ = 1$$

Sum	C	V
1011	0	1

e)

$M = 1 \rightarrow \text{subtractor.}$

$C_3=0$
 0000

$$\begin{array}{r} 1111 \\ \hline 0111 \end{array} \leftarrow 2\text{'s Comp.}$$

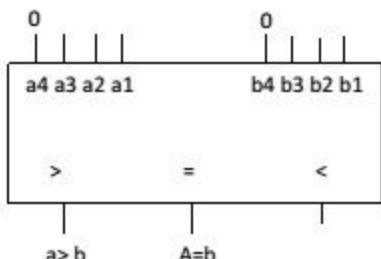
$\underbrace{}_{C_4} \quad \text{sum} +$

$$V = C_3 \oplus C_4 = \\ = 0 \oplus 0 = 0$$

$$\begin{array}{ccc} \text{Sum} & C & V \\ 1111 & 0 & 0 \end{array}$$

6. We need to determine whether a three-bit number, a_3, a_2, a_1 , is equal to another number, b_3, b_2, b_1 , or if it is greater than that number. (We do not need an output for less than.)

Implement this with AND and OR gates.



Here we will unwind the recursive definition of the $A > B$ for n bits number stated as:

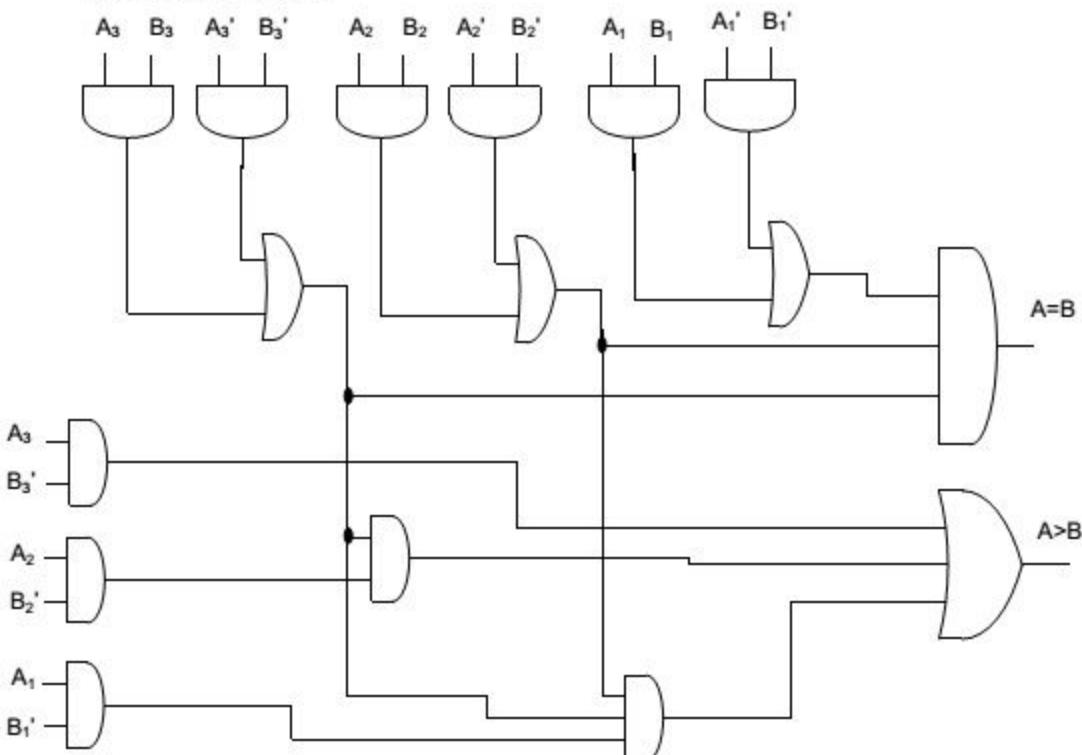
$$A > B = (a_n > b_n) \text{ or } [(a_{n-1} > b_{n-1}) \text{ and } (a_n > b_n)]$$

For three bits number:

$$A > B = (a_3 > b_3) + [(a_2 > b_2) \text{ and } (a_3 = b_3)] + [(a_1 > b_1) \text{ and } (a_2 = b_2) \text{ and } (a_3 = b_3)]$$

$$F(A=B) = A \text{ xnor } B = A B + A' B' = (A_1 B_1 + A'_1 B'_1)(A_2 B_2 + A'_2 B'_2)(A_3 B_3 + A'_3 B'_3)$$

Now build the circuit

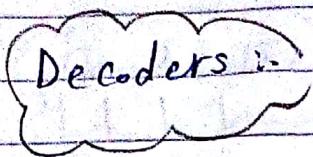


4.11**Using four half-adders**

- (a) Design a four-bit combinational circuit incrementer (a circuit that adds 1 to a four-bit binary number).
- (b) Design a four-bit combinational circuit decrementer (a circuit that subtracts 1 from a four-bit binary number).

The solution >> in Assignment 5 file
on the e-learning site

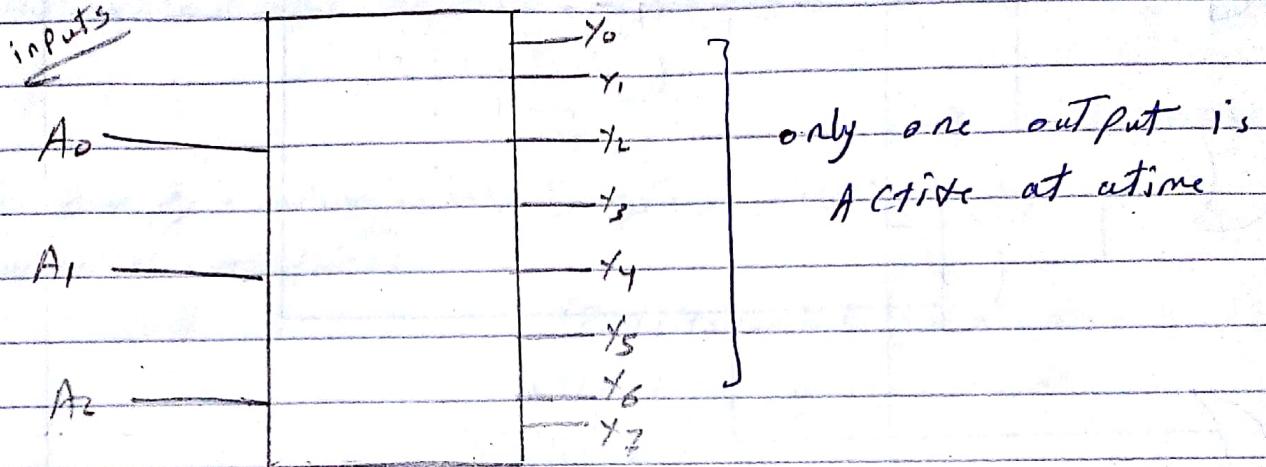
V



↳ Digital Circuits.

↳ Combinational Gates

↳ Converts input binary into single numeric output.



⇒ (n inputs) ⇒ (2ⁿ outputs)

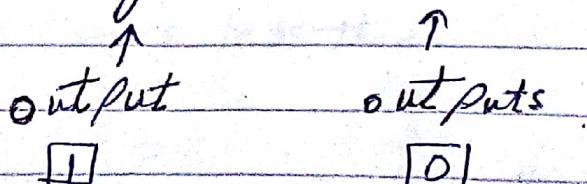
⇒ 3 inputs ⇒ $2^3 = 8$ outputs. From (0 → 7)
called

↳ 3 to 8 Decoder

or

↳ 1 of 8 ⇒ means that one of the outputs is Active.

⇒ The Active can be High or Low



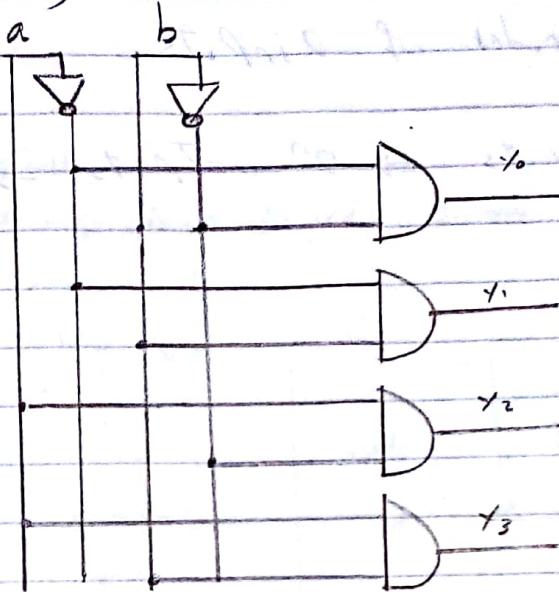
Active high Decoder (2 - 4)

$$y_0 = \bar{a} \bar{b}$$

$$y_1 = \bar{a} b$$

$$y_2 = a \bar{b}$$

$$y_3 = a b$$



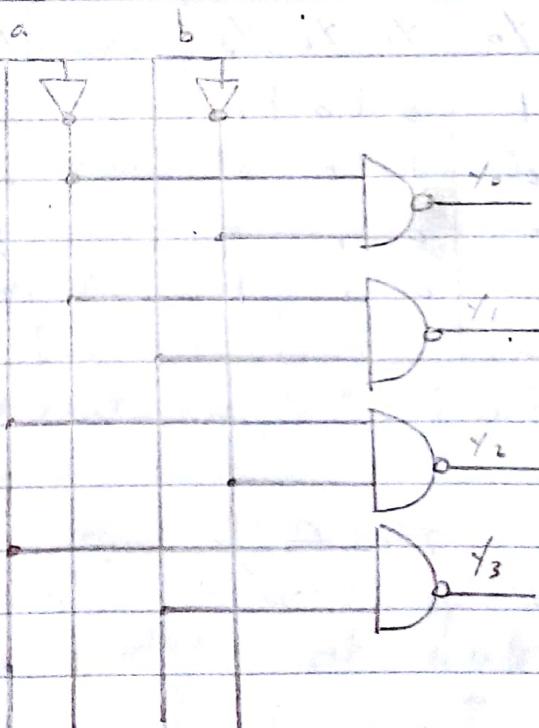
Active low Decoder (2 - 4)

$$y_0 = (\bar{a} \bar{b})' = (a + b)$$

$$y_1 = (\bar{a} b)' = (a + \bar{b})$$

$$y_2 = (a \bar{b})' = (\bar{a} + b)$$

$$y_3 = (a b)' = (\bar{a} + \bar{b})$$



Decoder & Enable.

