

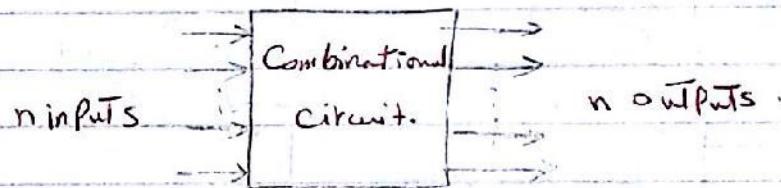
Chapter 4 From "Mano" book

Combinational logic

أمثلة على الـ logic gates مثل $NAND$, NO , OR و NOT كأنه تكون مثلاً $D \oplus D = D$ و $D \oplus D = \bar{D}$ و $D \oplus \bar{D} = D$ و $D \oplus \bar{D} = 1$ و $D \oplus 1 = \bar{D}$ و $D \oplus 1 = 1$ و $D \oplus 0 = D$ و $D \oplus 0 = 0$

\therefore negation (Combinational logic) \rightarrow \neg

inputs, logic gates, outputs.



(Block diagram of Combinational circuit.)

\therefore Combinational circuits \rightarrow $f = f(x_1, x_2, \dots, x_n)$

→ Adder

Subtractor

Competitor

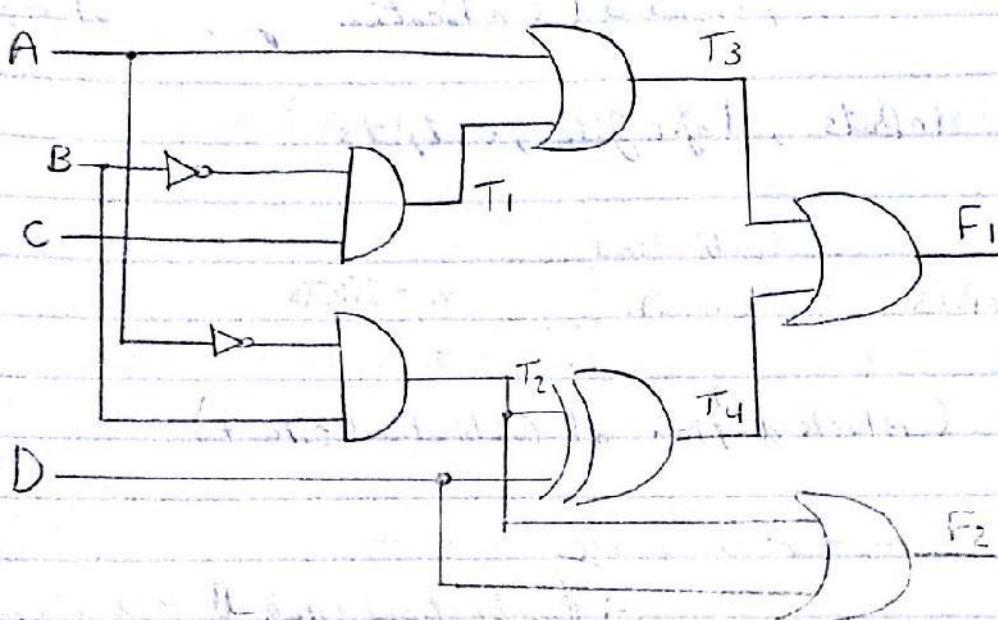
→ Decoder

\Rightarrow Encoded

→ Multiplexer

* Analysis of Combinational Logic :-

Q 4.1 >> p.191



- Derive the Boolean expressions for T_1 , T_2 , T_3 , T_4 , F_1 , and F_2 and Evaluate The outputs F_1 and F_2 as functions of the inputs.
- List the Truth Table (TT) With 16 Combinations of 4 input variables Then list the binary values for T_1 , T_2 , T_3 , T_4 and the outputs F_1 and F_2 in The Table.
- Plot the Boolean output functions obtained in part (b) on Maps and show that the Simplified Boolean expressions are equivalent To the one's obtained in Part (a).?!!

ANS

a)

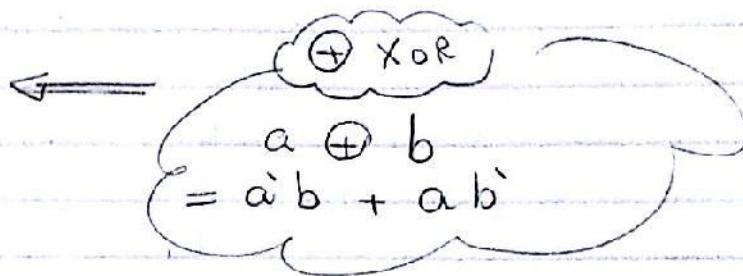
لـ نـعـنـ أـنـاـ خـصـلـ بـ مـعـاـدـرـتـ مـعـ الرـسـمـ

$$T_1 = \boxed{B'C}$$

$$T_2 = \boxed{\bar{A}B}$$

$$\begin{aligned} T_3 &= A + T_1 \\ &= \boxed{A + B'C} \end{aligned}$$

$$\begin{aligned} T_4 &= D \oplus T_2 \\ &= D \oplus (\bar{A}B) \\ &= D(\bar{A}B) + D(\bar{A}B') \\ &= \boxed{\bar{A}'B'D' + AD + DB'} \end{aligned}$$



$$\begin{aligned} F_1 &= T_3 + T_4 \\ &= \boxed{(A + B'C) + (\bar{A}'B'D' + AD + DB')} \\ &= \boxed{A(1+D) + B'C + \bar{A}'BD' + DB'} \end{aligned}$$

$$\begin{aligned} &= \boxed{A + B'C + \bar{A}'BD' + DB'} \quad \leftrightarrow \quad \boxed{A + B'C + \bar{A}'BD'} \\ &= \boxed{(A+A')(A+BD') + B'C + DB'} \\ &= \boxed{A + BD' + B'C + B'D} \end{aligned}$$

$$\begin{aligned} F_2 &= D + T_2 \\ &= \boxed{D + \bar{A}'B} \end{aligned}$$

b)