Decoder و میکرو کنترلر دارای فرمان (enable) می باشد
 inactive Active

(تخلیق) Active دستور Decoder می باشد (Active) دستور (enable) می باشد

Inactive دستور outputs می باشد (inactive) دستور (enable) می باشد
 inactive (decoder) می باشد

Active high دستور (Enable) می باشد

Active دستور ① می باشد (EN) در حالت بالا

Active low دستور می باشد

Active دستور دستور لیز می باشد (EN) در حالت پایین

enable دستور (enable) دستور (decoder) می باشد لیز می باشد
 \rightarrow Active low
 (EN)

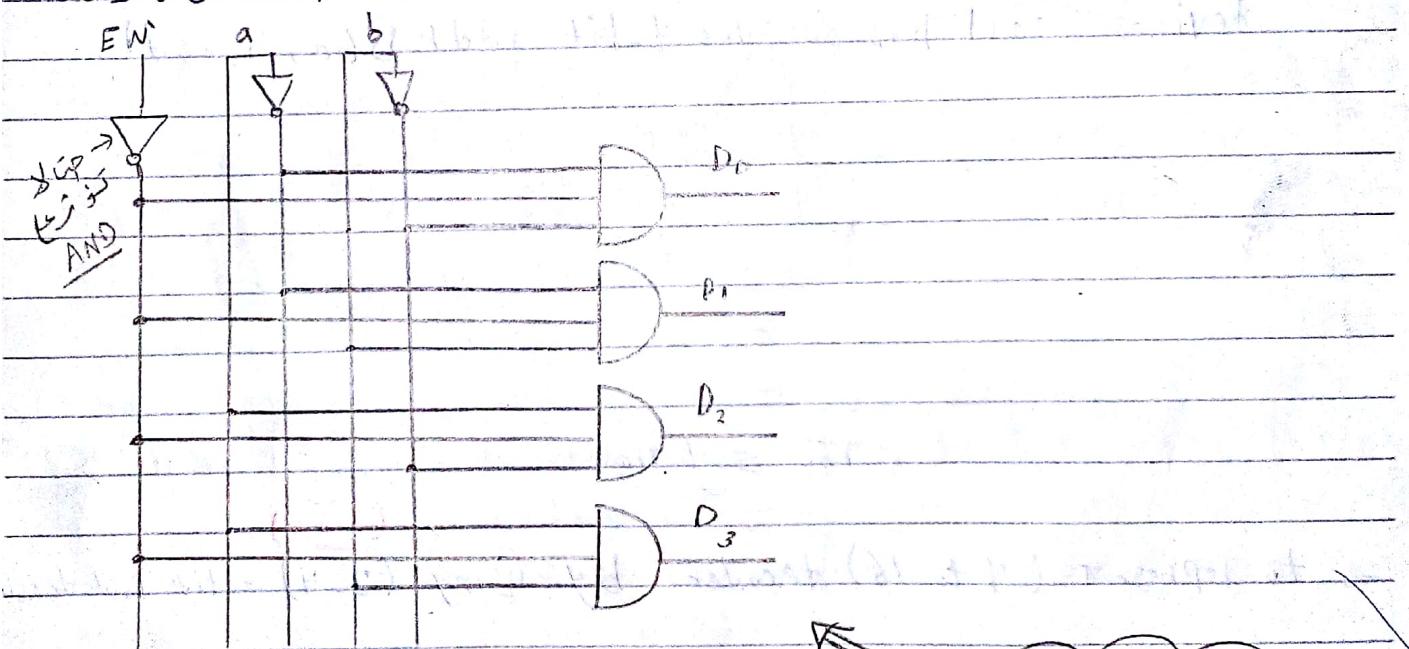


Truth Table

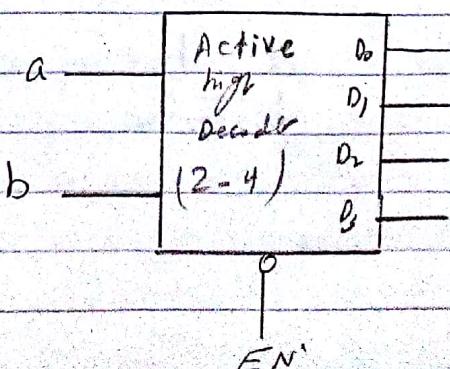
(Enable active low)

		2 inputs		Outputs			
\bar{EN}	a	b		D_0	D_1	D_2	D_3
1	x	x		0	0	0	0
0	0	0		1	0	0	0
0	0	1	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	1		0	0	0	1

$$D_0 = \bar{a}\bar{b}, D_1 = \bar{a}b, D_2 = a\bar{b}, D_3 = ab$$



Active high decoder with
Active low enable.



Applications of Decoder

(Decoder) is called ~~not~~

II Implement logic function.

Ex:

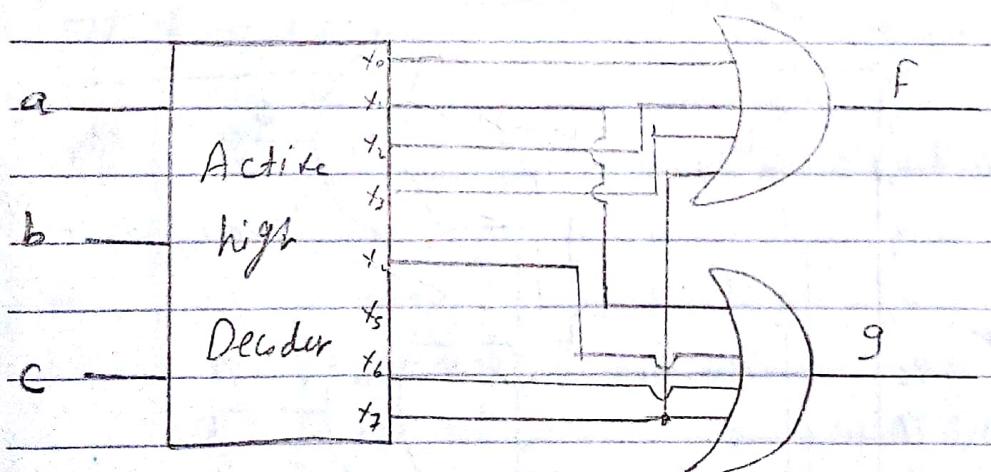
$$F(a, b, c) = \sum m(0, 2, 3, 7)$$

$$g(a, b, c) = \sum m(1, 4, 6, 7)$$

implement these functions by ^{the} Decoder (Active high)

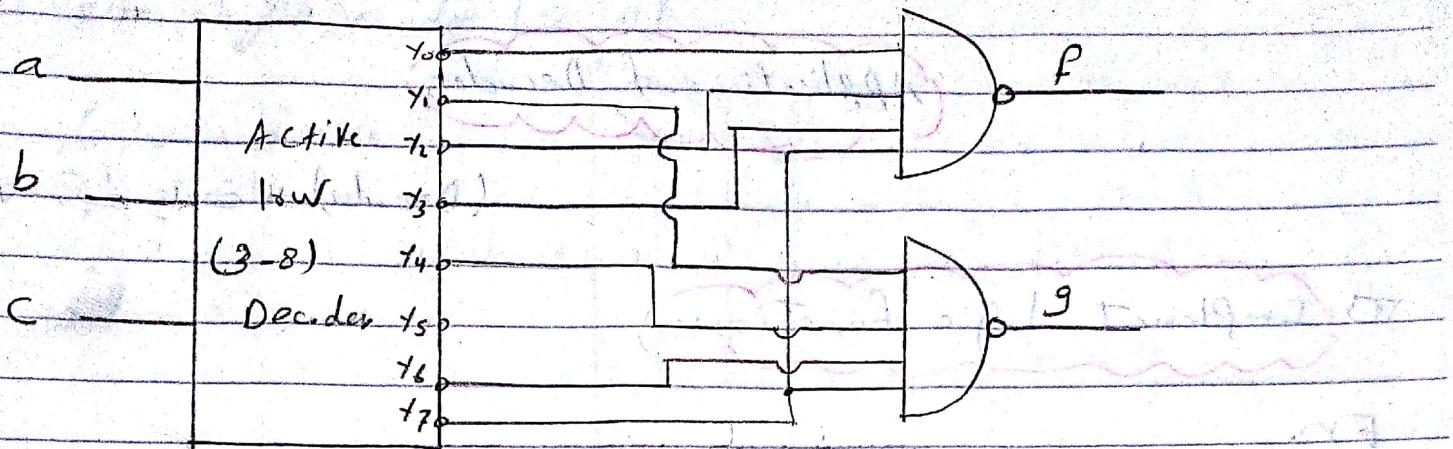
The Solution

$a, b, c \Rightarrow 3$ inputs $\Rightarrow 2^3 \Rightarrow 8$ outputs. We will use
 $(3 - 8)$ Decoder for each func



by Active low Decoder:





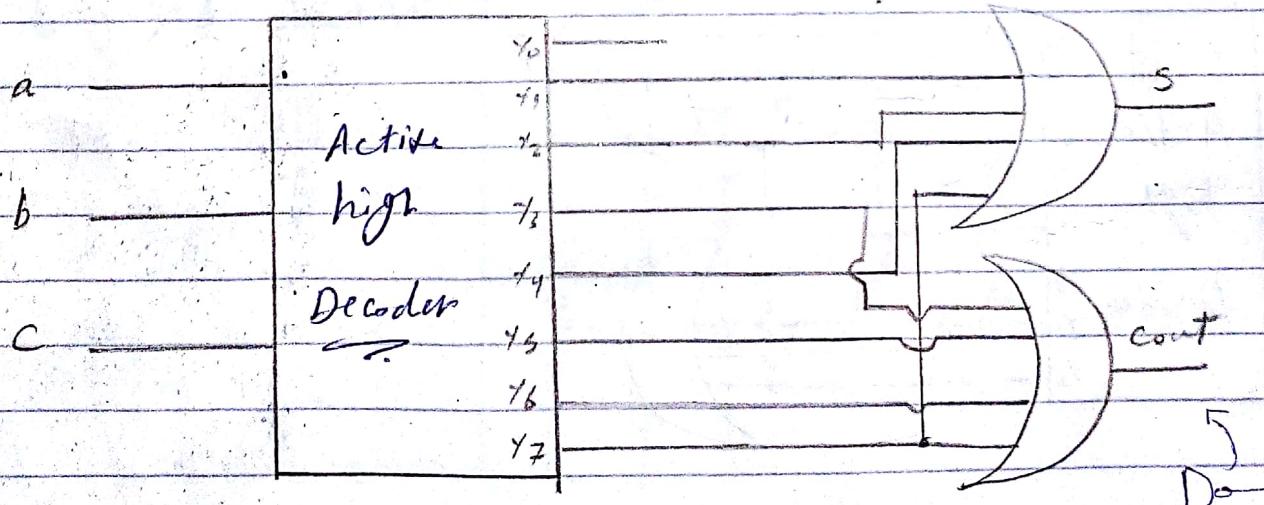
⇒ The Decoder can be used to implement any Combinational Function Circuits.

⇒ From the Truth table for Full adder:-

$$S(a, b, c) = \sum m(1, 2, 4, 7)$$

$$C_{out}(a, b, c) = \sum m(3, 5, 6, 7)$$

⇒ The implementation by using Decoders is :-

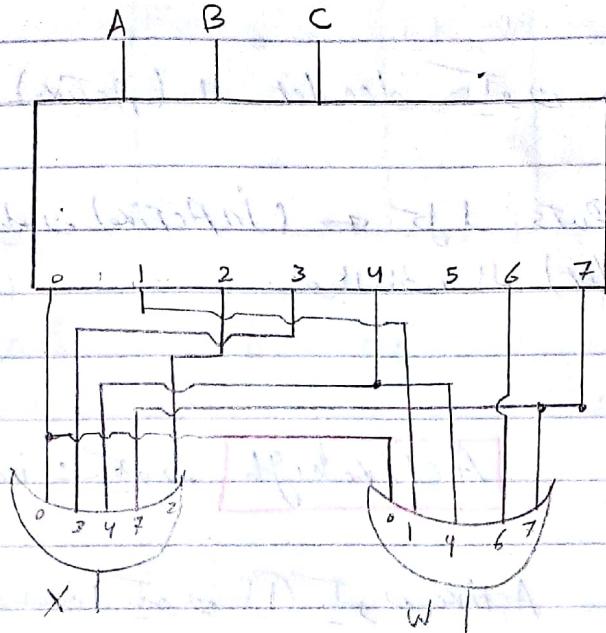


⇒ if we will use Active low Decoders, we use NAND OR gate

Ex:-

(in quiz 201)

Consider the following circuit with an active high output decoder
 Draw a truth table for X and W, then extract the switching formula for the begin in terms of A, B, C.



The solution

Truth Table :-

Switching formula \Leftrightarrow Algebraic expression

A	B	C	X	W
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

$$X(A, B, C) = A'B'C + A'BC' + AB'C + ABC'$$

$$W(A, B, C) = A'B'C + A'BC + AB'C + ABC$$

\Rightarrow question #5 (Final 2009)

A 1-bit full subtractor that has three inputs, a borrow "Bin", x and y , and produces two outputs (The difference D where $D = "x-y-Bin"$ and the new borrow $Bout$).

- Show the truth table for the full subtractor.
- use the truth table to find the expressions for D and $Bout$
- Minimize and implement the subtractor using 3-to-8 active-low decoder with one active low enable, by using OR gates

The Solution

- The truth table for the full subtractor:

x	y	Bin	$Bout$	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

b)

$$B_{out} = \bar{x}yB_{in} + \bar{x}yB_{in} + \bar{x}yB_{in} + xyB_{in}$$

$$D = \bar{x}yB_{in} + \bar{x}yB_{in} + \bar{x}yB_{in} + xyB_{in}$$

c) We will minimize the functions by K-map.

		B _{out}			
X _y	B _{in}	$\bar{x}y$	$\bar{x}y$	xy	xy'
Bin		1	2	3	4
Bin	1	1	1	1	0
Bin	2	0	1	0	0
Bin	3	0	0	1	0
Bin	4	0	0	0	1

		D			
X _y	B _{in}	$\bar{x}y'$	$\bar{x}y$	xy	xy'
Bin		1	2	3	4
Bin	1	1	0	0	0
Bin	2	0	1	0	0
Bin	3	0	0	1	0
Bin	4	0	0	0	1

$$B_{out} = \bar{x}y + \bar{x}B_{in} + yB_{in}$$

$$D = \bar{x}yB_{in} + \bar{x}yB_{in} + \bar{x}yB_{in} + xyB_{in}$$

$$B_{out} = \Sigma m(1, 2, 3, 7)$$

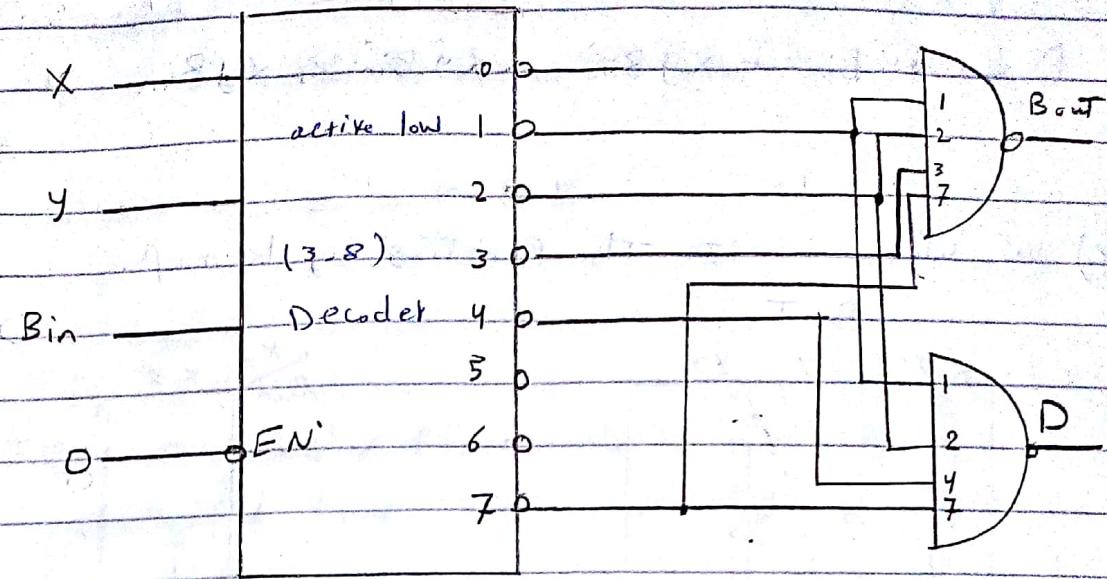
$$= \Sigma m(1, 2, 4, 7)$$

→ We need to implement these function by using (3-8) Active Low decoder with one active Low enable

By using OR gate

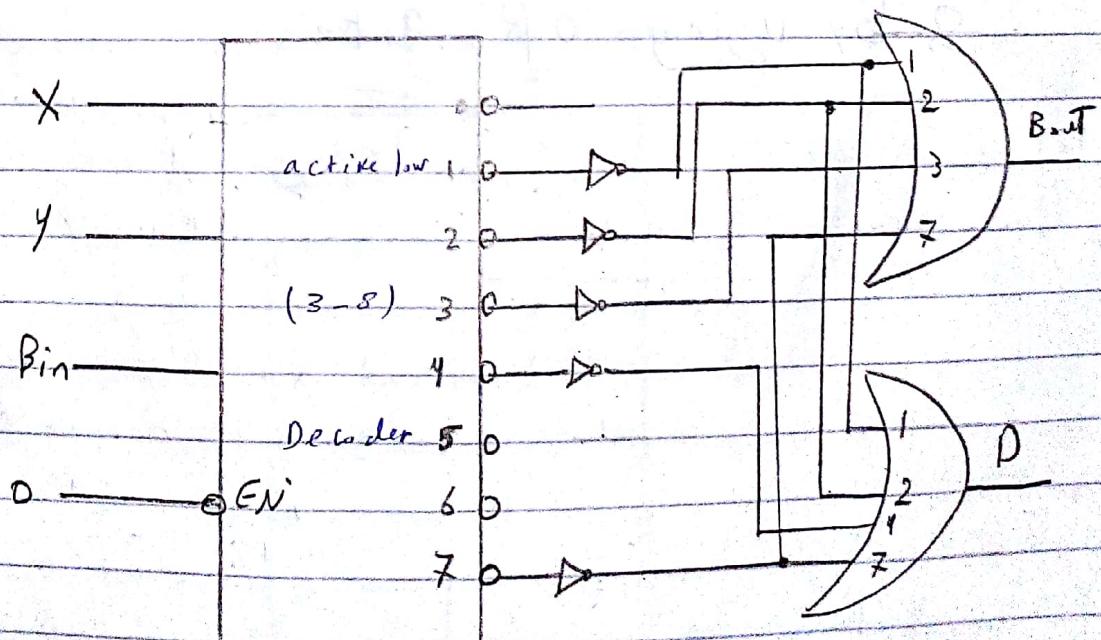


↳ Active Low Decoder Use NAND gates



$$B_{out} = \sum m(1, 2, 3, 7) \quad / \quad D = \sum m(1, 2, 4, 7)$$

decoder (دوال عاشر (NAND) و عاشر (OR) فی الحالات
OR (NOT) فی الحالات
Active low \Leftrightarrow (decoder) + دو



Applications of Decoders

Application [2] :

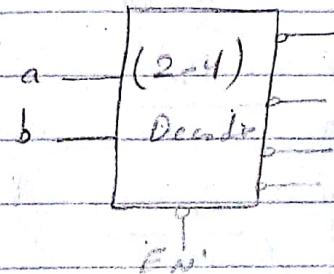
⇒ The Decoder helps to select one of many devices each of which has unique address.

⇒ The address is the input to the decoder.

⇒ One output is active to select one device that has this address.

Ex ::

⇒ Use (2-4) active low decoders to select one of 16 devices, each has unique 4-bit address (a, b, c, d)



The Solution

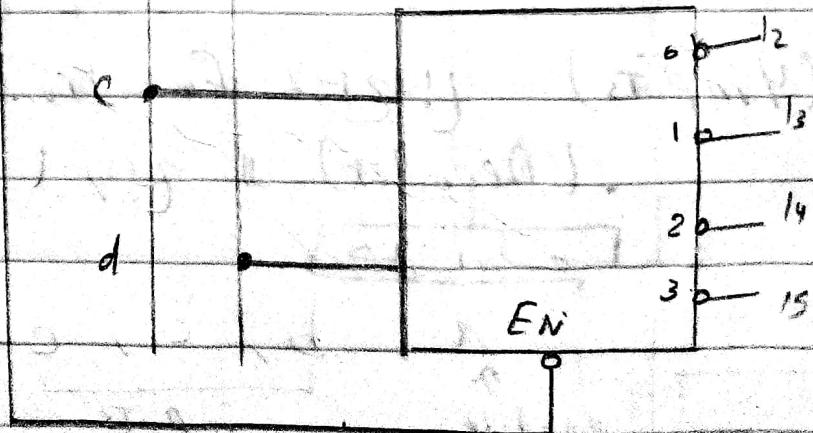
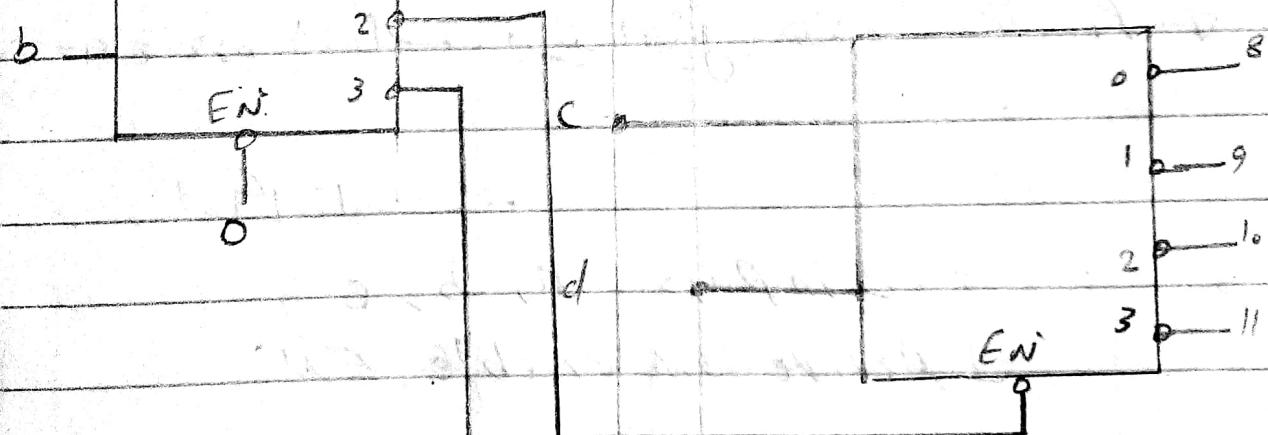
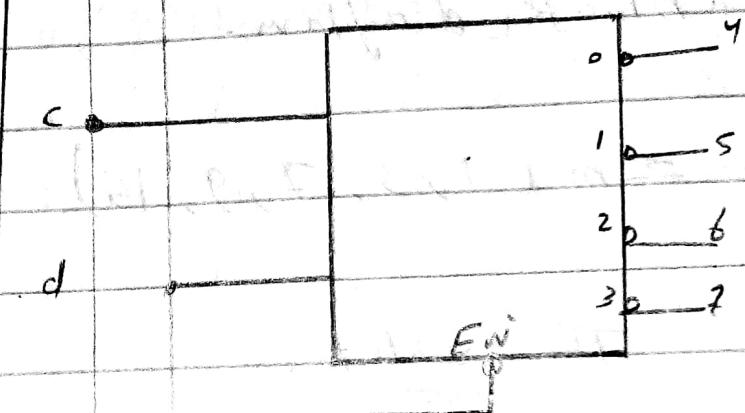
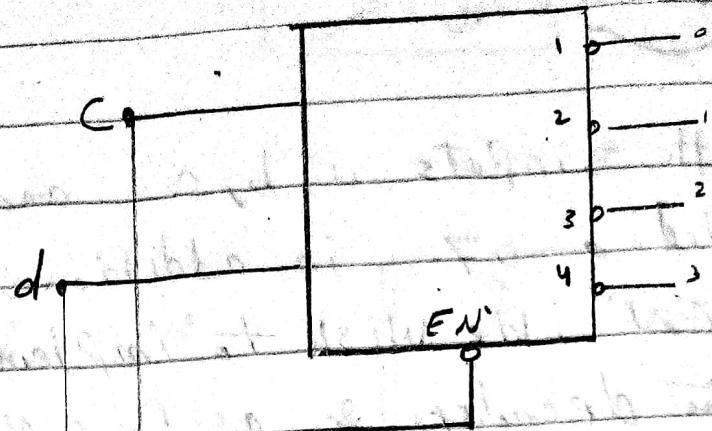
(3-8)

⇒ To represent (4 to 16) decoder by using (2-4) active low decoders

⇒ We need $16/4 \Rightarrow 4$ decoder (2-4) active low Decoder.

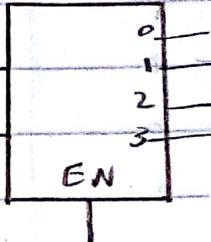
and one decoder to choose among other 4 decoders.
based on 2 of the inputs.