\downarrow \downarrow \downarrow \downarrow \downarrow

BC \bar{AB} $A + \bar{B}$ $D \oplus T_2$ $T_3 + T_4$ $D + T_2$

| A | B | C | D | T_1 | T_2 | T_3 | T_4 | F_1 | F_2 | m |
|---|---|---|---|-------|-------|-------|-------|-------|-------|----------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | m_0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | m_1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | m_2 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | m_3 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | m_4 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | m_5 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | m_6 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | m_7 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | m_8 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | m_9 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | m_{10} |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | m_{11} |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | m_{12} |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | m_{13} |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | m_{14} |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | m_{15} |

From TRut table.. $F_1 = \sum m(1, 2, 3, 4, 6, 8, 9, 10, 11, 12, \dots, 15)$

$F_2 = \sum m(1, 3, 4, 5, 6, 7, 9, 11, 13, 15)$

c) for F_1

| CD | $\bar{C}D$ | CD | $\bar{C}\bar{D}$ | CD' |
|---------------|------------|------|------------------|-------|
| $A\bar{B}$ | 0 | 1 | 1 | 1 |
| $A(\bar{B})$ | 1 | 1 | 0 | 1 |
| $B(\bar{A})$ | 1 | 0 | 1 | 1 |
| $A(\bar{A}B)$ | 0 | 1 | 1 | 1 |

$$F_1 = A + B\bar{D} + (CD) + \bar{B}D$$

$$(OR) = A + B\bar{D} + (\bar{B}C) + \bar{B}D$$

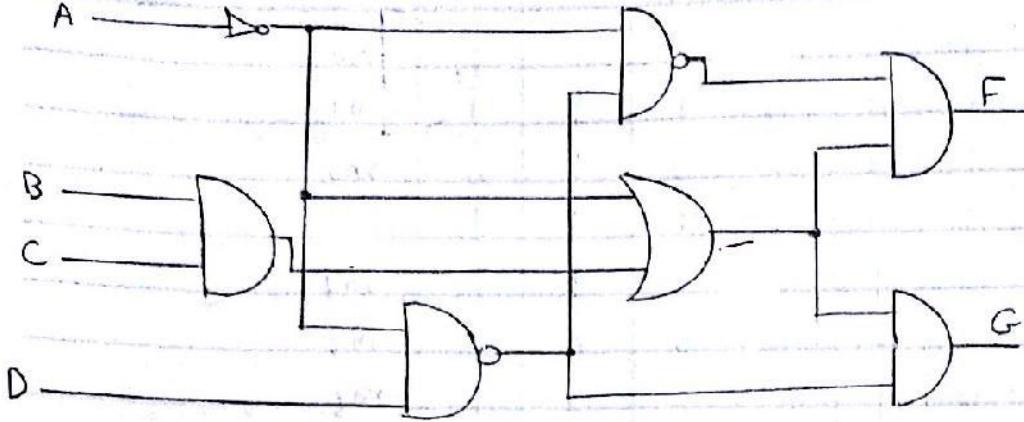
for F_2

| CD | $\bar{C}D$ | CD | $\bar{C}\bar{D}$ | CD' |
|---------------|------------|------|------------------|-------|
| $A\bar{B}$ | 0 | 1 | 1 | 2 |
| $A(\bar{B})$ | 1 | 1 | 0 | 1 |
| $B(\bar{A})$ | 1 | 0 | 1 | 1 |
| $A(\bar{A}B)$ | 0 | 1 | 1 | 1 |

$$F_2 = D + \bar{A}B$$

Je jis J3112
Partial

In the book P. 19] Q4.2



* obtain the simplified Boolean expressions for outputs F & G in terms of the input variables in the circuit.

Ans:

$$F = (A \cdot (A'D))' (A + BC)$$

الرسالة (output) من المدخل (input) \rightarrow للتحويل فرم \rightarrow gate form

$$F = T_3 + T_4$$

$$, G = \overline{T_4} \cdot T_2$$

$$\Rightarrow T_4 = B \cdot C$$

$$\Rightarrow T_2 = (A'D)' \\ = A + D$$

$$\Rightarrow T_4 = \overline{A} + T_1 \\ = \overline{A} + BC$$

$$\begin{aligned} \Rightarrow T_3 &= (A \cdot T_2)' \\ &= A + T_2 \\ &= A + (A'D)' \\ &= A + A'D \end{aligned}$$

→

$$F = T_3 \cdot T_4$$

$$= (A + \bar{A}D)(\bar{A} + BC)$$

$$= A\bar{A} + A\bar{B}C + \bar{A}\bar{A}D + \bar{A}BCD$$

$$= \cancel{A\bar{B}C} + \underbrace{\bar{A}D}_{A'D} + \underbrace{\bar{A}BCD}_{A'D}$$

$$= ABC + \bar{A}D(1 + BC)$$

$$= \boxed{ABC + \bar{A}D}$$

$$G = T_4 \cdot T_2$$

$$= (\bar{A} + BC)(\bar{A} + D')$$

$$= \cancel{\bar{A}\bar{A}} + \bar{A}D' + ABC + BCD'$$

$$= \underbrace{\bar{A}D'}_{1} + \underbrace{ABC}_{\cancel{BCD}}$$

(Congensus Term) = \cancel{BCD}

$$= \boxed{\bar{A}D' + ABC}$$

* Design of Combinational Logic

To Design The Combinational Circuit

- ① Construct Truth Table.
- ② Obtain the equations of the outputs in $\Sigma m = (\dots)$ numeric form
- ③ Put the function on the K-map & Simplify it/them.
- ④ Draw the Block diagram of each output function.