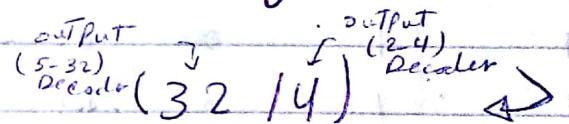
⇒ We wish to build a 32-Way active high decoder using only the four Way decoders shown below -

<u>EN</u>	<u>a</u>	<u>b</u>	0	1	2	3
0	X	X	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

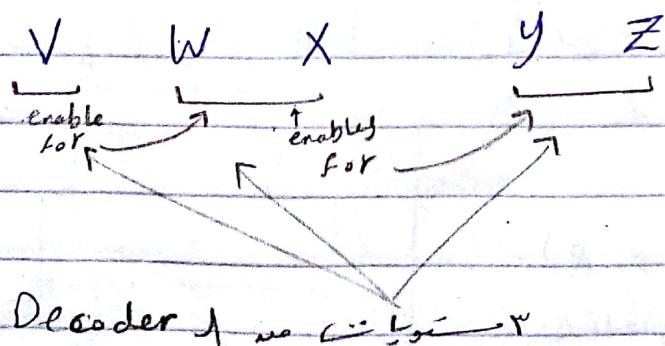


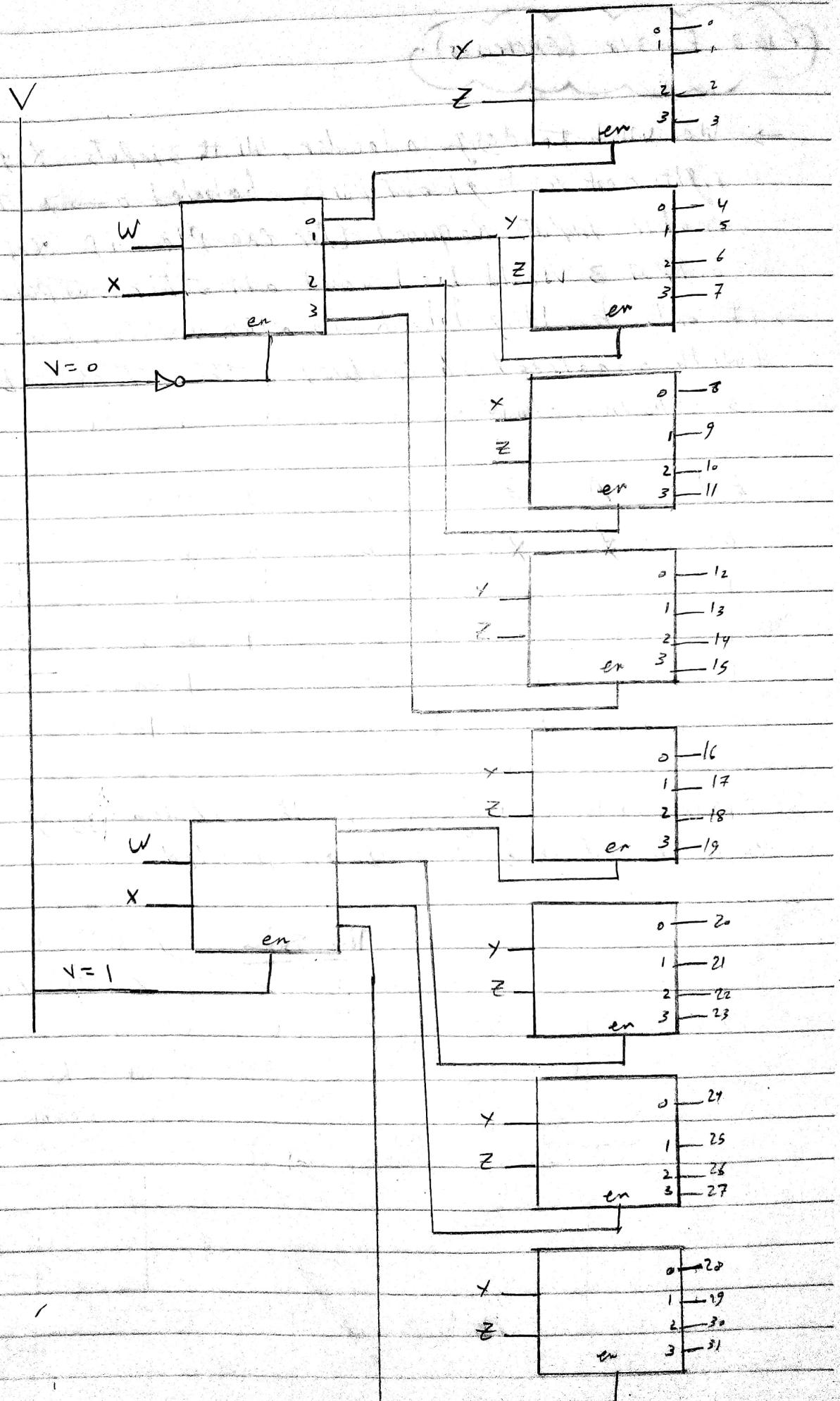
The solution

(Decoder) 1 input (32 outputs) & (5 inputs) → (Decoder) ~~five inputs~~
 (1 enable), ~~4 outputs~~ (2 inputs) if it will fit ~~in~~



= 8 Decoders (2-4)



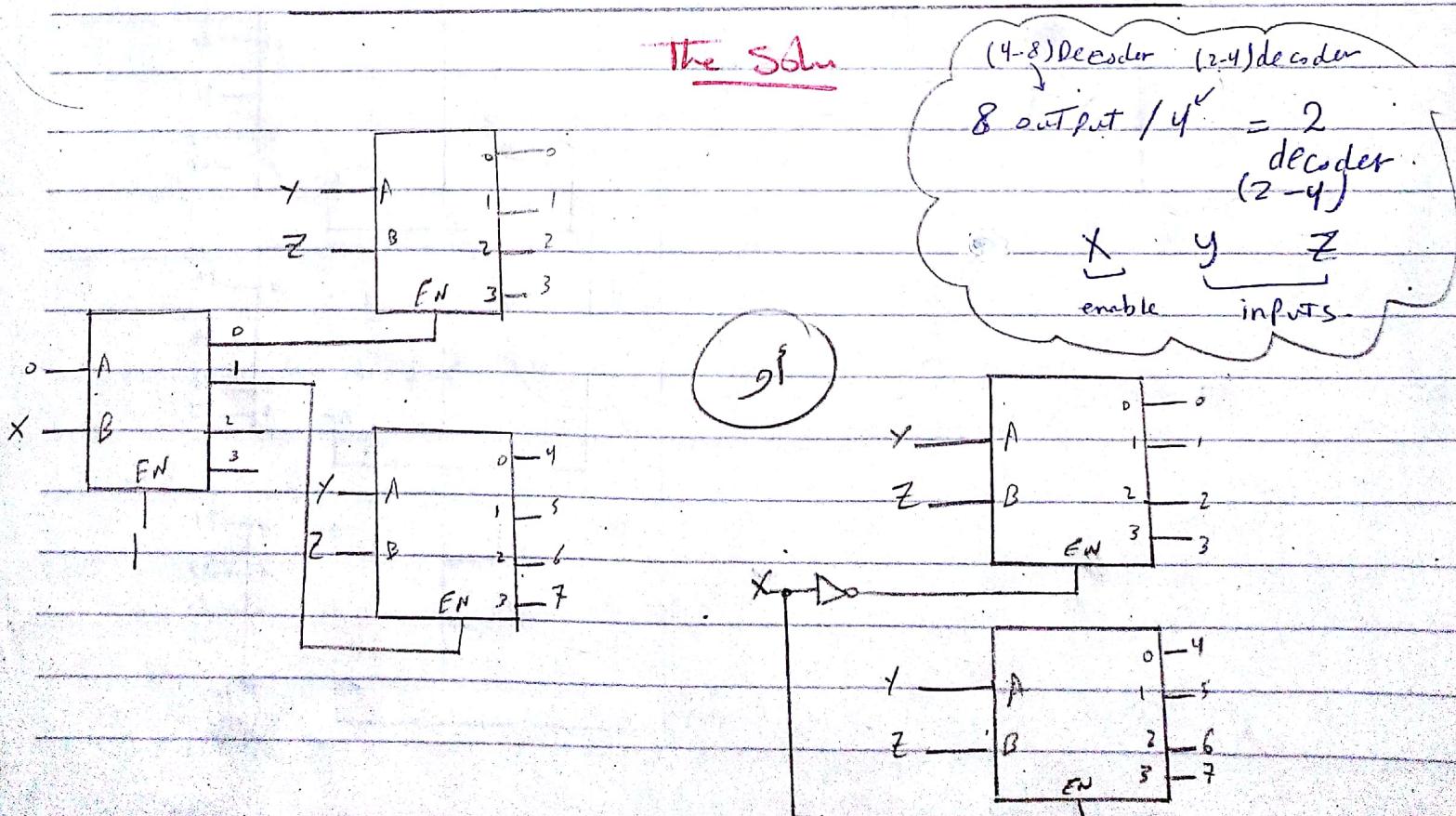


→ We wish to design a decoder, with 3 inputs X, Y, Z and eight active high outputs, labeled $0 \rightarrow 7$. There is no enable input required (for example if $XYZ = 011$, then output 3 would be 1 and all other outputs would be 0).

→ The only building block is a two-inputs, four-output decoder (with an active high enable), the truth table for which is shown below.

EN	A	B	0	1	2	3
0	*	*	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

→ Draw a block diagram of the system using as many of these building blocks as are needed.



Ex:

→ Implement by using 3 inputs, 8 outputs decoders with 2 enables EN_1, EN_2 :

$$F = \sum m(0, 4, 5, 6, 7, 12, 15)$$

$$G = \sum m(1, 3, 12, 13, 14, 15)$$

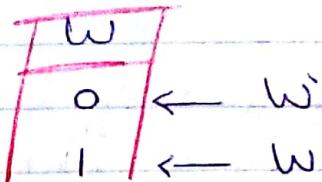
The Solution

w, x, y, z
enable 3 inputs

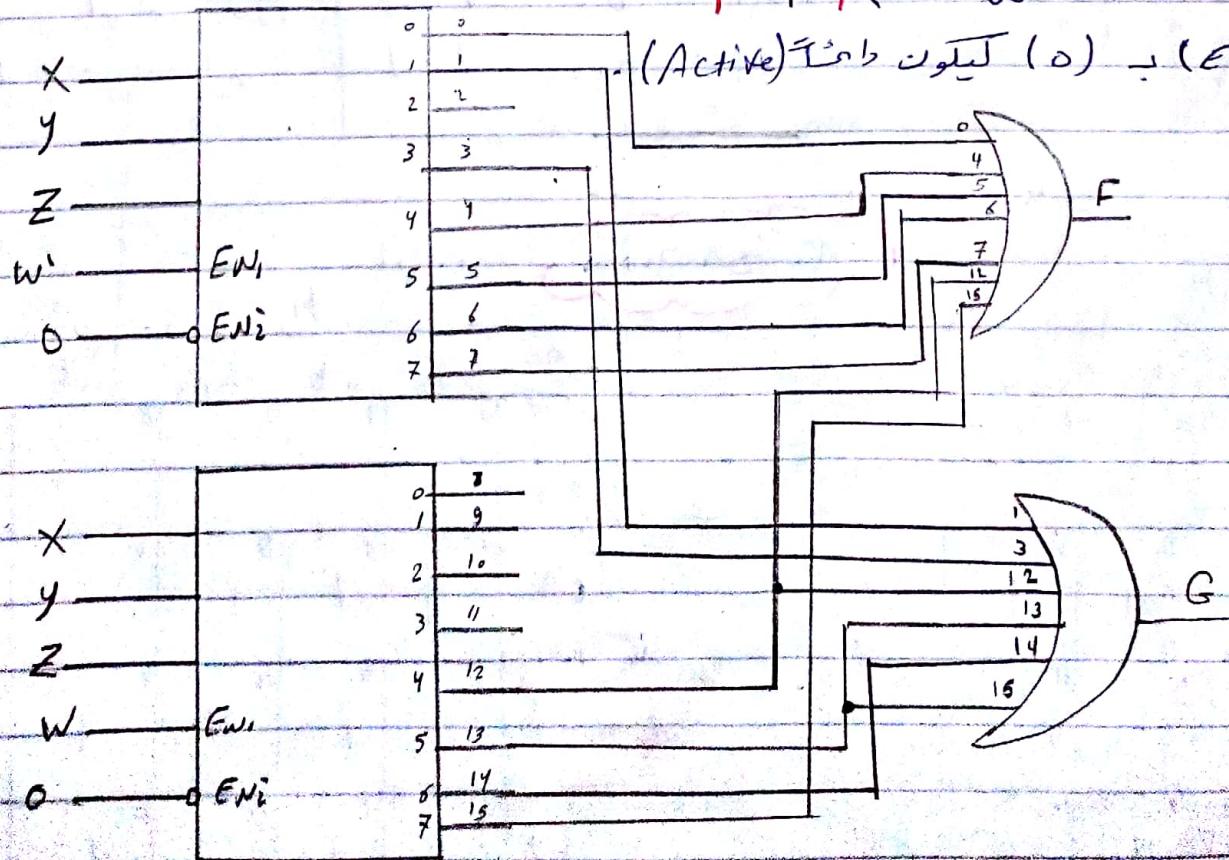
$x, y, z \Leftarrow (3 \text{ inputs})$ ا. ذیج.