→ Examples

Ex1: Q4.3 >> P.191

* Design a Combinational Circuit with 3 inputs and one output

The output is 1, when the binary value of the input is less than 3 and 0 otherwise.

Ans:-

① TT → Let the 3 inputs are a, b, c

& the output F

| | a | b | c | F |
|-------|-----|-----|-----|-----|
| m_0 | 0 | 0 | 0 | 1 |
| m_1 | 0 | 0 | 1 | 1 |
| m_2 | 0 | 1 | 0 | 1 |
| m_3 | 0 | 1 | 1 | 0 |
| m_4 | 1 | 0 | 0 | 0 |
| m_5 | 1 | 0 | 1 | 0 |
| m_6 | 1 | 1 | 0 | 0 |
| m_7 | 1 | 1 | 1 | 0 |

(Q4.3) (Ans)
 (b) i) negative "i.e. low" signal (back) (input) II
 receiving (X) no J1 binary per
 output get logic levels
 (j1)

$$② F = \Sigma m(0, 1, 2)$$

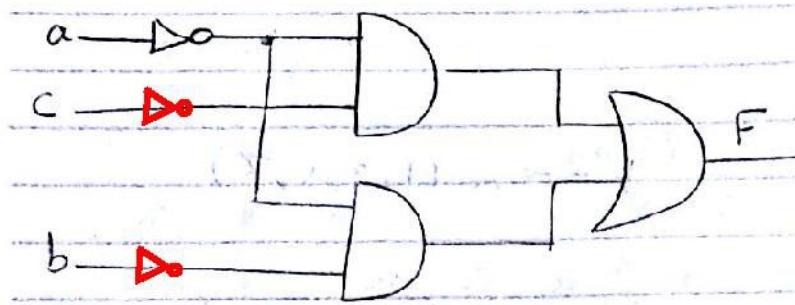


(3) K-map.

| $\bar{b} \bar{c}$ | $\bar{b} c$ | $b \bar{c}$ | $b c$ | $b \bar{c}$ |
|-------------------|-------------|-------------|-------|-------------|
| \bar{a} | 1 | 1 | 1 | 1 |
| a | 1 | 0 | 0 | 0 |
| \bar{a} | 4 | 5 | 7 | 6 |

$$F = a'c' + a'b'$$

(4) The block diagram by AND, OR



\rightarrow

EX2: From the book p.191
Q.4.3

* Design a Combinational circuit with 3 inputs & one output,
The output is one when the binary value of the inputs is an odd number.

Ans.

①

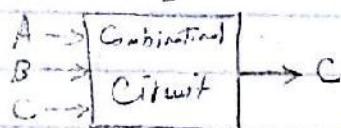
| | A | B | C | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 |

② $F = \Sigma_m (1, 3, 5, F)$

| | B_C | B_C | B_C | B_C | B_C |
|-----|-------|-------|-------|-------|-------|
| A | 0 | 1 | 1 | 0 | 1 |
| A' | 1 | 0 | 0 | 1 | 0 |
| A'' | 4 | 5 | 6 | 7 | 8 |

$F = C$

④ The Block diagram



Ex 3, From the book P. 191 Q4.

* Design a combinational circuit with 3 inputs $X, Y \& Z$ and three outputs A, B and C. When the binary input is 0, 1, 2 or 3 \rightarrow The binary output is two greater than the input and when the binary input is 4, 5, 6 or 7 \rightarrow The binary output is three less than the input.

Ans:

(i)

| Inputs | | | Outputs | | |
|--------|---|---|---------|---|---|
| X | Y | Z | A | B | C |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 1 |
| 5 | 1 | 0 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 | 1 |
| 7 | 1 | 1 | 1 | 0 | 0 |

②

Outputs

$$A = \Sigma m(2, 3, 7)$$

$$B = \Sigma m(0, 1, 5, 6)$$

$$C = \Sigma m(1, 3, 4, 6)$$

(ii) $A = \bar{X}Y + YZ$

| | $\bar{Y}Z$ | YZ | $\bar{Y}\bar{Z}$ | $Y\bar{Z}$ |
|-----------|------------|------|------------------|------------|
| \bar{X} | 0 | 1 | 0 | 1 |
| X | 1 | 0 | 1 | 0 |
| | 0 | 1 | 1 | 0 |

C

| | $\bar{Y}Z$ | YZ | $\bar{Y}\bar{Z}$ | $Y\bar{Z}$ |
|-----------|------------|------|------------------|------------|
| \bar{X} | 0 | 1 | 0 | 1 |
| X | 1 | 0 | 1 | 0 |
| | 0 | 1 | 1 | 0 |

$$C = \bar{X}Z + X\bar{Z}$$

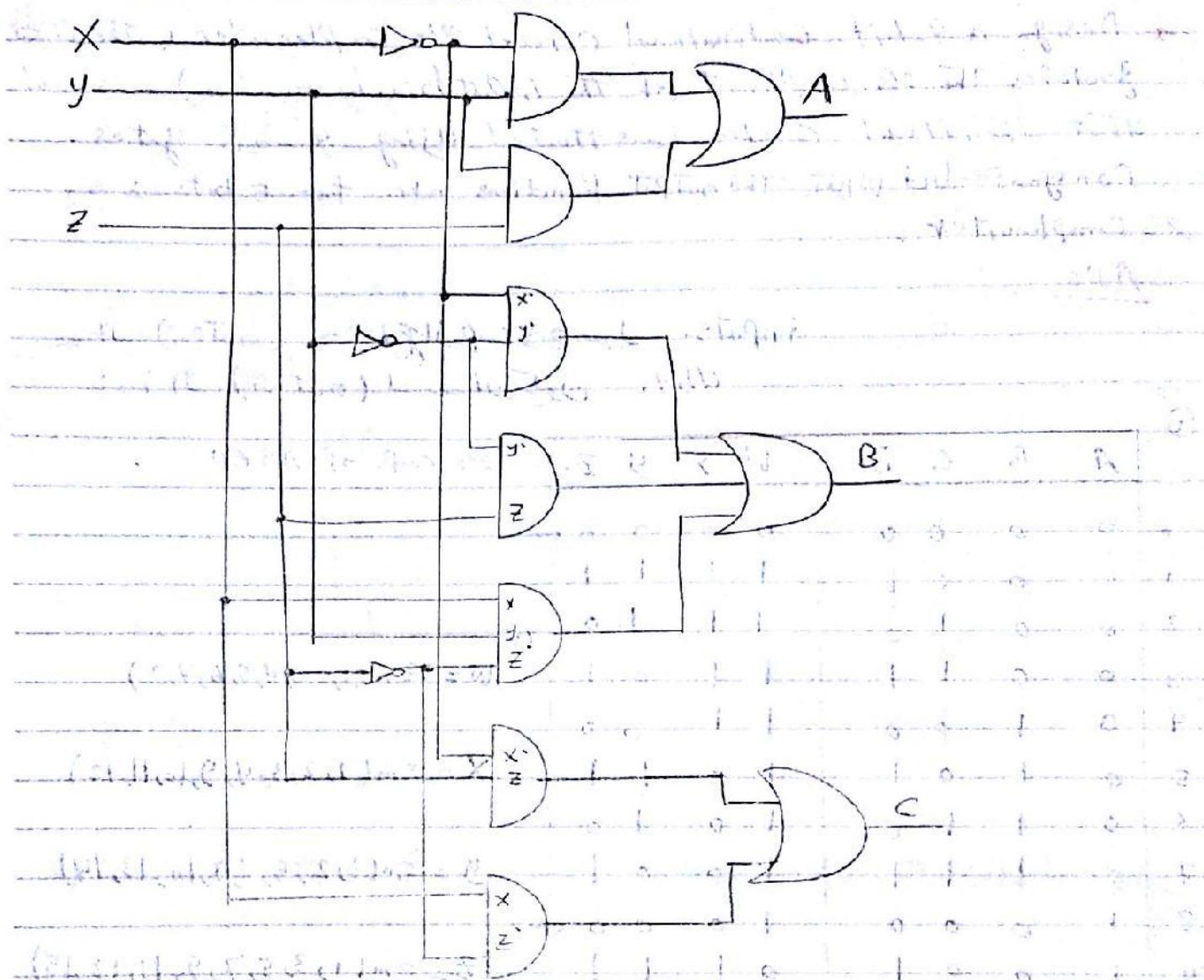
B

| | $\bar{Y}Z$ | YZ | $\bar{Y}\bar{Z}$ | $Y\bar{Z}$ |
|-----------|------------|------|------------------|------------|
| \bar{X} | 0 | 1 | 0 | 1 |
| X | 1 | 0 | 1 | 0 |
| | 0 | 1 | 1 | 0 |

$$B = \bar{X}Y + Y\bar{Z} + XY\bar{Z}$$

\Rightarrow

(4)



Ex 4: Q4.7 >> P.192

* Design a 4-bit Combinational Circuit 2's Complementer (The output generates the 2's complement of the input binary number). Show that the circuit can be constructed using X-OR gates. Can you predict what the output functions are for 5 bits 2's Complementer.

Aus

| ① | A | B | C | D | W | X | Y | Z | 2 ² s comp. of ABCD |
|----|---|---|---|---|---|---|---|---|--------------------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | |
| 4 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | |
| 8 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | |
| 11 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | |
| 12 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | |
| 14 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | |

$$W = \sum m(1, 8)$$

| | $C'D'$ | $C'D$ | $C'D'$ | CD | CD' |
|--------|--------|-------|--------|------|-------|
| $A'B'$ | 0 | 1 | 1 | 1 | 1 |
| $A'B$ | 1 | 1 | 1 | 0 | 1 |
| AB | 12 | 13 | 15 | 14 | |
| AB' | 8 | 1 | 9 | 11 | 10 |

$$\begin{aligned} W &= A'B + A'D + A'C + AB'CD' \\ &= \overline{A}(B+D+C) + A(B'C'D') \\ &= \boxed{A \oplus (B+C+D)} \end{aligned}$$

| | $C'D'$ | $C'D$ | $C'D'$ | CD | CD' |
|--------|--------|-------|--------|------|-------|
| $A'B'$ | 0 | 1 | 1 | 3 | 2 |
| $A'B$ | 4 | 5 | 1 | 7 | 6 |
| AB | 12 | 13 | 1 | 15 | 14 |
| AB' | 8 | 9 | 1 | 10 | 1 |

$$\begin{aligned} y &= C'D + CD' \\ &= \boxed{C \oplus D} \end{aligned}$$

(X)

| | $C'D'$ | $C'D$ | $C'D'$ | CD | CD' |
|--------|--------|-------|--------|------|-------|
| $A'B'$ | 0 | 1 | 1 | 3 | 2 |
| $A'B$ | 4 | 5 | 1 | 7 | 6 |
| AB | 12 | 13 | 1 | 15 | 14 |
| AB' | 8 | 9 | 1 | 11 | 10 |

$$\begin{aligned} X &= B'D + B'C + B\bar{C}D' \\ &= \overline{B}(D+C) + B(\bar{C}D') \\ &= \boxed{B \oplus (C+D)} \end{aligned}$$

(Y)

| | $C'D'$ | $C'D$ | $C'D'$ | CD | CD' |
|--------|--------|-------|--------|------|-------|
| $A'B'$ | 0 | 1 | 1 | 3 | 2 |
| $A'B$ | 4 | 5 | 1 | 7 | 6 |
| AB | 12 | 13 | 1 | 15 | 14 |
| AB' | 8 | 9 | 1 | 11 | 10 |

$$\begin{aligned} Z &= \boxed{D} \\ &\rightarrow D \oplus 0 \end{aligned}$$

(Z)

⇒ The equations of W, X, Y, Z show that the circuit can be constructed by X -OR gates.