2 enables $\Rightarrow (enable) \Leftrightarrow (w)$ و ا. ذیج.

Active high $\Leftrightarrow (EN_1)$ ا. ذیج



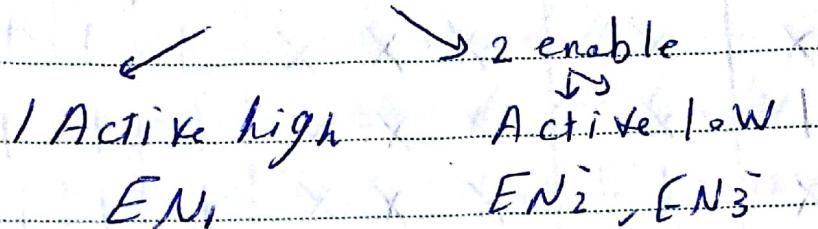
(Active) تابع کیلون (0) $\Rightarrow (EN_2)$ و نیت.



74138 Decadet

↳ is Active low outputs.

↳ has 3 Enable



↳ (3 - 8) Decadet

↳ A, B, C are inputs while C has "high order bit"

C		y_0
B		y_1
A	74138	y_2
EN_1		y_3
EN_2	=	y_4
Low active	EN_3	y_5
		y_6
		y_7

Truth table \Rightarrow P. 273.

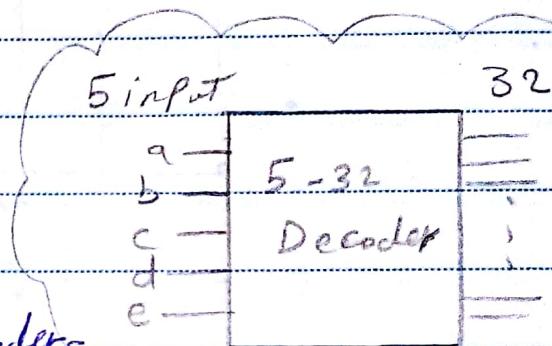
Ex1: By using 74138 decoders, select one of 32 devices (each has a 5-bit address abcde)

The Solution

- 74138 decoder is 3-8 Decoder

- So, we need $32 \text{ output} / 8 \Rightarrow 4$ decoders

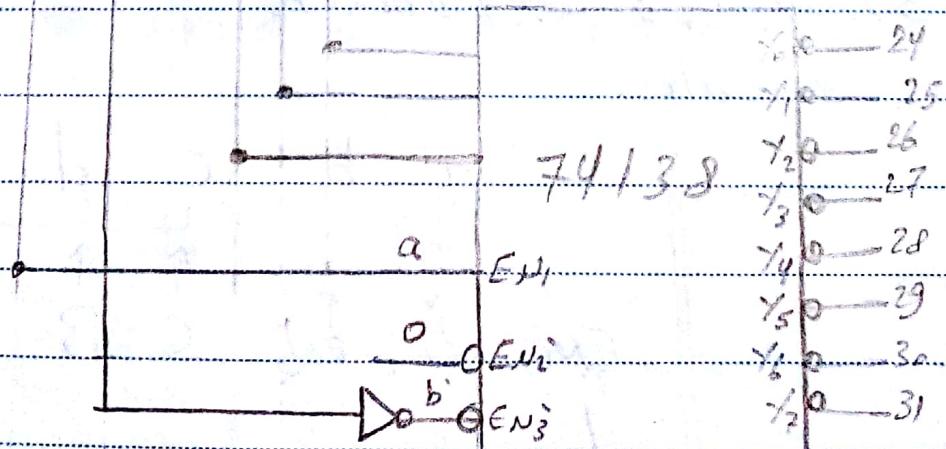
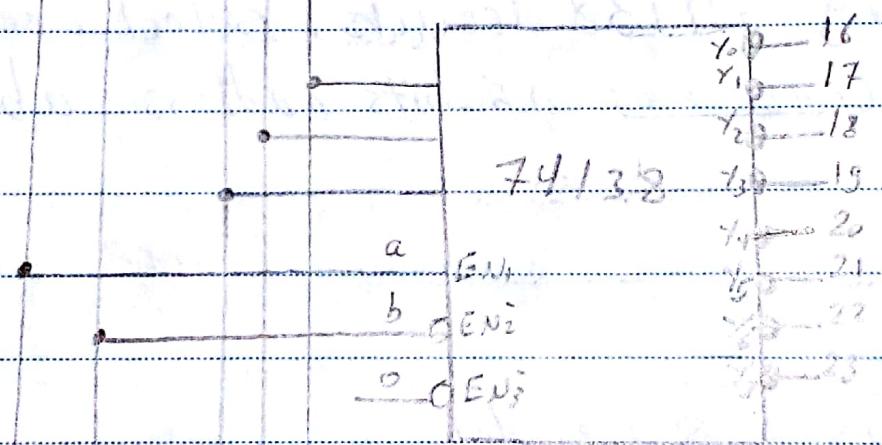
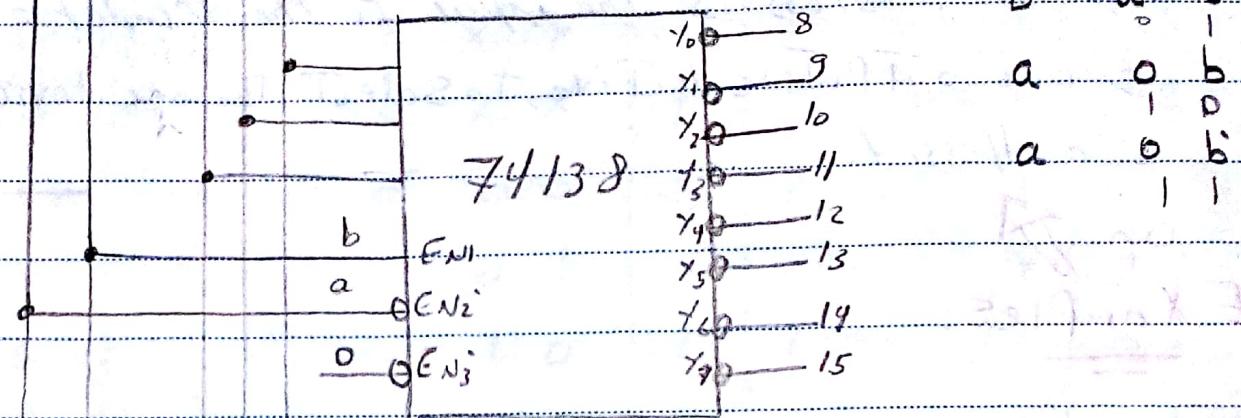
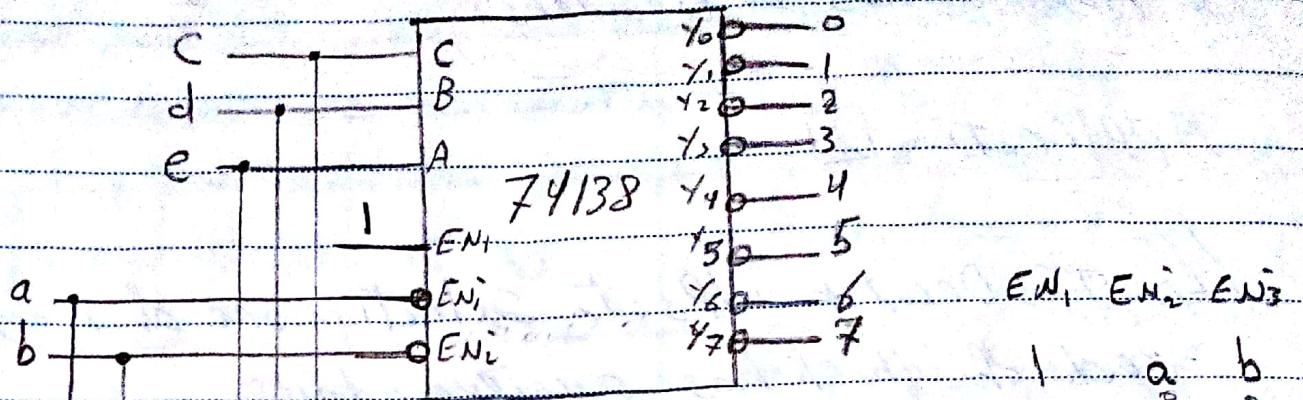
→ each, with 3 inputs, 3 enables.



EN ₁	EN ₂	EN ₃	a	b
1	0	0	a	b
0	1	0	b	a
0	0	1	a	b
1	1	1	b	a

a	b	c	d	e
EN ₁	EN ₂	EN ₃	Enables	
1	0	0	C	B
1	0	1	A	
1	1	0		
1	1	1		

(S)



* How To implement a Decoder To Convert Binary
To octal ?!

Ans :

By using (3-8) Decoder:

A	B	C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

~~~~~

Ex (midterm 2011) :-

→ Consider the following 2 functions F & H of the four variables a, b, c, and d, whose minimum solutions are listed below:-

$$F(a, b, c, d) = b'c'd + bc + abd$$

$$H(a, b, c, d) = ac + cd' + a'cd + bd$$

→ Implement them using only decoders of the type shown (as many as needed) and two OR gates.

|                 |  | EN <sub>1</sub> | EN <sub>2</sub> | A | B | 0 | 1 | 2 | 3 |
|-----------------|--|-----------------|-----------------|---|---|---|---|---|---|
| A               |  | 0               | X               | X | X | 0 | 0 | 0 | 0 |
| B               |  | 1               | X               | 1 | X | 0 | 0 | 0 | 0 |
| EN <sub>1</sub> |  | 2               | 1               | 0 | 0 | 1 | 0 | 0 | 0 |
| EN <sub>2</sub> |  | 3               | 1               | 0 | 0 | 0 | 1 | 0 | 0 |
|                 |  |                 | 1               | 0 | 1 | 0 | 0 | 0 | 1 |
|                 |  |                 | 1               | 0 | 1 | 1 | 0 | 0 | 1 |

F

The solution:

H

| ab' | ab | ab | ab | ab |
|-----|----|----|----|----|
| cd' | 0  | 4  | 12 | 8  |
| cd  | 1  | 5  | 13 | 9  |
| cd' | 3  | 7  | 15 | 11 |
| cd  | 2  | 6  | 14 | 10 |
| cd' | 1  | 1  | 1  | 1  |

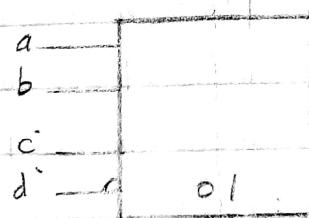
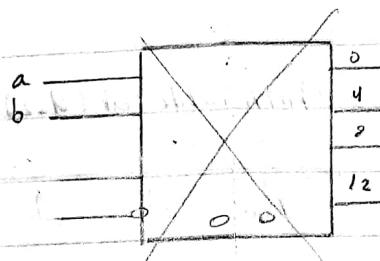
| ab' | ab | ab | ab | ab |
|-----|----|----|----|----|
| cd' | 0  | 4  | 12 | 8  |
| cd  | 1  | 5  | 13 | 9  |
| cd' | 3  | 7  | 15 | 11 |
| cd  | 2  | 6  | 14 | 10 |
| cd' | 1  | 1  | 1  | 1  |

From K-map:

$$F = \sum m(1, 6, 7, 9, 13, 14, 15)$$

$$H = \sum m(1, 2, 5, 6, 7, 10, 11, 13, 14, 15)$$

- $a' \oplus b' \oplus c'd' \Leftrightarrow 100$  ← Row 10 of K-map (Row 10 is best)  
 $100$  ~~is~~ ~~an~~ ~~enable~~ (decoder) ~~will~~ ~~enable~~ (enables)  $(c, d)$  ~~for~~ ~~the~~ ~~line~~  
- inputs  $m_0 (A, b)$  ~~is~~



Active 1

Active 0

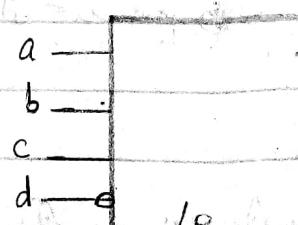
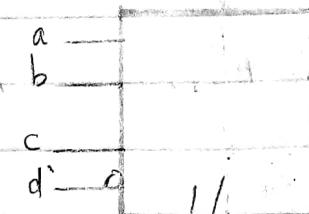
| EN <sub>1</sub> | EN <sub>2</sub> |
|-----------------|-----------------|
| C               | d               |

$$0 \oplus 0 \oplus X$$

$$1 \oplus d'$$

$$C \oplus 1 \oplus d$$

$$C \oplus 1 \oplus d'$$



## Implementing functions by using Decoders

Q: We have available as components three NAND gates (as many inputs as you need) and some active low input, active low enable decoders, as shown below (as many as you need).

| EN | a | b | 0     | 1 | 2 | 3 |  | a |  | 0 | 1 | 2 | 3 |
|----|---|---|-------|---|---|---|--|---|--|---|---|---|---|
| 1  | X | X | (1,1) | 1 | 1 | 1 |  |   |  | 0 | 1 | 2 | 3 |
| 0  | 0 | 0 | 0     | 1 | 1 | 1 |  | b |  | 1 | 0 | 0 | 0 |
| 0  | 0 | 1 | 1     | 0 | 1 | 1 |  |   |  | 0 | 1 | 2 | 3 |
| 0  | 1 | 0 | 1     | 1 | 0 | 1 |  |   |  | 0 | 0 | 1 | 2 |
| 0  | 1 | 1 | 1     | 1 | 1 | 0 |  |   |  | 0 | 0 | 0 | 1 |

Show diagram of circuit to implement these functions :  
(For each group of functions)

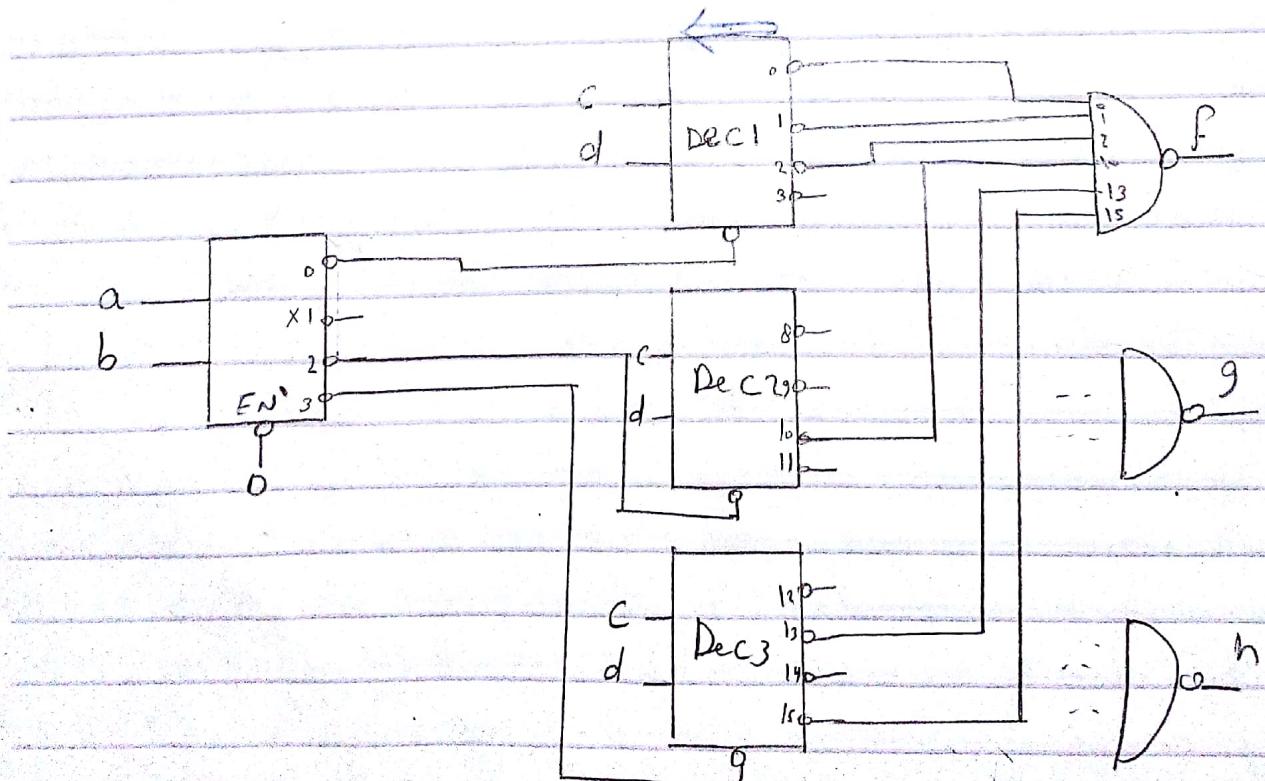
Decoders  $\Rightarrow$

a)

|            | ab | $\bar{a}b$ | $a\bar{b}$ | ab | ab |  | cd          | 0 | 1 | 4 | 12 | 8  |  | cd         | 0 | 1 | 4 | 12 | 8  |
|------------|----|------------|------------|----|----|--|-------------|---|---|---|----|----|--|------------|---|---|---|----|----|
| cd         | 0  | 1          | 4          | 12 | 8  |  | cd          | 0 | 1 | 5 | 13 | 9  |  | cd         | 0 | 1 | 5 | 13 | 9  |
| $\bar{c}d$ | 1  |            | 5          | 13 | 9  |  | $\bar{c}d'$ | 1 |   | 5 | 13 | 9  |  | $\bar{c}d$ | 1 |   | 5 | 13 | 9  |
| $\bar{c}d$ | 3  |            | 6          | 14 | 10 |  | $\bar{c}d'$ |   |   | 6 | 14 | 10 |  | $\bar{c}d$ |   |   | 6 | 14 | 10 |
| $\bar{c}d$ | 2  |            | 7          | 15 | 11 |  | $\bar{c}d'$ |   |   | 7 | 15 | 11 |  | $\bar{c}d$ |   |   | 7 | 15 | 11 |
| $\bar{c}d$ | 1  |            | 8          | 16 | 12 |  | $\bar{c}d'$ |   |   | 8 | 16 | 12 |  | $\bar{c}d$ |   |   | 8 | 16 | 12 |
| $\bar{c}d$ | 0  |            | 9          | 17 | 13 |  | $\bar{c}d'$ |   |   | 9 | 17 | 13 |  | $\bar{c}d$ |   |   | 9 | 17 | 13 |

$F = \sum m(0, 1, 2, 10, 13, 15)$  (Binary to 9 (max)) ملاحظات  
 $\Rightarrow (h \& g \& f)$  هي قيمة ( $\bar{a}b$ ) التي تُؤدي  
enables  $\Leftarrow (a, b)$  تُتيح  
inputs  $\Leftarrow (c, d)$

| Enable signal |   | inputs        | out puts          |
|---------------|---|---------------|-------------------|
| a             | b |               |                   |
| 0             | 0 | For Decoder 1 | cd 0, 1, 2, 3     |
| X             | X | (Deleted)     | cd                |
| 1             | 0 | For Decoder 2 | cd 8, 9, 10, 11   |
| 1             | 1 | For Decoder 3 | cd 12, 13, 14, 15 |



b)

| $wx$        | $w\bar{x}$ | $w\bar{x}$ | $wx$ | $wx$ | $wx$ |
|-------------|------------|------------|------|------|------|
| $y_2$       | 1          | 4          | 1    | 12   | 3    |
| $y_2$       | 1          | 5          | 1    | 13   | 9    |
| $\bar{y}_2$ | 3          | 7          | 15   | 11   |      |
| $y_2$       | 2          | 6          | 1    | 14   | 16   |
| $\bar{y}_2$ |            |            |      |      | 1    |

$$= \pm m(0, 3, 4, 5, 6, 8, 10, 12)$$

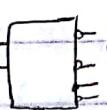
| $x$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |
|-----|-------|-------|-------|-------|-------|-------|
|     |       |       |       |       |       |       |
|     | 1     | 1     | 1     | 1     | 1     | 1     |
|     | 1     | 1     | 1     | 1     | 1     | 1     |
|     |       |       |       | 1     |       |       |

$$= \Sigma_m(1, 3, 5, 7, 9, 11, 13, 14)$$

| $w_x$ | 1 | 1 | 1 | 1 |
|-------|---|---|---|---|
| $y_2$ | 1 | 1 | 1 |   |
|       | 1 | 1 | 1 |   |
|       | 1 |   |   | 1 |

$$= \sum_m (x_1, 2, 4, 5, 7, 8, 10, 12, 13, 15)$$

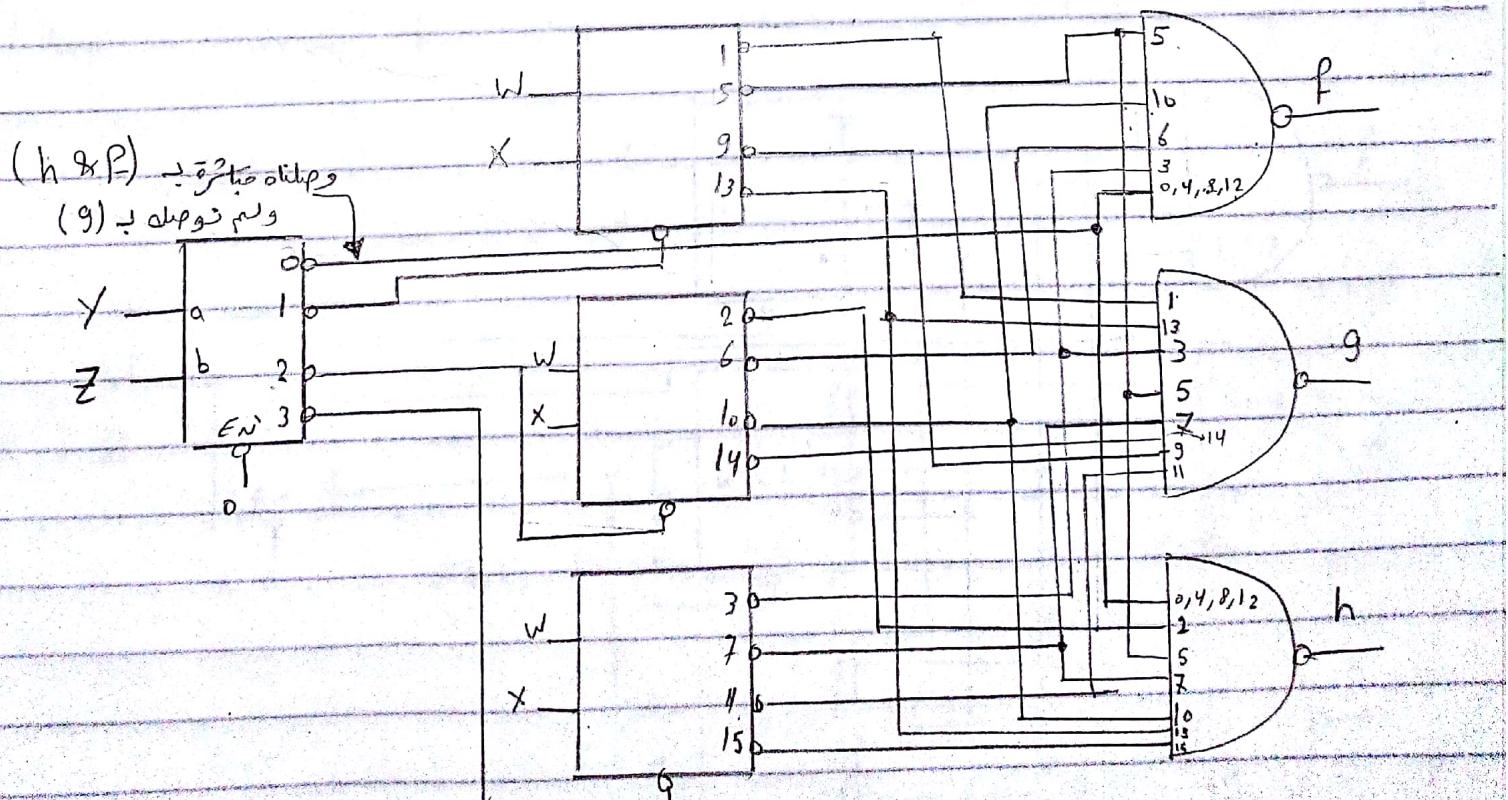
نحوه اسفل ذیل آن ( $2^3$ ) مختلف (کامپارایر) و خارج کاراکتر  $h_{SP}$  (enable) بعنوان (Decoder) نشان می‌گیرد و خواسته شده است که دستگاه  $S$  را باز کند.



ـ (Decoder) يحول احالة وظائفنا من  $\Sigma$  الى  $\Sigma$  . (Decoder) يحول احالة وظائفنا من  $\Sigma$  الى  $\Sigma$

وتوصل المخرج إلى معايرة (enable) لـ  $f \& g$ :  
 لـ  $f$  كـ  $f(w, x)$  وـ  $g$  كـ  $g(y, z)$

| Enable signal<br>Y | Z |           | Inputs | Output                |
|--------------------|---|-----------|--------|-----------------------|
| 0                  | 0 | (Deleted) | w x    | 0, 4, 8, 12    ↪ y z  |
| 0                  | 1 | Decoder1  | w x    | 1, 5, 9, 13    ↪ y z  |
| 1                  | 0 | Decoder2  | w, x   | 2, 6, 10, 14    ↪ y z |
| 1                  | 1 | Decoders  | w, x   | 3, 7, 11, 15    ↪ y z |



$$\sum_m = (1, 2, 3, 5, 7, 12, 14) \quad \Rightarrow \quad \sum_m (1, 3, 5, 7, 8, 9, 10, 15) \quad \Rightarrow \quad \sum_m (2, 4, 11, 12, 13, 14, 15)$$

العنوان (P & Q) في المجموعة {1, 3, 5, 7} هو 4

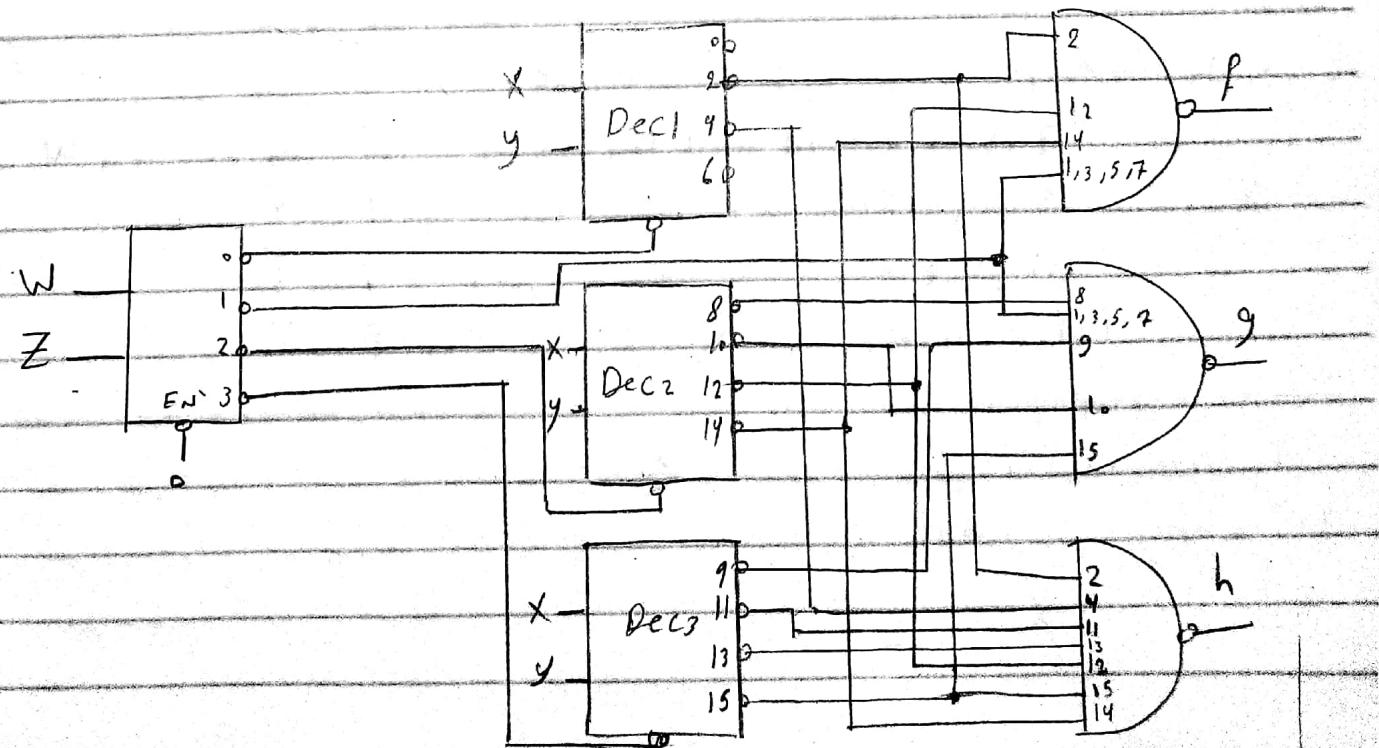
(W'Z)  $\Leftrightarrow (1, 3, 5, 7)$  جزء من المصفوفة (K-map) (نقطة خضراء)

$(w, z)$  و  $(enable)$   $S(w, z)$  فيما يلي

$\hookrightarrow$  Decoder  $\rightarrow$  it

(inputs)  $S(x, y)$  (پرتوں کا).

| Enable Signals |   | inputs    | outputs                  |
|----------------|---|-----------|--------------------------|
| W              | Z |           |                          |
| 0              | 0 | Decoder 1 | x y → 0, 2, 4, 6 ↪ WZ    |
| x              | 1 | (Deleted) | xxx → 1, 3, 5, 7 ↪ WZ    |
| 1              | 0 | Decoder 2 | x y → 8, 10, 12, 14 ↪ WZ |
| 1              | 1 | Decoders  | x y → 9, 11, 13, 15 ↪ WZ |



|       | $wx$ | $wxi$ | $wxk$ | $wx$ | $wx'$ |
|-------|------|-------|-------|------|-------|
| $y_3$ | 0    | 4     | 12    | 8    |       |
| $y_2$ | 1    | 5     | 13    | 9    |       |
| $y_1$ |      |       |       | 10   |       |
| $y_0$ | 3    | 7     | 15    | 11   |       |
| $y_2$ | 2    | 6     | 14    | 10   |       |
| $y_1$ |      |       |       |      |       |
| $y_0$ | 1    | 1     | 1     | 1    |       |

هناك بعض الحالات التي ينتج عنها مخرج متساوٍ

$w, x \rightarrow 00$  The output will be  $(0, 1, 2, 3)$ .

$(0, 1, 2, 1) 01 \Rightarrow (4, 5, 6, 7)$

$(1, 0, 1, 1) 10 \Rightarrow (8, 9, 10, 11)$

$11 \Rightarrow (12, 13, 14, 15)$

الحالات  $(5, x)$  تُمكّن  $S(Y, Z)$  من إنتاج

$y, z \rightarrow 00$  The output will be  $(0, 4, 8, 12)$

$(0, 1, 2, 1) 01 \Rightarrow (1, 5, 9, 13)$

$(1, 0, 1, 1) 10 \Rightarrow (2, 6, 10, 14)$

$(1, 1, 0, 1) 11 \Rightarrow (3, 7, 11, 15)$

$(0, 0, 0, 0) \Rightarrow 1$

: enables  $S(w, y)$  من إنتاج

$w, y \rightarrow 0^w 0^y$  The output will be  $(0, 1, 4, 5)$

$\Rightarrow (0, 1, 2, 3) \text{ حيث } 4, 5, 6, 7 \in S(X) \text{ حيث}$

$(0, 1, 2, 1) \text{ حيث } 5, 6, 7 \in S(X) \text{ حيث}$

$w^y \text{ يُمثل }(1, 2, 3, 0) \Rightarrow (10, 11, 14, 15)$

الحالات  $(0, 1, 2, 1) \Rightarrow (1, 2, 3, 0)$  هي الحالات التي

تحتاج إلى تبديل المدخلات (الحالات التي تُمكّن  $S(w, y)$  من إنتاج

الحالات المطلوبة).

(enables)  $S(w, z)$   $\leftarrow$  أوستاخم

$w, z \Rightarrow$

|                |                                               |
|----------------|-----------------------------------------------|
| $w = 0, z = 0$ | The output will be $\rightarrow (0, 2, 4, 6)$ |
| $w = 0, z = 1$ | " " $\rightarrow (1, 3, 5, 7)$                |
| $w = 1, z = 0$ | " " $\rightarrow (8, 10, 12, 14)$             |
| $w = 1, z = 1$ | " " $\rightarrow (9, 11, 13, 15)$             |

(enables)  $S(x, y)$   $\leftarrow$  أوستاخم

$x \times y \Rightarrow$   $x \times y \rightarrow 0, 0$  The output will be  $\rightarrow (0, 1, 8, 9)$

|             |     |                              |
|-------------|-----|------------------------------|
| $(0, 0, 1)$ | " " | $\rightarrow (2, 3, 10, 11)$ |
| $(1, 0, 2)$ | " " | $\rightarrow (4, 5, 12, 13)$ |
| $(1, 1, 3)$ | " " | $\rightarrow (6, 7, 14, 15)$ |

الآن  $S(x, z)$   $\leftarrow$  أوستاخم

$x, z \Rightarrow$   $0, 0$  The output will be  $\rightarrow (0, 2, 8, 10)$

|                |     |                              |
|----------------|-----|------------------------------|
| $(0, 1, 3, 5)$ | " " | $\rightarrow (1, 3, 9, 11)$  |
| $(1, 1, 5, 6)$ | " " | $\rightarrow (4, 6, 12, 14)$ |
| $1, 1$         | " " | $\rightarrow (5, 7, 13, 15)$ |

الآن  $S(y, w)$   $\leftarrow$  أوستاخم

$y, w \Rightarrow$   $0, 0$  The output will be  $\rightarrow (0, 2, 8, 10)$

لذلك  $x, z \rightarrow$  كود (Kode)  $\rightarrow$  بآلة نحوس (Output)  $\rightarrow$  بآلة ديجيت (Digital)

$(x, z) \rightarrow (y, w) \rightarrow (0, 0)$

$(2, 1, 1, 1) \rightarrow (0, 2, 8, 10)$   $\rightarrow$  بآلة ديجيت

Kode

لذلك  $(Decoder)$   $\leftarrow$  (Function)  $\rightarrow$  سؤال (Question)  $\rightarrow$  حل فيه.

جزء صناعي يعمد (Function)  $\rightarrow$  معرفة اليمان ويساعد على حل