⇒ if the inputs were 5 bits ($E A B C D$) → The outputs are the $^{2^{nd}}$ Complement of the inputs → if the outputs were ($\vee W X Y Z$)

→ We predict that $\vee = E \oplus (A+B+C+D)$
According to the equations of 4 bits where:

$$W = A \oplus (B+C+D)$$

$$X = B \oplus (C+D)$$

$$Y = C \oplus D$$

$$Z = D$$

Q3: Design a code converter that converts from BCD₈₄₂₁ to Excess3

→ Build the truth table.

→ Minimize the functions by using k-map.

Ans:

| Dec. | Inputs | | | | Outputs | | | | $(S+X)E$ |
|------|--------|---|---|---|---------|---|---|---|-----------------|
| | a | b | c | d | W | X | Y | Z | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | $S'E = (S+kX)E$ |
| 3 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | $(S+kX)E$ |
| 8 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | $S'E = (S+X)E$ |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | $S'E + XE$ |

$$W(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}\bar{b}cd + ab\bar{c}d + ab\bar{c}d \\ = \Sigma m(5, 6, 7, 8, 9) + \Sigma d(10, 11, 12, 13, 14, 15) - (S+kX)E$$

$$X(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}\bar{b}\bar{c}d + ab\bar{c}d \\ = \Sigma m(1, 2, 3, 4, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$$

$$Y(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + ab\bar{c}d \\ = \Sigma m(0, 3, 4, 7, 8) + \Sigma d(10, 11, 12, 13, 14, 15) \times$$

$$Z(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + ab\bar{c}d \\ = \Sigma m(0, 2, 4, 6, 8) + \Sigma d(10, 11, 12, 13, 14, 15)$$

⇒ Note → From (10 to 15) all don't care on k-map.

Karnaugh:

| | ab' | $a'b'$ | ab | $a'b$ | b' |
|------------|-------|--------|------|-------|------|
| cd' | 0 | 1 | 2 | 3 | 4 |
| $\bar{c}d$ | 5 | 6 | 7 | 8 | 9 |
| cd | 10 | 11 | 12 | 13 | 14 |
| $c'd$ | 15 | 16 | 17 | 18 | 19 |

w

$$W = a + bd + bc$$

| | ab' | $a'b'$ | ab | $a'b$ | b' |
|------------|-------|--------|------|-------|------|
| cd' | 0 | 1 | 2 | 3 | 4 |
| $\bar{c}d$ | 5 | 6 | 7 | 8 | 9 |
| cd | 10 | 11 | 12 | 13 | 14 |
| $c'd$ | 15 | 16 | 17 | 18 | 19 |

x

$$X = b\bar{c}d' + \bar{b}'d + \bar{b}'c$$

| | ab' | $a'b'$ | ab | $a'b$ |
|------------|-------|--------|------|-------|
| cd' | 0 | 1 | 2 | 3 |
| $\bar{c}d$ | 4 | 5 | 6 | 7 |
| cd | 8 | 9 | 10 | 11 |
| $c'd$ | 12 | 13 | 14 | 15 |

y

| | ab' | $a'b'$ | ab | $a'b$ |
|------------|-------|--------|------|-------|
| cd' | 0 | 1 | 2 | 3 |
| $\bar{c}d$ | 4 | 5 | 6 | 7 |
| cd | 8 | 9 | 10 | 11 |
| $c'd$ | 12 | 13 | 14 | 15 |

$$Y = \bar{c}d' + cd$$

$$Z = \bar{c}d' + c'd'$$

$$Z = 0$$

Q. Design a combinational circuit that converts a four bit binary number to a four bit Gray code.

(Implement the circuit with exclusive OR gates)

\oplus OR.

| | Binary number | | | | Gray code. | | | |
|-----|---------------|---|---|---|------------|---|---|---|
| Dec | a | b | c | d | w | x | y | z |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 3 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 11 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

$$\begin{aligned}
 w &= \Sigma m(8, 9, 10, 11, 12, 13, 14, 15) \\
 x &= \Sigma m(4, 5, 6, 7, 8, 9, 10, 11) \\
 y &= \Sigma m(2, 3, 4, 5, 10, 11, 12, 13) \\
 z &= \Sigma m(1, 2, 5, 6, 9, 10, 13, 14)
 \end{aligned}$$

K-maps \Rightarrow

K-maps

$$W = \sum m(8, 9, 10, 11, 12, 13, 14, 15)$$

| ab' | ab | ab' | ab | ab' |
|-------|------|-------|------|-------|
| cd | 0 | 9 | 12 | 8 |
| cd' | 1 | 5 | 13 | 1 |
| cd | 3 | 7 | 11 | 1 |
| cd' | 2 | 6 | 14 | 10 |

$$W = a$$

$$X = \sum m(4, 5, 6, 7, 8, 9, 10, 11)$$

| ab' | ab | $a'b$ | ab | ab' |
|-------|------|-------|------|-------|
| cd | 0 | 4 | 1 | 8 |
| cd' | 1 | 5 | 1 | 9 |
| cd | 3 | 7 | 1 | 14 |
| cd' | 2 | 6 | 1 | 15 |

$$X = \bar{a}b + a\bar{b} \\ = (a \oplus b)$$

$$Y = \sum m(2, 3, 4, 5, 10, 11, 12, 13)$$

| ab' | ab | $a'b$ | ab | ab' |
|-------|------|-------|------|-------|
| cd | 0 | 4 | 12 | 8 |
| cd' | 1 | 5 | 10 | 9 |
| cd | 3 | 7 | 15 | 11 |
| cd' | 2 | 6 | 14 | 10 |

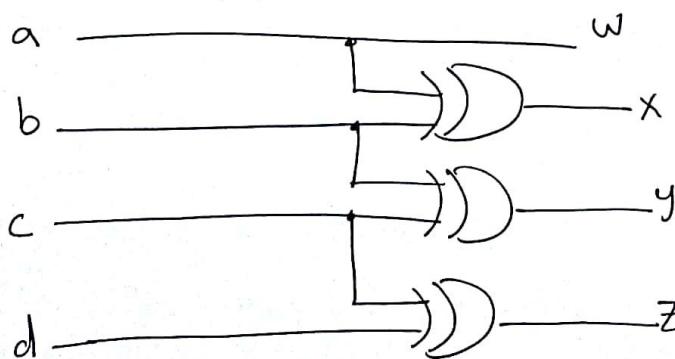
$$Y = \bar{b}c + b\bar{c} \\ = (b \oplus c)$$

$$Z = \sum m(1, 2, 5, 6, 9, 10, 13, 14)$$

| ab' | ab | $a'b$ | ab | ab' |
|-------|------|-------|------|-------|
| cd | 0 | 4 | 12 | 8 |
| cd' | 1 | 5 | 13 | 9 |
| cd | 3 | 7 | 15 | 11 |
| cd' | 2 | 6 | 14 | 10 |

$$Z = \bar{c}d + c\bar{d}' \\ = c \oplus d$$

⇒ Implementing the output functions using XOR gates.



Q) Design a code converter that converts from BCD₂₄₂₁ to BCD₈₄₂₋₁

Ans:

| DEC | 2 | 4 | 2 | 1 | 8 | 4 | 2 | -1 | W | X | Y | Z |
|-----|---|---|---|---|---|---|---|----|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 7 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 8 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| AB | AB | FB | AB | AB |
|----|----|----|----|----|
| CD | CD | 4 | 12 | 5 |
| CD | CD | 1 | 1 | 1 |
| CD | CD | 1 | 1 | 1 |
| CD | CD | 1 | 1 | 1 |

$$W = A$$

| AB | AB | X | AB | AB |
|----|----|---|----|----|
| CD | CD | 1 | 1 | X |
| CD | CD | 1 | X | X |
| CD | CD | 1 | X | X |
| CD | CD | 1 | X | X |

$$X = A'B + A'C + AD$$

$$W = ABCD + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D + ABC\bar{D}$$

$$= \Sigma m(11, 12, 13, 14, 15) + \Sigma d(5, 6, 7, 8, 9, 10)$$

| AB | AB | X | AB | AB |
|----|----|---|----|----|
| CD | CD | 1 | 1 | X |
| CD | CD | 1 | X | 1 |
| CD | CD | 1 | X | X |
| CD | CD | 1 | X | X |

$$X = A'\bar{B}'\bar{C}D + A'\bar{B}'\bar{C}D + A\bar{B}'\bar{C}D + A\bar{B}'\bar{C}D + A\bar{B}CD$$

$$= \Sigma m(1, 2, 11, 12, 15) + \Sigma d(5, 6, 7, 8, 9, 10)$$

| AB | AB | Z | AB | AB |
|----|----|---|----|----|
| CD | CD | 1 | 1 | X |
| CD | CD | 1 | X | 1 |
| CD | CD | 1 | X | 1 |
| CD | CD | 1 | X | X |

| AB | AB | Z | AB | AB |
|----|----|---|----|----|
| CD | CD | 1 | 1 | X |
| CD | CD | 1 | X | 1 |
| CD | CD | 1 | X | 1 |
| CD | CD | 1 | X | X |

$$Z = D$$

Q) Design a code converter that converts From BCD₅₄₂₁ to BCD₂₄₂₁

ANS:

| Dec | 5 | 4 | 2 | 1 | 2 | 4 | 2 | 1 |
|-----|---|---|---|---|---|---|---|---|
| Dec | A | B | C | D | W | X | Y | Z |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 6 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 7 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 8 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 9 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

$$W = A'B'C'D' + A'B'C'D + AB'C'D + AB'C'D'$$

$$= \Sigma m(8, 9, 10, 11, 12) + \Sigma d(5, 6, 7, 13, 14, 15)$$

$$X = A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D'$$

$$= \Sigma m(4, 9, 10, 11, 12) + \Sigma d(5, 6, 7, 13, 14, 15)$$

$$Y = A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D'$$

$$= \Sigma m(2, 3, 8, 11, 12) + \Sigma d(5, 6, 7, 13, 14, 15)$$

$$Z = A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D + A'B'C'D'$$

$$= \Sigma m(1, 3, 8, 10, 12) + \Sigma d(5, 6, 7, 13, 14, 15)$$

truth table (TT) :

توعية الترتيب

- Determine using truth table if these expressions are equal or no.

$$f = \underline{\quad}$$

$$g = \underline{\quad}$$

(Algebraic Form) عاى آخر يكتب! تناهون ال (Function)

- Write a sum of minterms functions in algebraic form or in (Disjunctive Normal Form (DNF))

او يطلبها في المترم المجموع (Logic form of numeric form)

- Write a sum of minterms in Numeric Form.

$$f = \Sigma m(\underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad})$$

صادر (1's)

(Product of maxterms) او يطلبها في المترم المنتج (Product of maxterms)

- Find a product of sums expression in a product of maxterms form or in (Conjunctive Normal Form) (CNF)

From (0's)