(enables)  $\leftarrow$  يحسن من

D)

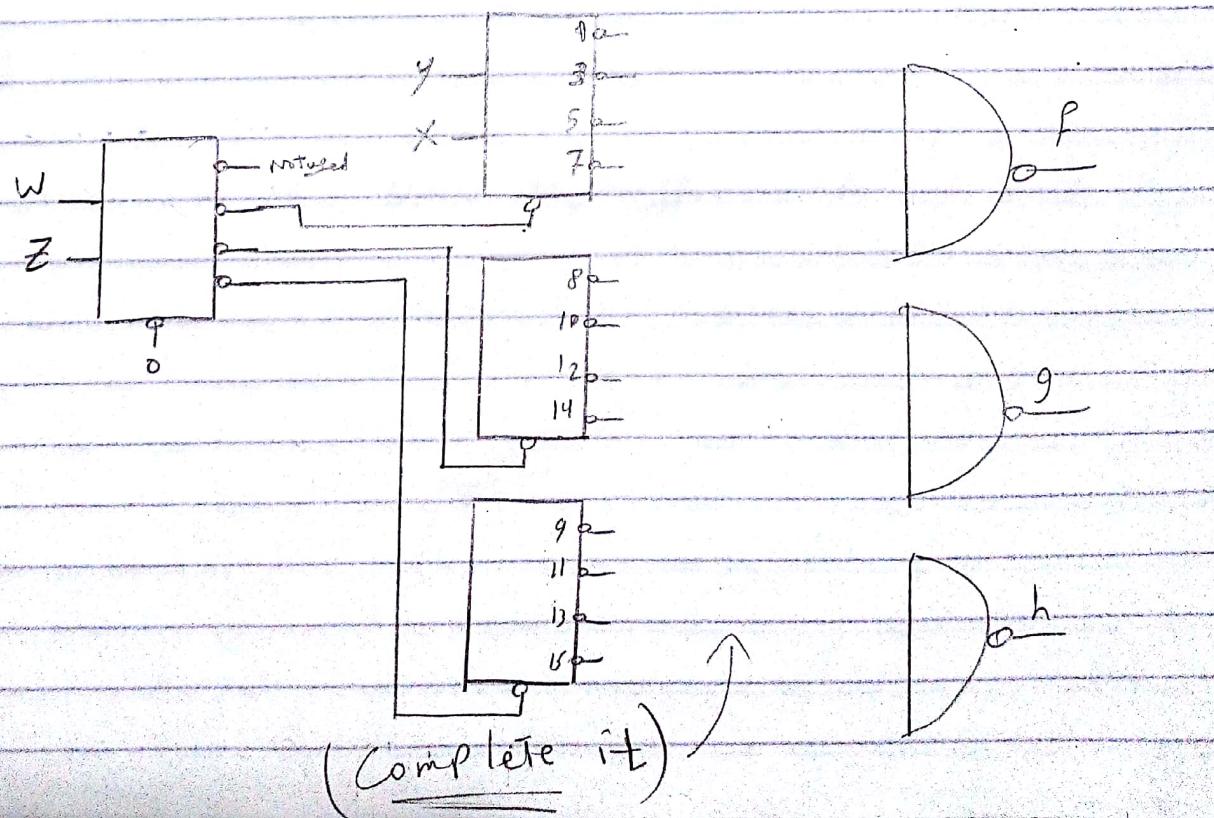
| $w_x$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $y_2$ | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
| $y_2$ | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    |
| $y_2$ | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
| $y_2$ | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    |
| $y_2$ | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    |
| $y_2$ | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    |
| $y_2$ | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    |
| $y_2$ | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    |
| $y_2$ | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    | 18    |
| $y_2$ | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    | 18    | 19    |
| $y_2$ | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    | 18    | 19    | 20    |

(functions)  $\{f_i\}_{i=0}^6$  plotted on  $y_2$  axis  
 $\Leftrightarrow$   $(w, z)$   $\Leftrightarrow$   $y_2$   $\Leftrightarrow$   $w, z$

(enables)  $\{f_i\}_{i=0}^6$  plotted on  $y_2$  axis

(inputs)  $\{y, x\}$

| Enable signals<br>$w, z$ |           | Inputs | Outputs                         |
|--------------------------|-----------|--------|---------------------------------|
| X 0 0                    | (Deleted) | $y, x$ | $0, 2, 4, 6 \leftarrow w, z$    |
| 0 1                      | Decoder 1 | $y, x$ | $1, 3, 5, 7 \leftarrow w, z$    |
| 1 0                      | Decoder 2 | $y, x$ | $8, 10, 12, 14 \leftarrow w, z$ |
| 1 1                      | Decoder 3 | $y, x$ | $9, 11, 13, 15 \leftarrow w, z$ |



e)

|       | $ab'$ | $ab$ | $ab'$ | $ab$ | $ab'$ |   |  |   |   |  |  |
|-------|-------|------|-------|------|-------|---|--|---|---|--|--|
| $cd'$ | 0     | 4    | 1     | 12   | 1     | 0 |  | 1 | 1 |  |  |
| $cd$  | 1     | 3    | 1     | 13   | 1     | 9 |  | 1 | 1 |  |  |
| $cd'$ | 3     | 7    | 14    | 11   | 1     |   |  |   |   |  |  |
| $cd$  | 2     | 1    | 6     | 15   | 10    |   |  | 1 | 1 |  |  |

(Functions)  $f_1$  è la  $\sum(4, 5, 12, 13)$  in K-map è best  
 $(b'c) \leftarrow$  summing  
(enables)  $S(b, c)$  piccoli valori  
inputs.  $S(a, d)$

| enable signals |     | inputs   | outputs                                |
|----------------|-----|----------|----------------------------------------|
| $b$            | $c$ |          |                                        |
| 0              | 0   | Decoder1 | a d      0, 1, 8, 9 $\leftarrow b'c$   |
| 0              | 1   | Decoder2 | a d      1, 3, 10, 11 $\leftarrow b'c$ |
| X              | 1   | Deleted  | a d      4, 5, 12, 13 $\leftarrow b'c$ |
| 1              | 1   | Decoder3 | a d      1, 7, 14, 15 $\leftarrow b'c$ |

⇒ You Draw the Design.

Ex: (Final)

We have found a minimum sum of products expressions for each of two functions F and G. For these functions, we have available as many of the decoders described below as are needed plus 2 eight-input OR gates. Show a block diagram for this implementation with only these components.

→ All inputs are available both uncomplemented and complemented.

→ 5 point Extra credit: show a diagram that uses only 2 eight-input OR gates and 3 of these decoders.

$$F = A'B + BD + AB'C'D'$$

$$G = \bar{B}\bar{D} + BC + ABD'$$

|                 |   |
|-----------------|---|
| A               | 0 |
| B               | 1 |
| EN <sub>1</sub> | 2 |
| EN <sub>2</sub> | 3 |

Solution

| AB               | $\bar{A}\bar{B}$ | $\bar{A}B$ | $AB$ | $A\bar{B}$ |
|------------------|------------------|------------|------|------------|
| CD               | 0                | 4          | 12   | 10         |
| $\bar{C}\bar{D}$ | 1                | 5          | 13   | 9          |
| CD               | 3                | 7          | 15   | 11         |
| $\bar{C}D$       | 2                | 6          | 14   | 10         |

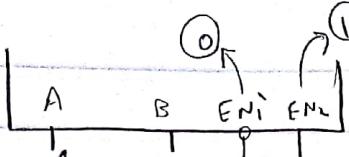
| AB               | $\bar{A}\bar{B}$ | $\bar{A}B$ | $AB$ | $A\bar{B}$ |
|------------------|------------------|------------|------|------------|
| CD               | 0                | 1          | 1    | 1          |
| $\bar{C}\bar{D}$ | 1                | 0          | 0    | 0          |
| CD               | 3                | 0          | 1    | 1          |
| $\bar{C}D$       | 2                | 1          | 1    | 1          |

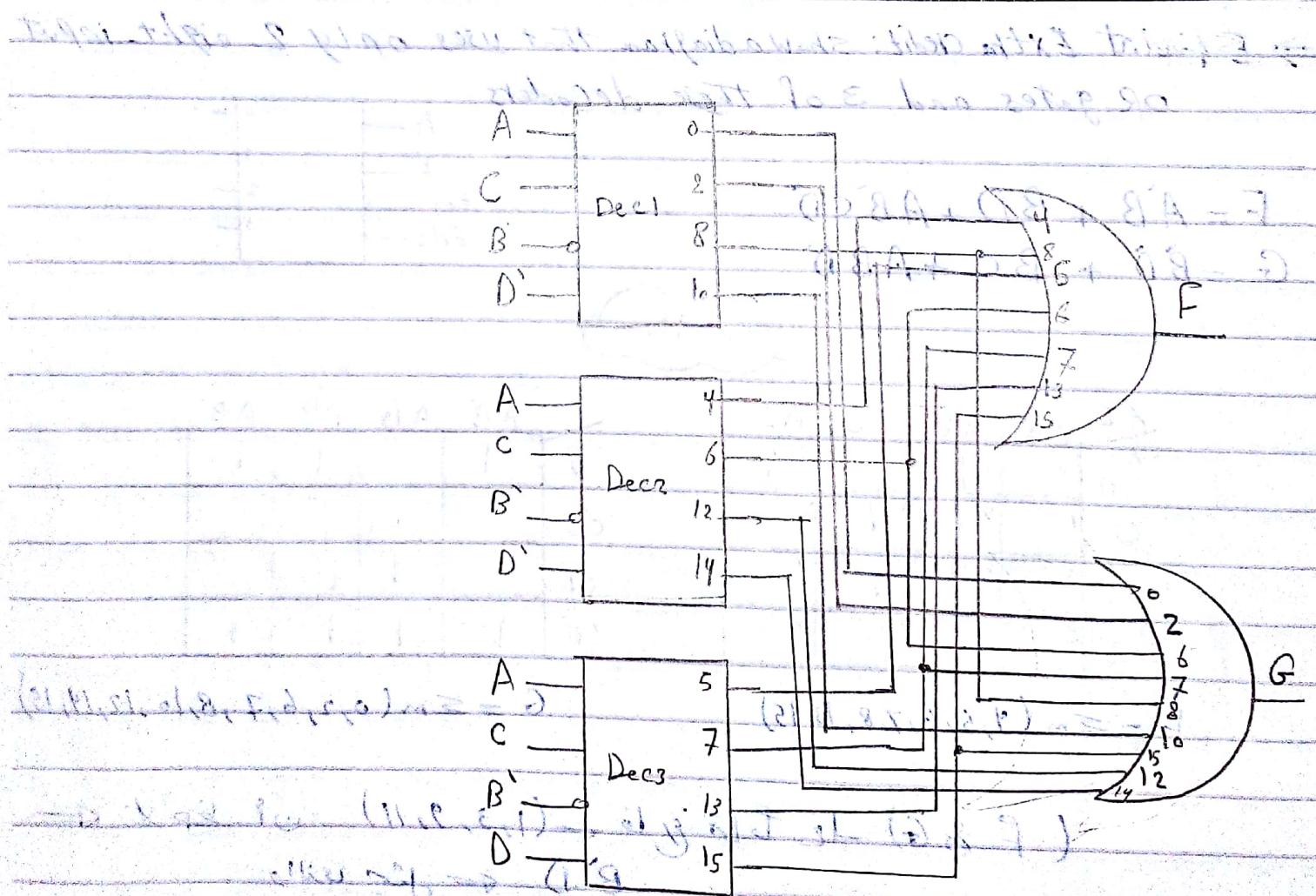
$$F = \sum m(4, 5, 6, 7, 8, 13, 15)$$

$$G = \sum m(0, 2, 6, 7, 8, 10, 12, 14, 15)$$

(F & G) ← فاعل ← (1, 3, 9, 11) ← بـ D ← والـ C ←  
 enables S(B, D) previous will  
 inputs. S(A, C) g



| Enable Signals |  | Outputs              |
|----------------|-----------------------------------------------------------------------------------|----------------------|
| B → D          | A → C → B → D                                                                     | (0, 2, 8, 10) ↪ B'D' |
| 0 (Deleted)    | A → C → B → D                                                                     | (4, 6, 12, 14) ↪ BD' |
| 1 (Dec2)       | A → C → B → D                                                                     | (5, 7, 13, 15) ↪ B'D |
| 1 (Dec3)       | A → C → B → D                                                                     |                      |



Counting sequence (0-8) ↪ (folded)

(2, 4) ↪ F.G