Kmap Rules ابزار (Simplification) تعلمها قبل تناهون

(OR, AND, NOT) ابزار (Block Diagram) او (Design) اعماليات انجعل

or (NAND) او بـ

(NOR) او بـ

(2 input NAND) او بـ

expand
(Complement of f) او (Function) للنهاية

Ex:

Determine, using truth tables, which expressions in the group of functions (are equal) :-

$$F = \bar{a}\bar{c} + bc + ab$$

$$g = b\bar{c} + \bar{a}\bar{c} + ac$$

$$h = b\bar{c} + ac + \bar{a}b$$

ANS.

| a | b | c | $\bar{a}\bar{c}$ | ac | bc | ab | $\bar{a}b$ | $\bar{b}\bar{c}$ | F | g | h |
|---|---|---|------------------|----|----|----|------------|------------------|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

$$(F = h) \neq g$$

Ex-

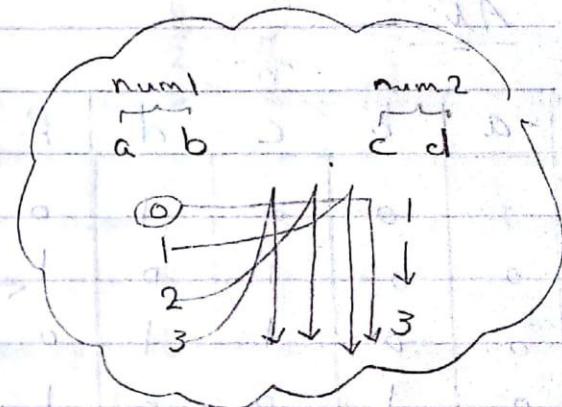
Show truth table for a system that has four inputs a, b, c, d and one output (f), the first 2 inputs (a, b) represent one binary number (in the range 0 to 3) and the last two (c, d) represent another number (in the range 1 to 3).

\Rightarrow The output (f) is one if the second number is at least two larger than the first.

(جواب "2"، لیکن دوست نمایی که این دستگاه (F=1) نباشد)

Ans.

| m | inputs | | | | output F |
|----|--------------|--------------|--------------|--------------|-------------|
| | number1 a | number1 b | number2 c | number2 d | |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 ← |
| 3 | 0 | 0 | 1 | 1 | 1 ← |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 ← |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |



\therefore (Function) f نامنوع است

① ~~Algebraic function :-~~

(sum of minterms) or

(sum of products).

$$f = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d$$

② ~~Numerical function or~~
~~(numerical form).~~

$$f = \sum m(2, 3, 7)$$

Ex.

Show the truth table for:

The system has four inputs a, b, c, d and one output "F".
The last 3 inputs (b, c, d) represent binary number " n "
in the range $0 \rightarrow 7$; however, the input 0 never occurs,
the first input " a " specifies which of two computations
is made.

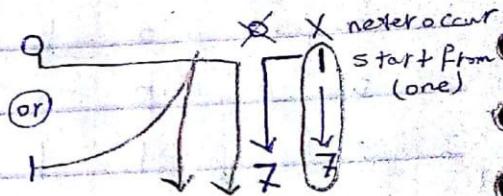
$a = 0$: "F" is 1 if " n " is a multiple of 2

$a = 1$: "F" is 1 if " n " is a multiple of 3

ANS.

| n | a | b | c | d | F. |
|-----|-----|-----|-----|-----|----|
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 0 |

$\overbrace{a}^{\text{or } 1}$ $\overbrace{b \ c \ d}^{(n)}$



(7 reg 1) no " n " is a multiple of 2.

($a=0$) used

($a=1$) used for multiples

Algebraic Function:-

$$F = a'b'cd' + a'b'cd' + a'b'cd' + ab'cd + ab'cd'$$

Numetic Function:-

$$F = \Sigma m(2, 4, 6, 11, 14)$$

Ex-

Show a truth table for a system has four inputs A, B, C, D
the first two inputs (A, B) represent a 2-bits unsigned binary number (in the range 0 to 3). the last two (C, D) represent a second unsigned binary number (in the same range). The output "Y" is "1" if and only if the two numbers differ by two or more.

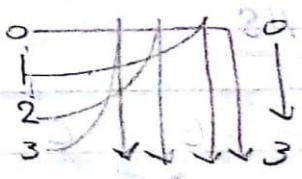
(جواب ۲ = مجموع دو عدد unsigned binary (Y=1) یا بیش از ۲)

Ans -

| m | A | B | C | D | Y |
|----|---|---|---|---|-----|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 ← |
| 3 | 0 | 0 | 1 | 1 | 1 ← |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 ← |
| 8 | 1 | 0 | 0 | 0 | 1 ← |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 ← |
| 13 | 1 | 1 | 0 | 1 | 1 ← |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

num₁
A, B

num₂
C, D



Algebraic Function :-

$$Y = \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \\ \bar{A}B\bar{C}D + A\bar{B}\bar{C}\bar{D} + \\ AB\bar{C}\bar{D} + A\bar{B}\bar{C}D$$

Numeric Form :-

$$X = \sum m (2, 3, 7, 8, \\ 12, 13)$$

Ex. (Final 2012)

Show the truth table for a system that has four inputs a, b, c, d and two outputs, f & g . The first two inputs (a, b) represent one binary number (in the range 0 to 3) & the last two (c, d) represent another number in the range 1 to 3. The output, f and g represent the magnitude of the difference of two binary numbers inputs ($|ab - cd|$)

ANS.

| num1 | | num2 | | ab - cd | |
|------|---|------|---|---------|---|
| a | b | c | d | f | g |
| 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

$$f = \bar{a}b'cd + \bar{a}b'cd + \bar{a}bcd + ab'cd$$

$$g = \bar{a}b'cd + \bar{a}b'cd + \bar{a}bcd + ab'cd + ab'cd + abcd$$

* Show a truth table for a system that convert from
BCDExcess 3 to BCD8421.

ANS.

| Inputs | | | | outputs | | |
|----------|---|---|---|-----------------|---|---------|
| Excess 3 | | | | 8 | 4 | 2 1 |
| A B C D | | | | W | X | Y Z |
| 0 | 0 | 1 | 1 | ← ³ | 0 | 0 0 0 0 |
| 0 | 1 | 0 | 0 | ← ⁴ | 0 | 0 0 0 1 |
| 0 | 1 | 0 | 1 | ← ⁵ | 0 | 0 1 0 0 |
| 0 | 1 | 1 | 0 | ← ⁶ | 0 | 0 0 1 1 |
| 0 | 1 | 1 | 1 | ← ⁷ | 0 | 1 0 0 0 |
| 1 | 0 | 0 | 0 | ← ⁸ | 0 | 1 0 0 1 |
| 1 | 0 | 0 | 1 | ← ⁹ | 0 | 1 1 0 0 |
| 1 | 0 | 1 | 0 | ← ¹⁰ | 0 | 1 1 1 0 |
| 1 | 0 | 1 | 1 | ← ¹¹ | 1 | 0 0 0 0 |
| 1 | 1 | 0 | 0 | ← ¹² | 1 | 0 0 0 1 |

$$W = \sum_m(11, 12) + \sum_d(0, 1, 2, 13, 14, 15)$$

$$X = \sum_m(7, 8, 9, 10) + \sum_d(0, 1, 2, 13, 14, 15)$$

$$Y = \sum_m(5, 6, 9, 10) + \sum_d(0, 1, 2, 13, 14, 15)$$

$$Z = \sum_m(4, 6, 8, 10, 12) + \sum_d(0, 1, 2, 13, 14, 15)$$

* Show a truth table for a system that converts from BCD₈₄₂₁ to BCD_{Excess-3}. Then show the switching algebra for the output functions.

\downarrow
Algebraic form

Ans.

| Inputs | | | | Outputs | | | |
|--------|---|----|----|----------|---|---|---|
| 8 | 4 | -2 | -1 | Excess 3 | | | |
| A | B | C | D | w | x | y | z |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

$$w = \sum m(8, 9, 10, 11, 15) + \sum d(1, 2, 3, 12, 13, 14)$$

=

$$x = \sum m(4, 5, 6, 7, 15) + \sum d(1, 2, 3, 12, 13, 14)$$

$$y = \sum m(0, 4, 5, 8, 9) + \sum d(1, 2, 3, 12, 13, 14)$$

$$z = \sum m(0, 4, 6, 8, 10) + \sum d(1, 2, 3, 12, 13, 14)$$

* You simplify them on the K-map.

Q2:

Develop a truth table for a system with 3 inputs a, b, c and four outputs W, X, Y and Z. The output is a binary number equal to the largest integer that meets the input conditions :-

$$a = 0 \quad ; \quad \text{odd}$$

$$a = 1 : \text{exten.}$$

b = 0 : prime

b=1 : notPrime -

$C = 0$: less than 8

C=1 : greater than 8

(The output is never all 0's)

ANS.

و^ن مدخلات a, b, c و^ن مدخلات w, x, y, z و^ن مخرجان o و^ن مخرجان o الـ 4 bits $(15 \leftarrow o)$ هـلـيـور (output) الـ 4 bits $(15 \leftarrow o)$ هـلـيـور (output)

$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline 0 & 1 & 1 \\ \hline \end{array}$$

مثال :- لو : $c=1$ و $b=1$ و $a=0$ ، $n > 8$ ، not prime ، odd first or second

و عندنا لـ ١٥ مم (٩، ١٠، ١١، ١٢، ١٣، ١٤، ١٥) فقط والباقي ٨ مم اختارتم

١٥) $\frac{11 \times 13}{15}$ وكمبر واجه صفحه (prime) ١١ (odd) - ١

$\begin{array}{cccc} w & x & y & z \\ 1 & 1 & 1 & 1 \end{array} \rightarrow (\text{binary}), \text{ and}$
4 bits.

| a | b | c | d | x | y | z |
|-----|-----------|----|---|---|---|---|
| odd | prime | <8 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| odd | prime | >8 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| odd | not prime | <8 | x | x | x | x |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| odd | N.P | >8 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | x | x | x | x |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 |

لیست کارهای ممکن

متغیر: ① or ③ or
⑤ or ⑦
لکن فقط ۷ تا ممکن است

لزمه تا اگر رسم صورت انتخاب

(Q2 - Final 2010) (12 Points)

Q: The months of the year are coded in four variables a, b, c & d such that January is 0001, February is 0010, ... and December is 1100. February of a leap year (in which February is 29 days) is coded as 0000.

Remember: April, June, September and November has 30 days
all other months except February has 31 days

- Show the truth table with five variables V, W, X, Y, Z , that indicates the number of days in the selected month.
- Write the switching formula for each of V, W, X, Y & Z .
- Minimize V, W, X, Y & Z , applying switching rules.

ANS.

a)

| months & days. | a | b | c | d | v | w | x | y | z |
|----------------|---|---|---|---|---|---|---|---|---|
| Feb (29) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| Jan (31) | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| Feb (28) | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| Mar. (31) | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| April (30) | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| May (31) | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| June (30) | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| July (31) | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Aug. (31) | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| Sep. (30) | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| Oct. (31) | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Nov. (30) | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| Dec. (31) | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| x | 1 | 1 | 0 | 1 | | | | | |
| x | 1 | 1 | 1 | 0 | | | | | |
| x | 1 | 1 | 1 | 1 | | | | | |

$$(29)_{10} \Rightarrow (11101)_2, (28)_{10} = (11100)_2, (30)_{10} = (11110)_2, (31)_{10} = (11111)_2$$